Ejercicios Circuitos Inductores Resistivos, Paralelo

Circuitos Eléctricos (Instituto Politécnico Nacional)

Para los siguientes circuitos, calcular:

a. Impedancia total y triángulo de impedancia.
b. Corriente total, ángulo de fase y el diagrama fasorial que muestre la relación de voltaje y corriente.
c. P, Q, S, FP y triángulo de potencias
d. Voltaje y corriente en cada elemento

Circuito 1:

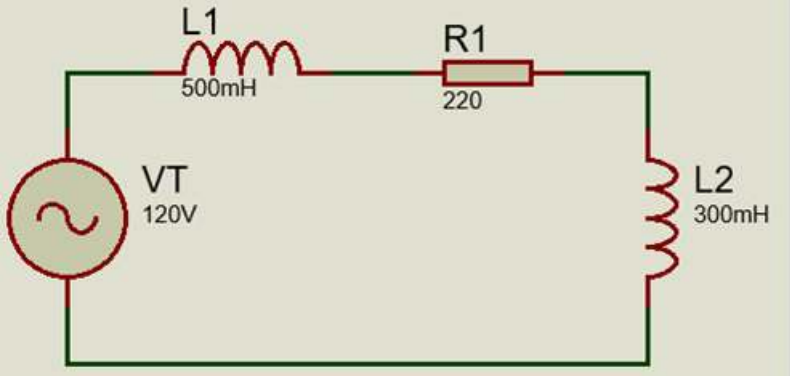


Figura 1. RL serie

Impedancia total

$$\boldsymbol{Z_t = R + j\, X_L}$$

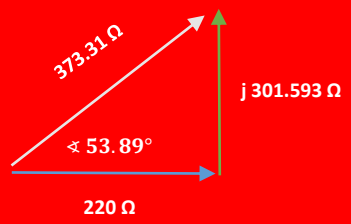
$$X_{L1} = 2\,\pi * (L_1) * f = 2\,\pi * (500\,mH) * 60Hz = 188.496\,\Omega$$
$$X_{L2} = 2\,\pi * (L_2) * f = 2\,\pi * (300\,mH) * 60Hz = 113.097\,\Omega$$

$$X_L = X_1 + X_2 = 188.496\,\Omega + 113.097\,\Omega = 301.593\,\Omega$$
$$R_1 = 220\,\Omega$$

$$Z_T = 220\Omega + j\,(301.593\,\Omega)$$

$$\underline{\boldsymbol{Z_T \rightarrow polar = 373.31 \measuredangle 53.89°\,\Omega}}$$

Triangulo de impedancia



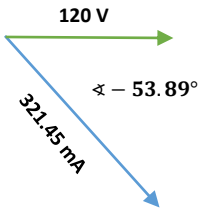
Corriente total, ángulo de fase.

$$I_T = \frac{V_T}{Z_T} = \frac{120 \measuredangle 0°\,V}{373.31 \measuredangle 53.89°\,\Omega}$$

$$\boldsymbol{I_T = 321.45 \measuredangle -53.89°\,mA}$$

$$\boldsymbol{\theta = 53.89°\ en\ atraso}$$

Diagrama fasorial que muestre la relación de voltaje y corriente.



P, Q, S, FP y triángulo de potencias

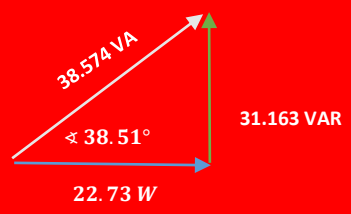
$$P = 120\ V * (321.45\ mA) * \cos 53.89° = 22.73\ W$$

$$Q = 120\ V * (321.45\ \ mA) * \sin 53.89°$$
$$Q\ = 31.163\ VAR$$

$$S = 120\ V * (321.45\ \ mA) = 38.574\ VA$$

$$FP = \cos 53.89 = 0.589$$

Triangulo de potencia



Voltaje en cada elemento

$$V_R = I_T * R_1$$
$$V_{R1} = (321.45 \measuredangle -53.89°\ mA) * (220 \measuredangle 0°\ \Omega)$$
$$\boldsymbol{V_{R1} = 70.719 \measuredangle -53.89°\ V}$$

$$V_{L1} = I_T * X_{L1}$$
$$V_{L1} = (321.45 \measuredangle -53.89°\ mA) * (188.496 \measuredangle 90°\ \Omega)$$
$$\boldsymbol{V_{L1} = 60.59 \measuredangle 36.11°\ V}$$

$$V_{L2} = I_T * X_{L2}$$
$$V_{L2} = (321.45 \measuredangle -53.89°\ mA) * (113.097 \measuredangle 90°\ \Omega)$$
$$\boldsymbol{V_{L2} = 36.36 \measuredangle 36.11°\ V}$$

Corriente en cada elemento

$$I_T\ = I_{R1} = I_{L1} = I_{L2}$$

$$I_T = 321.45 \measuredangle -53.89°\ mA$$

Por lo tanto;

$$I_{R1} = 321.45 \measuredangle -53.89°\ mA$$
$$I_{L1} = 321.45 \measuredangle -53.89°\ mA$$
$$I_{L2} = 321.45 \measuredangle -53.89°\ mA$$

Circuito 2:

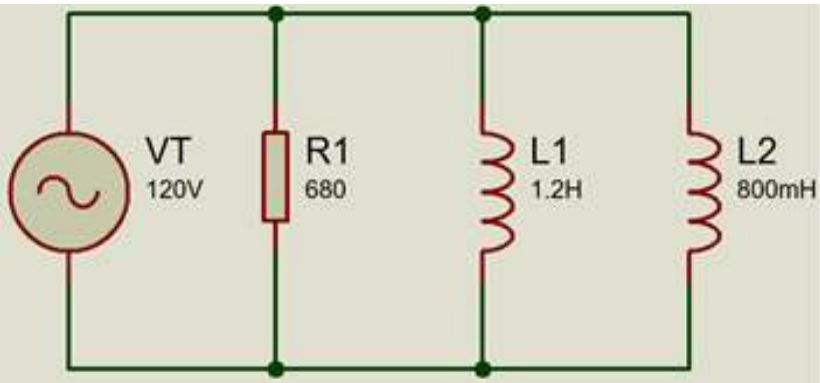


Figura 2. RL paralelo

Impedancia total

$$Z_t = R + j\,X_L$$

$$X_{L1} = 2\,\pi * (L_1) * f = 2\,\pi * (1.2H) * 60Hz = 452.39\,\Omega$$
$$X_{L2} = 2\,\pi * (L_2) * f = 2\,\pi * (800\,mH) * 60Hz = 301.59\,\Omega$$

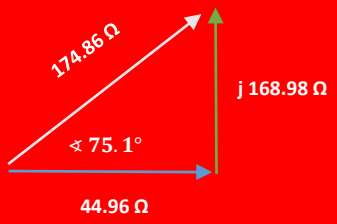
$$X_L = \frac{X_1 + X_2}{X_1 * X_2} = \frac{452.39\Omega * 301.59\,\Omega}{452.39\Omega + 301.59\,\Omega} = 180.95\,\Omega$$
$$R_1 = 680\,\Omega$$

$$Z_T = \frac{R1 * XL}{\sqrt{(R1)^2 + (XL)^2}} \sphericalangle tan^{-1}\left(\frac{R1}{XL}\right)$$
$$Z_T = \frac{(680\Omega) * (180.95\Omega)}{\sqrt{(680\Omega)^2 + (180.95\Omega)^2}} \sphericalangle tan^{-1}\left(\frac{680\,\Omega}{180.95\,\Omega}\right)$$

$$\boldsymbol{Z_T = 174.86\,\Omega \sphericalangle 75.1°}$$

$$Z_{T\,(rectangular)} = 44.96\Omega + j\,168.98\Omega$$

Triangulo de impedancia



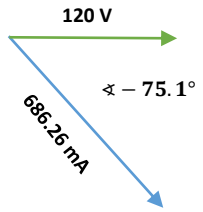
Corriente total, ángulo de fase.

$$I_T = \frac{V_T}{Z_T} = \frac{120 \measuredangle 0° \, V}{174.86 \, \Omega \measuredangle 75.1°}$$

$$\boldsymbol{I_T = 686.26 \measuredangle -75.1° \, mA}$$

$$\boldsymbol{\theta = 75.1° \, en \, atraso}$$

Diagrama fasorial que muestre la relación de voltaje y corriente.



P, Q, S, FP y triángulo de potencias

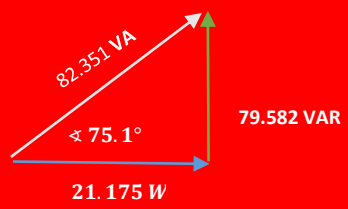
$$P = 120 \, V * (686.26 \, mA) * \cos 75.1° = 21.175 \, W$$

$$Q = 120 \, V * (686.26 \;\; mA) * \sin 75.1°$$
$$Q = 79.582 \, VAR$$

$$S = 120 \, V * (686.26 \;\; mA) = 82.351 \, VA$$

$$FP = \cos 75.1 = 0.257$$

Triangulo de potencia



Corriente en cada elemento

$$I_R = \frac{V_T}{R_1}$$
$$I_{R1} = \frac{120 \, V \measuredangle 0°}{680 \, \Omega \measuredangle 0°}$$
$$\boldsymbol{I_{R1} = 176.47 \measuredangle 0° \, mA}$$

$$I_{L1} = \frac{V_T}{X_{L1}}$$
$$I_{L1} = \frac{120 \, V \measuredangle 0°}{452.39 \Omega \measuredangle 90°}$$
$$\boldsymbol{I_{L1} = 265.26 \measuredangle 0° \, mA}$$

$$I_{L2} = \frac{V_T}{X_{L2}}$$
$$I_{L2} = \frac{120 \, V \measuredangle 0°}{301.59 \, \Omega \measuredangle 90°}$$
$$\boldsymbol{I_{L2} = 398.48 \measuredangle 0° \, mA}$$

Voltaje en cada elemento

$$V_T \;\; = V_{R1} = \; V_{L1} = V_{L2}$$

$$V_T = 120 \measuredangle 0° \, V$$

Por lo tanto;

$$\boldsymbol{V_{R1} = 120 \measuredangle 0° \, V}$$
$$\boldsymbol{V_{L1} = \;\; 120 \measuredangle 0° \, V}$$
$$\boldsymbol{V_{L2} = 120 \measuredangle 0° \, V}$$

Ejercicio 3:

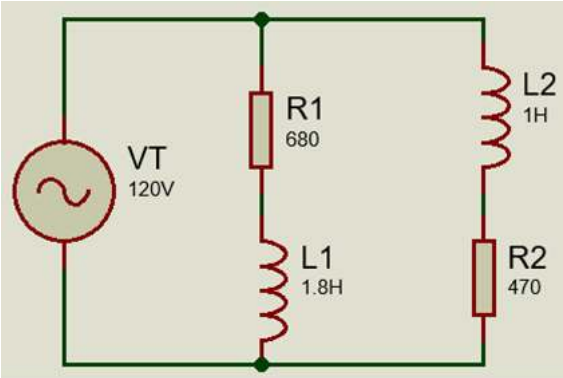


Figura 3. RL mixto

Impedancia total

$$Z_t = R + j\,X_L$$

Separando en RA y RB donde:

$$A \rightarrow R1 \text{ y } L1$$
$$X_{L1} = 2\,\pi * (L_1) * f = 2\,\pi * (1.8H) * 60Hz = 452.39\,\Omega$$
$$R_1 = 680\,\Omega$$

$$Z_A = 680\Omega + j\,(452.39\,\Omega)$$
$$Z_{A \rightarrow polar} = 816.74\,\Omega \measuredangle 33.63°$$

$$B \rightarrow R2 \text{ y } L2$$
$$X_{L2} = 2\,\pi * (L_2) * f = 2\,\pi * (1\,H) * 60Hz = 376.99\Omega$$
$$R_2 = 470\,\Omega$$

$$Z_B = 470\Omega + j\,(376.99\,\Omega)$$
$$Z_{B \rightarrow polar} = 602.51\,\Omega \measuredangle 38.73°$$

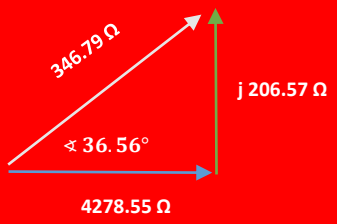
$$Z_T = \frac{Z_A * Z_B}{Z_A + Z_B}$$
$$Z_T = \frac{(816.74\,\Omega \measuredangle 33.63°) * (602.51\,\Omega \measuredangle 38.73°)}{(680\Omega + j\,(452.39\,\Omega)) + (470\Omega + j\,(376.99\,\Omega))}$$

$$Z_T = \frac{(492.094\ k\Omega \measuredangle 72.36°)}{(1150\Omega + j\,(829.38\,\Omega))}$$
$$\rightarrow \textit{convirtiendo el denominador a polar}$$
$$Z_T = \frac{(492.094\ k\Omega \measuredangle 72.36°)}{(1.419\,k\Omega \measuredangle 35.80°)}$$

$$\boldsymbol{Z_T = 346.79\ \Omega \measuredangle 36.56°}$$

Triangulo de impedancia

$$Z_{T\,(rectangular)} = 278.55\,\Omega + j\,206.57\,\Omega$$



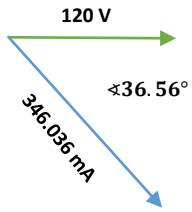
Corriente total, ángulo de fase.

$$I_T = \frac{V_T}{Z_T} = \frac{120 \sphericalangle 0° \, V}{346.79 \ \Omega \sphericalangle 36.56°}$$

$$\boldsymbol{I_T = 346.03 \sphericalangle -36.56° \, mA}$$

$$\boldsymbol{\theta = 36.56° \, en \, atraso}$$

Diagrama fasorial que muestre la relación de voltaje y corriente.



P, Q, S, FP y triángulo de potencias

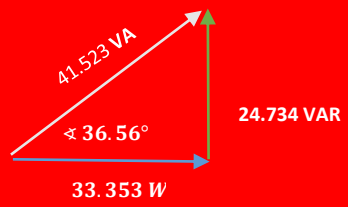
$$P = 120 \, V * (346.03 \ mA) * \cos 36.56°$$
$$P = 33.353 \, W$$

$$Q = 120 \, V * (346.03 \ mA) * \sin 36.56°$$
$$Q = 24.734 \, VAR$$

$$S = 120 \, V * (346.03 \ mA) = 41.523 \, VA$$

$$FP = \cos 36.56° = 0.803$$

Triangulo de potencia



Voltaje en cada elemento

$$V_T = V_B = V_A = 120 \sphericalangle 0° \, V$$

$$V_{R1} = I_A * R_1$$
$$V_{R1} = (146.93 \sphericalangle -33.63° \, mA) * (680 \sphericalangle 0° \, \Omega)$$
$$\boldsymbol{V_{R1} = 99.91 \sphericalangle -33.63° \, V}$$

$$V_{L1} = I_A * X_{L1}$$
$$V_{L1} = (146.93 \sphericalangle -33.63° \, mA) * (452.39 \sphericalangle 90° \, \Omega)$$
$$\boldsymbol{V_{L1} = 66.47 \sphericalangle 56.37° \, V}$$

$$V_{R2} = I_B * R_2$$
$$V_{R2} = (199.17 \sphericalangle -38.73° \, mA) * (470 \sphericalangle 0° \, \Omega)$$
$$\boldsymbol{V_{R2} = 93.61 \sphericalangle -38.73° \, V}$$

$$V_{L2} = I_B * X_{L2}$$
$$V_{L2} = (199.17 \sphericalangle -38.73° \, mA) * (376.99 \sphericalangle 90° \, \Omega)$$
$$\boldsymbol{V_{L2} = 75.09 \sphericalangle 51.27° \, V}$$

Corriente en cada elemento

$$I_A = \frac{V_A}{Z_A}$$
$$I_A = \frac{120 \, V \sphericalangle 0°}{816.74 \, \Omega \sphericalangle 33.63°}$$
$$I_A = 146.93 \sphericalangle -33.63° \, mA$$
$$I_A = I_{R1} = I_{L1}$$

$$\boldsymbol{I_{R1} = 146.93 \sphericalangle -33.63° \, mA}$$
$$\boldsymbol{I_{L1} = 146.93 \sphericalangle -33.63° \, mA}$$

$$I_B = \frac{V_B}{Z_B}$$
$$I_B = \frac{120 \, V \sphericalangle 0°}{602.51 \, \Omega \sphericalangle 38.73°}$$
$$I_B = 199.17 \sphericalangle -38.73° \, mA$$
$$I_B = I_{R2} = I_{L2}$$

$$I_{R2} = 199.17 \sphericalangle -38.73° \, mA$$
$$I_{L2} = 199.17 \sphericalangle -38.73° \, mA$$