Problem 1.- Solution

Variables:

$AM_1, AM_2, AM_3, AM_4, AM_5$: Quantities of wood send from place A to $M_1, M_2 \ldots M_5$

$BM_1, BM_2, BM_3, BM_4, BM_5$: Quantities of wood send from place B to $M_1, M_2 \ldots M_5$

$CM_1, CM_2, CM_3, CM_4, CM_5$: Quantities of wood send from place C to $M_1, M_2, \ldots M_5$

$$\begin{aligned}\text{Max Profit} = 1000 &(AM_1 + AM_2 + AM_3 + AM_4 + AM_5 + \\ &BM_1 + BM_2 + BM_3 + BM_4 + BM_5 + \\ &CM_1 + CM_2 + CM_3 + CM_4 + CM_5) \\ &- 51AM_1 - 62AM_2 - 35AM_3 - 45AM_4 - 56AM_5 \\ &- 59BM_1 - 68BM_2 - 50BM_3 - 39BM_4 - 46BM_5 \\ &- 49CM_1 - 56CM_2 - 53CM_3 - 51CM_4 - 37CM_5\end{aligned}$$

Subject to:

$$5000 \leq AM_1 + BM_1 + CM_1 \leq 27500$$

$$5000 \leq AM_2 + BM_2 + CM_2 \leq 30000$$

$$5000 \leq AM_3 + BM_3 + CM_3 \leq 22500$$

$$5000 \leq AM_4 + BM_4 + CM_4 \leq 25000$$

$$5000 \leq AM_5 + BM_5 + CM_5 \leq 20000$$

$$AM_1 + AM_2 + AM_3 + AM_4 + AM_5 \leq 30000$$

$$BM_1 + BM_2 + BM_3 + BM_4 + BM_5 \leq 50000$$

$$CM_1 + CM_2 + CM_3 + CM_4 + CM_5 \leq 30000$$

$AM_1 \geq 0$, $BM_1 \geq 0$, $CM_1 \geq 0$

$AM_2 \geq 0$, $BM_2 \geq 0$, $CM_2 \geq 0$.

$AM_3 \geq 0$, $BM_3 \geq 0$, $CM_3 \geq 0$

$AM_4 \geq 0$, $BM_4 \geq 0$, $CM_4 \geq 0$

$AM_5 \geq 0$, $BM_5 \geq 0$, $CM_5 \geq 0$

Problem 2.- Solution.

Pump type A:

$$\text{Net Present Value} = -1000 - \frac{1000}{(1.10)^2} - \frac{1000}{(1.10)^4} - 200 - \frac{200}{(1.10)^2} - \frac{200}{(1.10)^4}$$

$$NPV = -3011.4$$

Pump type B:

$$\text{Net Present value} = -1500 - \frac{1500}{(1.10)^3} - 250 - \frac{250}{(1.10)^3}$$

$$NPV = 3064.8$$

Pump type A is a slightly better option.

Problem 3:- Solution:

a) Data Reconciliation Problem:

$$\text{Min } J = \frac{(30.5 - \hat{F}_A)^2}{1} + \frac{(90.2 - \hat{F}_B)^2}{1} + \frac{(150.7 - \hat{F}_C)^2}{4}$$

Subject to

$$\hat{F}_A + \hat{F}_B - \hat{F}_C = 0$$

b) $\hat{F}_C = \hat{F}_A + \hat{F}_B$

$$\text{Min } J = (30.5 - \hat{F}_A)^2 + (90.2 - \hat{F}_B)^2 + \frac{(150.7 - \hat{F}_A - \hat{F}_B)^2}{4}$$

$$\frac{\partial J}{\partial \hat{F}_A} = -2(30.5 - \hat{F}_A) - \frac{2(150.7 - \hat{F}_A - \hat{F}_B)}{4} = 0$$

$$\frac{\partial J}{\partial \hat{F}_B} = -2(90.2 - \hat{F}_B) - \frac{2(150.7 - \hat{F}_A - \hat{F}_B)}{4} = 0$$

① $30.5 - \hat{F}_A + \frac{150.7 - \hat{F}_A - \hat{F}_B}{4} = 0$

② $90.2 - \hat{F}_B + \frac{150.7 - \hat{F}_A - \hat{F}_B}{4} = 0$

$$\Rightarrow 4(30.5) - 4\hat{F}_A + 150.7 - \hat{F}_A - \hat{F}_B = 0$$

$$4(90.2) - 4\hat{F}_B + 150.7 - \hat{F}_A - \hat{F}_B = 0$$

$$\Rightarrow 5\hat{F}_A + \hat{F}_B = 272.7 \quad ①$$

$$\hat{F}_A + 5\hat{F}_B = 511.5 \quad ②$$

Solving ① and ②:

$\hat{F}_A = 35.5$ $\quad \hat{F}_C = 130.7$

$\hat{F}_B = 95.2$

Problem 4 : Solution

a)

$$\min_{b1,b2,b3,b4} \chi^2 = \frac{(0.34 - 1/(b_1 - e^{-(0 b_2 + 0.001 b_3) b_4})^2}{0.0004} +$$

$$\frac{(0.44 - 1/(b_1 - e^{-(0.005 b_2 + 0.007 b_3) b_4})^2}{0.0004} +$$

$$\frac{(0.50 - 1/(b_1 - e^{-(0.008 b_2 + 0.008 b_3) b_4})^2}{0.0004} +$$

$$\frac{(0.59 - 1/(b_1 - e^{-(0.012 b_2 + 0.011 b_3) b_4})^2}{0.0002} +$$

$$\frac{(0.72 - 1/(b_1 - e^{-(0.015 b_2 + 0.017 b_3) b_4})^2}{0.0002} +$$

$$\frac{(0.93 - 1/(b_1 - e^{-(0.02 b_2 - 0.021 b_3) b_4})^2}{0.0002} +$$

$$\frac{(0.99 - 1/(b_1 - e^{-(0.03 b_2 - 0.028 b_3) b_4})^2}{0.0002}.$$

b) Declare a function in matlab with the expression above:

```
function LS = model(B)
LS = (0.34 - 1/(B(1) - exp(-(0*B(2) + 0.001*B(3))))^B(4)
/0.0004 --- etc.
end.
```

call fminunc with different starting values for B.-

```
B0 = [1,1,1,1];
X = fminunc(model, B0)
```

It is also possible to use fmincon, lsqcurvefit, etc.

Problem 5. Solution

a) Non dominated set:
A, D, E, G, H

b) Since the set contains fewer options, it is easier to make a solution. Besides, the student can be sure that these options "cover" all the options eliminated. The final desition should be made according with the criteria or criterias that the student value the most.