The phonon vacuum state in a Lipkin model

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The phonon vacuum state in a Lipkin model

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Abstract. The action of the long-range residual force on the expectation value of observables in the nuclear ground states is evaluated by finding optimal values for the coefficients of the canonical transformation which connects the phonon vacuum state with the independent-particle ground-state using the two-level Lipkin-Meshkov-Glick (LMG) model. After estimating the improvements over the predictions of the independent-particle approximation we compare the ground and first excited states wave functions, obtained using the presented approach, with those, obtained using the conventional random phase approximation (RPA) and its extended version. The problem with overbinding of the nuclear ground state calculated using the RPA is shown to be removed if one sticks to the prescriptions of the present approach. The reason being that the latter conforms to the original variational formulation.

1. Introduction

The purpose of this note is to perform a comparative study for the binding energies and the wave functions of the nuclear ground states approximated by the phonon vacuum state [1, 2, 3] using different methods for evaluating average values of quantum mechanical operators in a many-body wave function. All considerations are performed employing the LMG model [4] for reasons of simplicity. In doing this comparison we subject a variational approach to a test against the true LMG ground state as well as against well established approximations such as RPA[5] and one of its extended variants (ERPA)[6, 7, 8]. A special attention is given to the cases when the interaction between the particles exceeds the inherent to the RPA critical point.

The importance of this development to nuclear structure physics is that it provides a blueprint for a variational theory which is able to account for the effects of the long-range part of the residual nuclear forces in the ground states. In the spirit of the variational method we examine the convergence for some of the attributes of the ground states, with its energy being the most important one, when expanding the configurational space. Building over the obtained ground state using the featured method we head off to construct an one particle-hole excited state and examine its qualities. It is argued that the discrepancies for the first excited states in the strong interaction regime are due to the fact that configurations with more than one particle and one hole give major contribution to this state.

2. Model

2.1. LMG model basics

In order to access the utility of different approaches and to prove the usefulness of the proposed scheme we limit the configuration space and simplify the inter-nucleon interaction to monopole-monopole one as suggested by Lipkin, Meshkov and Glick. This setting permits comparisons between the rates of convergence of different approximation methods, including the hereby described, to the exact solution.

In this model the interaction of N particles on 2 quantum levels is presented by the following Hamiltonian

$$H = H_0 + V;\ H_0 = \varepsilon J_0;\ V = \frac{G}{2}(J_+ + J_-)^2, \tag{1}$$

where

$$J_+ = \sum_i a^\dagger_{1i} a_{-1i};\ J_- = \sum_i a^\dagger_{-1i} a_{1i};\ J_0 = \frac{1}{2}\sum_i (a^\dagger_{1i} a_{1i} - a^\dagger_{-1i} a_{-1i}) \tag{2}$$

are analogous to the raising, lowering and angular momentum' z-component of the quasi-spin algebra respectively, $a^\dagger$ represents the particle creation operator, the suffix ± 1 denotes the upper or lower level, ε is the energy gap between the two levels and G is the interaction strength.

The exact solution for this Hamiltonian is readily obtained after performing a simple diagonalization [9].

2.2. Phonon vacuum solution

The method that we examine is expressed in the language of the LMG model in the following way. The phonon vacuum wave function is taken in the form

$$|\rangle = N_0 e^{\frac{1}{2} S J_+^2}\rangle. \tag{3}$$

Up to arbitrary order $n \le N/2$ the ground-state energy is obtained to be

$$\langle|H|\rangle = N_0^2 \sum_n \frac{1}{(n!)^2}\left(\frac{S}{2}\right)^{2n} \langle J_-^{2n} H J_+^{2n}\rangle + 2N_0^2 \sum_n \frac{n}{(n!)^2}\left(\frac{S}{2}\right)^{2n-1} \langle J_-^{2n-2} H J_+^{2n}\rangle$$

with

$$N_0^2 = \left[\sum_n \frac{1}{(n!)^2}\left(\frac{S}{2}\right)^{2n} \langle J_-^{2n} J_+^{2n}\rangle\right]^{-1} \tag{4}$$

The variational equation $\partial_S \langle|H|\rangle = 0$ then yields the following problem

$$\begin{aligned} &N_0^2 \sum_n \frac{n}{(n!)^2}\left(\frac{S}{2}\right)^{2n-2} \left[\frac{S}{2}\langle J_-^{2n} H J_+^{2n}\rangle + (2n-1)\langle J_-^{2n-2} H J_+^{2n}\rangle\right] + \\ &\partial_S N_0^2 \sum_n \frac{1}{(n!)^2}\left(\frac{S}{2}\right)^{2n-1} \left[\frac{S}{2}\langle J_-^{2n} H J_+^{2n}\rangle + 2n\langle J_-^{2n-2} H J_+^{2n}\rangle\right] = 0. \end{aligned} \tag{5}$$

Respectively, the energy of the $1p-1h$ excited state is evaluated as

$$\omega = \psi^2 \langle|J_- H J_+|\rangle - 2\psi\varphi\langle|J_- H J_-|\rangle + \varphi^2\langle|J_+ H J_-|\rangle. \tag{6}$$

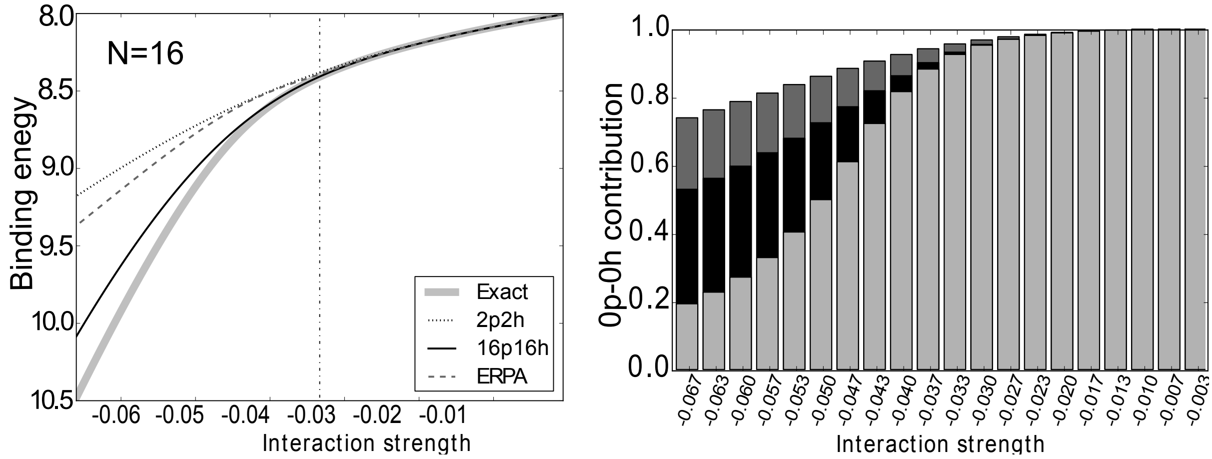

Figure 1. Binding energy (left panel) and contribution of the independent particle state to the correlated ground state (right panel) for a two-level LMG model system with $N = 16$ particles. The energy gap ε between the two LMG levels is set to 1 MeV. *Left panel.* Energies obtained using the phonon vacuum explicit variation with $2p - 2h$ and up $16p - 16h$ admixtures into the ground-state wave function along with the ERPA solution are compared to the exact one. The vertical dash-dotted line indicates the strength at which the RPA experiences a collapse. *Right panel.* The $0p - 0h$ contribution to the exact ground-state wave function (light gray), the phonon vacuum solution from Eq.(3) with $2p-2h$ admixtures (dark gray) and with correlations of order $N/2$ (black) as function of the interaction strength G.

The expressions for the Hamiltonian average values in Eqs.(5) and (6) are given in Ref. [10]. The forward and backward phonon amplitudes in Eq.(6) are obtained by applying the normalization condition for the one-phonon state

$$N_0^2\left(\psi^2 - \varphi^2\right)\sum_n \left(\frac{1}{n!}\right)^2 \left(\frac{S}{2}\right)^{2n} (N - 4n)\left\langle J_-^{2n} J_+^{2n}\right\rangle = 1, \tag{7}$$

along with the definitive equation

$$Q|\rangle = 0, \tag{8}$$

resulting in the relation

$$[\varphi - (N-1)\psi S]^2 + 6\psi^2 S^4 (N-1)(N-2) = 0. \tag{9}$$

Note that the latter relation is independent of the expansion order n.

3. Results

In Figs. 1 and 2 are present results for the energies and the contributions to the wave functions of the ground and first excited states respectively.

From the analysis of the plot in the left panel of Fig. 1 it is concluded that the explicit variation of the phonon vacuum with $2p-2h$ admixtures only in the ground state yields solution at any value of G. However, increasing the interaction strength beyond the RPA critical point (G_{crit}) causes progressive divergence of this solution from the exact one. Adding higher order terms to the energy functional, which account for further correlations effects, greatly improves the results bringing the energy of the phonon vacuum much closer to the exact value. It is also

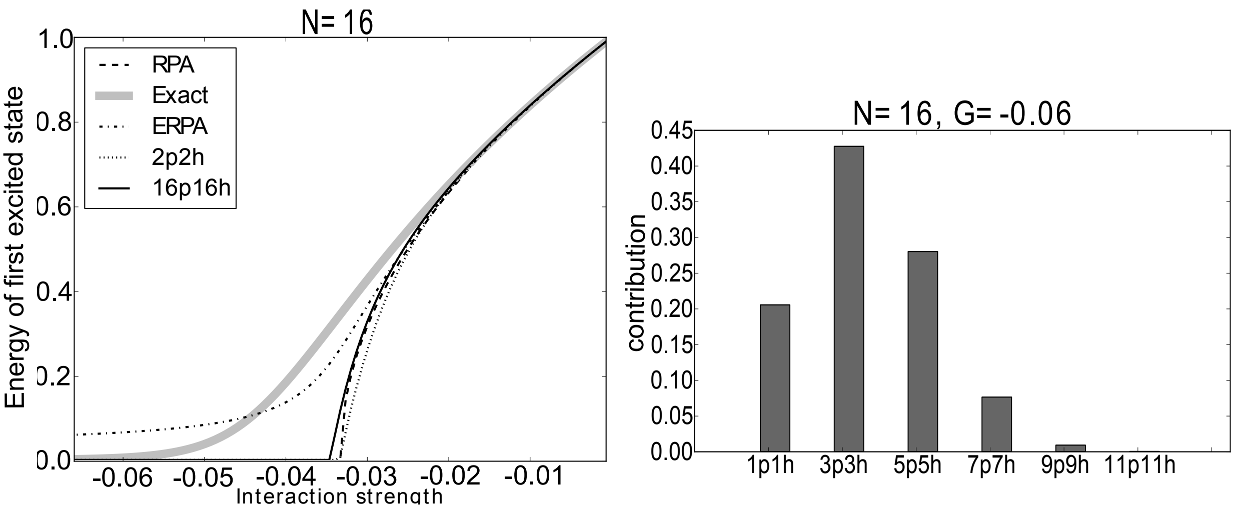

Figure 2. *Left panel.* Same as the left panel of Fig. 1 but for the first excited state with the RPA solution added. *Right panel.* The distribution of the contributions (squared) from different multi-particle-hole configurations to the first excited state in the strong interaction regime.

shown that the ERPA energy strongly underbinds the ground state beyond G_{crit}. However, as seen from the right panel of this figure, despite the reasonable agreement for the ground state energies in this regime the wave function of the phonon vacuum yields a substantially less correlated state than the true ground state.

In contrast, as seen in the left panel of Fig. 2, for the first excited state's energy the ERPA gives rather good results in the strong interaction regime whereas the phonon vacuum variational solution experiences a collapse. The reason for the behavior can be tracked in the right panel of Fig. 2 where it is seen that the higher order particle-hole admixtures contributing in the structure of the first excited state are giving a dominant contribution. Although the reproduction of the energy within the ERPA is good, the wave function of the first excited state happens to be almost orthogonal to the true ground state.

4. Conclusions

The explicit variational approach to the ground states using the phonon vacuum as a trial wave function proves a tractable method for reproducing the binding energies but it underestimates the degree of correlation in the strong interaction regime. Building a $1p1h$ excitations on top of the correlated ground state in order to reproduce the first excited state renders unsatisfactory results in this regime since configurations with a higher number of particles and holes give the major contribution to this state.

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