# Quantifiers Module Unit.

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## 1 Request for feedback

I am trying my best to get use to LaTex. I know there is a template provided in the resources download, however, it doesn't compile for me. Let me know if I need to resubmit with the correct template, or this one is okay. Cheers.

## 2 Response to feedback

Lets try to solve $\exists y \forall x : x^2 + y = 0$ (considering natural numbers domains). Here we try to solve it for false. Since $P(y)\forall x : x^2 + y = 0$ has the universal quantifier, we can take an example as proof. We also have to keep in mind that y uses the existential quantifier. Therefore $x = \sqrt{1-y}$. If we now plug in our x, we see that $\sqrt{1-y}^2 + y = 0$ which simplifies to $1 = 0$ which is obviously not true, meaning that no $x$ can satisfy the equation. This proves the statement as false.

## 3 Module Learning Objectives

1. Formulate complex statements using quantifiers and predicates
2. Nest and compound quantified statements
3. Prove or disprove quantified statements.

## 4 Quantifiers and Predicates

### 4.1 Predicates

Firstly, what are Predicates and Quantifiers? Well, how do we express statements such as $x \mod 2 = 2$ or $x > 2 \wedge x < 3$? We can see that these are propositions, however they are only true in certain cases when $x$ is a value that satisfies the statement. This is where Predicates come in handy. For something to become a proposition, we first need to assign it a value such as $x = 1$ or $x = 50$. This means that the expression now has a truth value, and therefore is a Proposition. In short, predicates allow us to represent propositions with variables.

Consider the example: Every element in the set $A = \{20, 30, 40, 50\} \quad \forall x \in A, x$ is less than 100. We can obviously see that this is true, however how can we express this in a mathematical and effective way?

### 4.2 Quantifiers

The quantifiers $\forall$ is called the Universal Quantifier and represent "for all", meaning that all of the values in a specific set are applied to a given proposition. On the other hand, $\exists$ is called the Existential Quantifier and is used to represent that there exists at least one element in the set that satisfies a given condition. If we bring back our example set $A$ ($A = \{20, 30, 40, 50\}$) and the condition of $x < 100$ then $\quad \forall x \in A : x < 100$. Similarly, we can also say that at least one of the elements in this set satisfies the condition, which can be denoted as $\exists x \in A, x < 100$. As you can see, this is obviously a much more formal and faster way to represent $x < 50 \wedge x < 40$ etc. Predicates can either be true or false, depending on the propositional requirements. For example, statements such as $x \geq 13$ and $x + 25 = 100$ can also be expressed using Predicates and a set. (hopefully you are starting to see the usefulness here) To denote statements like this, we can use $P(x)$ where $P$ is the proposition for the variable $x$. Let's consider the following: $P(x) \geq 55$, where $x$ is the subject (a person) and the predicate $\geq 55$ represents age. We also need a domain, which could be the set of people. As discussed before, we can then use $\forall$ and $\exists$ to represent this.

### 4.3 Proving or Disproving Quantifiers

To prove a $\exists$ statement is $true$, we need just one example. On the other hand, to prove it is false, we need a general statement because giving a false example would have no useful impact (since another element of the set could prove that it is true). Secondly, to prove a $\forall$ Quantifier, we need a general statement, as providing an example would produce no useful information. If we want to disprove the $\forall$ Quantifier, all we have to do is prove that one singular statement false (because if one is false, then they are not all $true$). As a note, a set does not have to have a discrete number of elements. For example, in this module, we work with $\mathbb{R}$ and $\mathbb{Z}$, which represent Real Numbers and Integers.

### 4.4 Why is this even useful?

Quantifiers are useful as they allow us to effectively communicate mathematical concepts in an effective way rather than writing out a large string of connectives such as $\wedge, \vee$, etc.

## 5 Nested Quantifiers

A statement where the predicate is also quantified, followed by being quantified by a main quantifier is know as a nested quantifier, these typically have multiple different variables. For example, for every real number ( $\mathbb{R}$ ) $x$, there exists ( $\exists$ ) a real number ( $\mathbb{R}$ ) $y$ such that $y = x^2$. This may look confusing at first, however, they are practically the same as Quantifiers that are NOT nested, the only difference is that the predicate itself is quantified, which is then quantified again by the main quantifier. Just to recap, a quantifier is just a the range of values the expression can accept. Similar to $\implies$ , the order of the qualifiers matters. We can see this if we look at the following example: $\forall x \exists y : P(x,y)$ where $P(x,y)$ means that person $x$ knows a person $y$ who is older than 18 years old. This means that every person knows someone who is above 18 years old. If we flip the quantifiers ( $\exists y \forall x : P(x,y$ ), the suddenly we are saying that there exists some person that is above 18 years old that in which everyone knows which doesn't make sense anymore.

# 6 Proving Nested Quantifiers

Proving nested quantifiers may seem tricky or confusing at first (as it was to me) however these can easily be solved if we remember what type of proof ( an example, or a example) to use and in what order the quantifiers are in. Keeping this is mind, proving nested quantifiers becomes almost as easy and solving just regular quantifiers. For example, just say we are trying to prove (or dis-prove) something like $\forall x \exists y : (y \neq 0, xy = 1)$ we first have to note what quantifier is in the scope of the other. We can see that $\forall x$ comes before $\exists y$ therefor we can re arrange the equation since $y$ depends on $x$, this looks like $y = 1/x$. When we re-arrange like this, we are saying that for all values of $x$, there must be some value $y$ that satisfies the equation. We can then prove or disprove this using our knowledge on what type of proof we need. In this example, we need can disprove this by giving an example of $x$ where $y$ does not work. Therefor, if we say $x = 0$, we have the equation $y = 1/0$, we know this is $false$ as no value of $y$ satisfy this, disproving the $\exists y$. This means we have successfully disproved the nested quantifier. Just to let you know, there are LOTS of examples here in my learning evidence notes. It may be useful to have them side by side to strengthen my explanation.

## 6.1 Negating quantifiers and nested quantifiers.

Similarly to predicates, nested quantifiers are just the inverse of their original statement. For example let's analyse $\not\forall x$. We know that when this statement is not negated, it means for all, or for every $x$. Similarly, $\not\forall x$ means not for all x, or "every x is not true" this is just the same as the exists quantifier. On the other hand, the $\exists$ quantifier can be thought of in the same way as we thought of the universal quantifier. Lets take $\nexists y$, this means that there is no y that exists. In short, this same logic applies for nested quantifiers. $\forall x \ \nexists y$ This means that for every $x$, there does not exist any $y$. We can then, for example, disprove this by providing a example of when $y$ does exist.

# 7 Reflection

## 7.1 What was the most important thing I learnt

The most important thing in this unit was learning about what quantifiers are and why we even use them in the first place. When linking back to my example in the beginning, it is very difficult to express large predicates with lots of connectives. For example, if we want to express the $\forall$ quantifier without actually using it, it might look something like this $A \geq 15B \geq 15C \geq 15$..... However, this is only possible with a discrete domain, for example a set of people, however, if we want to use express the $\forall$ quantifier without actually using it (expressing just as a predicate) it would be impossible when the domain is infinite such as ( $\mathbb{R}$). This highlights the importance of quantifiers.

## 7.2 How does this relate to what I already know

We explored propositions in the previous module, and based on my understanding, predicates are just an extra layer of complexity based on what we already know. Take the example about representing quantifiers as a predicate (just a bunch of and statements). Learning about predicates just built on my ability to express more complex propositions in an effective way.

## 7.3 Why do you think your course team wants you to learn the content of this module for your degree?

I think that the course team wants CS students to learn about predicates as it is a foundational concept in logic, which is key to programming, algorithms, and even more advanced topics like AI. For example, in my AI unit, we learn about AI "acting logically" which ties directly into this content. This type of mathematics also develops reasoning and problem solving which is key in a technological career path.

# 8 Feedback reflection

After some interaction with Julian, I learned that i was more focused on how to solve the problem than understanding how it works. This lead me to often make mistakes regarding which quantifier was nested within the other. This often left me confused as my "proof" would like it was almost true and false at the same time. However, after many back and fourths with my tutor and many practise problems from ChatGPT, I was able to understand that nested quantifiers were just predicates that also happened to be quantified. I developed a further understanding of the relationship between nested quantifiers and could reflect on the dependencies on had one the other (depending on the order of the quantifiers).