$$\hat{J}_x^2|j,m\rangle = \hat{J}_x \frac{1}{2}\left(\hat{J}_+ + \hat{J}_-\right)|j,m\rangle = \hat{J}_x \frac{1}{2}\left(\hat{J}_+|j,m\rangle + \hat{J}_-|j,m\rangle\right) =$$

$$= \hat{J}_x \frac{1}{2}\left[\hbar\sqrt{j(j+1)-m(m+1)}\,|j,m+1\rangle + \hbar\sqrt{j(j+1)-m(m-1)}\,|j,m-1\rangle\right]$$

$$= \frac{\hbar}{4}\left(\hat{J}_+ + \hat{J}_-\right)\left[\sqrt{j(j+1)-m(m+1)}\,|j,m+1\rangle + \sqrt{j(j+1)-m(m-1)}\,|j,m-1\rangle\right]$$

$$= \frac{\hbar}{4}\left[\sqrt{j(j+1)-m(m+1)}\,\hat{J}_+|j,m+1\rangle + \sqrt{j(j+1)-m(m-1)}\,\hat{J}_+|j,m-1\rangle + \sqrt{j(j+1)-m(m+1)}\,\hat{J}_-|j,m+1\rangle + \sqrt{j(j+1)-m(m-1)}\,\hat{J}_-|j,m-1\rangle\right]$$

* De todos sale un $\hbar$ *

$$= \frac{\hbar^2}{4}\left[\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+2)}\,|j,m+2\rangle + \sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-1+1)}\,|j,m-1+1\rangle + \sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+1-1)}\,|j,m+1-1\rangle + \sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,|j,m-2\rangle\right]$$

$$= \frac{\hbar^2}{4}\left[\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+2)}\,|j,m+2\rangle + [j(j+1)-m(m-1)]\,|j,m\rangle + [j(j+1)-m(m+1)]\,|j,m\rangle + \sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,|j,m-2\rangle\right]$$

$$\hat{J}_y^2|j,m\rangle = \hat{J}_y \frac{1}{2i}\left(\hat{J}_+ - \hat{J}_-\right)|j,m\rangle = \hat{J}_y \frac{1}{2i}\left(\hat{J}_+|j,m\rangle - \hat{J}_-|j,m\rangle\right)$$

$$= \hat{J}_y \frac{1}{2i}\left(\hbar\sqrt{j(j+1)-m(m+1)}\,|j,m+1\rangle - \hbar\sqrt{j(j+1)-m(m-1)}\,|j,m-1\rangle\right)$$

$$= \frac{\hbar}{4i^2}\left(\hat{J}_+ - \hat{J}_-\right)\left[\sqrt{j(j+1)-m(m+1)}\,|j,m+1\rangle - \sqrt{j(j+1)-m(m-1)}\,|j,m-1\rangle\right]$$

$$= -\frac{\hbar}{4}\left[\sqrt{j(j+1)-m(m+1)}\,\hat{J}_+|j,m+1\rangle - \sqrt{j(j+1)-m(m-1)}\,\hat{J}_+|j,m-1\rangle - \sqrt{j(j+1)-m(m+1)}\,\hat{J}_-|j,m+1\rangle + \sqrt{j(j+1)-m(m-1)}\,\hat{J}_-|j,m-1\rangle\right]$$

* De todos sale un $\hbar$

$$= -\frac{\hbar^2}{4}\left[\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+2)}\,|j,m+2\rangle - [j(j+1)-m(m-1)]\,|j,m\rangle - [j(j+1)-m(m+1)]\,|j,m\rangle + \sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,|j,m-2\rangle\right]$$

$$\langle j,k|\hat{H}|j,m\rangle = \langle j,k|\left[\hat{J}_z + \left(\frac{\gamma_x}{2j-1}\right)\hat{J}_x^2 + \left(\frac{\gamma_y}{2j-1}\right)\hat{J}_y^2\right]|j,m\rangle$$

$$= \langle j,k|\hat{J}_z|j,m\rangle + \left(\frac{\gamma_x}{2j-1}\right)\langle j,k|\boxed{\hat{J}_x^2|j,m\rangle} + \left(\frac{\gamma_y}{2j-1}\right)\langle j,k|\boxed{\hat{J}_y^2|j,m\rangle}$$

$$= m\hbar\langle j,k|j,m\rangle + \frac{\hbar^2}{4}\left(\frac{\gamma_x}{2j-1}\right)\langle j,k|\Big[\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+2)}\,|j,m+2\rangle + [j(j+1)-m(m-1)]|j,m\rangle +$$

$$+ [j(j+1)-m(m+1)]|j,m\rangle + \sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,|j,m-2\rangle\Big] +$$

$$-\frac{\hbar^2}{4}\left(\frac{\gamma_y}{2j-1}\right)\langle j,k|\Big[\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+2)}\,|j,m+2\rangle - [j(j+1)-m(m-1)]|j,m\rangle +$$

$$- [j(j+1)-m(m+1)]|j,m\rangle + \sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,|j,m-2\rangle\Big]$$

$$= m\hbar\,\delta_{km} + \frac{\hbar^2}{4}\left(\frac{\gamma_x}{2j-1}\right)\Big[\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+2)}\,\delta_{k,m+2} + [j(j+1)-m(m-1)]\,\delta_{k,m} +$$

$$+ [j(j+1)-m(m+1)]\,\delta_{km} + \sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,\delta_{k,m-2}\Big] +$$

$$-\frac{\hbar^2}{4}\left(\frac{\gamma_y}{2j-1}\right)\Big[\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+2)}\,\delta_{k,m+2} - [j(j+1)-m(m-1)]\,\delta_{km} +$$

$$- [j(j+1)-m(m+1)]\,\delta_{km} + \sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,\delta_{k,m-2}\Big]$$

En general, esta es la componente que depende de $j, m, k, \gamma_x, \gamma_y$,

- j es fijo
- γ_x va de -5 a 0
- $\gamma_y = 3\gamma_x$
- m y k se varían de $-j$ a j

Para lograr el gráfico haré la simplificación

$\gamma_y = 3\gamma_x$

$\hbar = 1$

y se agrupará lo posible

Simplificación : $\gamma_y = 3\gamma_x$, $\hbar = 1$, $\epsilon_0 = 1$

$$\langle j,k|\hat{H}|j,m\rangle = m\delta_{km} + \frac{1}{4}\left(\frac{\gamma_x}{2j-1}\right)\Big[\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+2)}\,\delta_{k,m+2} + [j(j+1)-m(m-1)]\delta_{km} + [j(j+1)-m(m+1)]\delta_{km} + \sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,\delta_{k,m-2} - 3\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+2)}\,\delta_{k,m+2} + 3[j(j+1)-m(m-1)]\delta_{km} + 3[j(j+1)-m(m+1)]\delta_{km} - 3\sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,\delta_{k,m-2}\Big]$$

$$\langle j,k|\hat{H}|j,m\rangle = m\delta_{km} + \frac{1}{4}\left(\frac{\gamma_x}{2j-1}\right)\Big\{-2\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+2)-(m+1)(m+2)}\,\delta_{k,m+2} - 2\sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,\delta_{k,m-2} + 4\left[j(j+1)-m(m-1)+j(j+1)-m(m+1)\right]\delta_{km}\Big\}$$

$$\langle j,k|\hat{H}|j,m\rangle = m\delta_{km} - \frac{1}{2}\left(\frac{\gamma_x}{2j-1}\right)\Big\{\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+2)}\,\delta_{k,m+2} + \sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,\delta_{k,m-2} - 2\Big[2j(j+1)\underbrace{-m\left(m-1+m+\cancel{1}\right)}_{-2m^2}\Big]\delta_{km}\Big\}$$

$$\boxed{\langle j,k|\hat{H}|j,m\rangle = m\delta_{km} - \frac{1}{2}\left(\frac{\gamma_x}{2j-1}\right)\Big\{\sqrt{j(j+1)-m(m+1)}\sqrt{j(j+1)-(m+1)(m+2)}\,\delta_{k,m+2} + \sqrt{j(j+1)-m(m-1)}\sqrt{j(j+1)-(m-1)(m-2)}\,\delta_{k,m-2} - 4\left[j(j+1)-m^2\right]\delta_{km}\Big\}}$$