Public announcement implementation of a game of sleuth.

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1 Introduction

Sleuth is a card game. There are 2 types of cards. The first type are the diamond playing cards. Each diamond card has 2 parameters: one of three possible colours(red, green, blue) and 1..3 diamonds. Each card is unique so in total there are $3 * 3 = 9$ different diamond cards. In the initial state of the game one random diamond card is removed from the deck and the other cards are evenly distributed between players. If it is the case that the cards cannot be evenly distributed the excess cards are shown to all players. The goal of each player is to find out which card is hidden from the deck. The way to do that is to ask other players about the type of diamond cards they have. Questions are asked using question cards, from which each players gets 4 at the start of the game. There are a different kind of questions like for example "how many blue cards you have". When a question card is played it is removed and a new question card is drawn from the stack. In the original game asking there are questions where the person being asked only shows the answer to the person asking the question. Since we only use public announcements in our implementation we presume that the person being asked the question will just show the cards to all the players.

2 Model

To make an implementation of this variance of sleuth first all the rules need to be specified. For the formal description of the cards in the model we use the following notation xCy where $x \in \{1, 2, 3, H, O\}$ represent player 1..3, Hidden and Open, $C \in \{R, G, B\}$ represent the 3 colors Red, Green and Blue and lastly $y \in \{1, 2, 3\}$ represents the amount of diamonds on the card.
The are two different Search cards:

1. Ask for a question regarding the amount a player has, e.g. *"how many red cards do you have"*.The player who was asked this question answers out loud so everybody can hear how many cards he or she has. Formally this can be described as: The same can be asked regarding how many cards with a specific amount of diamonds a player has, e.g. *how many cards with 2 diamonds do you have?"*

2. Ask for a specific question with 2 parameters: *"Do you have the red card with 1 diamond?"*. Then the player who answers will show the cards to every player in the game.

These 2 types of question cards can be formally defined as:

Question 1: If a player announces how many red cards he has this becomes common knowledge.
Player 1 announces that he has 0 red cards:
$[1R^0]\neg(1R1 \vee 1R2 \vee 1R3)$
Player 1 announces that he has 1 red card:
$[1R^1](1R1 \vee 1R2 \vee 1R3) \wedge \neg(1R1 \wedge 1R2) \wedge \neg(1R1 \wedge 1R3) \wedge \neg(1R2 \wedge 1R3)$
Player 1 announces that he has 2 red cards:
$[1R^2](1R1 \wedge 1R2 \wedge \neg 1R3) \vee (1R1 \wedge \neg 1R2 \wedge 1R3) \vee (\neg 1R1 \wedge 1R2 \wedge 1R3)$
The same goes for R2,R3,G1,G2,G3,B1,B2,B3.

Question 2: If a player announced how many cards of a specif amount of diamonds he has this becomes common knowledge.
Player 1 announces that he has 0 cards with 1 diamond:
$[1R^0]\neg(1R1 \vee 1R2 \vee 1R3)$
Player 1 announces that he has 1 card with 1 diamond:
$[1R^1](1R1 \vee 1R2 \vee 1R3) \wedge \neg(1R1 \wedge 1R2) \wedge \neg(1R1 \wedge 1R3) \wedge \neg(1R2 \wedge 1R3)$
Player 1 announces that he has 2 cards with 1 diamond:
$[1R^2](1R1 \wedge 1R2 \wedge \neg 1R3) \vee (1R1 \wedge \neg 1R2 \wedge 1R3) \vee (\neg 1R1 \wedge 1R2 \wedge 1R3)$
The same goes for R2,R3,G1,G2,G3,B1,B2,B3.

Furthermore some general rules apply for this game.

Rule 1 Every card in the deck is unique. In other words; for example card R1 can only be in 1 of the positions:
$(1R1 \vee 2R1 \vee 3R1 \vee HR1 \vee OR1) \wedge \neg(1R1 \wedge 2R1) \wedge \neg(1R1 \wedge 3R1) \wedge \neg(1R1 \wedge HR1) \wedge \neg(1R1 \wedge OR1) \wedge \neg(2R1 \wedge 3R1) \wedge \neg(2R1 \wedge HR1) \wedge \neg(2R1 \wedge OR1) \wedge \neg(3R1 \wedge HR1) \wedge \neg(3R1 \wedge OR1) \wedge \neg(HR1 \wedge OR1)$

3 Implementation

The most important part of the game is of course not only asking the right questions to the right person yourself but also looking at what the other players are saying to each other. Thus, is seems possible to describe the model of this game in terms of public announcement logic. For example all propositional atoms can be represented as p[i,j] = player[i] has diamond card[j] and each turn can be represented as public announcement.

In terms of Kripke models we have to model a couple of things. Firstly we need to model that if an agent responds, all the epistemic alternatives that become impossible are removed from the model. Let's say that agent 1 responds that he has two red cards. For the model this means that all the worlds where the agent does not have 2 cards are invalid and thus can be discarded. The model does not have to keep track if it knows which is the card that is hidden because at the end of the game only 1 world will remain possible and the hidden card in that world is the actual hidden card.

4 Discussion