

OPEN ACCESS

Quantum phase transitions in the LMG model by means of quantum information concepts

To cite this article: O Castaños *et al* 2012 *J. Phys.: Conf. Ser.* **387** 012021

View the [article online](#) for updates and enhancements.

You may also like

- [Global quantum discord in the Lipkin–Meshkov–Glick model at zero and finite temperatures](#)
Jia Bao, Yan-Hong Liu and Bin Guo
- [The extended Heine–Stieltjes polynomials associated with a special LMG model](#)
Feng Pan, Lina Bao, Liyuan Zhai et al.
- [Eliminating the dose-rate effect in a radiochromic silicone-based 3D dosimeter](#)
E M Høye, P Balling, E S Yates et al.



The Electrochemical Society
Advancing solid state & electrochemical science & technology

241st ECS Meeting

Vancouver, BC, Canada. May 29 – June 2, 2022



ECS Plenary Lecture featuring
Prof. Jeff Dahn,
Dalhousie University



Register now!



Quantum phase transitions in the LMG model by means of quantum information concepts

O Castaños, R López-Peña, E Nahmad-Achar, and J G Hirsch

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México
Apartado Postal 70-543, 04510 México, D. F., México

E-mail: ocasta@nucleares.unam.mx

Abstract.

Quantum phase transitions are currently studied in several fields of physics, as examples one finds: in nuclear physics the shape phase transitions within the Interacting Boson Model; in quantum optics two level atoms interacting with a one mode electromagnetic radiation in a cavity; and in condensed matter in the analysis of the behavior of spin systems. In this contribution we employ the fidelity and its susceptibility, two concepts widely used in quantum information theory, to determine the quantum phase transitions of the Lipkin-Meshkov-Glick (LMG) model together with its scaling properties. By means of these concepts, we propose a quantum method to determine the crossings and anti-crossings present in a model Hamiltonian as a function of the control parameters of the model. A review of the separatrix of the LMG model is done to compare the results obtained by means of quantum information concepts with those related with the singular behavior of the energy surface of the model, which is the expectation value of the Hamiltonian with respect to spin coherent states.

1. Introduction

The quantum phase transitions of many body systems are characterized by a sudden change in the ground state properties. They are different to the thermal phase transitions because they are due entirely to quantum fluctuations. They are described typically as due to a non-analytic behavior of the ground state properties when the Hamiltonian system crosses a transition point. Although one of the most important manifestations of a quantum phase transition occurs in the energy spectra of the physical systems, it can be observed also in many other observables.

The crossing of levels in the spectrum of physical system is an indication of a first order transition, while the second order ones correspond to other causes and they are continuous. This latter type of transitions can be characterized by means of the spontaneous symmetry breaking formalism developed by Ginzburg, Landau and Wilson, where the order parameters play a crucial role. In particular the Ginzburg-Landau theory is related with the phase transitions of a material between the superconductor state to the normal one [1]. A manner to relate a variety of different phenomena in condensed matter theory is through the renormalization group, which originated to reconcile divergences in continuous theories with the finite experimental results. This is done by means of a cutoff procedure which in the modern vision lets us separate the modes of the physical system [2].

The study of complex systems can be simplified by analyzing their static and dynamic entanglement properties. When a physical system approximates a critical point, the structure

of the entanglement suffers a transition. For bipartite systems characterized by pure states the best measure of entanglement is provided by the von Neumann entropy, while for physical systems described by a mixed state one uses other quantities like the formation entanglement (concurrence), and the relative entropy [3].

In the literature there are two main forms of characterizing a quantum phase transition, the one introduced by Ginzburg and Landau where the catastrophe formalism can be very important and more recently those related with the use of entanglement entropy, fidelity and fidelity susceptibility. The fidelity approach is an alternative procedure to understand the critical phenomena, that is the physics associated to the critical points in the thermodynamical sense. It includes relations between different quantities, power laws of the quantities described by critical exponents and universality [4].

2. Description of phase transitions

The classical theory of phase transitions is adjusted to the framework of elementary phase transitions, for describing a physical system as a family of potentials $V = V(x; c)$, which is dependent of n real variables or order parameters x , and k real control parameters c . It is generally assumed that the physical system is described for the value x that minimizes the potential, at least locally. Then the study of the system is reduced to the equilibrium and stability properties of the family of potentials. In general, almost for all values of the control parameters the potential will have isolated critical points. The relations $\frac{\partial V}{\partial x_k} = 0$ constitute a system of n equations that can be considered state equations and they let us get $x^p(c)$, the critical points in terms of the control parameters. The local stability of one of these points is determined by using the hessian matrix evaluated at $(x^p(c), c)$. A phase transition happens when the point x describing the ground state of the physical system jumps from a critical point to another [5].

Usually the phase transitions occur when the control parameters are varied, frequently the time is chosen as the parametric variable, and then one has $c_\alpha \rightarrow c_\alpha(t)$, $x^p(c_\alpha) \rightarrow x^p(c_\alpha(t))$ and $V^p(x^p(c_\alpha), c) \rightarrow V^p(t)$. In the catastrophe formalism a phase transition occurs when the curve $c_\alpha(t)$ crosses a component of the so called *separatrix*. It is defined as the union of the Maxwell S_M and bifurcation S_B sets which are formed by the critical points of the potential energy [6].

The determinant of the hessian matrix, evaluated at the critical points,

$$V_{i,k} = \frac{\partial^2 V}{\partial x_i \partial x_k} \bigg|_{x^p(c)}, \quad (1)$$

allows a determination of the degenerated critical points. This region of points defines the bifurcation sets while the Maxwell sets are determined by means of the Clausius-Clapeyron equations [6]

$$V^p = V^{p+1}, \quad \left(\frac{\partial V^p}{\partial c_\alpha} - \frac{\partial V^{p+1}}{\partial c_\alpha} \right) \delta c_\alpha = 0. \quad (2)$$

In the delay convention the physical system remains in a stable or metastable equilibrium state until that state disappears while in the Maxwell convention the physical system is the one that globally minimizes the potential energy. The order of a phase transition is given according to the Ehrenfest classification, that is one gets a phase transition of order m if

$$\lim_{\epsilon \rightarrow 0} \frac{d^j V^p(t)}{dt^j} \bigg|_{t=t_0+\epsilon} = \lim_{\epsilon \rightarrow 0} \frac{d^j V^p(t)}{dt^j} \bigg|_{t=t_0-\epsilon} \quad (3)$$

is satisfied for all $j = 0, 1, 2, \dots, m-1$ but not for $j = m$. The phase transition is called local or soft if the following relation is satisfied

$$\lim_{\epsilon \rightarrow 0} (x^p(t_0 + \epsilon) - x^q(t_0 - \epsilon)) = 0, \quad (4)$$

where x^p and x^q denote the values of x at which V attains its minimum, after and before the transition, respectively; otherwise is called hard or non-local [6].

To show the procedure, one can consider the potential energy surface associated to the cusp catastrophe [5, 6], which has one order variable and two control parameters

$$V(x; a, b) = \frac{1}{4}x^4 + \frac{a}{2}x^2 + bx. \quad (5)$$

This potential energy describes an anharmonic oscillator. It is immediate that the critical points are determined by solving the third order algebraic equation

$$x^3 + ax + b = 0. \quad (6)$$

whose solutions can be given in terms of Cardan's formulas,

$$\begin{aligned} x_1 &= \left(-\frac{b}{2} + \sqrt{D}\right)^{1/3} + \left(-\frac{b}{2} - \sqrt{D}\right)^{1/3}, \\ x_2 &= \xi \left(-\frac{b}{2} + \sqrt{D}\right)^{1/3} + \xi^2 \left(-\frac{b}{2} - \sqrt{D}\right)^{1/3}, \\ x_3 &= \xi^2 \left(-\frac{b}{2} + \sqrt{D}\right)^{1/3} + \xi \left(-\frac{b}{2} - \sqrt{D}\right)^{1/3}, \end{aligned} \quad (7)$$

where we have defined the discriminant $D = \left(\frac{b}{2}\right)^2 + \left(\frac{a}{3}\right)^3$ and the complex parameter $\xi = \exp(-i\frac{2\pi}{3})$.

In Fig. 1 the bifurcation and Maxwell sets are plotted in the control parameter space. The bifurcations are defined when the discriminant takes the value zero, that is when there are degenerated critical points, while the Maxwell set corresponds to the case of two critical points with the same value for the potential energy surface. The latter happens for the line $b = 0$ with $a \leq 0$, to allow for the existence of the critical point. In the region of points above the fold lines (the points where the manifold of critical points is singular) of the critical points, the potential energy surface has only one minimum, deformed to the left for $b > 0$ and to the right for $b < 0$. On the bifurcation set the minima take place away from $x = 0$, for negative x values when $b < 0$ and positive x values for $b > 0$. Inside the bifurcation set the potential energy has three critical points, one maximum and two minima, and exactly at the Maxwell set the minima for positive and negative values of the order variable are of equal depth. At the points on the bifurcation the phase transitions are of order zero, the crossing through the Maxwell set yields a phase transition whose order generally it depends of the curve used to cross it. Finally by going directly along the line $b = 0$ the transition is of order two.

The quantum mechanical study of the cusp potential energy surface has been done by Gilmore et al [7]. The spectrum presents two different regimes: at low excitation energies is linear in the number of phonon excitations n , while at high excitation energies the spectrum behaves like $n^{4/3}$. The spectrum has single and double mode regions, the one-mode case occurs for $a > 0$ where the spectrum is equally spaced, while for $a < 0$ it is again equally spaced but there is a softening of the spring constant at middle energies. The bimodal case takes place inside the fold

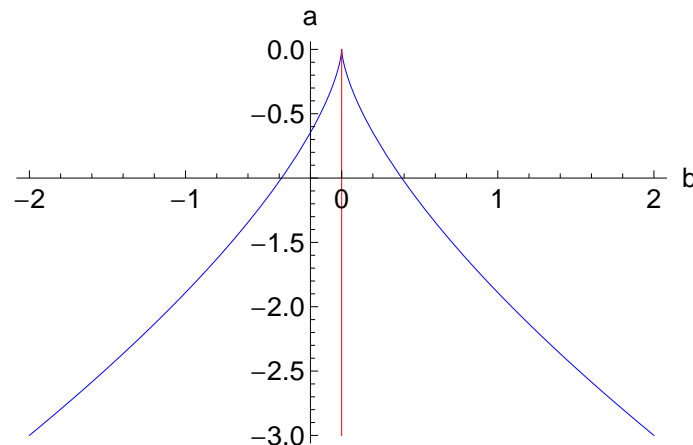


Figure 1. The separatrix for the cusp catastrophe. The blue line (cusped) corresponds to the bifurcation set while the red (vertical) line to the Maxwell set. The separatrix distinguishes physical systems with mixtures of two phases below the fold lines and a pure phase above them.

lines of the separatrix and there are two bands with a harmonic behavior at low energies and a softening of the spring constant gradually until the saddle region, after which the levels become more spaced going to the limit mentioned above. Thus one can see that the procedure indicated describe the main characteristics of the energy spectrum of the system.

3. Summary of the Lipkin-Meshkov-Glick model

The LMG model was introduced in nuclear physics because it is simple enough to be solved exactly but is yet non-trivial [8] and it has been used to test approximation methods of many body systems [9]. Also, it has played an important role in other fields of physics like, for example, in condensed matter studies to study the size scaling for infinitely coordinate systems [10, 11], in the description of a two-mode Bose-Einstein condensates [12], and in quantum optics by Kitagawa and Ueda [13] to produce spin squeezed states. In the context of condensed matter describing mutually interacting 1/2 spins embedded in a magnetic field, entanglement entropy and phase transitions in collective spin systems, in particular in the LMG model has attracted much attention. This quantum information concept (entanglement entropy) displays a singularity at the critical point as a function of the magnetic field, the anisotropy parameter, and the size of the system [14]. Using the Holstein-Primakoff representation and the continuous unitary transformation technique finite size scaling exponents for several observables of the LMG model were calculated [15, 16, 17, 18]. Entanglement properties of the ground state and its dynamics have been analyzed under different approaches, remarking important properties of the system [19, 20, 21, 22, 23].

The model Hamiltonian is given by [8]

$$H = \epsilon J_0 + \frac{\lambda}{2}(J_+^2 + J_-^2) + \frac{\gamma}{2}(J_+ J_- + J_- J_+) , \quad (8)$$

where the λ -term annihilates pairs of particles in one level and creates pairs in the other level while the γ -term scatters one particle up and another one down. The Hamiltonian can be rewritten in the form

$$H = \epsilon J_0 + \bar{\gamma}_x J_x^2 + \bar{\gamma}_y J_y^2 , \quad (9)$$

where $\bar{\gamma}_x = \gamma + \lambda$ and $\bar{\gamma}_y = \gamma - \lambda$.

As it is usual, for simplicity only the case with maximum symmetry $J = N/2$, with N the total number of particles is considered. Exact eigenvalues and eigenvectors of the Hamiltonian (9), can be obtained in the basis $\{|N n\rangle\}$, with n varying from 0 to N , defined as

$$|N n\rangle = \sqrt{\frac{(N-n)!}{N!n!}} J_+^n |N 0\rangle . \quad (10)$$

The state $|N 0\rangle$ is the unperturbed ground state, i.e. the ground state when $\lambda = \gamma = 0$ because we are considering the parameter $\epsilon > 0$. In this case all the particles occupy the lowest single particle state. Since the Hamiltonian (9) has non-vanishing matrix elements between states (10) with $n \rightarrow n, n \pm 2$ its eigenfunctions can be written in terms of linear combinations of states with even (e) or odd (o) values of n (e.g. the number of particle-hole pairs), namely:

$$|\alpha, e\rangle = \sum_{i=0}^N C_{i,e}^\alpha |N 2i\rangle , \quad |\alpha, o\rangle = \sum_{i=0}^{N-1} C_{i,o}^\alpha |N 2i+1\rangle . \quad (11)$$

In this notation α is the eigenvalue-index, thus $|\alpha = 1, e\rangle$ and $|\alpha = 1, o\rangle$ are the exact ground state and the exact first excited state, respectively.

Following the method mentioned in the previous section, in [24, 25] the separatrix of the system was found taking the expectation value of the Hamiltonian with respect to the SU(2) coherent states, i.e.,

$$\mathcal{H} = \langle \zeta | H | \zeta \rangle ,$$

where $|\zeta\rangle = (1 + |\zeta|^2)^{-J} \exp(\zeta J_+) |0\rangle$, and its stability behavior was determined through the catastrophe formalism. \mathcal{H} represents the classical Hamiltonian function of the model in the limit when $N \rightarrow \infty$. A detailed analysis of the classical and quantum dynamics of the LMG model can be found in [25].

The separatrix of the LMG model is given in Fig. 3, and the behavior of the corresponding eigenstates is also indicated with the names collective (bimodal) and independent particle (unimodal). In the independent particle regime the ground state is unique, while in the collective one has degeneracy two. The arrows indicate trajectories in the control parameter space which cross quantum phase transitions, they are of second order when the separatrix is crossed from the collective regime to the single particle one and of first order for the crossing between two collective ones. Another fact related with the LMG model is that the crossing of the point $(\gamma_x = -1, \gamma_y = -1)$, which is not shown in Fig. 3, along the straight line $\gamma_y = -\gamma_x - 2$ constitutes a third order phase transition. In that point a convergence of second phase transitions are happening, which it is currently under investigation.

It is important to mention that trial states with good parity yield a better description of the ground and first excited states of the LMG model. These trial states are constructed as Schrödinger cat states in terms of combinations of spin coherent states [25]. The calculation of even and odd energy surfaces were shown to closely reproduce the exact quantum behavior of the ground and first excited states, and their crossings. The intersection of these even and odd energy surfaces yields another Maxwell set $\gamma_{y0} = (1/\gamma_x)$; in the region between the branches of the hyperbola there are no crossings between energy levels, whereas in the exterior part degeneracy is present.

4. Fidelity

In classical information theory emerges the concept of *fidelity*. It is a basic ingredient of communication theory, which measures the accuracy of a transmission. For example, one can transmit a sequence of N signals and it is important to know the probability, over all possible

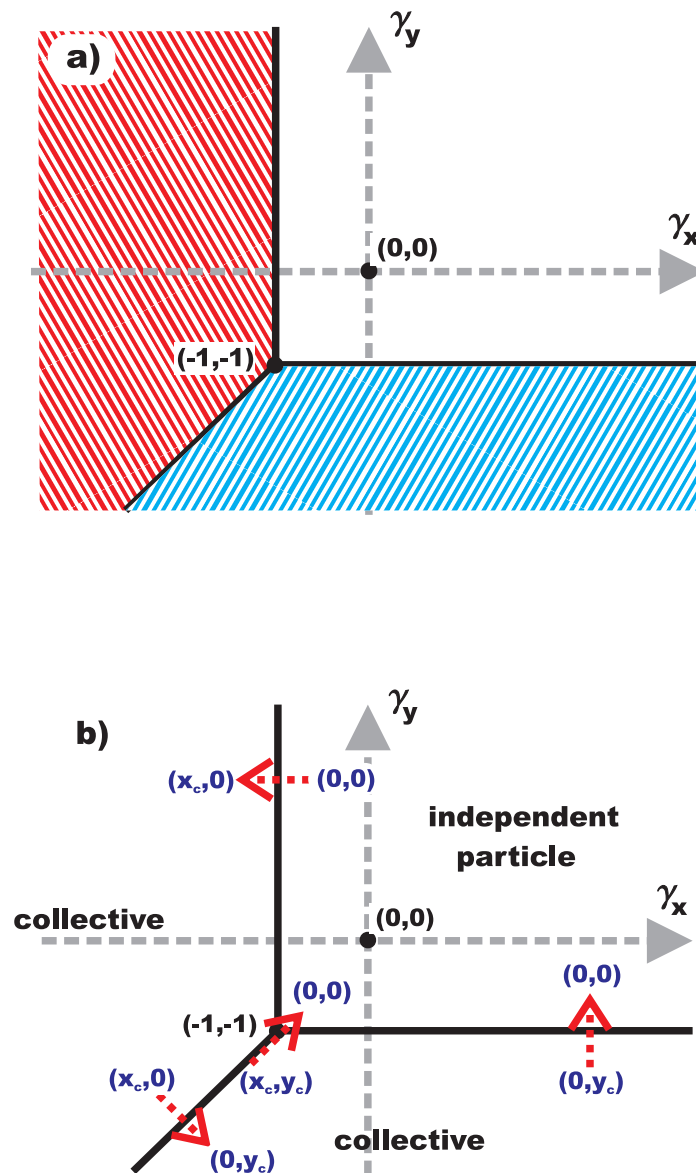


Figure 2. a) Bifurcation sets are shown with black continuous lines. The Maxwell sets are displayed filled up with negative slope (red) lines for equally deep minima along the axes x , $(\pm x_c, 0)$, and with positive slope (blue) lines for minima along the y axis, $(0, \pm y_c)$. b) The shape phase diagram for the LMG model is established and the quantum phase transitions indicated. There are second order transitions for $(0, 0) \leftrightarrow (x_c, 0)$, $(0, y_c) \leftrightarrow (0, 0)$, and $(x_c, y_c) \leftrightarrow (0, 0)$; and first order transitions for the case $(x_c, 0) \leftrightarrow (0, y_c)$. Notice that $\gamma_x \equiv \frac{2J-1}{\epsilon} \bar{\gamma}_x$ and $\gamma_y \equiv \frac{2J-1}{\epsilon} \bar{\gamma}_y$.

N -sequences, that the received signals coincide with the sent one; this is the definition of the fidelity. This concept is also closely related with Shannon's noiseless coding theorem and channel capacity, which play a fundamental role in classical information theory. The first one answers the question: how much can a message be compressed while still obtaining the same information?, being the answer the calculation of the corresponding Shannon's entropy [3]. In 1994, a similar theorem was developed by Schumacher for quantum communication as an extension of Shannon's noiseless coding theorem [26], where the von Neumann entropy plays the role of the Shannon entropy for the classical case.

For two pure states described by the density matrices $\rho_1 = |\Psi_1\rangle\langle\Psi_1|$ and $\rho_2 = |\Psi_2\rangle\langle\Psi_2|$ the fidelity between them is defined as follows [27]

$$F = |\langle\Psi_1|\Psi_2\rangle|^2. \quad (12)$$

In quantum physics, an overlap between two quantum states denotes the transition amplitude from one state to other, while in the information formalism it measures the similarity between two states. Then the overlap between the input and output states becomes a useful measure of the loss of information during the transportation of a message.

Recently [28], the fidelity has been proposed as a tool to determine the quantum phase transitions of a physical system, the motivation being related directly with the meaning of a quantum phase transition. A phase transition in the control parameter space takes place when the ground state of the system changes significantly when one or several of the control parameters are varied. Therefore, one can fix all the control parameters except one, and the quantum phase transition will be localized at point where

$$F(\gamma, \gamma + \delta\gamma) = |\langle\Psi(\gamma)|\Psi(\gamma + \delta\gamma)\rangle|^2 \quad (13)$$

reaches its minimum value.

In this work, we generalize the definition of the fidelity by considering the overlap between the exact quantum state and an arbitrary $|\Phi\rangle$ representative state of the Hilbert space of the physical system,

$$F(\Phi, \Psi(\gamma)) = |\langle\Phi|\Psi(\gamma)\rangle|^2. \quad (14)$$

By means of this calculation we are able to determine the position in the varying control parameter related with the quantum phase transitions due to crossings or anticrossings of the energy levels in a quantum system. As an example of the procedure, we are going to show this for the LMG model.

First of all, we consider the diagonal case of the LMG model, that is $\gamma_x = \gamma_y$. It has been shown that there are crossings of the energy spectra for the values [25]

$$\gamma_k^{(1)} = -\frac{2J-1}{2J-1-2k}, \quad \gamma_k^{(2)} = -\frac{2J-1}{2J-2-2l}, \quad (15)$$

with $k = 0, 1, \dots, 2J-1$ and $l = 0, 1, \dots, 2J-2$. Then, if for example we consider $N = 20$ particles distributed into the two-level system one has two sets of crossings, the first one associated to the crossings between the ground and first excited states, and the second one between the ground and second excited states,

$$\begin{aligned} S_1 &= \left\{ -1, -\frac{19}{17}, -\frac{19}{15}, -\frac{19}{13}, -\frac{19}{11}, -\frac{19}{9}, -\frac{19}{7}, -\frac{19}{5}, -\frac{19}{3}, -19 \right\}, \\ S_2 &= \left\{ -\frac{19}{18}, -\frac{19}{16}, -\frac{19}{14}, -\frac{19}{12}, -\frac{19}{10}, -\frac{19}{8}, -\frac{19}{6}, -\frac{19}{4}, -\frac{19}{2} \right\}, \end{aligned}$$

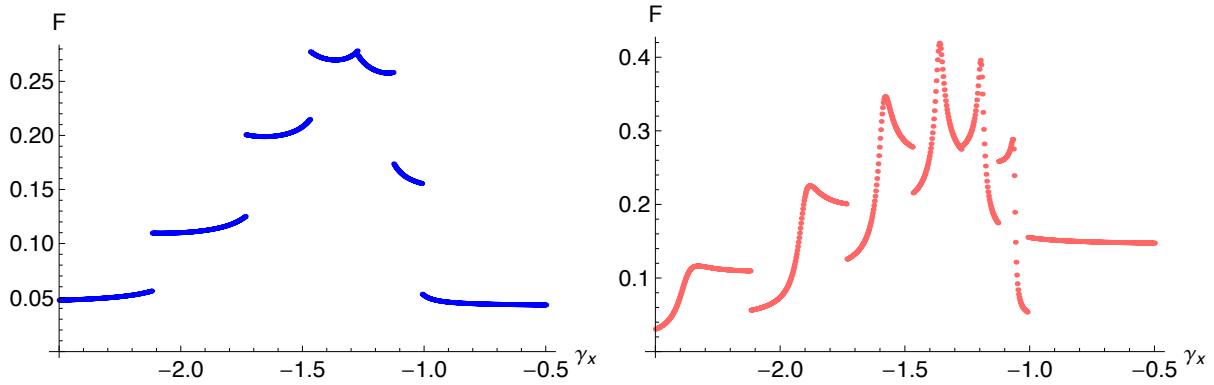


Figure 3. At the left, the fidelity between the even ground state of the LMG model with a spin coherent state with $\zeta = 0.9$ is shown. At the right, the fidelity between the odd ground state of the model with the same spin coherent state is displayed.

and their corresponding positive values because the model Hamiltonian is symmetric under the interchange of sign of the control parameter γ_x .

Fig. 3 shows six sudden changes in the values of the fidelity between a spin coherent state, with $\zeta = 0.9$, and the ground state of the even parity case. We propose that these changes are defining quantum phase transitions in the LMG model for the diagonal case. Comparing these with the known crossings above, one finds that there is an exact agreement with the crossings indicated by the set S_1 , in the region $-2.5 \leq \gamma_x \leq -0.5$. For the overlap between the same spin coherent state with the odd eigenstate of the LMG model one finds twelve sudden changes, and again one has a complete agreement with the crossings between the energy levels indicated in the sets S_1 and S_2 .

For the non-diagonal case, we evaluate the fidelity between the (trivial) test functions for $N = 20$ particles of the form

$$\begin{aligned} |\phi_T\rangle_{\text{even}} &= \frac{1}{1540} (0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20), \\ |\phi_T\rangle_{\text{odd}} &= \frac{1}{1330} (1, 3, 5, 7, 9, 11, 13, 15, 17, 19). \end{aligned}$$

with respect to the ground and first excited states of the even and odd parity cases. We selected $\gamma_y = \gamma_x + 0.015$, and the fidelities between the even and odd first excited states with respect to the corresponding test functions are exhibited in Fig. 4. From the results, one notices immediately the localization of four sudden changes in the value of the fidelity for the even ground state while for the first excited state there are seven sudden changes in the fidelity. One can also see that the even first excited state detects the quantum phase transitions present in the even and odd ground states of the LMG model. Therefore, the calculation of the fidelity is able to detect all the phase transitions present in the LMG model for the diagonal and non-diagonal cases.

We have shown that the fidelity is a good tool to determine the localization of the quantum phase transitions, when a reasonable test function is used to calculate the fidelity of the quantum ground state with respect to it, and determine the localization of the quantum phase transitions.

It is also interesting to calculate the fidelity in the form $F(\gamma, \gamma + \delta\gamma)$ and look for the minima. This implies the determination of the second derivative of the fidelity as follows

$$\chi_F = 2 \frac{1 - F(\gamma, \gamma + \delta\gamma)}{\delta\gamma^2}, \quad (16)$$

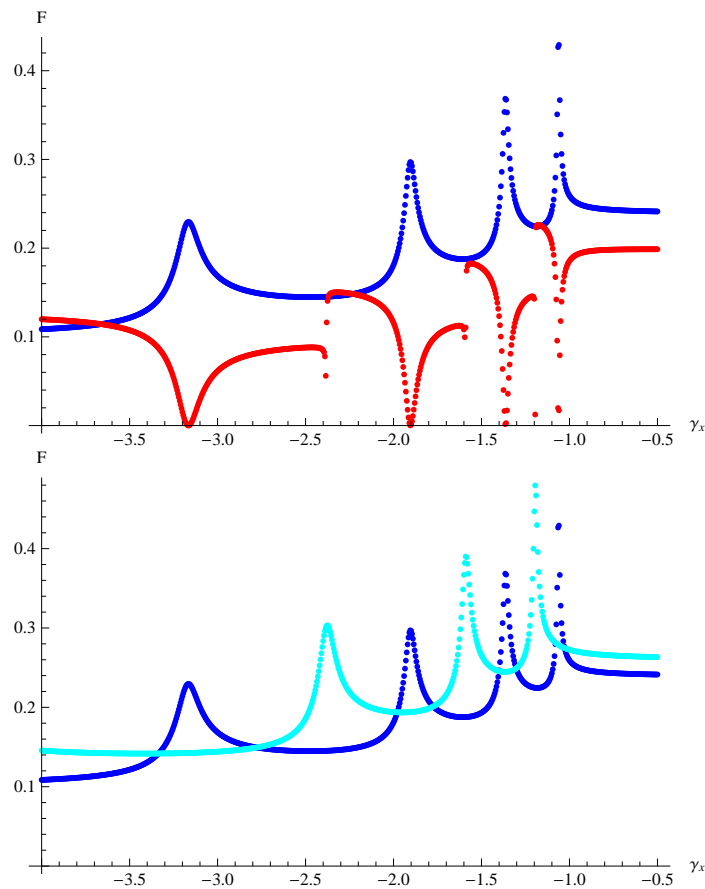


Figure 4. At the top, the fidelity between the even ground state, upper (blue) line, and even first excited state, lower (red) line, of the LMG model with respect to the $|\phi_T\rangle_{\text{even}}$ test function is shown. At the bottom, the fidelity between the even parity, lower (blue) lines, and odd parity, upper (cyan) line, ground states of the model with respect to the $|\phi_T\rangle_{\text{odd}}$ test function is displayed.

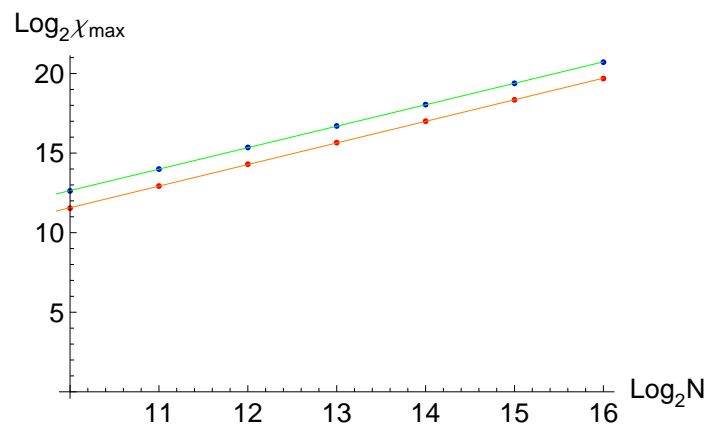


Figure 5. We display the plots $\text{Log}_2 \chi_{Fmax}^e$ vs $\text{Log}_2 N$, upper (green) line and $\text{Log}_2 \chi_{Fmax}^o$ vs $\text{Log}_2 N$, lower (orange) line, for $\gamma_y = 0.5$. The lines indicate the corresponding best fits, and one can see that they have approximately the same slope.

which expresses the answer of the physical system to the interaction term associated to the control parameter γ , and for that reason it has been called the fidelity susceptibility. The maximum of the fidelity susceptibility determines the exact localization of the quantum phase transition together with critical exponents of the quantum phase transition associated to the corresponding interaction term of the Hamiltonian [29].

This new concept can be used to determine the scaling behavior and universality of the quantum phase transition [4, 29]. Following the procedure indicated in [15] the LMG model Hamiltonian can be expanded in terms of powers of N^{-1} , $N^{1/2}$ and N^0 by means of the Holstein-Primakoff representation of the angular momentum generators and determine exact expressions for the fidelity susceptibility in both sides of the quantum phase transition. These results let us determine the scaling properties and universality of the quantum phase transitions. In the case of the first order phase transitions the fidelity is zero and then the procedure cannot be applied while for the second order quantum phase transitions it works perfectly. To show this, we study the LMG model for $\gamma_y = -0.5$ and change the parameter γ_x associated to the component J_x^2 of the Hamiltonian in a range close to the quantum phase transition determined previously with an arbitrary test function as it was mentioned above. In this way one is able to establish the range of variation for the control parameter to $-1.03 \leq \gamma_x \leq 0.998$.

As we are interested in finding the scaling properties of the quantum phase transition associated to the interaction Hamiltonian J_x^2 , the LMG model Hamiltonian is diagonalized for a number of particles 2^N with $N = \{10, 11, 12, 13, 14, 15, 16\}$. Then we calculate the fidelity susceptibility for the even and odd parity cases, by means of the expression (16), for $\gamma_y = 0.5$ and $-1.03 \leq \gamma_x \leq -0.998$. The results are displayed in Fig. 5, where the best fits of the corresponding even and odd fidelity susceptibilities are indicated by the upper (green) and lower (orange) lines, respectively. For the even case the slope is given by 1.348 and the ordinate to the origin by -0.836 , while for the odd case one gets the slope 1.356 and the ordinate -1.997 . Notice that for a small number of particles of the order of 2^8 and smaller there are deviations of this universal rule.

5. Conclusions

We have made a brief description of phase transitions in classical mechanics, and how it can be extended to characterize the quantum phase transitions by means of the catastrophe formalism together with the spin coherent states. We have shown that the quantum information concepts as the fidelity and fidelity susceptibility are very useful to determine the quantum phase transitions present in the LMG model together with universal properties of for a very large number of particles. We concluded also that by means of almost *any* test function, the fidelity can be used to determine the approximate values of the control parameters that yield quantum phase transitions for a finite number of particles. This lets us determine the position in the control parameter space of the crossings and anti-crossings present in the LMG model, for a finite number of particles. Thus, it will be important to study the behavior of the different physical observables of the LMG model at these finite quantum phase transitions.

The scaling behavior of a quantum transition can be estimated for second order quantum phase transitions using the fidelity susceptibility concept. This cannot be done for first order quantum phase transitions because no matter how small is the difference $\delta\gamma$ the states are orthogonal. This is known in the literature of many-body physics as the Anderson orthogonality catastrophe [30]. The scaling properties of the quantum phase transitions are shown in Fig. 5 for the parameter value $\gamma_y = 0.5$; analogous results have been found for $\gamma_y = -0.5$, and in the vicinity of the phase transition $\gamma_x = -1$.

The specific susceptibility defined as

$$\chi_{spe} = \frac{\chi_{Fmax}}{\chi_F(\gamma_x)} - 1, \quad (17)$$

can be plotted in terms of the product of the control parameter $(\gamma_x - \gamma_x^c)N^{2/3}$. It exhibits a universal behavior for any large number of particles, as it was shown in [29], although in our case a different interaction term of the LMG model Hamiltonian was considered.

Acknowledgments

This work was partially supported by CONACyT-México, with reference number 101541, and by DGAPA-UNAM under project IN102811.

References

- [1] Gor'kov L P 1959 *Soviet Physics JETP* **9** 1364
- [2] Wilson K G 1975 *Rev Mod Phys* **47** 773
- [3] Benenti G, Casati G, and Strini G 2007 *Principles of quantum computation and information* Vol II (World Scientific, Singapore)
- [4] Gu S H 2010 *Int J Mod Phys B* **24** 4371
- [5] Arnol'd V I 1992 *Catastrophe Theory* (Springer-Verlag)
- [6] Gilmore R 1981 *Catastrophe Theory for Scientists and Engineers* (Wiley, New York)
- [7] Gilmore R, Kais S, and Levine R 1986 *Phys Rev A* **42** 2442
- [8] Lipkin H J, Meshkov N, and Glick A J 1965 *Nucl Phys* **62** 188, 199, 211
- [9] Hirsch J G, Civitarese O, and Reboiro M 1999 *Phys Rev C* **60** 24309
- [10] Botet R, Jullien R, and Pfeuty R 1982 *Phys Rev Lett* **49** 478
- [11] Botet R and Jullien R 1983 *Phys Rev B* **28** 3955
- [12] Pérez C, González-Alonso J R, Castaños O, and López-Peña R 2010 *Ann Phys* **325** 325
- [13] Kitagawa M and Ueda M 1993 *Phys Rev A* **47** 5138
- [14] Latorre J I, Orus R, Rico E, and Vidal J 2005 *Phys Rev A* **71** 064101
- [15] Dusuel S and Vidal J 2004 *Phys Rev Lett* **93** 237204; 2005 *Phys Rev B* **71** 224420
- [16] Dusuel S, Vidal J, Arias J M, Dukelsky J, and García-Ramos J E 2005 *Phys Rev C* **72** 064332
- [17] Leyvraz F and Heiss W D 2005 *Phys Rev Lett* **95** 050402
- [18] Heiss W D, Scholtz F G, and Geyer H B 2005 *J Phys A: Math Gen* **38** 1843
- [19] Wu L-A, Sarandy M S, and Lidar D A 2004 *Phys Rev Lett* **93** 250404
- [20] Wu L-A, Sarandy M S, Lidar D A and Sham L J 2006 *Phys Rev A* **74** 052335
- [21] Vidal J, Palacios G and Aslangul C 2004 *Phys Rev A* **70** 062304
- [22] Menotti C, Anglin J R, Cirac J I and Zoller P 2001 *Phys Rev A* **63** 023601
- [23] Micheli A, Jaksch D, Cirac J I and Zoller P 2003 *Phys Rev A* **67** 013607
- [24] Castaños O, López-Moreno E and López-Peña R 2003 *Rev Mex Fís.* **49** S4, 15
- [25] Castaños O, López-Peña R, Hirsch J G and López-Moreno E 2005 *Phys Rev B* **72** 012406; 2006 *Phys Rev B* **74** 104118
- [26] Schumacher B 1995 *Phys Rev A* **51** 2738
- [27] Jozsa R 1994 *J Mod Optics* **41** 2315
- [28] Zanardi P and Paunkovic N 2006 *Phys Rev E* **74** 031123
- [29] Kwok H, Ning W, Gu S and Lin H 2008 *Phys Rev E* **78** 032103
- [30] Anderson P W 1967 *Phys Rev Lett* **18** 1049