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Maximum entanglement of quantum-dot systems with single excitation in the spin van der Waals model

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Time evolution of entanglement of N quantum dots are analyzed within the spin-1/2 van der Waals (or Lipkin-Meshkov-Glick) XY model. It is shown that, for a single dot initially excited and disentangled from the remaining unexcited dots, the maximum bipartite entanglement can be obtained in the systems of $N = 2, \cdots, 6$ dots only.

Introduction. Entanglement is one of the most profound concepts of quantum mechanics. But only recently it has been recognized as a possible resource for quantum information processing including quantum computation, quantum teleportation, quantum dense coding, or certain types of quantum key distributions [1].

To describe a bipartite entanglement of quantum-dot systems in a pure state, given by $\hat{\rho}_{AB} = (|\psi\rangle\langle\psi|)_{AB}$, we apply the von Neumann reduced entropy defined as [2]

$$E[\hat{\rho}_{AB}] = -\mathrm{Tr}\{\hat{\rho}_A \log_2 \hat{\rho}_A\} = -\mathrm{Tr}\{\hat{\rho}_B \log_2 \hat{\rho}_B\}$$

where $\hat{\rho}_A = \mathrm{Tr}_B\{\hat{\rho}_{AB}\}$ and $\hat{\rho}_B = \mathrm{Tr}_A\{\hat{\rho}_{AB}\}$ are the reduced density matrices.

Model. We analyze entanglement in a system of N two-level quantum dots described by the Pauli spin operators $(\hat{\sigma}_n^x, \hat{\sigma}_n^y, \hat{\sigma}_n^z)$ for each $(n = 1, \cdots, N)$ lattice site by assuming that each dot is coupled to all others with the same strength independent of separation distance and nature. Such a model is referred to as the spin van der Waals (SVW) [3], Lipkin-Meshkov-Glick [4], equivalent-neighbor or infinitely-coordinated spin system. Some aspects of entanglement in the SVW systems in different contexts have already been analyzed in Refs. [5–8].

In particular, the quantum SVW system in the XY-like regime can be defined by the interaction Hamiltonian [3]:

$$\hat{H}_{\mathrm{int}} = \kappa \sum_{n \neq m} \left(\hat{\sigma}_n^+ \hat{\sigma}_m^- + \hat{\sigma}_n^- \hat{\sigma}_m^+\right).$$

The creation and annihilation spin operators are given by $\sigma_n^{\pm} = \hat{\sigma}_n^x \pm \hat{\sigma}_n^y$, respectively. Let's assume that the initial state describing a system of M $(M = 0, \cdots, N)$ dots excited and $N - M$ dots in the ground state is given as $|\psi(0)\rangle = \{|1\rangle^{\otimes M}\}_A \{|0\rangle^{\otimes (N-M)}\}_B$. Then, the solution of the Schrödinger equation of motion for the SVW model reads as

$$|\psi(t)\rangle = \sum_{m=0}^{M'} C_m^{NM}(t) \{|1\rangle^{M-m}|0\rangle^m\}_A \{|1\rangle^m|0\rangle^{N-M-m}\}_B$$

where $M' = \min(M, N - M)$. The states in curly brackets, $\{|1\rangle^{\otimes(M-m)}|0\rangle^{\otimes m}\}$, denote a sum of all possible [i.e., $\binom{M-m}{m}$] M-dot states with the excitation number $(M - m)$. The time-dependent superposition coefficients, $C_m^{NM}(t)$, are given by

$$C_m^{NM}(t) = \sum_{n=0}^{M'} b_{mn}^{NM} \exp\left\{\mathrm{i}[n(N+1-n) - M(N-M)]\kappa t\right\}$$

with

$$b_{mn}^{NM} = \sum_{k=0}^{m} (-1)^k \binom{m}{k} \binom{N-2k}{M-k}^{-1} \left\{\binom{N+1-2k}{n-k} - 2\binom{N-2k}{n-k-1}\right\}$$

where $\binom{x}{y}$ are binomial coefficients. In the following we will focus on the evolution of the system of N dots in which a single dot $(M = 1)$ is initially excited.

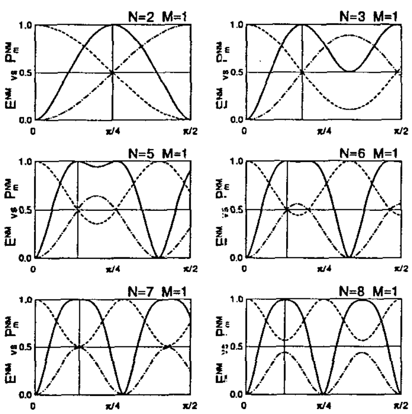

FIG. 1. Evolution of the entanglement, E^{NM} (solid curves), and the Schmidt coefficients: P_1^{NM} (dot-dashed curves) and P_2^{NM} (dashed curves) for systems of $N = 2, \cdots, 8$ quantum dots with only one $(M = 1)$ of them initially excited. It is seen that the maximum entanglement corresponds to the Schmidt coefficients mutually equal or, in general, the least different.

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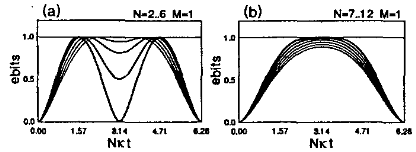

FIG. 2. Entanglement as a function of the rescaled time $N\kappa t$. It is seen that one ebit of entanglement can be achieved in systems with the total number of dots ranging from two (thickest curve in fig. (a)) up to six (thinnest curve in fig. (a)) only. For $N > 6$, the maximum entanglement decreases with increasing N. The evolution of the systems with higher N is marked by thinner curves.

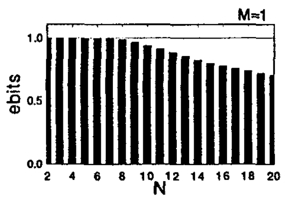

FIG. 3. The maximum values of the entanglement $E^{N,1}$ as a function of the total number N of quantum dots in systems with single excitation. On the scale of the figure, $\max_t E^{7,1}(t) = 0.9997$ does not differ from unity. However, for higher values of N, a monotonous decrease of the maximum entanglement is clearly visible.

Entanglement. The Schmidt coefficients for our solutions are given by [9]

$$P_m^{NM}(t) = \binom{M}{m}\binom{N-M}{m}|C_m^{NM}(t)|^2.$$

They enable the direct calculation of the bipartite entanglement via the Shannon entropy

$$E^{NM}(t) \equiv E[|\psi(t)\rangle\langle\psi(t)|] = -\sum_m P_m^{NM}\log_2 P_m^{NM}.$$

In the simplest nontrivial case, for $M = 1$, the Schmidt coefficients reduce to

$$P_1^{N,1}(t) = 4\tfrac{(N-1)}{N^2}\sin^2(Nt/2)$$

and $P_2^{N,1}(t) = 1 - P_1^{N,1}(t)$. The evolution of the Schmidt coefficients, $P_m^{N,1}(t)$, and the entanglement, $E^{N,1}(t)$, are depicted in Fig. 1.

The quantum-dot systems evolve into the maximum entangled state for times given by

$$0 = \dot{E}^{NM} = 2\tfrac{N-1}{N}\sin(N\kappa t)\log_2\left[\tfrac{N^2}{4(N-1)}\csc^2(\tfrac{N}{2}\kappa t) - 1\right]$$

with the roots

$$\kappa t' = \tfrac{2}{N}\operatorname{arccsc}(\tfrac{2}{N}\sqrt{2(N-1)})$$

and $\kappa t'' = \frac{\pi}{N}$. We find that the maximum of entanglement, equal to $E^{N,1}(t') = 1$ ebit, can be achieved for the evolution time t' for $N \le 6$ only. For $N > 6$, real solution for t' does not exist. This conclusion can also be drawn by analyzing Figs. 1 and 2.

Entanglement for $N > 6$ reaches its maximum at the evolution times t''. This maximum value, given by

$$E^{N,1}(t'') = \tfrac{2}{N^2}\{N^2\log_2 N - (N-2)^2\log_2(N-2) \\ -2(N-1)\log_2[4(N-1)]\} < 1,$$

monotonically decreases with increasing N as clearly depicted in Figs. 2 and 3.

Conclusion. We have studied evolution of quantum dots in the spin van der Waals XY model. We have applied its Schmidt decomposition to investigate information-theoretic entropic properties and entanglement of the quantum dot ensembles. Closer analysis of our solution shows that the quantum dots evolve periodically into the bipartite entangled states. We have found for the systems of N dots with a single dot initially excited and disentangled from the remaining unexcited $(N-1)$ dots that the maximum bipartite entanglement can be observed in the systems of $N = 2, \cdots, 6$ dots only. It is worth noting that the same "magic" number $N = 6$ has been found in a different context of the equivalent-neighbor entangled webs in Ref. [7].

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