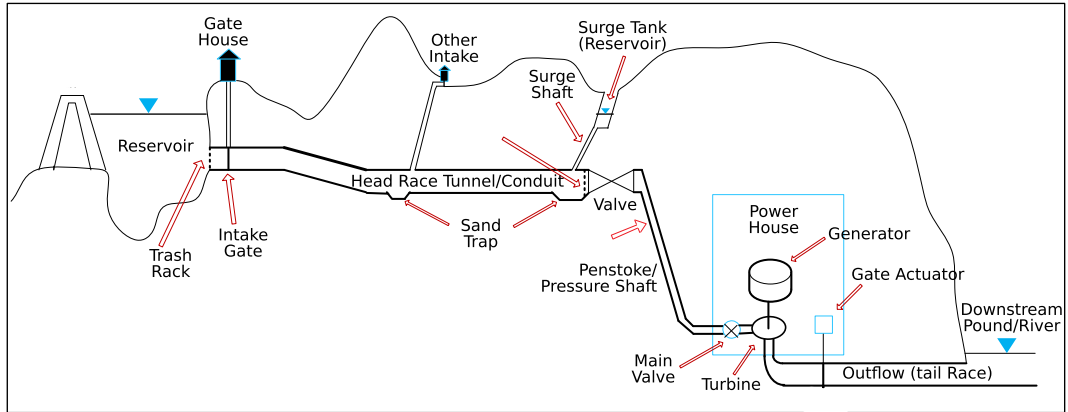


① Tegn HPS vann veien

# Overview

## Typical high-head hydro power system



## Gross quantities

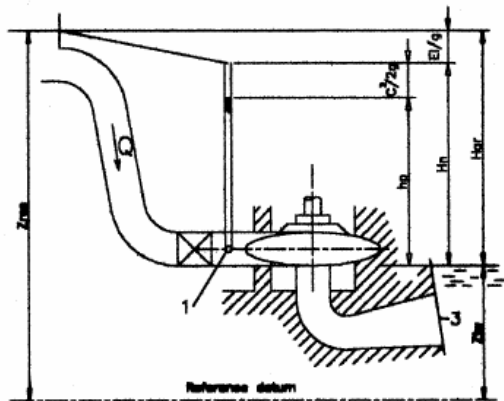


$$H_{gr} = Z_{res} - Z_{tw}$$

$$P_{gr} = \rho Q g H_{gr} \quad (1)$$

# General theory

## Net quantities



## Definition (Head and Energy)

$$\begin{aligned} H_n &= H_{gr} - \frac{E_L}{g} = H_{gr} - H_L \\ &= h_p + \frac{c^2}{2g} \\ E_n &= gH_n \end{aligned}$$

## Definition (Power)

$$P_n = \rho Q g H_n \quad (2)$$

L&S oppgave:

$$Q = 1 \text{ m}^3/\text{s} \quad H = 100 \text{ m}$$

Hva er Power?

# Turbine types

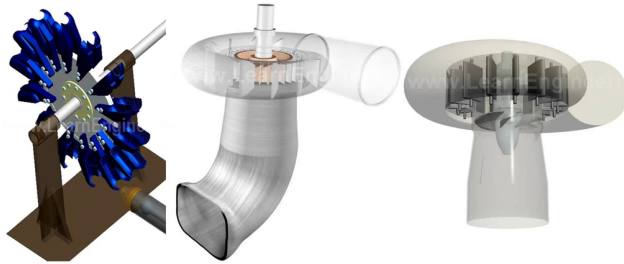


Figure: <https://youtu.be/koBLOKEZ3KU>

# Hydraulic efficiency

## Part 1

### Definition (Total available power)

$$P_n = \rho Q g H_n \quad (3)$$

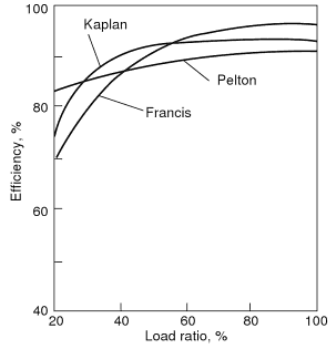
### Definition (Power of the runner)

$$P_R = \rho Q (u_1 c_{u1} - u_2 c_{u2}) \quad (4)$$

### Definition (Hydraulic efficiency)

$$\eta_h = \frac{P_R}{P_n} = \frac{1}{g H_n} (u_1 c_{u1} - u_2 c_{u2}) \quad (5)$$

# Turbine efficiencies



Variation of hydraulic efficiency for various types of turbine over a range of loading, at constant speed and constant head.

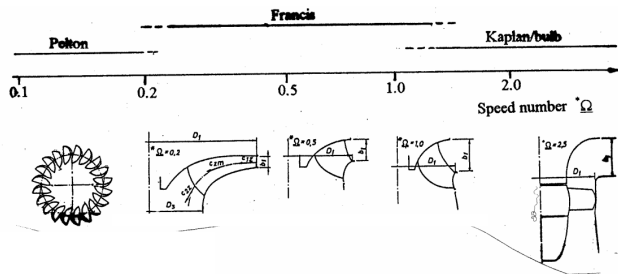


# Turbine Classification

## Speed Number and Specific Speed

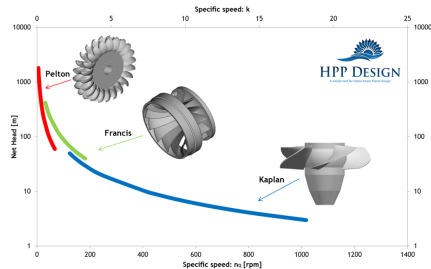
### Speed Number

$$\frac{n_q}{89} = \frac{\Omega}{\omega \sqrt{Q}} = \frac{\pi n \sqrt{Q}}{30 \sqrt[4]{(2gH_n)^3}}$$



### Specific Speed

$$n_q = \frac{n \sqrt{Q}}{\sqrt[4]{H_n^3}} \quad k = \frac{\omega \sqrt{Q}}{\sqrt[4]{gH_n^3}}$$



# Lecture Outline

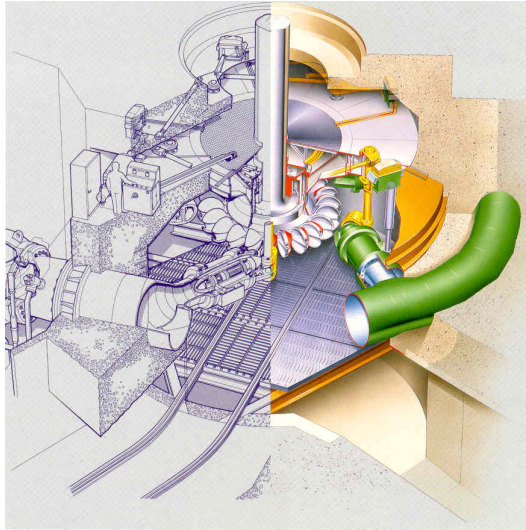
## Slides

- Design of a Pelton turbine
  - Calculation of the different parameters
  - Two example calculation

## Black board

- Introduction of a “Control Volume”
- In- and extensive properties
- General formulation of change of properties
- Lumped parameter model

# Design of Pelton turbines



# Design of Pelton turbines

Speed number range for an **ideal** Pelton turbine

## Minium speed number

For the diameter  $D = 10 \cdot d_s$  and one nozzle  $z = 1$ :

$$\underline{\Omega} = \frac{d_s}{D} \sqrt{\frac{\pi \cdot z}{4}} = \frac{1}{10} \sqrt{\frac{\pi \cdot 1}{4}} = 0.09$$

The minimum speed number for a Pelton turbine with one nozzle is:

$$\underline{\Omega}_{\min} = 0.09$$

## Maximum speed number

For the diameter  $D = 10 \cdot d_s$  and six nozzles  $z = 6$ :

$$\underline{\Omega} = \frac{d_s}{D} \sqrt{\frac{\pi \cdot z}{4}} = \frac{1}{10} \sqrt{\frac{\pi \cdot 6}{4}} = 0.22$$

The maximum speed number for a Pelton turbine with one nozzle is:

$$\underline{\Omega}_{\max} = 0.22$$

# Active power and frequency control

## Frequency response

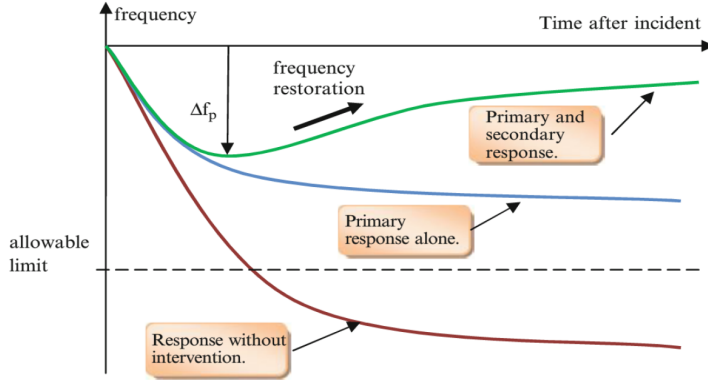


Figure: Planned frequency profile in response to large and sudden loss of generation

# Active power and frequency

## Primary Control

### Droop: slope of frequency-power characteristic

$$R_{gi} = -\frac{\Delta f / f_r}{\Delta P_{gi} / P_{gi,r}}$$

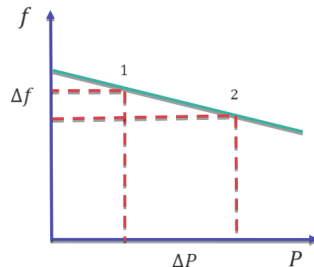
$R_{gi}$  droop (regulation) of generator  $i$  [pu]

$\Delta f$  change in system frequency [Hz]

$f_r$  nominal rated frequency [Hz]

$\Delta P_{gi}$  change in active power of generator  $i$  [MW]

$P_{gi,r}$  nominal rated power of generator  $i$  [MW]

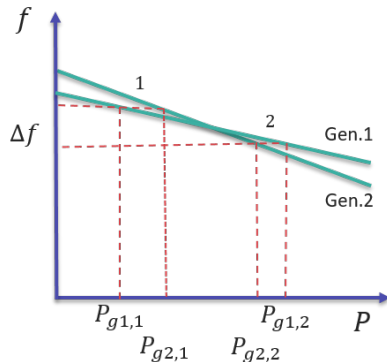


# Active power and frequency: Primary Control

Two generators in parallel

## Example

- $P_{g1r} = P_{g2r} = 200 \text{ MW}$ ,  $R_{g1} = 0.02$ ,  $R_{g2} = 0.06$
- Balance (1):  $f_r = 50 \text{ Hz}$
- Sudden increase in of load  $\Delta P \rightarrow$  frequency drop  $\Delta f$
- Power distributed over two generators:  
$$\frac{\Delta P_{g1}}{\Delta P_{g2}} = \frac{R_{g2}}{R_{g1}} = 3$$
- Power increase in ratio 3:1 (75%  $G_1$  and 25%  $G_2$ )



# Active power and frequency: Primary Control

Many generators in parallel

## Network Power-Frequency Characteristic

$$\lambda = -\frac{\Delta P[\text{MW}]}{\Delta f[\text{Hz}]} \quad (\text{bias factor})$$

$$\Delta P = \sum_i \Delta P_{g_i} = \sum_i \left( -\frac{1}{R_{g_i}} \cdot \frac{P_{g_{i,r}}}{f_r} \cdot \Delta f \right)$$
$$\Rightarrow \lambda = -\frac{\Delta P}{\Delta f} = \sum_i \left( \frac{1}{R_{g_i}} \cdot \frac{P_{g_{i,r}}}{f_r} \right)$$



# Active power and frequency: Primary Control

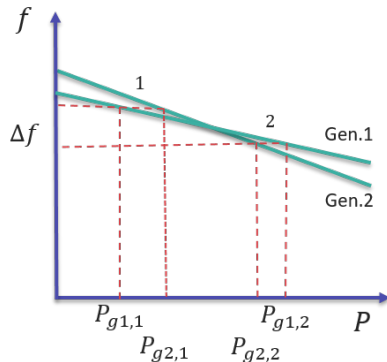
Many generators in parallel

## Example

- $P_{g1r} = P_{g2r} = 200 \text{ MW}$ ,  $R_{g1} = 0.02$ ,  $R_{g2} = 0.06$

$$\Rightarrow \lambda = \frac{1}{f_r} \sum_i \frac{P_{gi,r}}{R_{gi}} = \frac{1}{50 \text{ Hz}} \left( \frac{200 \text{ MW}}{0.02} + \frac{200 \text{ MW}}{0.06} \right)$$
$$= \frac{13,333}{50 \text{ Hz}} = 266.6 \frac{\text{MW}}{\text{Hz}}$$

$$\Delta P = 133.3 \text{ MW} \Rightarrow \Delta f = -0.5 \text{ Hz}$$



# Active power and frequency: Primary Control

## Self-regulation

### Definition of Self-regulation

Motors cause the system load to be slightly frequency dependent.

$$\lambda = -\frac{\Delta P}{\Delta f} = \sum_i \left( \frac{1}{R_{gi}} \cdot \frac{P_{gi,r}}{f_r} \right) + \frac{\mu}{100} \cdot P$$
$$\mu = -\frac{\Delta P'/P}{\Delta f} \cdot 100 \%$$

$\mu$  the self regulation effect of the load  $P$  [%/Hz]

$\Delta P'$  the change in the power consumption of the loads because of the change in frequency  $\Delta f$  [MW]

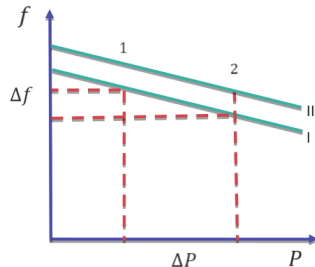
$P$  the value of the original load of the system plus the change  $\Delta P$  at the original system frequency [Hz]

# Active power and frequency: Secondary Control

## Load Frequency Control (LFC)

### Principle

1. Droop regulation – from point 1 to 2  
→ Restore balance of active power
2. LFC – from characteristic I to II  
→ Increase frequency (power remains constant)



# Active power and frequency: Tertiary Control

## Area Control Error (ACE)

### Definition

$$ACE_i = (P_{a_i} - P_{s_i}) + \lambda_i(f_a - f_s) = \Delta P_i + \lambda_i \Delta f$$

$ACE_i$  Area Control Error of control area  $i$  [MW],

$P_{a_i}$  actual power export of control area  $i$  [MW],

$P_{s_i}$  scheduled power export of control area  $i$  [MW],

$\lambda_i$  network  $P/f$  characteristic of control area  $i$  [MW/Hz],

$f_a$  actual frequency [Hz],

$f_s$  scheduled frequency [Hz].

ACE should be zero for scheduled power exchange at nominal frequency.

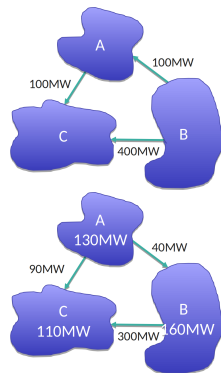
# Active power and frequency: Tertiary Control

## Area control

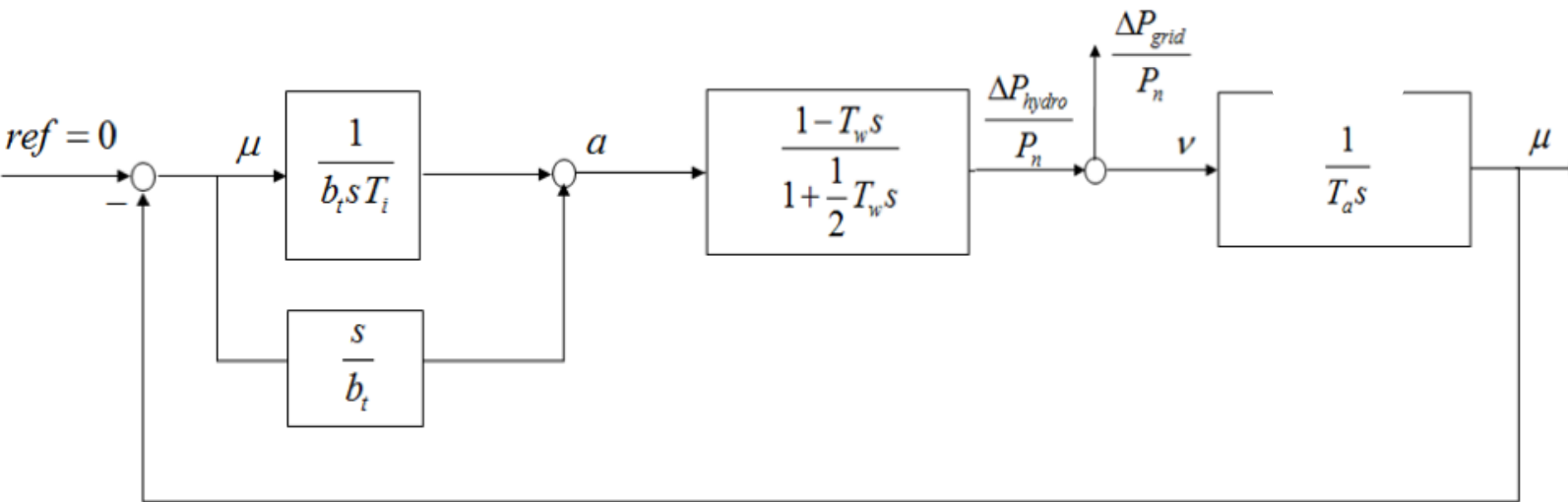
### Example

Example for three areas A, B, C:

- Scheduled  $B \rightarrow C = 400 \text{ MW}$ ,  $B \rightarrow A = A \rightarrow C = 100 \text{ MW}$
  - $\lambda_A = 13 \text{ GW/Hz}$ ,  $\lambda_B = 16 \text{ GW/Hz}$ ,  $\lambda_C = 11 \text{ GW/Hz}$
  - System  $\lambda = 40 \text{ GW/Hz}$
  - Sudden drop in frequency  $\Delta f = -0.01 \text{ Hz}$
- 
- Generation in area B should be increased by 400 MW



The transfer function of the hydro power system can be represented by:



For the determining the time constants of the transfer-function based model we need to determine the following two time constants:

- $T_a = \frac{J \cdot \omega_n^2}{P_n}$
- $T_w = \frac{Q_n}{g \cdot H_n} \left( \frac{L}{A} \right)$

We do not worry about the controller parameters since we are going to use a standard PID model from the standard library.