Hand-in 2

a) Known:

$l = 0.16$ m $\qquad a = 0,4\,l$

$\alpha_1' = 48^\circ$ $\qquad$ compressor cascade

$\alpha_2' = 16^\circ$ $\qquad$ design point

$i = 0^\circ$

Solution:

$$\varepsilon = \alpha_1 - \alpha_2, \quad \alpha_1 = \alpha_1' + i, \quad \alpha_2 = \alpha_2' + \delta$$

$$(i=0) \Rightarrow \varepsilon = \alpha_1' - \alpha_2' - \delta$$

$$\delta = m\,\theta\sqrt{\frac{s}{l}}, \qquad \theta = \alpha_1' - \alpha_2'$$

$$m = 0,23\left(\frac{2a}{l}\right)^2 + \frac{\alpha_2}{500} = 0,23\left(\frac{2a}{l}\right)^2 + \frac{\alpha_2' + \delta}{500}$$

$$\Rightarrow \delta = \frac{0,23\left(\frac{2a}{l}\right)^2 + \frac{\alpha_2'}{500}}{1 - \frac{1}{500}\theta\sqrt{\frac{s}{l}}}\,\theta\sqrt{\frac{s}{l}} = \delta\left(\frac{s}{l}\right)$$

$$\Rightarrow \varepsilon = \alpha_1' - \alpha_2' - \delta\left(\frac{s}{l}\right) \quad \checkmark$$

$$\zeta = \frac{\Delta p_0}{\frac{1}{2}\rho c_m^2 l}, \quad D = s\,\Delta p_0 \cos(\alpha_m), \quad D = \theta_2 \rho c_2^2$$

$$\Rightarrow \zeta = \frac{2\theta_2}{s\cos(\alpha_m)\,l}\,\frac{c_2^2}{c_m^2} = \left\{\begin{matrix} c_m = c_x/\cos(\alpha_m) \\ c_2 = c_x/\cos(\alpha_2) \end{matrix}\right\} =$$

$$= \frac{2\left(\frac{\theta_2}{l}\right)}{s/l\;\,l}\,\frac{\cos(\alpha_m)}{\cos^2(\alpha_2)} = \left\{ \alpha_m = \tan^{-1}\left(\frac{\tan(\alpha_1) + \tan(\alpha_2)}{2}\right)\right\} =$$

$$= \frac{2\frac{\theta_2}{l}}{s/l\;\,l}\,\frac{1}{\cos(\alpha_2)\sqrt{\frac{1}{4}(\tan(\alpha_1) + \tan(\alpha_2))^2 + 1}} \quad \checkmark$$

where:

$$\frac{\theta_2}{l} = \frac{0,004}{1 - 1.17\cdot\ln\left(\frac{c_{max,s}}{c_2}\right)}$$

Hand-in 2 cont.

$$\frac{c_{max,s}}{c_2} \cong \frac{\cos(\alpha_2)}{\cos(\alpha_1)}\left(1{,}12 + 0{,}61\frac{s}{l}\cos^2(\alpha_1)(\tan(\alpha_1) - \tan(\alpha_2))\right)$$

As for the physical reasons for the relations, flow deflection decreases with an increasing pitch-chord ratio, as there are less geometric features per unit length to turn the flow. The pressure loss initially decreases with a decreasing pitch-chord ratio since there's less surface area where losses due to boundary layers emerge, but then increases again. The increase is due to the large wake formations, subsequent from a large deviation angle.

b) The assumptions made to find the pressure loss, Liebleins empirically established relationships, assume in themselves an incompressible flow case. This entails that a higher maximum speed would decrease the accuracy of the assumption. Now, assuming a largely constant outflow speed c_2, the fractions:

$$\left(\frac{c_{max,s}}{c_2}\right)_{s/l=1} \cong 1{,}817 \quad , \quad \left(\frac{c_{max,s}}{c_2}\right)_{s/l=2} \cong 1{,}996$$

tell us that $c_{max,s}$ is higher for the second case and therefore the pressure loss prediction is less accurate.