

# DH2323 Lab2 TrackA

## Raytracing

V2.0 SDL2

### 1 Introduction

In this lab you will implement a Raytracer, which draws images of 3D scenes by tracing light rays reaching the simulated camera. The lab is divided into several steps and tasks. To get something on the screen, you first need to learn how to:

- Represent 3D scenes using triangular surfaces.
- Trace the ray of each pixel in the camera image into the scene.
- Compute ray-triangle intersections, to find out which surface a ray hit.

Then, you'll be able to write program that draws image screen to the left of Figure 1. Next, extend the program by adding.

- Camera motion.
- Simulation of light sources.
- Compute ray-triangle intersections, to find out which surface a ray hit.

The results can be seen in Figure 1

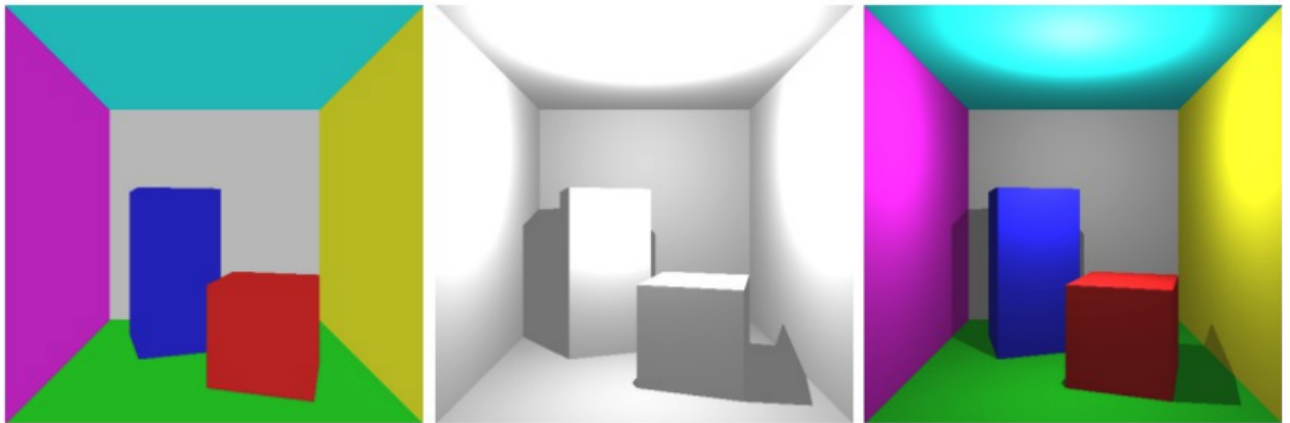


Figure 1: Expected final output for the lab

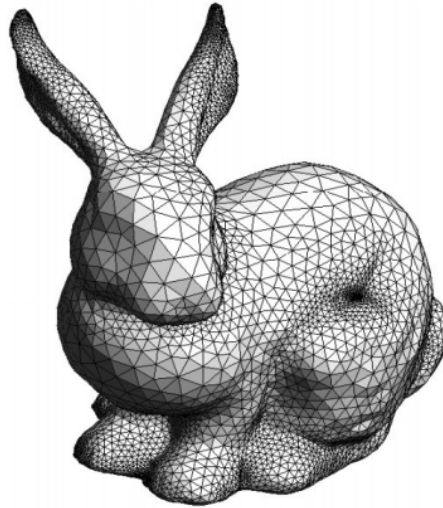


Figure 2: 3D model represented by triangular surfaces. This bunny is a common test model in computer graphics. Ref: <http://graphics.stanford.edu/data/3Dscanrep/>

## 2 Representing Surfaces

In this lab you will learn how to represent and draw surfaces, instead of points like you did in the first lab with the star-field. The simplest surface to describe mathematically is a triangular surface because it's planar and its geometrical properties can be described by only three 3D points: its vertices. For instance, from the three vertices we can compute the normal  $\mathbf{N}$  of the surface, i.e. a vector pointing out from the surface, being orthogonal to the surface itself. See Figure 2 for an example of a 3D model represented by triangular surfaces.

### Defining a *struct* for Triangles

When describing a surface for computer graphics we are also interested in its appearance. The simplest way to describe the appearance of a triangle is with just one color. We will use the following data structure (see *TestModel.h*) to represent a triangle:

```
struct Triangle
{
    vec3 v0;
    vec3 v1;
    vec3 v2;
    vec3 normal;
    vec3 color;

    //constructor and code for computing normal here
};
```

Although the normal can be computed from the vertices, it can be convenient to have it in the data structure as well. Then the normals  $\mathbf{N}$  can be computed once and then stored so that we can easily access them later whenever we want, without any computations.

### Task 2.1 Create a global vector for the Triangles in a model

A vector of triangles define a single model. In this case, we will use the global vector *triangles* to store our model:

```
vector<Triangle> triangles;
```

Add this code as a global variable in the *skeleton.cpp* file.

### A loading function for Triangles

To fill this vector with the triangles representing a specific 3D model, we have defined this function:

```
LoadTestModel( vector<Triangle>& triangles );
```

The function `LoadTestModel` is defined in the header file `TestModel.h` which is included near the beginning of the `skeleton.cpp` file. This function will fill the vector it takes as an argument with the triangles describing the model seen in Figure 1.<sup>1</sup>

### 3 Intersection of Ray and Triangle

In our raytracer we want to trace rays for each pixel of the image, to see where they come from. Since we use triangular surfaces to represent our model, this corresponds to finding the first intersection between a triangle and a ray. We will now derive equations for this computation.

#### Triangles in 3D

$v_0, v_1, v_2$  are real 3D vectors representing the vertices of the triangle, i.e. they are elements of  $R^3$ . To describe a point in the triangle we construct a coordinate system that is aligned with the triangle. Let  $v_0$  be the origin of the coordinate system and we let the axis be parallel to two of the edges of the triangle:

$$e_1 = v_1 - v_0 \quad (1)$$

$$e_2 = v_2 - v_0 \quad (2)$$

$e_1 \in R^3$  is then a vector that is parallel to the edge of the triangle between  $v_0$  and  $v_1$ . Similarly,  $e_2 \in R^3$  is parallel to the edge between  $v_0$  and  $v_2$ . Any point  $r \in R^3$  in the plane of the triangle can then be described by two scalar coordinates  $u \in R$  and  $v \in R$  such that its position in 3D is:

$$r = v_0 + ue_1 + ve_2 \quad (3)$$

This holds for all points  $r$  that are in the plane of the triangle. For points that are not only in the same plane, but actually within the triangle we also have that:

$$0 < u \quad (4)$$

$$0 < v \quad (5)$$

$$u + v < 1 \quad (6)$$

#### Rays in 3D

Next, we need a way to describe points that are on a ray. By a ray we mean a segment of a line in 3D. We define a 3D line by a vector representing its start position  $s \in R^3$  and a vector representing its direction  $d \in R^3$ . All points  $r$  lying on the line can then be written:

$$r = s + td \quad (7)$$

where  $r \in R$  is a scalar coordinate describing the position on the line. It is the signed distance, in units of  $\|d\|$ , to the position  $r$  from the start position  $s$ . If we do not put any constraint on  $t$  we are describing a line that extends infinitely in both directions. However, by a ray we only mean the points that comes after the start position. For the ray we thus require:

$$0 \leq t \quad (8)$$

#### Intersection

To get the intersection between the plane of the triangle and the line of the ray, we can insert the equation of the plane (3) into the equation of the line (5) and solve for the coordinates  $t, u, v$ :

$$v_0 + ue_1 + ve_2 = s + td \quad (9)$$

This is an equation of the 3D vectors. We thus have one equation for each vector component, i.e. three. And since we have 3 unknowns  $(t, u, v)$  it can be solved. To find the solution we re-formulate the equation using matrices, as one typically does when solving linear systems of equations:

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<sup>1</sup>This is a famous test model in computer graphics. You can read more about it at <http://www.graphics.cornell.edu/online/box/>. It is a cubic room with dimensions  $-1 \leq (x, y, z) \leq 1$

$$v_0 + ue_1 + ve_2 = s + td \quad (10)$$

$$td + ue_1 + ve_2 = s - v_0 \quad (11)$$

$$(-de_1e_2)(tuv)^T = s - v_0 \quad (12)$$

This might still look a bit weird. To express the linear equation in a standard way we use a  $3 \times 3$  matrix:

$$A = (-de_1e_2) \quad (13)$$

where the vectors  $-d, e_1, e_2$  are the columns of the matrix. We also give a new name for our right hand side vector:

$$b = s - v_0 \quad (14)$$

and we put our unknowns in the vector:

$$x = (tuv)^T \quad (15)$$

The linear Equation 10 can be written better like:

$$Ax = b \quad (16)$$

and the solution  $x$  can be found by multiplying both sides with the inverse of the matrix  $A$ :

$$Ax = b \quad (17)$$

$$A^{-1}Ax = A^{-1}b \quad (18)$$

$$Ix = A^{-1}b \quad (19)$$

$$x = A^{-1}b \quad (20)$$

## Intersection computations

If we can compute the inverse of the matrix  $A$  we can easily find the intersection point  $x$ . Luckily, GLM has both a class to represent  $3 \times 3$  matrices (`glm::mat3`) and a function to compute matrix inverses. GLM has defined the operators  $+, -, *, /$  so that they can be used directly with `glm::vec3` and `glm::mat3`. We can do the intersection computations by writing:

```
using glm::vec3;
using glm::mat3;
vec3 v0 = triangle.v0;
vec3 v1 = triangle.v1;
vec3 v2 = triangle.v2;
vec3 e1 = v1 - v0;
vec3 e2 = v2 - v0;
vec3 b = s - v0;
mat3 A( -d, e1, e2 );
vec3 x = glm::inverse( A ) * b;
```

This concise syntax is one of the reasons why we are using C++ for these labs, rather than e.g. Java or C#, and also because it is often faster than Java and especially Python.

### Task 3.1 Write the ClosestIntersection function and Intersection struct

Now we know the coordinates  $t, u, v$  for the intersection point. We can then check the Inequalities 4, 5, 6, 8 to see whether the intersection occurred within the triangle and after the beginning of the ray. You should write a function that does the following:

```
bool ClosestIntersection(
    vec3 start,
    vec3 dir,
    const vector<Triangle>& triangles,
    Intersection& closestIntersection
);
```

it takes the start position of the ray, its direction and a `std::vector` of triangles as input. It should then check for intersection against all these triangles. If an intersection occurred, it should return true. Otherwise, return false. In the case of an intersection it should also return some information about the closest intersection. Define the following data structure to store this information:

```
struct Intersection
{
    vec3 position;
    float distance;
    int triangleIndex;
};
```

### Task 3.2 Update argument sent to closestIntersection with the 3D position

The argument `closestIntersection` sent to the function should be updated with the 3D position of the closest intersection. Its distance is the one from the start of the ray and the index of the triangle that was intersected. When writing the function `ClosestIntersection` you might need to know the largest value ("limits") in the beginning of your code and then write:

```
float m = std::numeric_limits<float>::max();
```

Note: To gain access to the `max()` function, include the *algorithm* library:

```
#include <algorithm>
```

### Task 3.3 Calculate solution for inverse of matrix (Optional)

If you are into linear algebra a good optional exercise is to calculate a closed form solution for the inverse of the matrix, instead of just using `glm::inverse`. This can be done with Cramer's rule. We need this result if we would like to increase the speed of the intersection computations, which will be the bottleneck of our program. Then, when we have the closed form solution we can postpone the computation of some of the matrix elements, until they are needed. We start by just computing the row needed to compute the distance `t`. Then, if `t` is negative, there is no need to continue analyzing that intersection, as it will not occur.

## 4 Tracing Rays

Now that you have a function that takes a ray and finds the closest intersecting geometry, you have all you need to start rendering an image with raytracing.

The idea is that you trace a ray for each pixel of the image. You then need to compute for each pixel what ray it corresponds to. Initially we assume that the camera is aligned with the coordinate system of the model with x-axis pointing to the right and y-axis pointing down and z-axis pointing forward into the image. The start position for all rays corresponding to pixels is the camera position. A vector  $d \in R^3$  pointing in the direction where the light reaching pixel (x,y) comes from can then be computed as:

$$d = (x - \frac{W}{2}, y - \frac{H}{2}, f) \quad (21)$$

where `W` and `H` are the width and height of the image and `f` is the focal length of the camera (in units of pixel). We strongly encourage you to draw a Figure of the pinhole camera and make sure that you understand this equation.

### Task 4.1 Declare focal length value and camera position

The focal length can be thought of as the distance from the hole in the pinhole camera to the image plane. Add the following global variables to store the camera parameters:

```
float focalLength = ...;
vec3 cameraPos( ... , ... , ... );
```

Fill in the dots above with good values for these variables so that the model can be seen from the camera position you choose.

**Question:** what will the horizontal and vertical field of view be for the focal length you have chosen?

## Task 4.2 Create Draw function

You are now ready to write the Draw function. In it you should loop through all pixels and compute the corresponding ray direction. Then, call the function `ClosestIntersection` to get the closest intersection in that direction. If there was an intersection, the color of the pixel should be set to the color of that triangle. Otherwise it should be black. The result should be similar to Figure 3.

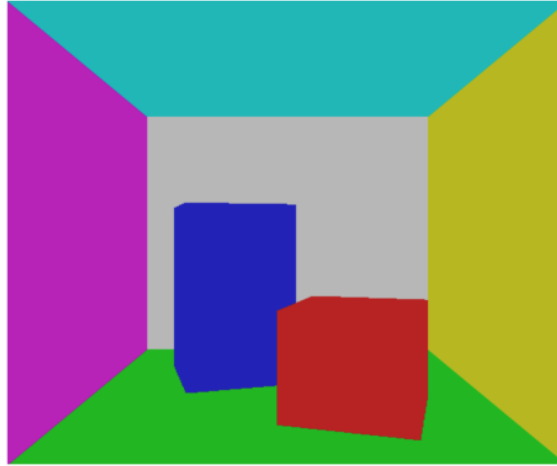


Figure 3: Tracing a light ray for each pixel to see which surface it first intersects.

## 5 Moving the Camera

We now have a simple visualization of our 3D scene. However, it would be nice if we could move the camera around.

### Task 5.1 Implement translation of the camera

First try to implement translation of the camera by updating the global variable `cameraPos` if the user presses the arrow keys. You can use SDL to check if a key is pressed in this way:

```
void Update()
{
    // Compute frame time:
    int t2 = SDL_GetTicks();
    float dt = float(t2-time);
    time = t2;
    cout << "Render time: " << dt << " ms." << endl;
    Uint8* keystate = SDL_GetKeyboardState( NULL );
    if( keystate[SDL_SCANCODE_UP] )
    {
        // Move camera forward
    }
    if( keystate[SDL_SCANCODE_DOWN] )
    {
        // Move camera backward
    }
    if( keystate[SDL_SCANCODE_LEFT] )
    {
        // Move camera to the left
    }
    if( keystate[SDL_SCANCODE_RIGHT] )
    {
        // Move camera to the right
    }
}
```

## Task 5.2 Translate the camera

Add code that translates the camera. Since raytracing is slow you will need to set a low width and height of the rendered image to get real-time performance, e.g. try 100x100 pixels.

## Task 5.3 Rotate camera around y-axis

When you got that working you should add a global variable that controls the rotation of the camera. Use a `glm::mat3` to represent the rotation matrix of the camera:

```
mat3 R;
```

Then make the left and right arrow keys rotate the camera around the y-axis.

## Task 5.4 Rotation of the camera

Add another global variable that stores the angle which the camera should be rotated around the y-axis:

```
float yaw;
```

When the left and right arrow keys are pressed this angle should be updated and then the rotation matrix  $R$  should be updated to represent a rotation around the y-axis with this angle. If you do not remember how such a matrix looks you can have a look in your linear algebra book or at Wikipedia.

If the camera is rotated by the matrix  $R$  then vectors representing the right (x-axis), down (y-axis) and forward (z-axis) directions can be retrieved as:

```
vec3 right( R[0][0], R[0][1], R[0][2] );
vec3 down( R[1][0], R[1][1], R[1][2] );
vec3 forward( R[2][0], R[2][1], R[2][2] );
```

## Task 5.5 Model a rotating camera

To model a rotating camera you need to use these directions both when you move the camera and when you cast rays. Implement this. As mentioned previously, you probably need to set a low width and height of the image for the algorithm to be fast enough for real-time motion. Since raytracing is slow it is typically not used for games and other interactive real-time applications. It is mainly used to render single images and special effects for movies. In lab 3 we will implement another algorithm, rasterization, which is faster and thus more suitable for interactive applications. After you got the camera motion working you can change back to a higher image resolution when doing the rest of the lab.

# 6 Illumination

You should now have a raytracer in which you can move the camera around. However, so far all points of a surface has exactly the same color. To add some more realism to the image we will add a light source.

## Task 6.1 Adding new light source

We will model an omni light. Such a light spreads light equally in all directions from a single point in space. The light can be defined by two global variables: its position and power for each color component:

```
vec3 lightPos( 0, -0.5, -0.7 );
vec3 lightColor = 14.f * vec3( 1, 1, 1 );
```

The color vector describes the power  $P$ , i.e. energy per time unit  $\frac{E}{t}$  of the emitted light for each color component. Each component thus has the physical unit  $W = \frac{J}{s}$ . The light will spread uniformly around the point light source, like an expanding sphere. Therefore, if a surface is further away from the light source it will receive less light per surface area. To derive a formula for the received light at a distance  $r$  from the light source we can think about a sphere with radius  $r$  centered at the light source. The emitted light will be spread uniformly over the whole surface area of the sphere which is:

$$A = 4\pi r^2 \quad (22)$$

The power per area breaching any point on the sphere is then:

$$B = \frac{P}{A} = \frac{P}{4\pi r^2} \quad (23)$$

When working with illumination this physical quantity is often used. Since it describes power per area, it has the unit  $\frac{W}{m^2} = Js^{-1}m^{-2}$ . Equation 23 describes the power per area reaching any surface point directly facing the light source. However, most of the time we will have surfaces that do not directly face the light source. Let  $\hat{n}$  be a unit vector describing the normal pointing out from the surface and let  $\hat{r}$  be a unit vector describing the direction from the surface point to the light source. Then if  $B$  is the power of light reaching a virtual surface area directly facing the light source at the surface point, then the power per real surface  $D$  will just be a fraction of this:

$$D = B * \max(\hat{r} * \hat{n}, 0) = \frac{(P * \max(\hat{r} * \hat{n}, 0))}{4\pi r^2} \quad (24)$$

We get the fraction between  $D$  and  $B$  by the projection of  $\hat{r}$  on  $n\hat{n}$ , i.e. a scalar product of two unit vectors. Since we do not allow negative light we take the max of the projected value and zero. If the angle between the surface normal and direction to the light source is larger than 90 degrees it does not receive any direct illumination. One way to think about this projection factor is to consider light as a stream of particles. If the particles hit the surface from an angle, instead of straight ahead, they will spread out more over the surface and less particles will hit a fixed area over a fixed time.

## Task 6.2 Create direct light given an intersection point

If it is raining and you place a bucket outside it will fill more quickly if it is raining straight from above than if the wind is blowing and the rain comes from the side. Now we know how much light that reaches any surface point directly from the light source, i.e. without reflection via another surface. You should then implement the function:

```
vec3 DirectLight( const Intersection& i );
```

It takes an intersection, which gives us the position where we want to know the direct illumination, as well as the index to the triangle that should get illuminated, therefore we also know the normal of the surface. The function should then return the resulting direct illumination described by Equation 24.

## Task 6.3 Add function DirectLight to Draw function

Modify the Draw function such that it uses the function DirectLight to compute the color for every pixel, after you have computed the intersection it corresponds to. You should then get the result of Figure 4.

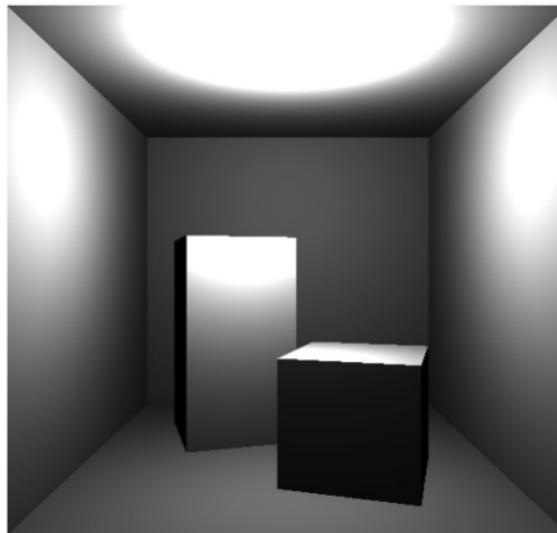


Figure 4: The incoming light.

Then you should also make it possible to move the light source, using the keys W and S to translate it forward and backward and A and D to translate it left and right and Q and E to translate it up and down. Do this in the Update function by writing something like:

```
if ( keystate[SDL_SCANCODE_W] )
    lightPos += forward;
```



However, this result does not correspond to how we would perceive the surface in the real world. It just tells how much light that reaches a surface point. What we actually see is the fraction of this that gets reflected in the direction of the ray. We thus need a model for this.

### Task 6.4 Model surfaces for diffuse light

We will model all surfaces as ideally diffuse (see Figure 5). Such surfaces are simple to simulate because they reflect light equally in all directions. The viewing angle does not influence the perceived color of such a surface point. It will look the same from all directions, having no specular high lights. In that sense, it is the opposite of a mirror, which will look completely different depending on the viewing angle. A mirror has an ideally specular surface (see Figure 5).

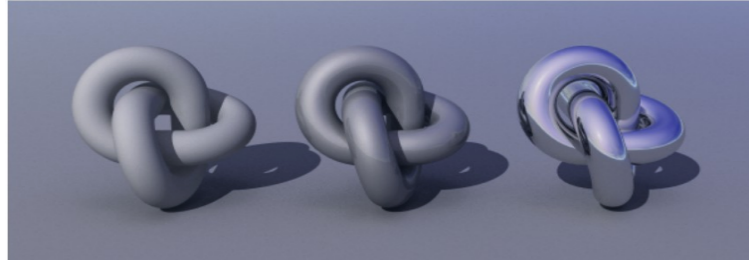


Figure 5: The left object is ideally diffuse. The right object is ideally specular. The middle object is somewhere in between.

Let  $\rho = (\rho_r, \rho_g, \rho_b)$  describe the fraction of the incoming light that gets reflected by the diffuse surface for each color component. Then the light that gets reflected is:

$$R = \rho * D = (\rho * P \max(\hat{r} \cdot \hat{n}, 0)) / 4\pi r^2 \quad (25)$$

where the operator  $.$  denotes element-wise multiplication between two vectors. If we use `glm::vec3` for 3D vectors we can compute this element-wise multiplication by just writing:

```
vec3 A(1,2,3);
vec3 B(4,5,6);
vec3 C = A*B; // C will be (4,10,18)
```

since GLM has overloaded the operator `*` for two `glm::vec3` to represent element-wise multiplication. Extend the `Draw`-function such that only the reflected light gets drawn. The color stored in each triangle is its  $\rho$ . This should give the result seen in Figure 6.

### Direct Shadows

So far a surface gets no direct illumination if it does not face the light source. However, we would also like to simulate shadows if another surface intersects the ray from the light source to the surface. To simulate this we can use the `ClosestIntersection` function that we use to cast rays.

### Task 6.5 Add surface light ray to `DirectLight` function

When illuminating a surface point we cast another ray from it to the light source. Then we check the distance to the closest intersecting surface. If that is closer than the light source the surface is in shadow and does not receive any direct illumination from the light source. Add this to your `DirectLight` function. If it works you should see shadows like in Figure 7.

### Indirect Illumination

Just implementing the direct illumination will make some surface points appear pitch black. It only simulates one bounce of light from the light source via a surface point to the camera. In the real world light keeps reflecting multiple times and therefore reaches all surfaces at some point. At each reflection some of the light is absorbed by the surface, as specified by its  $\rho$ . This indirect illumination results in the smooth shadows that we see in the real world.

It is very computationally expensive to simulate the multiple bounces of light required for accurate computation of indirect illumination. For this lab it is not necessary. Instead, we will use a very simple approximation to model the indirect illumination. We will just assume that the indirect illumination is the same for every

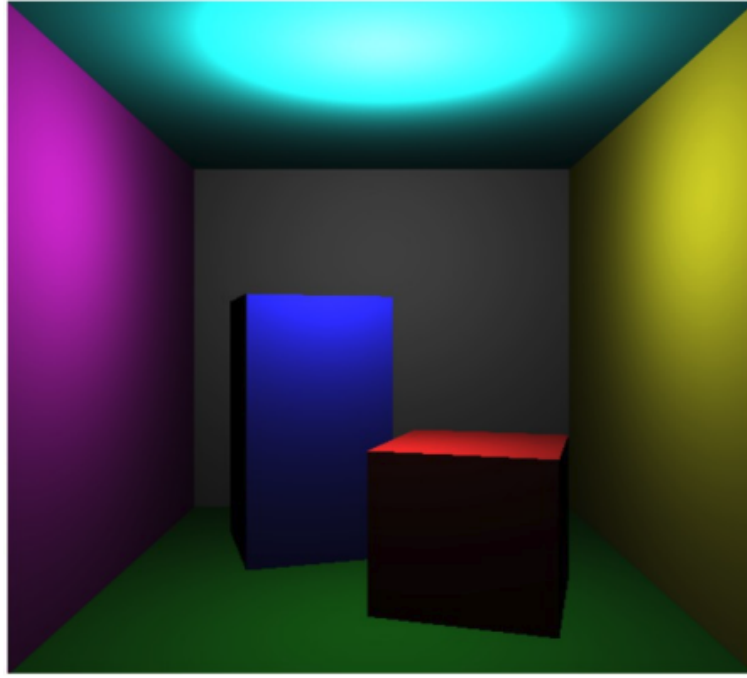


Figure 6: Direct illumination, without shadows.

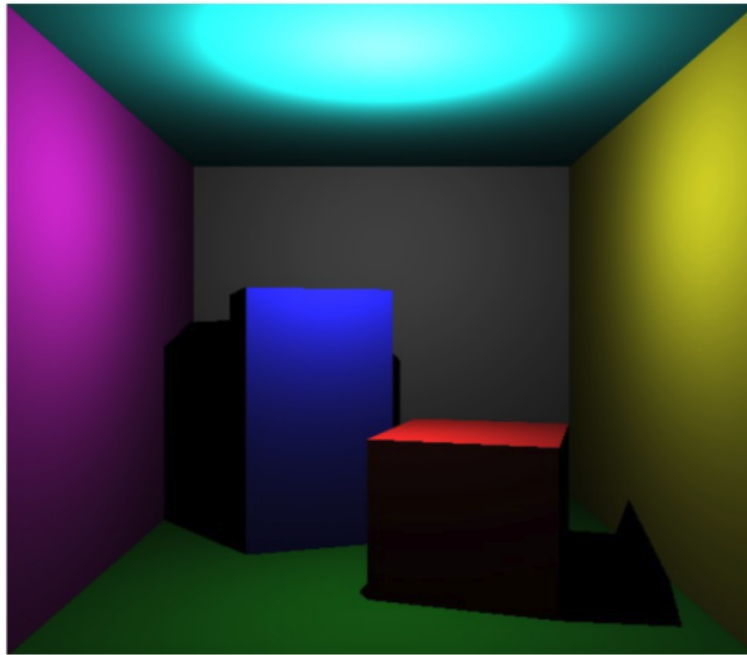


Figure 7: Direct illumination, with shadows.

surface of the scene. Let  $N$  be the power per surface area of this constant indirect illumination. Then the total incident illumination of a surface point is:

$$T = D + N \quad (26)$$

and the reflected light is:

$$R = \rho * T = \rho * (D + N) \quad (27)$$

### Task 6.6 Create indirect light

First create a new global variable to store the constant approximation of the indirect illumination:

```
vec3 indirectLight = 0.5f*vec3( 1, 1, 1 );
```

Then implement this illumination model, Equation 27, in your Draw function. You should not alter the function DirectLight as it should only return the direct illumination which it already does. The result is an image where no surface is completely black, like in Figure 8.

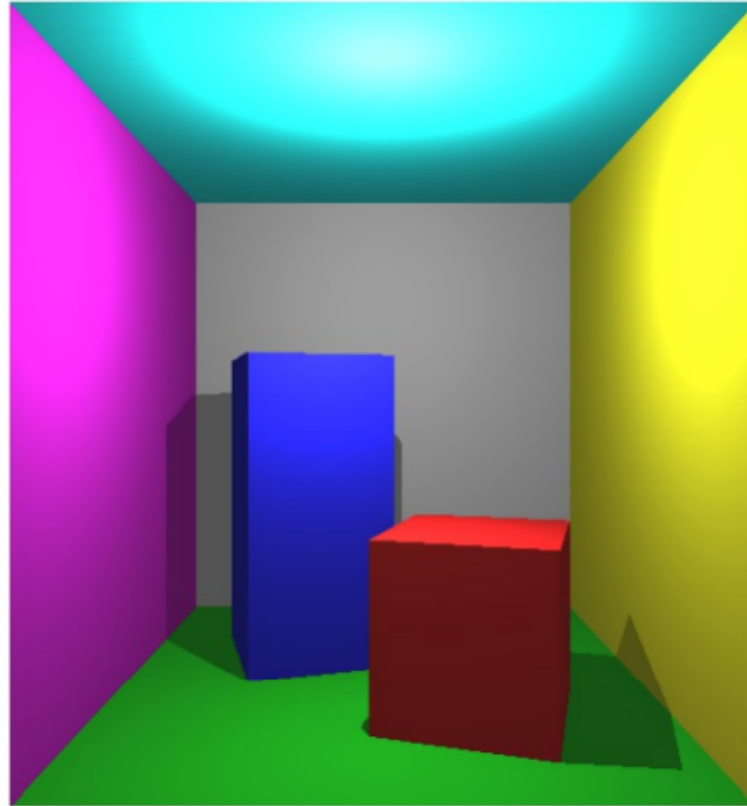


Figure 8: Direct illumination, with shadows.

You have now written a raytracer, that handles direct illumination and shadows and approximates indirect illumination with a constant term.

## 7 Extending to a Project

In the individual project, you have the opportunity to improve your raytracer. If you want you could then add things like:

- Soft edges (anti-aliasing) by sending multiple slightly perturbed rays for each pixel.
- Depth of field, by modeling the camera lens. Includes soft edges.
- Textures. Can be used for color/reflectance and/or normal maps, and/or parallax mapping.
- Specular materials. For example you could simulate water with normal maps.