Turbomachinery assignment 3

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Known

The known quantities to this problem are:
$DF_{\text{rotor}} = 0.45$
$DF_{\text{stator}} = 0.45$
$\psi = 0.5894$
$R = 0.5$
$\phi = 0.7156$ or $0.5 < \phi < 1$

Target

The target to this problem is:
η_{tt}

Solution

From the previous hand in, the profile losses can be found through correlations with the pitch-chord ratios and the blade in/out angles, as

$$\zeta = f_1(s/l, \alpha_{\text{in}}, \alpha_{\text{out}}), \tag{1}$$

which with the assumptions given in this assignment, this being low speed incompressible flow, is an okay approximation. Since only profile loss is considered in this problem, the pressure losses can be expressed as

$$\begin{aligned} Y_{\text{p,rotor}} &= f_1(s/l_{\text{rotor}}, \beta_1, \beta_2), \\ Y_{\text{p,stator}} &= f_1(s/l_{\text{stator}}, \alpha_2, \alpha_3), \end{aligned} \tag{2}$$

due to the correlations being found in cascades and therefore applicable with relative angles. The pitch-chord ratios are found from the diffusion factors, which are expressed as

$$\begin{aligned} DF_{\text{rotor}} &= 1 - \frac{w_2}{w_1} + \frac{\Delta w_\theta}{2w_1} s/l_{\text{rotor}}, \\ DF_{\text{stator}} &= 1 - \frac{c_3}{c_2} + \frac{\Delta c_\theta}{2c_2} s/l_{\text{stator}}. \end{aligned} \tag{3}$$

The pitch-chord ratios are then found as

$$\begin{aligned}
s/l_{\text{rotor}} &= \frac{2w_1}{\Delta w_\theta}\left(\text{DF}_{\text{rotor}} + \frac{w_2}{w_1} - 1\right), \\
s/l_{\text{stator}} &= \frac{2c_2}{\Delta c_\theta}\left(\text{DF}_{\text{stator}} + \frac{c_3}{c_2} - 1\right).
\end{aligned} \tag{4}$$

The velocity relations can be rewritten as trigonometric expressions, as

$$\begin{aligned}
\frac{w_2}{w_1} &= \frac{\cos(\beta_1)}{\cos(\beta_2)}, \\
\frac{c_3}{c_2} &= \frac{\cos(\alpha_2)}{\cos(\alpha_3)}, \\
\frac{\Delta w_\theta}{2w_1} &= 1/2\cos(\beta_1)\left(\tan(\beta_1) - \tan(\beta_2)\right), \\
\frac{\Delta c_\theta}{2c_2} &= 1/2\cos(\alpha_2)\left(\tan(\alpha_2) - \tan(\alpha_3)\right).
\end{aligned} \tag{5}$$

Note the sign change on the delta expression in the stator, since swirl is measured through the energy involved and therefore is positive. The flow angles are found through the stage loading, reaction degree and flow coefficient, as

$$\begin{aligned}
\alpha_1 = \alpha_3 &= \tan^{-1}\left(\frac{1}{\phi}(1 - \text{R} - \psi)\right), \\
\alpha_2 &= \tan^{-1}\left(\frac{\psi}{\phi} + \tan(\alpha_1)\right), \\
\beta_2 &= \tan^{-1}\left(\frac{1-\psi}{\phi} - \tan(\alpha_1)\right), \\
\beta_1 &= \tan^{-1}\left(\frac{\psi}{\phi} + tan(\beta_2)\right).
\end{aligned} \tag{6}$$

Since the compressor is operating at low speeds, the total-to-total efficiency can be estimated as

$$\eta_{tt} = 1 - \frac{1}{2}\frac{w_1^2 \text{Y}_{\text{p,rotor}} + c_2^2 \text{Y}_{\text{p,stator}}}{\Delta h_0}. \tag{7}$$

Formulating the total enthalpy change as $\Delta h_0 = \psi/\phi^2 c_x^2$, the total-to-total efficiency is then expressed as

$$\eta_{tt} = 1 - \frac{1}{2}\frac{\phi^2}{\psi}\left(\frac{\text{Y}_{\text{p,rotor}}}{\cos^2(\beta_1)} + \frac{\text{Y}_{\text{p,stator}}}{\cos^2(\alpha_2)}\right). \tag{8}$$

For the flow coefficient set as 0.7156, the total-to-total efficiency comes out as $\eta_{tt} = 0.9099$. The distributions for a flow coefficient between 0.5 and 1 can be found in figure 1.

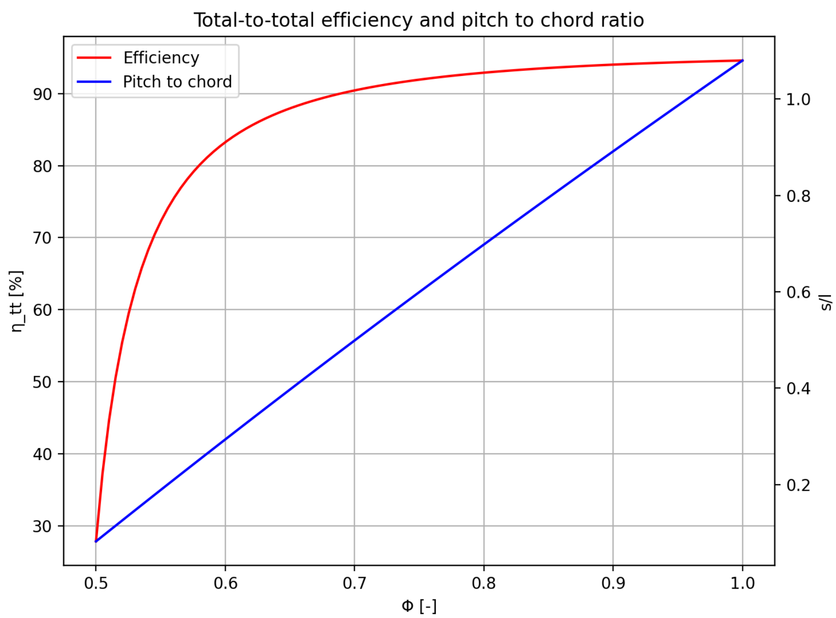

Figure 1: The total-to-total efficiency of the turbomachine stage (left axis), along with the pitch to chord ratios for both the stages (right axis), for a varying flow coefficient.

Physical observations

When the flow coefficient is increased, the pitch to chord ratio is increased, meaning that the number of blades is decreased. The physical explanation would be that as flow coefficient is increased, the amount of blocking in the gas canal, here blades, would have to be decreased.

When the flow coefficient is increased, the efficiency of the compressor is increased. This would be due to a higher pitch to chord ratio, meaning fewer sources of profile loss (blades).

If the rotational speed is held fixed, a high flow coefficient would mean significant axial velocity. If such a velocity would be the case, you would first and foremost see compressibility effects, which are not accounted for in the assumptions made. Second, the assumption that the diffusion factor is 0.45 would be hard to back up, since there would be significant deviation in the stage.