Hand-In problem 1

a) Known:

$\dot{m}_{des} = 60\ kg/s$ $\quad$ $\frac{P_{02\ des}}{P_{01}} = 20$

$T_{01} = 300\ K$ $\quad$ $P_{01} = 10^5\ Pa$

$\gamma = 1,4$ $\quad$ $M_2 = 1$ (Choked)

$\eta_{c,des} = 0,8$ $\quad$ $\eta_{c,opt} = 0,825$

$\frac{P_{02,opt}}{P_{01}} = 17$ $\quad$ $\dot{m}_{opt} = 48\ kg/s$

First, find the design point area. Use conservation of mass:

$$\frac{\dot{m}\sqrt{RT_{02,des}}}{P_{02,des}A_{des}} = \sqrt{\gamma}\, M_2\left(1+\frac{\gamma-1}{2}M_2^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}} =$$

$$\hat{=}\ 0,6847315\ldots$$

$$\Rightarrow A_{des} = \frac{\dot{m}\sqrt{RT_{02,des}}}{P_{02,des}\cdot 0,6847315} \quad (*)$$

Use isentropic efficiency relations to find $T_{02,des}$

$$\frac{T_{02,des}-T_{01}}{T_{01}} = \frac{1}{\eta_{c,des}}\left[\left(\frac{P_{02,des}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

$$\Leftrightarrow T_{02,des} = T_{01}\left(\frac{1}{\eta_{c,des}}\left[\left(\frac{P_{02,des}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}-1\right]+1\right) \quad (**)$$

$$\hat{=}\ 807.580\ldots\ K$$

Find $P_{02,des}$ as:

$$P_{02,des} = \frac{P_{02,des}}{P_{01}}\cdot P_{01} = 2\cdot 10^6\ Pa$$

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Use the previous solutions to find the design point area (*):

$$A_{des} \cong 0{,}0210428\ldots \ m^2$$

Now use the equations (*) and (**) but with optimal point quantities:

$$T_{02,\,opt} \cong 753{,}368\ldots \ K$$

$$A_{opt} \cong 0{,}0191742\ldots \ m^2$$

Giving the final area ratio:

$$\frac{A_{opt}}{A_{des}} \cong 90{,}90\ \%$$

$\boxed{\quad}$

b Given geometrically similar engines, the quantity:

$$\frac{\dot{m}\sqrt{R\,T_0}}{P_0\,D^2}$$

will remain constant. Using the subscripts S and L for small and large, the expression for the diameter is found as:

$$\frac{\dot{m}_S\sqrt{R\,T_{0,S}}}{P_{0S}\,D_S^2} = \frac{\dot{m}_L\sqrt{R\,T_{0,L}}}{T_{0L}\,D_L^2}$$

$$\Leftrightarrow D_S^2 = \frac{\dot{m}_S}{\dot{m}_L}\sqrt{\frac{T_{0,S}}{T_{0,L}}}\,\frac{P_{0,L}}{P_{0,S}}\,D_L^2$$

$$\Rightarrow D_S \cong 0{,}52811\ldots \ m$$

$\boxed{\quad}$