

DEPARTMENT OF ECONOMICS AND MANAGEMENT

Analytics and Data science for Economics and Management

OPTIMIZATION OF AN INTERRAIL ITINERARY

Course: Optimization Methods in Business Analytics

Prof. Archetti Claudia

Luigina Bertolotti (726865)

Isabella Cappiello (731302)

Martina Pagan (744788)

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1 INTRODUCTION

CRITERIA AND REASONS FOR CHOOSING THE TOPIC

Every year, millions of young Europeans choose Interrail to explore the continent in a sustainable, flexible and cultural way. However, planning an optimal itinerary that enhances the tourism experience while respecting constraints related to time, budget and travel mode is a complex and multifaceted challenge.

The present project stems from the interest in two complementary aspects: on the one hand, the desire to promote and enhance the cultural heritage of Southern Europe; on the other hand, the desire to concretely apply the mathematical optimization skills acquired in the academic course to a real problem with numerous variables and constraints. Indeed, the problem lends itself to the use of integer linear programming models and sensitivity analysis, effective tools for dealing with realistic and customizable scenarios.

PROJECT CONTEXT AND PROBLEM DESCRIPTION

In recent years, the tourism industry has undergone profound transformations, influenced by digitization, the emergence of new preferences among travelers, and a growing focus on sustainability. In this new scenario, it becomes essential to plan intelligently, optimizing the use of available resources, limiting environmental impact and maximizing the travel experience.

The use of mathematical models can provide valuable support in designing efficient rail routes that take into account factors such as time available, transportation and accommodation costs, distance between destinations, and the cultural value of the destinations. Planning in an optimized manner allows not only to avoid waste and unforeseen events, but also to make the trip more consistent with individual preferences, richer in content, and above all more sustainable.

The Interrail Pass presents an attractive opportunity for large-scale travel with some flexibility, but it introduces natural constraints that make planning a combinatorial optimization problem. Specifically, the project focuses on the Global Pass in the "7 days of travel in 1 month" formula, which allows unlimited travel by train for seven days, freely distributed over thirty. The challenge is to select a subset of cities to visit, define the length of stay at each stop, and determine the optimal order of destinations, taking into account constraints on total time, number of travel days allowed by the Pass, overall costs, and transportation constraints.

GEOGRAPHIC CHOICES AND DESTINATION SELECTION CRITERIA

The project focuses on southern Europe, selecting a set of cities accessible via the interrail rail network and characterized by high historical, cultural and logistical attractiveness. The destinations were chosen based on criteria of heritage value, rail connectivity and tourism potential.

In Portugal, for example, Lisbon perfectly embodies the meeting of past and present, with historic neighborhoods such as Alfama and iconic monuments such as the tower of Belém. Porto, Faro, Coimbra and Fátima are also included for their cultural and spiritual distinctiveness.

In Spain, the itinerary touches cities such as Madrid, the country's institutional and cultural heart, and Barcelona, famous for the works of Gaudí and its cosmopolitan atmosphere. Valencia, Zaragoza and Seville add further variety through their combination of tradition, architecture and urban vitality.

In italy, Rome, Florence, Venice, Naples and Milan represent must-see stops for those who wish to immerse themselves in Italian history, art and identity.

Finally, in France, Paris, Lyon, nice, Bordeaux, and Toulouse offer a wide range of cultural, artistic, and gastronomic experiences, all easily accessible by train.

All these cities are connected by train routes compatible with the interrail global pass, which covers major European networks (Trenitalia, Renfe, SNCF). It is important to take into account in planning the need for mandatory reservations and additional costs on some high-speed routes (e.g., Tgv, Frecciarossa, Ave).

AIMS AND PERSPECTIVES OF THE PROJECT

This work demonstrates how mathematical modeling, and combinatorial optimization can be effective tools for solving complex tourism planning problems.

Through the development of a realistic mathematical model, the project aims to make a concrete contribution to the organization of sustainable cultural travel, supporting the traveler in the efficient management of resources and preferences, while providing an application cue for the integration of tourism and data science.

2 DESCRIPTION OF THE OPTIMIZATION PROBLEM

The main objective of this project is to define an optimal travel itinerary in Southern Europe (Spain, Portugal, France and Italy), taking advantage of the Interrail pass. The problem is to plan a route that allows visitors to visit a subset of selected cities while maximizing the overall tourism value, within the time, economic, and logistical constraints imposed by the pass and the resources available to the traveler.

Specifically, the project addresses a combinatorial optimization problem, in which it is necessary to decide which cities to visit, in what order, and for how many days to stay in each. The solution aims to maximize the traveler's cultural and personal experience. The complexity of the problem necessitates the use of mathematical models capable of supporting optimal choices in realistic scenarios.

2.1 PROBLEM SCENARIO: INTERRAIL TRIP IN SOUTHERN EUROPE

In order to build an effective and sustainable travel itinerary, it is necessary to consider three basic, closely interrelated decisions that are fundamental to the entire planning process.

- Which cities to visit: the starting point is the selection of cities to include in the itinerary, chosen from a predefined set of destinations spread across four countries Spain, Portugal, France, and Italy. This choice must consider several factors. First, the cultural and historical richness of the cities, to ensure meaningful and enriching tourism experiences. At the same time, the logistical feasibility of the route must be assessed, considering the distance between destinations and the availability of efficient rail connections. For example, visiting Lisbon and Porto in Portugal or Rome and Florence in Italy can offer a balanced mix of art, history and striking landscapes. However, it is critical to avoid excessively long or time-consuming travel, which could reduce the actual time spent visiting and increase costs.
- Length of stay in each city: once the cities to be visited have been identified, the next step is to decide how many days to spend at each destination. This is crucial to maxime the overall tourism value. Staying too little in a city can prevent you from fully appreciating its main attractions, while staying too long limits the number of possible stops on the trip. Therefore, a balance must be struck that allows each place to be fully enjoyed without sacrificing the variety and richness of the overall itinerary. In addition, the management of stay days must respect the time constraints imposed by the Interrail pass and the total length of the trip, making it necessary to carefully balance time and number of destinations.
- **Visiting order**: Finally, it is essential to define the sequence with which to visit the chosen cities. The order of visitation has a direct impact on the number of train travel days used, the associated costs, and the overall convenience of the trip. A well-planned itinerary favors direct and fast train

connections, minimizing downtime and unnecessary travel. This not only optimizes the use of the pass and economic resources, but also improves traveler comfort, reducing fatigue and allowing the best use of each day devoted to discovery. Determining the optimal order requires careful analysis of rail networks, schedules, and distances between cities.

2.2 CONSTRAINTS TO BE RESPECTED

While the previously defined constraints strictly limit the scope of feasible itineraries, several other factors can strongly influence the overall quality and realism of the trip. These elements do not constitute hard constraints within the optimization model, but they are crucial for shaping a solution that is not only mathematically optimal, but also practically and experientially valuable. Among these:

- Total trip duration: the Global pass requires that the trip must be completed within a maximum period of 30 consecutive days. This time limit includes both the days spent in the various cities, devoted to sightseeing and accommodation, and those actually used for rail travel. Consequently, careful and efficient management of the total time available is essential to ensure that this deadline is met, avoiding delays or overlaps that could compromise the validity of the pass or the viability of the entire itinerary.
- Maximum number of days of rail travel: the Interrail pass in the "7 days within 1 month" formula allows rail travel for a maximum of seven days within the 30-day period. This constraint requires extremely careful planning of rail travel, as each activated travel day must be used strategically to cover one or more transfers. Therefore, waste or unnecessary use must be avoided in order to maximize the effectiveness of the available travel days and ensure optimal route coverage.
- Available Budget: the total cost of the trip must remain within a pre-determined financial limit and includes various expense items such as accommodation, meals, and any rail surcharges not covered by the pass (e.g., compulsory reservations on high-speed or international trains or subways). Financial management is a crucial constraint that directly affects the selection of cities to visit, the length of stay, and the choice of means of transportation used. Planning must therefore take into account not only economic sustainability, but also the overall efficiency of the trip.

2.3 CHALLENGES OF THE PROBLEM

Planning an optimal route with the Interrail pass has a high inherent complexity, resulting from the need to consider multiple heterogeneous aspects simultaneously. The main challenges can be summarized as follows:

Balancing objectives and constraints: the problem requires maximizing the overall tourism value of
the trip while ensuring that stringent constraints are met in terms of maximum duration, limited number
of train travel days, and available economic budget. This multidimensionality requires a delicate
balance between quality of experience and operational feasibility.

- **Presence of mixed decisions**: the nature of the problem includes both discrete decisions (which cities to visit) and continuous decisions (how many days to spend in each city), increasing computational and modeling complexity. The combination of variables of different nature makes the formulation and resolution of the problem nontrivial.
- Logistical constraints and realism of solutions: optimal solutions, to be useful and applicable, must be not only theoretically sound, but also practically feasible. The actual rail network, availability of connections, travel time, and incidental expenses must be integrated into the planning so that the resulting route is actually viable.

3 DATA COLLECTION AND PREPARATION

3.1 CANDIDATE CITIES: SELECTION CRITERIA AND FEATURES

To create the best itinerary, it was essential to pinpoint a group of potential cities to choose from for the final route. The emphasis was on locations situated in the four southern European nations encompassed by the Interrail Global Pass: Portugal, Spain, France, and Italy.

Forty cities were selected, divided equally among the four countries (10 for each), based on the following criteria:

- Cultural and tourist relevance, assessed by considering the cities' artistic, historical and architectural
 heritage.
- Rail accessibility, i.e., the presence of efficient connections compatible with the Interrail network.
- **Popularity among travelers**, inferred from sources such as travel guides and review platforms.

The goal was to promote diversity in the tourism offerings, encompassing both major urban areas (such as Madrid, Barcelona, Rome, and Paris) and smaller cities or those with a distinct identity (like Coimbra, Faro, Zaragoza, and Lyon), which can provide more genuine and less commercialized cultural experiences.

This selection serves as the foundation for making planning decisions, influencing both the choice of destinations and the assessment of the overall quality of the travel route.

3.2 TOURISM VALUE FOR EACH CITY

To quantify the tourism value associated with each city, only data from the TripAdvisor platform were used, with a focus on the overall ratings (in stars) of major tourist attractions.

For each city, the five most representative attractions were identified, selected based on their popularity and cultural relevance. The average score (between 1 and 5 stars) was recorded for each attraction, along with the total number of reviews received.

The overall tourism value of each city was then calculated as a weighted average of the attraction scores, using the number of reviews as the weight. In this way, a synthetic index is obtained that reflects both the perceived quality of the attractions and their popularity with visitors.

This index was adopted as a proxy for tourism value in the optimization model, with the aim of targeting the selection of cities toward those that offer more meaningful tourism experiences.

3.3 AVERAGE DAILY COST FOR EACH CITY

The average daily cost is a crucial parameter in defining the optimal itinerary, as it directly affects compliance with the overall budget constraint imposed on the trip. Accurate estimation of this cost allows for realistic modeling of the daily expenses faced by a traveler and to guide optimization model choices toward solutions that are also economically sustainable.

For each candidate city, a synthetic indicator of daily expenditure was calculated, obtained by consulting publicly available sources, such as tourism portals (e.g., BudgetYourTrip¹ and other sites specializing in travel cost comparisons), traveler blogs and reports of tourist experiences. The objective was to obtain a representative estimate of the cost that an average tourist may incur in each location during a typical day.

The estimate includes three main expense items:

- Lodging: the average cost of a night's stay at a mid-range budget hotel (such as a well-regarded hostel or a hotel with two or three stars) was taken into account. This is a reasonable option for many Interrail passengers.
- **Meals**: the average cost of two main meals per day was included, considering the availability of cheap options (such as trattorias, street food or tourist menus).
- Local transportation: finally, an average daily expense was considered for any urban transportation, such as by metro, bus or streetcar, especially in larger or extended cities.

The result is an estimated value of the average daily cost for each city, expressed in euros, which was then used within the optimization model as a binding parameter for budget management. The goal is to ensure that the total sum of expenses resulting from staying in the different destinations does not exceed the economic limit imposed on the trip, while at the same time ensuring a consistent distribution of resources among the various stops on the route.

The following tables summarize, for each city considered in the project, the tourist interest value and the estimated average daily cost used as key parameters in the optimization model as described in sections 3.1 and 3.2.

TABLE 1 – DATA OF CITIES IN PORTUGAL

City	Interest per city	Daily cost per city				
Lisbon	4,37	€130				
Porto	4,52	€100				
Faro	4,19	€120				
Coimbra	4,50	€110				
Fatima	4,59	€90				

¹ Source: BudgetYourTrip official website https://www.budgetyourtrip.com/

TABLE 2 - DATA OF CITIES IN SPAIN

City	Interest per city	Daily cost per city
Madrid	4,60	€130
Barcelona	4,55	€150
Zaragoza	4,56	€100
Valencia	4,46	€110
Seville	4,70	€110

TABLE 3 – DATA OF CITIES IN FRANCE

City	Interest per city	Daily cost per city
Paris	4,62	€155
Lyon	4,58	€110
Nice	4,53	€130
Bordeaux	4,35	€110
Toulouse	4,29	€100

TABLE 4 – DATA OF CITIES IN ITALY

City	Interest per city	Daily cost per city
Milan	4,61	€155
Venice	4,58	€155
Rome	4,56	€145
Florence	4,59	€135
Naples	4,68	€110

3.4 TRAVEL TIMES AND COSTS

In the optimization model, direct rail transportation costs between cities were not considered, since the trip is made using the Interrail pass. As mentioned above, this pass allows unlimited travel by train for a set number of days (in our case, 7 days within a maximum period of 30 days), thus eliminating the need to individually calculate transportation costs between locations visited.

The main constraint related to travel is therefore the maximum number of travel days allowed by the pass. Each day on which a train is taken is counted as a travel day, regardless of the number of trips made, so planning must optimize the use of these days to cover the necessary trips.

To estimate travel times between cities, updated data were collected from the official Interrail website, in the "Train times" section², which provides the average durations of the most relevant train routes for our itinerary.

² Source: Interrail Train times section https://www.interrail.eu/en/plan-your-trip/interrail-timetable#/

These travel times were incorporated into the model to ensure that time constraints were met, allowing realistic itinerary planning that considers the time required for travel between destinations.

Table 5 in the appendix shows a complete 20×20 matrix of travel times expressed in minutes, representing the estimated duration of rail travel between all city pairs considered. This set of information makes it possible to accurately estimate the required travel days, thus conditioning the formulation of time constraints in the optimization model.

3.5 TOTAL BUDGET AND FIXED COSTS

To define the overall travel budget constraint, we set a total amount of 2,500 euros, which includes all necessary expenses for the entire stay and rail transportation. This value was chosen taking into consideration the typical profile of a young traveler, under 27 years old, who takes advantage of the reduced Interrail pass fare and prefers an economically sustainable but quality travel experience.

The budget includes:

- The cost of the Interrail "7 days in 1 month" pass, which is 286 euros for travelers under 27 (figure updated to 2025 and retrieved from the official Interrail website).
- The travel insurance, estimated at about 55 euros, is based on online policies with adequate coverage for a trip of this duration and characteristics.
- The variable daily expenses, which include lodging, meals, local transportation and other necessities, are estimated through an analysis of average costs in the different candidate cities.

The overall budget constraint is therefore tight, forcing the model to plan a balanced itinerary that reconciles visits to cities of high tourist interest but high costs, with stops at cheaper locations. This choice reflects the goal of ensuring a rich and varied travel experience while maintaining economic sustainability.

It should also be considered that the budget given represents an optimistic and restrained estimate, typical of a young, budget-conscious traveler. In a real-world setting, any needs for increased comfort, extra activities, or unforeseen events may require an increase in the budget. However, the model aims to offer a travel plan that maximizes tourism value while respecting available resources, fostering a sustainable and affordable experience for the under-27 target audience.

4 MATHEMATICAL FORMULATION

The model adopted is an Integer Linear Programming (ILP) problem developed to plan an optimal itinerary through southern Europe, with the goal of maximizing overall tourism value. The formulation takes into account realistic constraints related to maximum travel time, overall available budget, and the operation of the Interrail pass, which allows for a limited number of days of rail travel.

The following paragraphs present and analyze the main components of the model: indices, parameters, decision variables, objective function and constraints.

4.1 INDEXES AND SET

The following indices are used in the optimization model to represent the cities involved in the trip:

- $i, j \in C$: indices that identify candidate cities within the model. Specifically, i and j are used to describe the possible pairs of cities involved in the shifts.
- *C*: set of candidate cities, i.e. the complete set of destinations considered for itinerary construction (selected as described in section 3.1).

4.2 PARAMETERS

The following parameters are introduced into the optimization model, representing the known and fixed data used to guide the choice of the optimal route:

- a_i : index of tourist interest associated with the city i, calculated as described in Section 3.2 and representative of perceived attractiveness from the traveler's perspective.
- k_i : estimated average cost for a day's stay in the city i, including accommodation, meals and local transportation, as discussed in Section 3.3.
- F: fixed cost of the Interrail "7 days in 1 month" pass for travelers under 27, which is 286 euros (updated figure to 2025, as reported on the official Interrail website).
- f: fixed cost of travel insurance, estimated at 55 euros based on common bids for policies of the appropriate duration and coverage.
- t_{ij} : average travel time expressed in minutes between the city i and city j, estimated from the data available in the "Train Times" section of the official Internail website, as described in Section 3.4.
- *B*: total budget available for the entire trip, set at 2,500 euros and including fixed costs and daily expenses (see section 3.5).
- T: maximum number of total days available for travel, including both days spent in cities and days spent traveling. The value is 30, based on the Global pass considered.
- V: maximum number of travel days allowed by the Interrail pass, which is 7 days out of 30.

- D_{max} : maximum number of days that can be spent in a single city during the trip. The value is set at 5.
- λ: parameter representing the weight (penalty) given to travel time. It is used in the objective function
 to balance tourist interest with the logistical efficiency of the itinerary.
- α : bonus awarded for each additional city visited. This parameter provides an incentive for travel variety within the model.

4.3 VARIABLES

The optimization model is based on the introduction of decision variables that represent the choices to be made to construct the optimal route. They are described below:

- $x_i \ (\ge 0)$: number of days spent in the city *i*. This integer variable indicates the length of stay in each selected city, contributing to the overall cost calculation and management of available time.
- $z_i \in \{0,1\}$: binary variable that takes value 1 if city i is included in the itinerary, and 0 otherwise. It is used to identify the cities you actually visited during the trip.
- y_{ij} ∈ {0,1}: binary variable that takes value 1 if the route involves a direct move from city i to city j, and 0 otherwise. This variable allows reconstructing the sequence of moves and the structure of the route.
- $g \ge 0$: integer variable representing the actual number of travel days, i.e., the number of rail routes used during the entire itinerary. Its value must meet the constraint imposed by the Interrail Pass (maximum 7 days).
- u_i ($0 \le u_i \le n-1$): auxiliary variable introduced for the formulation of the subtour elimination constraint according to the Miller-Tucker-Zemlin (MTZ) method. These variables ensure the consistency of the route and the absence of disconnected loops in the itinerary.

4.4 OBJECTIVE FUNCTION

The goal of the model is to maximize the overall interest of the traveler, which depends on the cities selected and the days spent in each of them. In other words, interest accumulates only if the city is actually visited and time is spent there.

The objective function is therefore defined as:

$$\max \sum_{i} a_i * x_i + \alpha * \sum_{i} z_i - \lambda \sum_{i} \sum_{j} t_{ij} * y_{ij}$$

• The term $\sum_i a_i * x_i$ quantifies the total value resulting from time spent in each city, weighted by the specific tourist interest of each location. The greater the number of days spent in a city with high interest, the greater the positive contribution to the objective function.

- The term $\alpha * \sum_i z_i$ represents an incentive to include more cities in the itinerary. This reflects a preference for more diverse itineraries, rewarding visiting new locations regardless of the length of stay.
- Finally, the term $\sum_i \sum_j t_{ij} * y_{ij}$ imposes a penalty proportional to the total time spent traveling between cities. This avoids itineraries with excessive or long transfers, which would reduce the time available for sightseeing and negatively impact the overall quality of travel.

4.5 CONSTRAINTS

To ensure the validity and feasibility of the optimized route, the Integer Linear Programming model is subject to a set of constraints that reflect the actual operational and temporal constraints of the trip.

The constraints implemented in the model are analyzed below.

4.5.1 Total duration

$$\sum_{i} x_i + g \le T \ (\le 30)$$

This constraint ensures that the total duration of the trip – including both days spent in the cities (variable x_i) and of the days devoted to travel (variable g) – does not exceed the maximum limit of available days, set at 30. This ensures the temporal feasibility of the entire itinerary, respecting the maximum allowable travel time.

4.5.2 Maximum number of travel days

$$g \leq V$$

This constraint imposes an upper limit on the number of days devoted to train travel, tying them to a maximum of 7 days, in accordance with the conditions of the Global Pass Interrail used. This restriction is a key operational constraint imposed by the pass, directly affecting route planning.

4.5.3 Total available budget

$$\sum_{i} k_i \cdot x_i + F + f \le B$$

This constraint ensures that the total cost of the trip does not exceed the maximum available budget, set at 2,500 euros. In calculating the total cost, the following are considered:

- The daily variable costs for the stay in each city k_i (lodging, meals, local transportation, etc.)
- The fixed cost of the Interrail pass F
- The cost of travel insurance f

This constraint ensures the economic sustainability of the planned itinerary.

4.5.4 Allocation of days only to selected cities

$$x_i \le T * z_i, \quad \forall i$$
 $x_i \ge z_i, \quad \forall i$
 $x_i \le D_{max} * z_i, \quad \forall i$

This constraint ensures consistency between the choice to visit a city, and the number of days assigned to it. Specifically:

- If the city i is not selected in the route $(z_i = 0)$, then no dwell days can be assigned $(x_i = 0)$.
- If instead the city i is included in the itinerary ($z_i = 1$), the model can assign a number of days x_i between 0 and the maximum total available T.

This prevents allocation of days in unvisited cities.

4.5.5 Flow conservation

$$\sum_{j \in C, j \neq i} y_{i,j} = z_i, \qquad \forall i$$

$$\sum_{j \in C, j \neq i} y_{j,i} = z_i, \qquad \forall i$$

$$y_{ii} = 0, \qquad \forall i$$

These constraints ensure consistency between the decision to visit a city, and the number of days assigned to stay there. Specifically:

- If a city i is not selected for travel $(z_i = 0)$, then the number of days assigned to that city must necessarily be zero $(x_i = 0)$.
- If, on the other hand, the city i is included in the itinerary ($z_i = 1$), then the model can assign a number of days x_i between a minimum of 1 and a predetermined maximum (e.g., the total available T or a reasonable daily maximum D_{max}).
- In addition, travel from a city to itself is prohibited, so it is imposed that $y_{ii} = 0$ for each city.

This formulation avoids attributing days to cities that are not actually visited, ensuring the logical and operational validity of the resulting itinerary.

4.5.6 Departure and return from Milan

$$\sum_{j} y_{Milano,j} = 1$$

$$\sum_{j} y_{j,Milano} = 1$$

$$\sum_{j} y_{j,Milano} = 1$$

These constraints stipulate that Milan represents both the starting and ending points of the journey.

- The first requires that exactly one train departs from Milan to one of the selected cities.
- The second dictates that a route ends in Milan, thus sanctioning a return to the point of origin.

Together, these constraints ensure that the itinerary constitutes a loop route that begins and ends in the same city. The choice of Milan as the central node of the trip reflects a realistic and logistically advantageous condition, consistent with the idea of circular and efficient tour planning.

4.5.7 Travel days and arcs

$$\sum_{i} \sum_{j} y_{ij} = g$$

This constraint stipulates that the total number of train routes traveled (i.e., the arcs y_{ij} activated in the city graph) must be equal to the actual number of train travel days, represented by the variable g.

In other words, each route activated in the model corresponds to a day on which the Interrail Pass is used. This bound ensures a one-to-one correspondence between actual trips and the days the pass is used, allowing the model to strictly adhere to the time constraints imposed by the train ticket.

This constraint is crucial for correctly representing the trip structure within the optimization.

4.5.8 **Subtour elimination (MTZ)**

$$u_i - u_j + 1 \le (n - 1) \cdot (1 - y_{ij}), \quad \forall i \in \{2, ..., n\} \, \forall j \in \{2, ..., n\}$$

$$u_1 = 0$$

This constraint is used to prevent the formation of disconnected loops (known as subtours), which might otherwise emerge in the planned route between cities. To ensure that the tour is a single, continuous loop, socalled MTZ constraints (Miller-Tucker-Zemlin) are used, which introduce auxiliary variables u_i associated with each city (except the starting city, Milan).

The variables u_i define an implicit ordering among the visited cities and allow the model to check for path consistency. The idea is that if there is an active arc between the city i and the city j, then the position in the tour of j must be later than that of i. In the absence of the arc, the inequality is automatically satisfied due to the multiplicative term.

To anchor the sequence, we fix the value of the auxiliary variable associated with Milan, setting $u_1 = 0$, which defines it as the starting point of the ordering.

This mechanism prevents multiple distinct unconnected loops from being generated, instead ensuring that all selected cities are part of a single connected itinerary, as required for a realistic and consistent travel plan.

4.5.9 Link between routes and selected cities

$$y_{ij} \le z_i \, \forall i, j \in C, i \ne j$$

 $y_{ij} \le z_i \, \forall i, j \in C, i \ne j$

This constraint ensures consistency between the selected cities and the travel routes included in the itinerary. These inequalities state that a travel arc y_{ij} – representing a direct route between two cities – can only be included in the itinerary if both cities involved (departure city i and arrival city j) are selected for the trip (i.e., if $z_i = 1$ and $z_j = 1$).

Without this constraint, the model might include connections between cities that are not actually visited, resulting in logistically inconsistent solutions. This condition therefore ensures that each travel route considered in the plan is logically tied to the effective inclusion of both cities on the tour.

5 IMPLEMENTATION OF THE MODEL

5.1 TECHNOLOGICAL CHOICES AND TOOLS USED

The implementation of the model was carried out entirely in the Python language, taking advantage of its versatility and the wide availability of libraries dedicated to mathematical optimization. In particular, it was decided to use Pyomo, an open-source framework widely used for the algebraic modelling of linear and integer optimization problems. Its syntax allows all the components of the model - variables, constraints, objective function - to be described in a clear and modular manner, while maintaining good code readability.

The GLPK solver, a free exact solver compatible with Pyomo, was used to solve the problem. Although it is not among the best performing on the market (such as Gurobi or CPLEX, for example), GLPK proved to be suitable for the scale of the problem tackled, guaranteeing short calculation times and good solution quality.

5.2 DATA ORGANIZATION AND MANAGEMENT

The structure of the code was organized to facilitate data reading and management. The initial data – including the list of cities, interest scores, daily costs and travel times – were read from an external Excel file, divided into two separate sheets: one containing the characteristics of each city, the other the travel time matrix. The extraction and preparation of this data was entrusted to a dedicated function, making the code reusable and easily adaptable.

5.3 CONSTRUCTION OF THE OPTIMIZATION MODEL

The central phase of the implementation was the construction of the optimization model, in which all the decision variables were defined: the days to be spent in each city, the selection of the cities to be visited, the connections between cities, the number of actual travel days and the variables necessary for the elimination of subtours. The constraints described in the theoretical phase were also included, including those relating to the maximum duration of the trip (30 days), the limit of travel days (7), the available budget, the mandatory departure and return from Milan, and logical constraints to ensure consistency between choices.

Once the model was defined, the problem was solved by assigning a time limit of 600 seconds to the solver, sufficient to find an optimal solution in our case.

5.4 EXTRACTION AND VISUALIZATION OF RESULTS

Next, the resulting solution was processed to extract the complete, ordered route. As the model simply returns which cities are connected to each other (via the variables y_{ij}), a sorting function had to be implemented to reconstruct the exact sequence of the journey starting from Milan, taking into account flow constraints.

Finally, a graphic visualization of the optimal route was provided, realized with the NetworkX library. This simple but effective representation shows the cities visited as nodes and the links as directional arcs, highlighting the ring structure of the journey (departure and return from Milan). Milan is visually distinguished from the other cities to highlight its central role in the itinerary.

5.5 Final considerations on the implementation

On the whole, the implementation made it possible to transform a theoretical formulation into an effective operational tool, capable of returning an optimal itinerary in a few seconds while respecting all the constraints imposed. The integration between mathematical modelling and open-source programming tools proved particularly effective, offering flexibility, reproducibility and the possibility of future extension. The structural clarity of the code and the modular approach adopted facilitated not only the development, but also the subsequent analysis of the results, paving the way for more complex and customizable experiments, which will be explored in the following chapters.

6 RESULTS

The implementation and resolution of the mathematical model made it possible to generate an initial optimal solution for the basic scenario outlined in the project: an intermodal journey through Southern Europe, lasting 30 days, with a maximum of 7 days of train travel, a total budget of 2,500 euros (including the cost of the Interrail pass and travel insurance) and the objective of maximizing overall tourist interest.

This chapter presents the main results obtained from the model, focusing on the optimal itinerary, the distribution of days, the choice of cities, and the visualization of the route.

6.1 VALUE OF THE OBJECTIVE FUNCTION

The optimal solution generated by the model corresponds to an objective function value of 62.98. This score represents the weighted combination of two key components:

- The total tourist interest accumulated through the selected cities and the number of days spent in each.
- The penalty associated with train travel times between cities.

This value quantifies the global efficiency of the itinerary in balancing enjoyment (tourist value) and operational constraints (travel time). It allows objective comparison with alternative scenarios, parameter settings, or constraint relaxations that might be tested in subsequent analyses.

6.2 OPTIMAL ITINERARY GENERATED BY THE MODEL

The output of the model produced an itinerary fully consistent with the constraints imposed and the objective of maximizing the overall tourist score. The journey begins and ends in Milan, a city chosen as a reference point due to its strategic location and easy access to numerous rail connections.

The cities selected in the optimal solution are:

- Milan 1 day
- Nice 1 day
- Zaragoza 5 days
- Barcelona 1 day
- Madrid 1 day
- Toulouse 5 days
- Lyon 5 days

The selection of these seven destinations, distributed between Italy, France and Spain, reflects a balance between tourist value (estimated on the basis of TripAdvisor ratings), daily subsistence costs, and the train

distance between cities. Each choice is functional to optimizing the overall experience, avoiding excessively long transfers or destinations with low returns in terms of utility.

6.3 A CIRCULAR AND EFFICIENT STRUCTURE

In terms of route structure, the itinerary follows a clear ring pattern. After departing from Milan, the journey proceeds westwards via Nice, enters the Iberian Peninsula with Zaragoza, Barcelona and Madrid, and re-enters France via Toulouse and Lyon, ending again in Milan.

This sequence is not random: it responds to the need to optimize transport time, making full use of the 7 days of rail travel allowed by the Interrail pass. All connections were selected taking into account the travel time and the weight that each journey has on the entire itinerary. The aim was to avoid detours and excessively long routes, ensuring a continuous and rational flow of travel.

6.4 DISTRIBUTION OF DAYS PER CITY

One of the most interesting aspects of the output concerns the distribution of days spent in the various cities. The model allocated time strategically, rewarding destinations that offer high tourist value with low costs and good rail connections.

The cities with extended stopovers are Zaragoza, Toulouse and Lyon, with 5 days each. These destinations represent, from the model's point of view, highly efficient nodes: they maximize the tourist score per euro spent while still maintaining good accessibility.

In contrast, cities such as Milan, Nice, Barcelona and Madrid were included with short stopovers (1 day), probably for logistical reasons (obligatory points of passage) or for variety of overall experience, even though they were not priorities in terms of length of stay.

This distribution demonstrates the model's ability to flexibly adapt the length of stay according to the specific characteristics of each city, optimizing the use of available time.

6.5 VISUALIZATION OF THE ROUTE

In order to reinforce the understanding of the route identified by the model and to verify its logistical coherence, a graphical representation was created using the NetworkX library. The graph obtained represents each city as a node and each railway transfer as a directional arc, following exactly the sequence generated by the optimal solution.

In this visualization, Milan - departure and return point of the journey - has been graphically highlighted in a distinct manner, so that its central role in the overall structure is immediately recognizable. The itinerary forms

a well-defined closed loop, clearly conveying the circular logic of the journey: one starts in Milan, passes through selected major cities in France and Spain, and returns to the starting point, without unnecessary detours or redundant routes.

The ring structure emerges clearly, underlining how the chosen connections fully respect the constraint of the 7-day train journey required by the Interrail pass. Each section represents a direct and rational connection between two cities, built to minimize travel time and costs.

The graphic visualization proved particularly useful not only to verify the correctness of the solution, but also to communicate it in an immediate and intuitive way. Through a simple graph, it becomes possible to grasp at a glance the efficiency and coherence of the planned route, appreciating the balance between variety of destinations, geographical compactness and compliance with operational constraints.

FIGURE 1 – OPTIMAL ROUTE



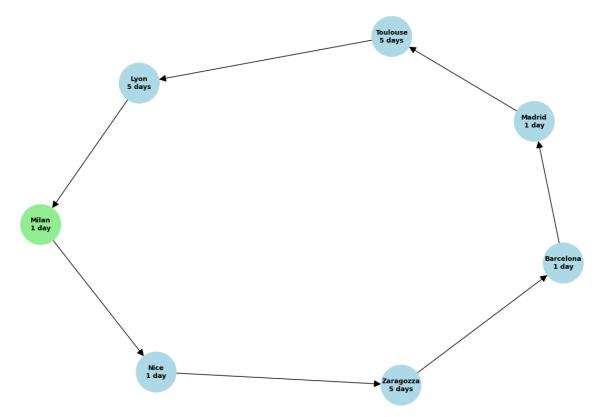


FIGURE 2 – OPTIMAL ROUTE ON MAP



6.6 EXCLUDED CITIES AND SELECTION LOGICS

The initial dataset of the project included a rather large set of potentially visitable destinations, mainly distributed among France, Spain, Italy and Portugal. However, the final itinerary generated by the model did not include all candidate cities. Some locations, although considered in the preliminary data collection phase, were excluded from the optimal solution.

This exclusion is not the result of an arbitrary choice, but the result of a rigorous optimization process, guided by well-defined constraints and objectives. In fact, the model favored destinations that offered an advantageous balance between tourist attractiveness, rail accessibility and economic sustainability. On the contrary, those cities were automatically excluded which, due to too high daily cost, a geographic location particularly distant from the other selected destinations, a relatively low tourist attractiveness score or an incompatibility with the imposed time constraints (such as the maximum total duration of the trip or the maximum number of travel days), could not compete with the cheaper alternatives.

This selection logic highlights the "rational" nature of the approach adopted. The model does not simply generate any set of destinations, but carries out a true optimization of available resources, proposing a route that maximizes overall utility within the constraints imposed. The final outcome is thus the result of a comparative and automated evaluation of possible options, which confirms the validity of the mathematical approach in the construction of customized, coherent and efficient travel solutions.

6.7 CONCLUDING REMARKS ON THE RESULTS OBTAINED

The results obtained demonstrate the validity and effectiveness of the approach adopted. The model produced a realistic, sustainable and optimized itinerary, capable of satisfying economic, time and logistical constraints at the same time.

The choices made, both in terms of cities visited and length of stay in each, show a well-balanced balance between exploration and in-depth study. Each destination was selected for the contribution it makes to the overall value of the trip, and the route structure reflects a clear and justifiable logic.

In the next chapter, the sensitivity of the model to variations in key parameters will be analyzed to verify how stable and adaptable the proposed solution is, and to explore alternative travel scenarios.

7 SENSITIVITY ANALYSIS

Sensitivity analysis is a fundamental tool for understanding how the model reacts to variations in key parameters and, more generally, for assessing its flexibility and robustness to different scenarios. In an application context such as the planning of an interrail trip, this phase is not limited to an academic exercise: rather, it offers concrete insights to customize choices according to individual preferences, economic availability or logistical constraints.

This chapter analyses the effects produced by modifying three strategic parameters of the model: the available budget (B), the weight associated with travel time (λ) , and the bonus linked to the number of cities visited (α) . For each of these, we observed how the composition of the itinerary, the objective function, the length of the stay and the geographical distribution of the selected destinations changed.

In the code the execution time has been limited to 60 for convenience, increasing it improves the solution.

7.1 EFFECT OF BUDGET VARIATION

Among the various constraints that affect the composition of the itinerary, the available budget is certainly one of the most influential. It directly establishes which destinations are accessible, for how many days it is possible to stay there, and how many stops one can realistically include. In the base model, the budget was set at 2,500 euros, including both the cost of the Interrail pass and travel insurance.

To assess the sensitivity of the model to this constraint, three scenarios were simulated: a reduced budget at 2,000 euros, the base budget at 2,500 euros, and finally an extended budget at 3,000 euros.

- In the first case, with a budget of 2,000 euros, the resulting solution is significantly smaller. The model selects a limited number of cities, all located in Italy (Milan, Florence, Rome, Naples), with average short stays and minimal travel. The final itinerary results in a target value of 55.58, reflecting the fewer opportunities in terms of tourist variety and quality.
- In the base case, with 2,500 euros, the model has greater freedom of action: the itinerary expands beyond Italian borders to include cities in France and Spain such as Nice, Zaragoza, Barcelona, Madrid, Toulouse, and Lyon. The increase in possibilities is reflected in a more varied and geographically broad solution, with the target value rising to 62.98.
- Finally, with a higher budget of 3,000 euros, the model can offer an even richer experience. Stays become longer and more comfortable, in cities of higher tourist value such as Nice, Lyon, Naples and Florence, with a route that also includes Rome. The overall quality of the itinerary improves significantly, and the target value reaches 82.66.

These results confirm the model's ability to adapt consistently and flexibly to different economic availabilities. Even with strict constraints, the model always manages to build sensible and optimized itineraries. Budget is a key element that changes travel arrangements, influencing their duration, quality, and variety.

TABLE 6 – BUDGET SENSITIVITY ANALYSIS

Budget	Visited Cities (Days of stay)	Total Days	Objective Value
2,000	Milan (3), Florence (5), Rome (1), Naples (3)	12	55.58
2,500	Milan (1), Nice (1), Zaragoza (5), Barcelona (1), Madrid (1), Toulouse (5), Lyon (5)	26	62.98
3,000	Milan (1), Nice (5), Lyon (5), Rome (1), Naples (5), Florence (4)	26	82.66

7.2 EFFECT OF LAMBDA VARIATION

The lambda parameter (λ) plays a key role in guiding the choice of the optimal route, modulating the penalty related to travel time between cities. Different values of λ significantly influence the structure of the trip: with a low value, the model tolerates long transfers in favor of a greater variety of destinations, while higher values favor more compact and less dispersed itineraries.

- In the first analyzed case ($\lambda = 0.01$), the penalization is minimal, and the model includes distant destinations, maximizing the overall tourist value. The itinerary crosses most of Spain, with cities such as Seville, Valencia, Barcelona, Madrid and Zaragoza, before returning to France via Lyon, for a total of 26 days. The high target value (77.54) reflects the richness of the experience, despite the longer transfer times, which can be more tiring.
- With λ equal to 0.02, the target value adopted in the model, the trade-off between quality of destinations and convenience of travel is balanced. The itinerary focuses on cities between France and Spain such as Nice, Zaragoza, Barcelona, Madrid, Toulouse and Lyon, maintaining good rail connections and short travel times. Again, the total duration is 26 days, but the target value is reduced to 62.98, signaling a moderate renunciation of variety in favor of logistical efficiency.
- Finally, with a high value of λ (0.05), the model strongly penalizes long transfers and generates a geographically compact itinerary. The selected cities Milan, Florence, Nice and Lyon are all close to each other, significantly reducing travel time and bringing the trip to a total duration of 20 days. However, this choice leads to a considerable reduction in the target value (20.29), due to the lesser variety and overall attractiveness of the destinations visited.

In summary, the λ parameter proves to be a powerful tool for adjusting the itinerary based on the traveler's comfort with transfers. Lower values encourage exploration of distant and varied destinations, while higher values favor a more compact and relaxed route. This makes the model highly adaptable, aligning the optimized plan with individual travel preferences.

TABLE 7 – LAMBDA SENSITIVITY ANALYSIS

Lambda	Visited Cities (Days of stay)	Total Days	Objective Value
0.01	Milan (1), Seville (5), Valencia (1), Barcelona (1), Zaragoza	26	77.54
	(5), Madrid (1), Lyon (5)		
0.02	Milan (1), Nice (1), Zaragoza (5), Barcelona (1), Madrid (1),	26	62.98
	Toulouse (5), Lyon (5)		
0.05	Milan (2), Florence (5), Nice (5), Lyon (4)	20	20.29

7.3 EFFECT OF ALPHA VARIATION

The alpha parameter (α) plays a key role in the objective function of the model, as it introduces a direct incentive to visit more cities. Each destination added to the itinerary activates a bonus proportional to α , rewarding more varied itineraries at the expense, possibly, of the length of stay in each location. This parameter thus allows the degree of travel variety to be modulated according to the traveler's preferences.

Three scenarios were simulated to analyze the impact of parameter α : a low value ($\alpha = 1$), an intermediate value ($\alpha = 3$, used as a reference) and a high value ($\alpha = 5$).

- In the first case ($\alpha = 1$), the model adopts a more selective strategy: the cities included in the tour are still numerous, but the stays are more unbalanced. For example, cities such as Valencia and Lyon host the traveler for five days, while Zaragoza, Madrid, and Toulouse are only short stops. The target value is 38.12, and the total trip duration is 25 days, reflecting a preference for more settled tourism, with long stops in a few central locations.
- At $\alpha = 3$, the model is more balanced between number of stops and length of stays. The itinerary becomes broader, including seven cities spread across Italy, France and Spain, with short but frequent stops. The target value rises to 62.98, and the trip is spread over 26 days. This solution represents a good compromise between variety and quality of experience.
- Finally, with a high incentive (α = 5), the stops are always seven cities with a clear fragmentation of the stay: several destinations are visited for only one day (such as Barcelona or Madrid), while others such as Toulouse and Zaragoza host the traveler for five days. The target value rises to 81.17, signaling a strong push toward intensive exploration, with the maximum number of stops allowed by budget and time constraints.

These results highlight how the α parameter can be used as a veritable "behavioral lever" to shape the type of travel experience: with low values, the model favors longer and more relaxed stays; with high values, it incentivizes frequent travel and a more dynamic pace. Alpha proves to be an effective tool for tailoring the itinerary to personal preferences, making the model flexible and customizable even if limited by other parameters such as budget and lambda.

TABLE 8 – ALPHA SENSITIVITY ANALYSIS

Alpha	Visited Cities (Days of stay)	•	Objective Value
1	Milan (1), Lyon (5), Zaragoza (1), Madrid (1), Valencia (5), Toulouse (1), Nice (4)		38.12
3	Milan (1), Nice (1), Zaragoza (5), Barcelona (1), Madrid (1), Toulouse (5), Lyon (5)		62.98
5	Milan (1), Lyon (4), Toulouse (5), Madrid (1), Zaragoza (5), Barcelona (1), Nice (2)	26	81.17

7.4 FINAL QUALITATIVE CONSIDERATIONS

The sensitivity analysis showed how the main parameters of the model – available budget, travel time penalty (λ) and variety incentive (α) – decisively influence itinerary composition.

Larger budgets allow for more extensive itineraries and longer stops, while smaller budgets impose more essential trips. The λ parameter regulates the level of tolerance for long transfers: high values favor compact itineraries; low values extend the range of exploration. The parameter α , on the other hand, determines the degree of variety desired: a high value favors many short stages, a low value longer stays in fewer cities.

Overall, the model proves not only effective but also highly adaptable: it allows tailor-made itineraries to be built in line with the individual traveler's preferences and travel style, thus offering a flexible and customized planning tool.

8 CONCLUSION AND FUTURE DEVELOPMENT

The objective of this work was to design, formulate and implement a mathematical optimization model aimed at the effective planning of an Interrail trip across Southern Europe. The proposed model is based on an integer linear programming approach, designed to maximize the overall tourism value of the travel experience, taking into account real and significant constraints such as the available budget, the maximum duration of the trip and the limits of rail pass usage.

The realization of the project demonstrated how it is possible to translate the problem of planning a complex trip into a formalized language, transforming it into a model that can be solved using optimization tools. The approach adopted made it possible to generate optimal itineraries capable of balancing logistical efficiency with the quality of the tourist experience, returning coherent, realistic and customizable solutions.

8.1 SUMMARY OF THE OPTIMAL DECISIONS OBTAINED

The analysis of the solution generated for the base scenario showed the effectiveness of the model in constructing a route that simultaneously respects all the constraints imposed while optimizing the objective function. The proposed route, starting and returning from Milan, includes a selected set of cities – including Nice, Zaragoza, Barcelona, Madrid, Toulouse and Lyon – chosen for their ability to offer a good compromise between tourist value, daily costs and travel time.

The distribution of the days spent in each location is not uniform but reflects a strategic balance: cities with a higher quality/price ratio in terms of tourism have been assigned longer stays, while others, although interesting, have been visited more briefly, in order not to compromise the overall balance of the trip. The result is a structured itinerary, efficient and in line with the expectations of a traveler attentive to the value of the experience but also to its economic and logistical sustainability.

8.2 CUSTOMIZATION AND CONTROL PARAMETERS

One of the most relevant aspects that emerged during the sensitivity analysis concerns the possibility of customizing the model through two key parameters: α (the bonus attributed to each city visited) and λ (the penalty coefficient associated with the time spent travelling).

These parameters make it possible to modulate the objective function according to the traveler's subjective preferences. In particular, a high value of α tends to incentivize itineraries that include a greater number of destinations, reflecting a propensity towards variety and the discovery of new locations. Conversely, an increased value of λ pushes the model towards more compact and geographically consistent solutions, favoring itineraries that reduce travel time and longer trips, in line with a more relaxed and concentrated travel style.

This possibility of control makes the model an extremely flexible tool, capable of adapting to different traveller profiles and priorities, while maintaining a rigorous and consistent approach.

8.3 LIMITATIONS OF THE MODEL

Despite its demonstrated effectiveness, the model has some structural limitations that deserve to be highlighted. Firstly, the static nature of the data used means that the model does not take into account temporal or contextual variables that may significantly affect the effectiveness of the proposed itinerary. For example, cultural events, local festivities, tourist seasonality and weather conditions are not currently taken into account, although they represent determining factors in the quality of the travel experience.

Secondly, the traveler's personal preferences are represented indirectly, through weights and bonuses, but are not made explicit in terms of specific attractions, cultural, gastronomic or natural interests. As a result, the model optimizes tourist interest in aggregate terms, without full individualization of the experience.

Finally, the model is currently constrained to the rail transport mode, in line with the objective of simulating an Interrail trip. However, this assumption limits the variety of possible itineraries and may penalize, in certain cases, destinations that are not easily accessible by train.

8.4 FUTURE DEVELOPMENTS AND DIRECTIONS FOR EXTENSION

In light of the above considerations, it is possible to outline some promising directions for the future development of the model. The first extension concerns the integration of events, seasonality and dynamic data, which would allow for a planning more adherent to the real context, improving the quality of the experience and the temporal coherence of the trip.

A second area of enhancement concerns the advanced personalization of preferences: through the construction of user profiles, preference questionnaires, or recommendation mechanisms, the model could be enriched by a deeper level of individualization, making solutions even more tailor-made.

Finally, while remaining focused on the Interrail context, the model could be extended to evaluate alternative scenarios involving forms of multi-modality, such as bus, regional flights or maritime routes, in order to analyze hybrid solutions in which the rail component continues to play a central, but not exclusive role.

8.5 OVERALL CONCLUSION

In conclusion, the optimization model developed in this project proves to be an effective and flexible tool for designing an Interrail trip that is both efficient and tailored to real-world constraints. Built around the logic of the Global Pass for travelers under 27, the model considers realistic limitations − such as a total budget of €2,500, a 30-day duration, and a 7-day rail travel allowance − to generate itineraries that balance cost, comfort, and tourist value.

In the base scenario, the optimized route achieves a strong objective function value of 62.98, showing how mathematical formulation can translate into a high-quality, feasible, and enjoyable travel plan. The possibility to tune parameters such as α and λ further enhances the model's adaptability, making it suitable for different travel styles, whether relaxed, exploratory, or efficiency driven.

Although further improvements are possible, the model provides a robust foundation for building smart travel planning tools that combine optimization with personalization, offering young travelers more control and insight into how to make the most of their Interrail experience.

APPENDIX

TABLE 5 – TIME MATRIX

Toul	Bord	Nice	Lyon	Paris	Naples	Flore	Rome	Venice	Seville	Vale	Zara	Barc	Madrid	Fatima	Coimbra	Faro	Porto	Lisbon	Milan	Time
oulouse	Bordeaux			S	es	lorence	ē	се	lle	alencia	aragozza	Barcelona	rid	na	nbra		•	on	=	ime Matrix Milan
1680	1150	300	420	480	257		186	129	1380	1728	1354	1228	1422	2225	1835	2205	2510	1620		Milan
						93 2						8		5	5	5	.0	ŏ	0 1	Lisbon
1500	2520	2040	1800	1860	2800	2760	2820	2760	684	1320	1371	792	528	90	124	180	195	0	1620	Porto
1380	2628	1860	1620	1620	2640	2580	2640	2560	684	1107	1181	1568	633	180	69	410	0	195	2510	
1740	1860	2220	2040	2040	3000	2940	3060	3000	1459	1680	1470	1554	1469	331	330	0	410	180	2205	Faro
														1	0		0			Coimbra
1463	2000	1860	1680	1740	2760	2580	2700	2640	732	1200	1136	1357	562	86	0	330	69	124	1835	ra Fatima
1560	1860	2040	1800	1920	2880	2640	2760	2760	660	1320	1374	1467	569	0	86	331	180	90	2225	
258	1162	1098	600	720	1800	1500	1680	1500	144	120	90	200	0	569	562	1469	633	528	1422	Madrid
			4		1500	1278	1380	1320						1467		1554	1568		1228	Barcelo
369	514	558	420	420	00	78	80		360	190	90	0	200	67	1357	54		792		Barcelona Zaragozza Valencia
246	1074	660	480	540	1560	1396	1500	1440	240	300	0	90	90	1374	1136	1470	1181	1371	1354	ozza Va
600	738	730	609	730	1800	1680	1560	1440	318	0	300	190	120	1320	1200	1680	1107	1320	1728	
720	1380	1380	720	840	1920	1740	1860	1740		318	240	360	144	660	732	1459	684	684	1380	Seville
								0	0 1											Venice
1260	840	480	504	618	318	120	180	0	1740	1440	1440	1320	1500	2760	2640	3000	2560	2760	129	Rome
1362	1423	540	600	720	144	96	0	180	1860	1560	1500	1380	1680	2760	2700	3060	2640	2820	186	
1257	1318	510	495	628	180	0	96	120	1740	1680	1396	1278	1500	2640	2580	2940	2580	2760	93	Florence
1452	1573	1128	690	755		18	144	318	1920	1800	1560	1500	1800	2880	2760	3000	2640	2800	257	Naples
52	73	28	90	55	0	180	4	8	20	0	80	0	0						57	Paris
397	370	360	117	0	755	628	720	618	840	730	540	420	720	1920	1740	2040	1620	1860	480	Lyon
245	491	274	0	117	690	495	600	504	720	609	480	420	600	1800	1680	2040	1620	1800	420	
503	1108		274	360	1128	510	540	480	1380	730	660	558	1098	2040	1860	2220	1860	2040	300	Nice
	8	0 1																		Bordea
190	0	1108	491	370	1573	1318	1423	840	1380	738	1074	514	1162	1860	2000	1860	2628	2520	1150	Bordeaux Toulouse
0	190	503	245	397	1452	1257	1362	1260	720	600	246	369	258	1560	1463	1740	1380	1500	1680	ouse

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