INTERRAIL

OPTIMIZATION METHODS IN BUSINESS ANALYTICS

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WHAT IS AN INTERRAIL?

Interrail is a train pass available to European residents that allows unlimited travel across 33 countries within a set period.

Global Pass: travel across multiple countries

One Country Pass: explore a single country

Is popular among young travelers and students who want to explore Europe in an affordable, eco-friendly, and adventurous way.

THE PASS

WHAT WE CHOOSE:

Global Pass – 7 days in 1 month, Youth version (for travelers under 27)

Valid for **unlimited train rides** on 7 days of choice within 30 consecutive days

Covers **33 European countries**

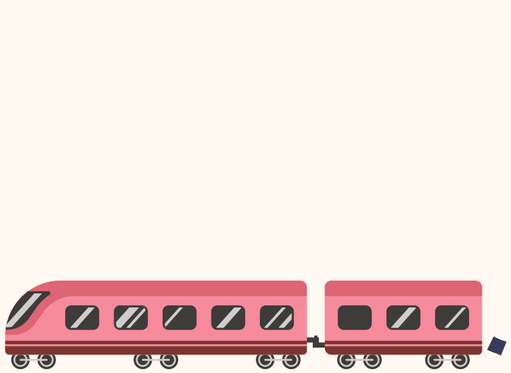
Flexible, no need to buy individual tickets (except for some reservations)

PROJECT DESCRIPTION

Object: design an optimal Interrail itinerary in Southern Europe.

Main goals:

- Select a subset of cities to visit
- Decide how many days to spend in each city
- Determine the optimal visiting order
- Maximize total tourist value



POSSIBLE CITIES

ITALY

- Milan
- Venice
- Rome
- Florence
- Naples

SPAIN

- Madrid
- Barcelona
- Zaragoza
- Valencia
- Seville

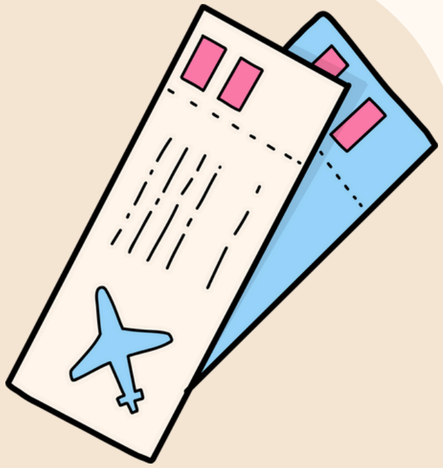
PORTUGAL

- Lisbon
- Porto
- Faro
- Coimbra
- Fatima

FRANCE

- Paris
- Lyon
- Nice
- Bordeaux
- Toulouse

PARAMETERS



- a_i • Interest associated with city i
- K_i • One-day stay cost in city i
- F • Cost of the Interrail pass
- f • Cost of travel insurance
- t_{ij} • Travel time (in minutes) from city i to city j
- B • Total available budget
- T • Total number of available days
- V • Maximum number of travel days allowed
- λ • Penalty for time spent travelling
- α • Bonus awarded for each additional city visited
- D_{max} • Maximum number of days that can be spent in a single city

INDECES, SET and VARIABLES

$$i, j \in C$$

$$C$$

- Cities eligible for the Interrail trip
- Eligible city set for the Interrail journey

$$x_i \geqslant 0$$

$$y_{ij} \in \{0, 1\}$$

$$g \geqslant 0$$

$$z_i \in \{0, 1\}$$

$$u_i \geqslant 0$$

$$0 \leqslant u_i \leqslant (n-1)$$

- Integer variable: number of days stayed per city
- Binary variable: 1 if travel from city i to city j, 0 otherwise
- Integer variable: number of actual travel days
- Binary variable: 1 if city i is visited, 0 otherwise
- Continuous (real) variable used for subtour elimination (MTZ).

OBJECTIVE FUNCTION

It aims to maximize the total value of the Interrail itinerary by combining three key components:

$$max \sum_i a_i \times x_i + \alpha \times \sum_i z_i - \lambda \sum_i \sum_j t_{ij} \times y_{ij}$$

Total interest gained from visited cities

A bonus for each additional city visited

A penalty for time spent travelling between cities

CONSTRAINTS

1. Total duration

$$\sum_i x_i + g \leqslant T$$

The total number of days, including stays and travel days, must not exceed the total available days T

2. Maximum number of travel days

$$g \leqslant V$$

Limits the number of days spent travelling to the maximum allowed V

CONSTRAINTS

3. Total available budget

$$\sum_i k_i \times x_i + F + f \leqslant B$$

The combined cost of accommodation, Interrail pass, and insurance must remain within the total available budget.

4. Travel days and arcs

$$\sum_i \sum_j y_{ij} = g$$

Each connection (from city i to city j) represents one travel step. The model counts how many of these steps are used, and store this value in variable g.

CONSTRAINTS

5. Visit logic

$$x_i \leqslant T \times z_i$$

$$x_i \geqslant z_i$$

$$x_i \leqslant z_i \times D_{max}$$

It ensures that:

- If city i is not visited ($z_i = 0$), no stay is allowed ($x_i = 0$).
- If visited ($z_i = 1$), the stay must be at least one day and no more than a maximum duration Dmax.

CONSTRAINTS

6. Flow conservation

$$\sum_{j \neq i} y_{ij} = z_i, \forall i$$

$$\sum_{j \neq i} y_{ji} = z_i, \forall i$$

$$y_{ii} = 0, \forall i$$

Ensures a valid tour structure: for every visited city, there must be exactly one inbound and one outbound connection. No city connects to itself.

CONSTRAINTS

1. Departure and return from Milan

$$\sum_{j \neq 1} y_{1j} = 1; \quad \sum_{j \neq 1} y_{j1} = 1$$

The combined cost of accommodation, Interrail pass, and insurance must remain within the total available budget.

8. Link between routes and selected cities

$$y_{ij} \leqslant z_i, \forall i \neq j$$

$$y_{ij} \leqslant z_j, \forall i \neq j$$

Travelling from city i to city j is allowed only if both cities i and j are visited

CONSTRAINTS

9. Subtour elimination constraint (MTZ)

$$u_i - u_j + 1 \leqslant (n-1) \times (1 - y_{ij}), \forall i \neq j, i \neq 1, j \neq 1$$

$$u_1 = 0$$

These constraints prevent the formation of subtours (disconnected loops) using the Miller–Tucker–Zemlin formulation, ensuring a single connected route.

MODEL RESULT

62.98
Objective function

€2500
Budget

0.002
Lambda

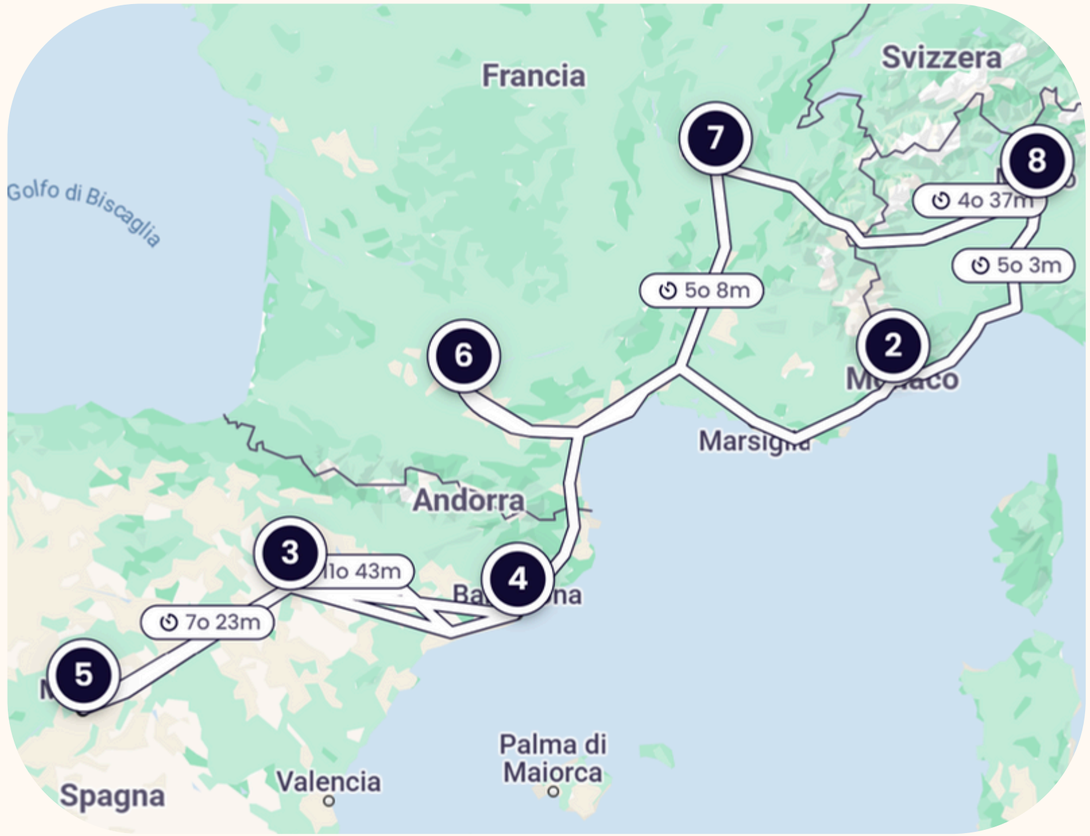
3
Alpha

7
Visited cities

26
Total days of travel

MODEL RESULT

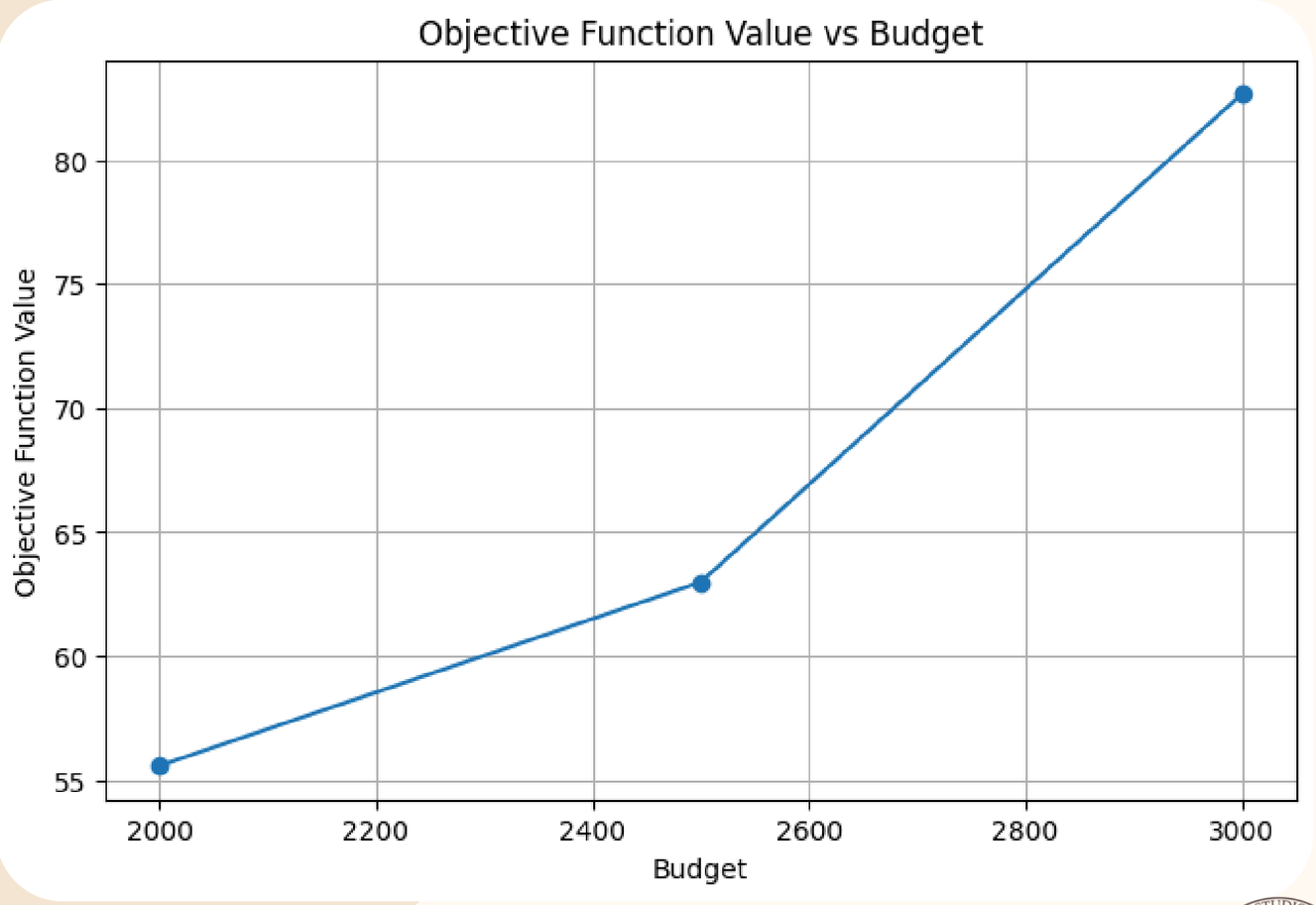
1. Milan: 1 day
2. Nice: 1 day
3. Zaragoza: 5 days
4. Barcelona: 1 day
5. Madrid: 1 day
6. Toulouse: 5 days
7. Lyon: 5 days
8. Return to Milan



SENSITIVITY ANALYSIS: budget

To analyze the behavior of the total value of the Interrail itinerary as the budget varies, the model was applied to different budget levels.

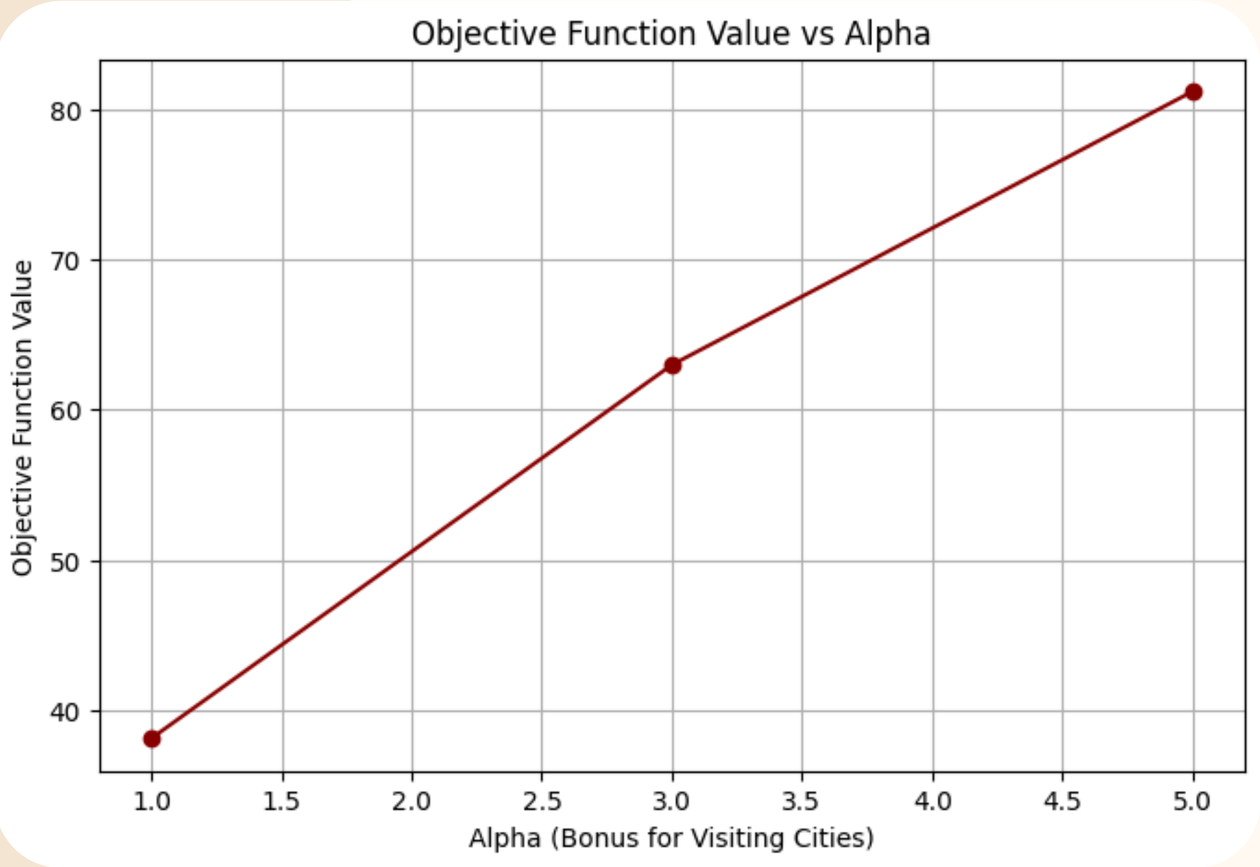
As illustrated in the graph, **an increase in the available budget leads to a corresponding increase in the objective function value**.



SENSITIVITY ANALYSIS: alpha

A sensitivity analysis was subsequently conducted on the parameter α.

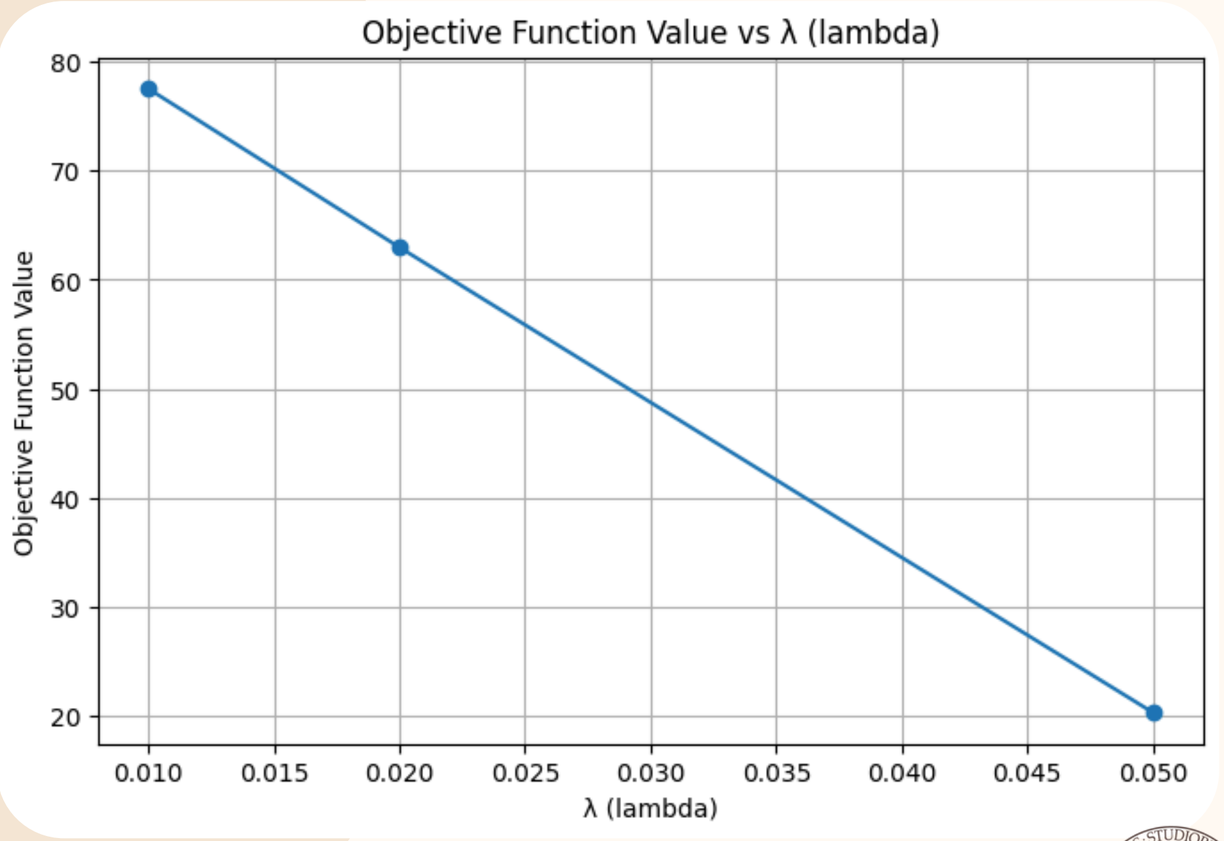
As expected, **an increase in α resulted in a higher objective function value**. This behavior is consistent with the model's formulation, where the objective function is designed to be directly proportional to the number of cities visited and, consequently, to the value of α.



SENSITIVITY ANALYSIS: lambda

A sensitivity analysis was conducted on the parameter λ, which introduces a penalty for the number of travel days.

The results show that **as λ increases, the value of the objective function decreases.** This inverse relationship, clearly observable in the graph, aligns with expectations, as higher penalties discourage frequent travel, thereby reducing the total value of the itinerary.



CONCLUSIONS

Model Flexibility

The model is easily customizable by adjusting input data (cities, budget, interests), enabling quick adaptation to different scenarios.

Sensitivity Analysis

Results show robustness: higher budgets and α increase the objective value, promoting longer and richer itineraries; higher λ reduces it, reflecting travel penalties—confirming expected model behavior.

Model limitations

Static Data

The model relies on fixed input data and does not account for dynamic factors such as local events, high or low tourist seasons, or weather conditions, which can significantly influence the real travel experience.

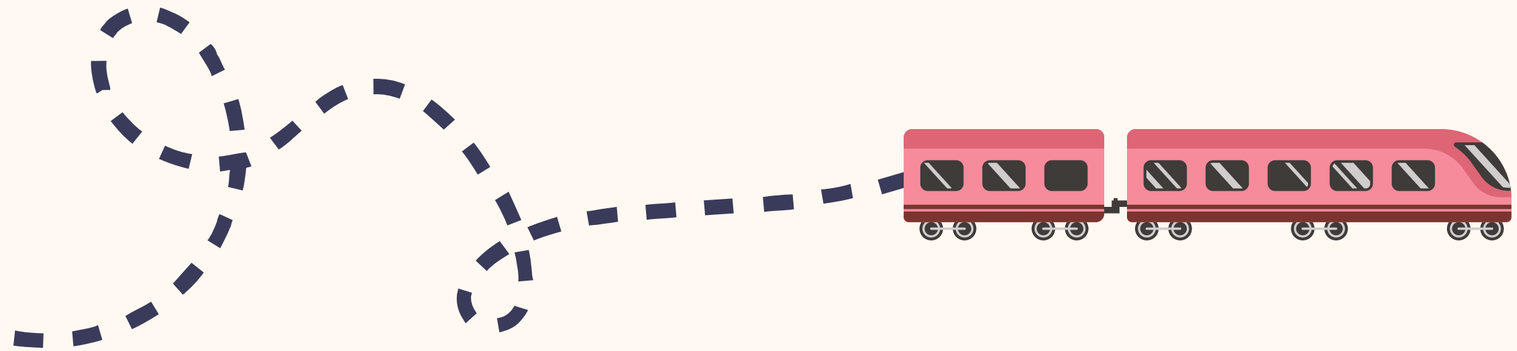
Limited Personalization

Traveller preferences are only captured indirectly through general scores and bonuses. There is no explicit customization for specific interests (e.g., art, nature, gastronomy), which limits the ability to tailor the itinerary to individual needs.

Model limitations

Transport Constraints

The model assumes rail as the only mode of transport, in line with the Interrail context. However, this restricts flexibility and may exclude or penalize attractive destinations not well connected by train.



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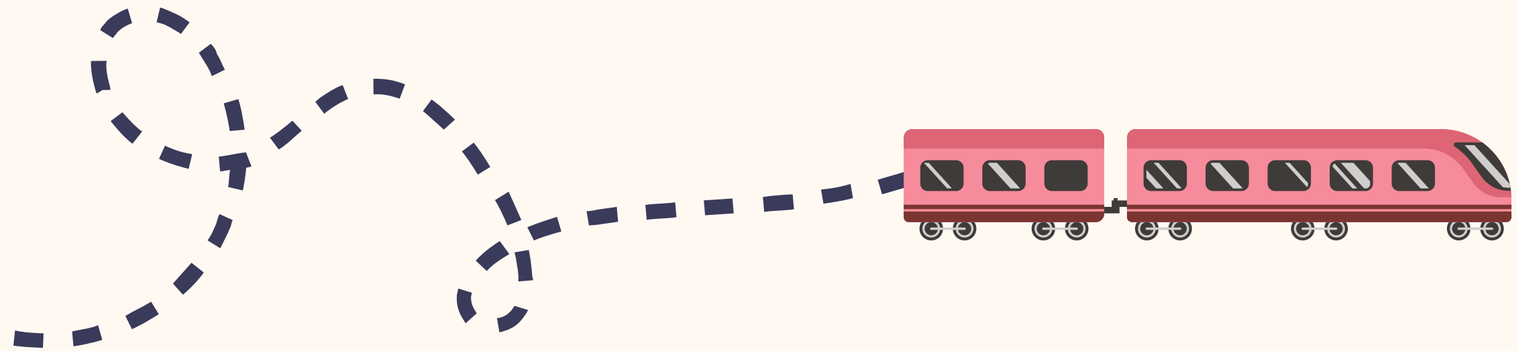

Thank You