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Problems

12.1 A booster stage produces 190,400 lb of thrust at sea level and 242,500 lb at altitude. Assuming the momentum thrust remains nearly constant with altitude, estimate the nozzle exit diameter D_2 for the booster.

12.2 RD-170 is a Russian launch vehicle booster. It has a nozzle throat diameter of 235.5 mm and exit diameter of 1430 mm. Propellant flow rate is 2393 kg/s and the nozzle exit static pressure is 7300 Pa. RD-170 develops a vacuum-specific impulse of 337 s. Calculate

(a) nozzle area ratio, A_2/A_{th}
(b) effective exhaust speed c in m/s
(c) vacuum thrust in MN
(d) pressure thrust in vacuum in kN

12.3 A rocket engine has a propellant mass flow rate of 1000 kg/s and an effective exhaust speed of $c = 3500\,\text{m/s}$. Calculate

(a) rocket thrust F in kN
(b) specific impulse I_s in seconds

12.4 A rocket engine has a chamber pressure of $p_c = 1000$ psia and the throat area is $A_{th} = 1.5\,\text{ft}^2$. Assuming that the nozzle is perfectly expanded with the gas ratio of specific heats $\gamma = 1.2$ and the ambient pressure of $p_0 = 14.7$ psia, calculate

(a) optimum thrust coefficient $C_{F,opt}$
(b) thrust F in lbf
(c) nozzle exit Mach number M_2
(d) nozzle area expansion ratio A_2/A_{th}

12.5 Consider a fuel-rich combustion of the hydrocarbon fuel known as RP-1 and oxygen in a liquid propellant rocket according to

$$2CH_{1.953} + n_{O_2}O_2 \longrightarrow CO_2 + (1.953/2)H_2O + CH_{1.953}$$

Calculate

(a) number of moles of oxygen n_{O2}
(b) the mixture ratio r, i.e., the oxidizer-to-fuel ratio
(c) the molecular weight of the mixture of gases in the products of combustion

12.6 The average specific impulse of a chemical rocket is 360 s. The rocket is in a zero gravity vacuum flight. Calculate and graph vehicle terminal speed ΔV for the propellant fraction ζ, that ranges between 0.80 and 0.95.

12.7 A rocket is vertically launched and operates for 60 s and has a mass ratio of 0.05. The (mean) rocket-specific impulse is 375 s. Assuming the average gravitational acceleration over the burn period is 9.70 m/s^2, calculate the terminal velocity of the rocket with and without gravitational effects. Neglect the effect of aerodynamic drag in both cases.

12.8 A rocket has a mass ratio of MR = 0.10 and a mean specific impulse of 365 s. The flight trajectory is described by a constant dynamic pressure of $q_0 = 50\,\text{kPa}$. The mean drag coefficient is approximated to be 0.25, the vehicle initial mass is $m_0 = 100,000\,\text{kg}$, and the vehicle (maximum) frontal cross-sectional area A_f is $5\ \text{m}^2$. For a burn time of 100 s, calculate the rocket terminal speed while neglecting gravitational effect.

12.9 In comparing the flight performance of a single-stage with a two-stage rocket, let us consider the two rockets have the same initial mass m_0, the same payload mass m_L, and the same overall structural mass m_s. The structural mass fraction ε, which is defined as the ratio of stage structural mass to the initial stage mass, is also assumed to be the same for the single-stage rocket and each of the two stages of the two-stage rocket. For the effective exhaust speed of 3500 m/s be constant for the single-stage and each stage of the two-stage rocket, calculate the terminal velocity for the two rockets in zero gravity and vacuum flight, for

$$\begin{aligned} m_0 &= 100,000\,\text{kg} \\ m_L &= 500\,\text{kg} \\ m_s &= 10,000\,\text{kg} \\ \varepsilon_{\text{single-stage}} &= 0.1 \\ \varepsilon_{\text{stage-1}} &= \varepsilon_{\text{stage-2}} = 0.1 \end{aligned}$$

12.10 A liquid propellant rocket uses a hydrocarbon fuel and oxygen as propellant. The heat of reaction for the combustion is $Q_R = 18.7\,\text{MJ/kg}$. The specific impulse is 335 s and the flight speed is 2500 m/s. Neglecting the propellant kinetic power at the injector plate, calculate

(a) effective exhaust speed c in m/s
(b) propulsive efficiency η_p
(c) overall efficiency η_o

12.11 An injector plate uses an unlike impingement design. The fuel and oxidizer orifice discharge coefficients are $C_{df} = 0.80$ and $C_{do} = 0.75$. The static pressure drop across the injector plate for both oxidizer and fuel jets is the same, $\Delta p_f = \Delta p_o = 180\,\text{kPa}$. The fuel and oxidizer densities are $\rho_f = 325\,\text{kg/m}^3$ and $\rho_o = 1200\,\text{kg/m}^3$ and the oxidizer-fuel mass ratio is $r = 3.0$. Calculate

(a) oxidizer-to-fuel orifice area ratio A_o/A_f
(b) oxidizer and fuel jet velocities v_o and v_f

12.12 A solid rocket motor has a design chamber pressure of 10 MPa, an end-burning grain with $n = 0.4$ and $r = 3$ cm/s at the design chamber pressure and design grain temperature of 15°C. The temperature sensitivity of the burning rate is $\sigma_p = 0.002/°\text{C}$, and chamber pressure sensitivity to initial grain temperature is $\pi_K = 0.005/°\text{C}$. The nominal effective burn time for the rocket is 120 s, i.e., at design conditions. Calculate

(a) the new chamber pressure and burning rate when the initial grain temperature is 75°C
(b) the corresponding reduction in burn time Δt_b in seconds

12.13 Extract the erosive burning parameter k from the data of Figure 12.30 for the two solid propellants shown.

12.14 A regeneratively cooled rocket thrust chamber has its maximum heat flux of 15 MW/m^2 near its throat. The hot gas stagnation temperature is 3000 K and the local gas Mach number is assumed to be ~1.0. The gas mean molecular weight is MW = 23 kg/kmol and the ratio of specific heats is $\gamma = 1.24$.
Calculate

(a) gas static temperature T_g in K
(b) gas speed near the throat in m/s
(c) gas-side film coefficient h_g for $T_{wg} \sim 1000$ K

12.15 A rocket combustion chamber is designed for a chamber pressure of $p_c = 50\,\text{MPa}$. The combustion gas has a ratio of specific heats $\gamma = 1.25$. If this rocket is to operate between sea level and 200,000 ft altitude, calculate the range of area ratios in the nozzle that will lead to perfect expansion at all altitudes. Assume isentropic flow in the nozzle.

12.16 The propellant flow rate in a chemical nozzle is 10,000 kg/s, the nozzle exhaust speed is 2200 m/s, and the nozzle exit pressure is $p_2 = 0.01\,\text{atm}$. Assuming the nozzle exit diameter is $D_2 = 2$ m, calculate

(a) the pressure thrust (in MN) at sea level
(b) the effective exhaust speed c (in m/s) at sea level

12.17 A solid propellant rocket motor uses a composite propellant with 16% aluminum. The same propellant with 18% aluminum enhances the combustion temperature by 5.7%. Assuming in both cases that the solid particles are fully accelerated (i.e., $V_s = V_g$) in the nozzle but the solid temperature remains constant (i.e., T_s = constant), calculate the ratio of specific impulse in the two cases. Aluminum specific heat is $c_s = 903\ \text{J/kg}\cdot\text{K}$, and the specific heat at constant pressure for the gas is $c_{pg} = 2006\,\text{J/kg}\cdot\text{K}$.

12.18 The coefficient of linear thermal expansion for a solid propellant grain is $1.5 \times 10^{-4}/°\text{C}$.Calculate the change of length ΔL for a 1 m long propellant grain that experiences a temperature change from −30°C to + 70°C.

12.19 A ramjet has a maximum temperature $T_{t4} = 2500\,\text{K}$. The inlet total pressure recovery π_d varies with flight Mach number according to

$$\pi_d = 1 - 0.075\,(M_0 - 1)^{1.35}$$

The ramjet burns hydrogen fuel with $Q_R = 120,000\,\text{kJ/kg}$ and combustor efficiency and total pressure ratio are $\eta_b = 0.99$ and $\pi_b = 0.95$, respectively. The nozzle is perfectly expanded and has a total pressure ratio $\pi_n = 0.90$. Assuming a calorically perfect gas with $\gamma = 1.4$ and $R = 287\,\text{J/kg}\cdot\text{K}$, use a spreadsheet to calculate the ramjet (fuel)-specific impulse, propulsive, thermal, and overall efficiencies over a range of flight Mach number starting at $M_0 = 3$ up until ramjet ceases to produce any thrust. Altitude pressure and temperature are 15 kPa and 250 K, respectively.