Assignment 3

torsdag 30. september 2021 11:55

1.

$$\left.\begin{array}{l} \text{Article 1} : x_1 \\ \text{Article 2} : x_2 \\ \text{Article 3} : x_3 \end{array}\right\} \text{Integers}$$

$$\begin{aligned} \text{Milling machine} &= 8 \cdot x_1 + 2x_2 + 3x_3 \\ \text{Lathe} &= 4 \cdot x_1 + 3 \cdot x_2 \\ \text{Grinder} &= 2 \cdot x_1 + x_3 \end{aligned}$$

Constraints for available machine hours:

- Milling machine ≤ 200
- Lathe ≤ 100
- grinder ≤ 50
- $x_3 \leq 20$

Objective function: Maximize profit

$$\text{Profit} = 200 \cdot x_1 + 60 \cdot x_2 + 80 \cdot x_3$$

Results from Python code:

```
--------- APM Model Size ------------
Each time step contains
  Objects      :            0
  Constants    :            0
  Variables    :            7
  Intermediates:            0
  Connections  :            0
  Equations    :            4
  Residuals    :            4

Number of state variables:              6
Number of total equations: -            3
Number of slack variables: -            3
----------------------------------------
Degrees of freedom       :              0

----------------------------------------------
Steady State Optimization with APOPT Solver
----------------------------------------------

show more (open the raw output data in a text editor) ...

Solution time  :   1.469999999972060E-002 sec
Objective      :   -5240.00000000000
Successful solution
---------------------------------------------------
```

```
x1, x2, x3
```

[5] ✓ 0.2s

```
... ([14.0], [14.0], [20.0])
```

$$\boxed{\begin{array}{l} \text{Profit} = 5240 \\ \text{Articles}: \\ \cdot \text{Article 1} = 14 \\ \cdot \text{Article 2} = 14 \\ \cdot \text{Article 3} = 20 \end{array}}$$

2.

A)

Variables:
- Ration of milk $= x_1$
- Ration of beef $= x_2$
- Ration of egg $= x_3$

$\mathbb{R}$

Objective: Minimize cost

$$\text{Cost} = 1 \cdot x_1 + 2 \cdot x_2 + 0{,}5 \cdot x_3$$

Equations:

$$A = x_1 + x_2 + 10 \cdot x_3$$

$$C = 100 x_1 + 10 x_2 + 10 x_3$$

$$D = 10 \cdot x_1 + 100 x_2 + 10 \cdot x_3$$

Constraints:

$A \geq 1$ mg

$C \geq 50$ mg

$D \geq 10$ mg

Results from Python Code

```
--------- APM Model Size ------------
Each time step contains
  Objects      :            0
  Constants    :            0
  Variables    :            7
  Intermediates:            0
  Connections  :            0
  Equations    :            4
  Residuals    :            4

Number of state variables:              6
Number of total equations: -            3
Number of slack variables: -            3
---------------------------------------
Degrees of freedom       :              0

**********************************************
Steady State Optimization with Interior Point Solver
**********************************************

show more (open the raw output data in a text editor) ...

Solution time  :   7.899999996880069E-003 sec
Objective      :   0.722223927269482
Successful solution
---------------------------------------------------
```

```
x1, x2, x3
[2] ✓ 0.4s
... ([0.44444519953], [5.4144125028e-07], [0.55555528972])
```

- Cost $= 0{,}72$
- Rations:
  - Milk $= 0{,}44$
  - Beef ≈ 0
  - Egg $= 0{,}556$

```
x1, x2, x3
[2] ✓ 0.4s
... ([0.44444519953], [5.4144125028e-07], [0.55555528972])
```

B) • Same setup, variables are integers.

Result from Python code

```
--------- APM Model Size ------------
Each time step contains
  Objects      :            0
  Constants    :            0
  Variables    :            7
  Intermediates:            0
  Connections  :            0
  Equations    :            4
  Residuals    :            4

Number of state variables:              6
Number of total equations: -            3
Number of slack variables: -            3
---------------------------------------
Degrees of freedom       :              0

----------------------------------------------
Steady State Optimization with APOPT Solver
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show more (open the raw output data in a text editor) ...

Solution time  :   1.660000000265427E-002 sec
Objective      :    1.00000000000000
Successful solution
---------------------------------------------------

x1, x2, x3
✓ 0.6s
([1.0], [0.0], [0.0])
```

Cost = 1

Rations:

- Milk = 1
- Beef = 0
- Eggs = 0

3.

Objective: Maximize profit; Oil income − Constituent cost

- Constituent cost = $5{,}5 \cdot A + 4{,}5 \cdot B + 3{,}5 \cdot C - (3{,}5 \cdot C_1 + 6 \cdot C_2 + 4 \cdot C_3 + 5 \cdot C_4)$
- Oil income = $5.5 \cdot A + 4{,}5 \cdot B + 3{,}5 \cdot C$

Variables: $C_{n,i}$ $\quad n$ = Constituent $[1, 4] \cdot \mathbb{R}$, $\quad i$ = Oil grade $[A, B]$

$A = C_{1A} + C_{2A} + C_{3A} + C_{4A}$

$B = C_{1B} + C_{2B} + C_{3B} + C_{4B}$

$C = C_{1C} + C_{2C} + C_{3C} + C_{4C}$

Constraints:

- $C_{1A} + C_{1B} + C_{1C} \leq 3000$
- $C \quad + C \quad + C \quad < 2000$

Constraints:

- $C_{1A} + C_{1B} + C_{1C} \leq 5000$
- $C_{2A} + C_{2B} + C_{2C} \leq 2000$
- $C_{3A} + C_{3B} + C_{3C} \leq 4000$
- $C_{4A} + C_{4B} + C_{4C} \leq 1000$

$C = C_{1C} + C_{2C} + C_{3C} + C_{4C}$

AC%

- $\dfrac{C_{1A}}{(C_{1A} + C_{2A} + C_{3A} + C_{4A})} \leq 0{,}3$
- $\dfrac{C_{2A}}{(C_{1A} + C_{2A} + C_{3A} + C_{4A})} \geq 0{,}4$
- $\dfrac{C_{3A}}{(C_{1A} + C_{2A} + C_{3A} + C_{4A})} \leq 0{,}5$
- $\dfrac{C_{1B}}{(C_{1B} + C_{2B} + C_{3B} + C_{4B})} \leq 0{,}5$
- $\dfrac{C_{2B}}{(C_{1B} + C_{2B} + C_{3B} + C_{4B})} \geq 0{,}1$
- $\dfrac{C_{1C}}{(C_{1C} + C_{2C} + C_{3C} + C_{4C})} \leq 0{,}7$

Result from Python

```
from gekko import GEKKO ...

apm 109.247.50.146_gk_model3 <br><pre> ----------------------------------------------------------
 APMonitor, Version 1.0.1
 APMonitor Optimization Suite
 ----------------------------------------------------------


 --------- APM Model Size ------------
 Each time step contains
   Objects      :            0
   Constants    :            0
   Variables    :           23
   Intermediates:            0
   Connections  :            0
   Equations    :           11
   Residuals    :           11

 Number of state variables:             22
 Number of total equations: -           10
 Number of slack variables: -           10
 ---------------------------------------
 Degrees of freedom       :              2

 ****************************************************
 Steady State Optimization with Interior Point Solver
 ****************************************************

show more (open the raw output data in a text editor) ...

 Solution time  :   3.220000000146683E-002 sec
 Objective      :   -7166.66696673977
 Successful solution
 ---------------------------------------------------


   var
 ✓ 0.3s
[[[863.00136429], [1466.66675], [1336.9988524], [0.0]],
 [[2136.9986357], [533.33325], [2663.0011476], [0.0]],
 [[0.0], [0.0], [0.0], [0.0]]]
```

- Profit = 7166
- A = 3665
- B = 5332,3
- C = 0

4. Objective: Profit = $800 \cdot A + 600 \cdot B + 100 \cdot C$

Variables: Crop A for the 3 farms

$$\left.\begin{aligned} A &= A_1 + A_2 + A_3 \\ B &= B_1 + B_2 + B_3 \\ C &= C_1 + C_2 + C_3 \end{aligned}\right\} \mathbb{R}$$

Constraints:

Crop Area:
- $A_1 \leq 350$, $A_2 \leq 350$, $A_3 \leq 350$
- $B_1 \leq 400$, $B_2 \leq 400$, $B_3 \leq 400$
- $C_1 \leq 150$, $C_2 \leq 150$, $C_3 \leq 150$

Water Consumption:
- $1{,}6 \cdot A_1 + 1{,}3 \cdot B_1 + C_1 \leq 250$
- $1{,}6 \cdot A_2 + 1{,}3 B_2 + C_2 \leq 333$
- $1{,}6 \cdot A_3 + 1{,}3 B_3 + C_3 \leq 150$

Area:

$$A_1 + B_1 + C_1 \leq 200$$
$$A_2 + B_2 + C_2 \leq 150$$
$$A_3 + B_3 + C_3 \leq 300$$

Used Area:

$$\frac{A_{used}}{A_{tot}} = \frac{B_{used}}{B_{tot}} = \frac{C_{used}}{C_{tot}}$$

Python script

```
--------- APM Model Size ------------
Each time step contains
  Objects      :            0
  Constants    :            0
  Variables    :           25
  Intermediates:            0
  Connections  :            0
  Equations    :           22
  Residuals    :           22

Number of state variables:             24
Number of total equations: -           21
Number of slack variables: -           15
---------------------------------------
Degrees of freedom       :           -12

* Warning: DOF <= 0
----------------------------------------------
Steady State Optimization with APOPT Solver

show more (open the raw output data in a text editor) ...

Solution time   :   1.510000000052969E-002 sec
```

Profit = 215384

Crop 1 = 153

Crop 2 = 115,4

Crop 3 = 80

```
--------- APM Model Size ------------
Each time step contains
  Objects      :            0
  Constants    :            0
  Variables    :           25
  Intermediates:            0
  Connections  :            0
  Equations    :           22
  Residuals    :           22

Number of state variables:             24
Number of total equations: -           21
Number of slack variables: -           15
---------------------------------------
Degrees of freedom       :            -12

* Warning: DOF <= 0
--------------------------------------------
Steady State Optimization with APOPT Solver

show more (open the raw output data in a text editor) ...

Solution time  :   1.510000000052969E-002 sec
Objective      :   -215384.615384615
Successful solution
---------------------------------------------------
```

```
var
✓ 0.2s
[[[27.59271657], [111.48123353], [60.926049903]],
 [[126.25343728], [3.9033818579], [19.843180866]],
 [[0.0], [0.0], [150.0]]]
```

Profit = 215384

Crop1 = 153

Crop2 = 115.4

Crop3 = 80