



University of  
South-Eastern Norway

FM3217 – Object-oriented Modelling of  
Hydro Power Systems  
L2: Energy Conversion

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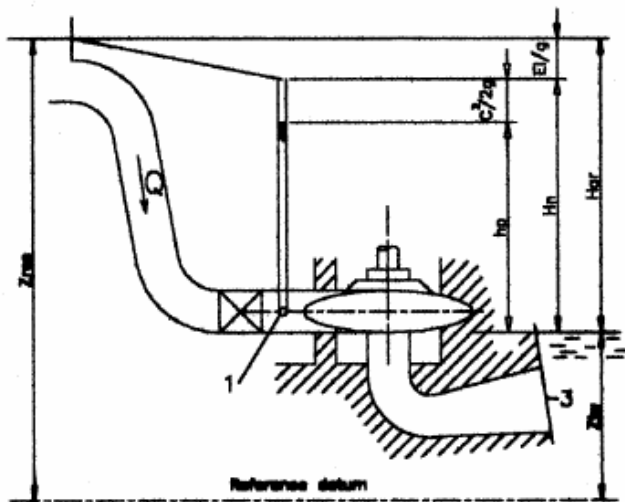
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## Lecture outline

- General theory
- Turbine types
- Energy equations for the different turbine types
- Operation areas

## General theory

### Gross quantities



### Definition (Head and Energy)

$$H_{gr} = z_{res} - z_{tw}$$

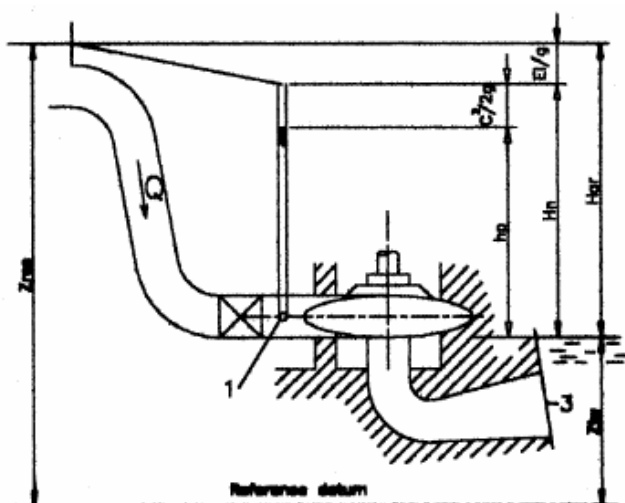
$$E_{gr} = gH_{gr}$$

### Definition (Power)

$$P_{gr} = \rho Q g H_{gr} \quad (1)$$

## General theory

### Net quantities



### Definition (Head and Energy)

$$H_n = H_{gr} - \frac{E_L}{g} = H_{gr} - H_L$$

$$= h_p + \frac{c^2}{2g}$$

$$E_n = gH_n$$

### Definition (Power)

$$P_n = \rho Q g H_n \quad (2)$$

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# General theory

Turbomachinery – video

## TURBOMACHINERY

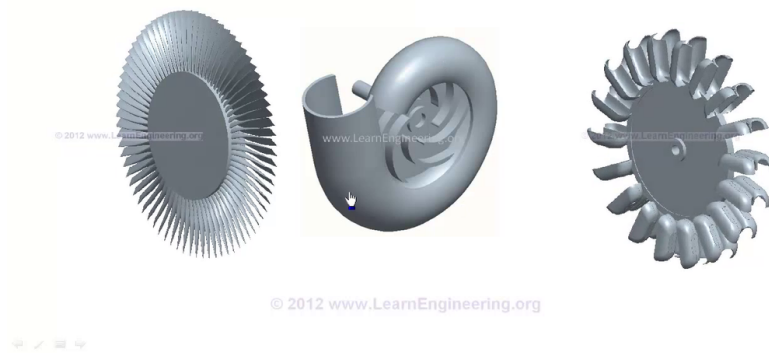


Figure: <https://youtu.be/473XQrJjDZE>

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# Turbine types

Based on the conversion principle

1. Impulse type (partial turbine):
  - Pelton turbines
  - Crossflow turbines
2. Reaction type (full turbine)
  - Francis turbines
  - Kaplan turbines
  - Bulb turbines

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# Turbine types

Comparison of Pelton, Francis and Kaplan – video

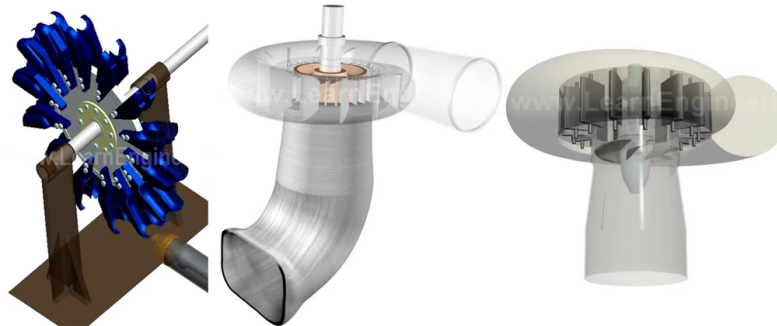
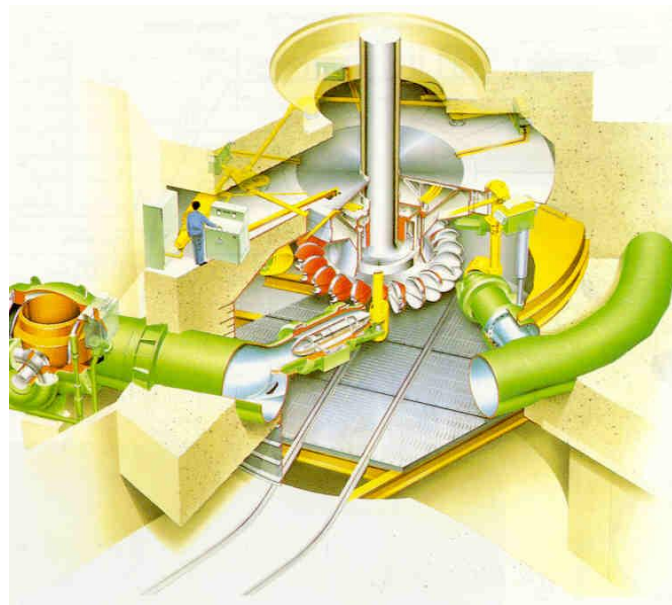


Figure: <https://youtu.be/k0BLOKEZ3KU>

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## Impulse turbines

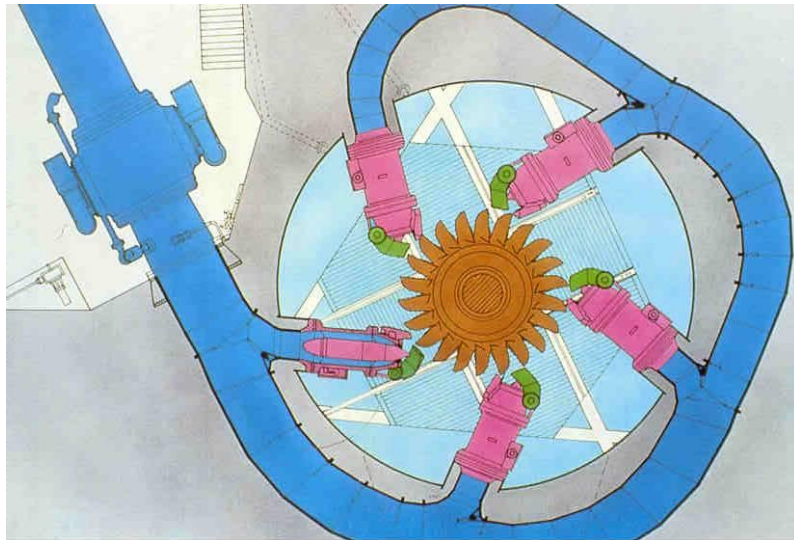
Pelton – physical installation



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# Impulse turbines

Pelton – physical installation (top view)



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# Impulse turbines

Pelton – video

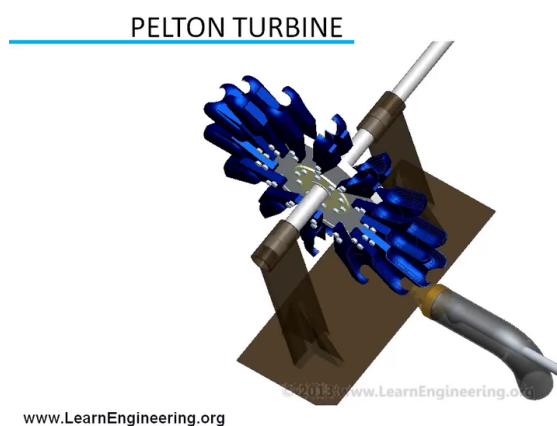
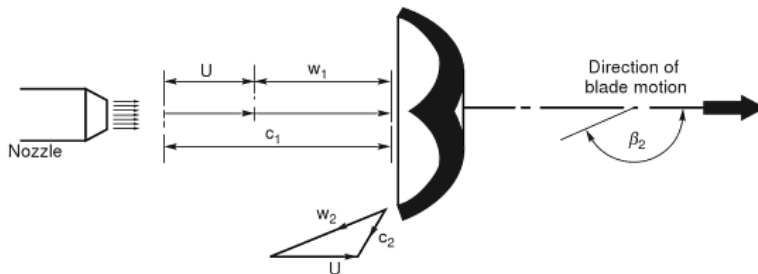


Figure: <https://youtu.be/rf9meqw2SQA>

# Impulse turbines

Pelton – flow diagram



## Definition (Runner Power)

$$P_R = \rho Q u (c_{u1} - c_{u2})$$

General:

$$P_R = \rho Q (u_1 c_{u1} - u_2 c_{u2})$$

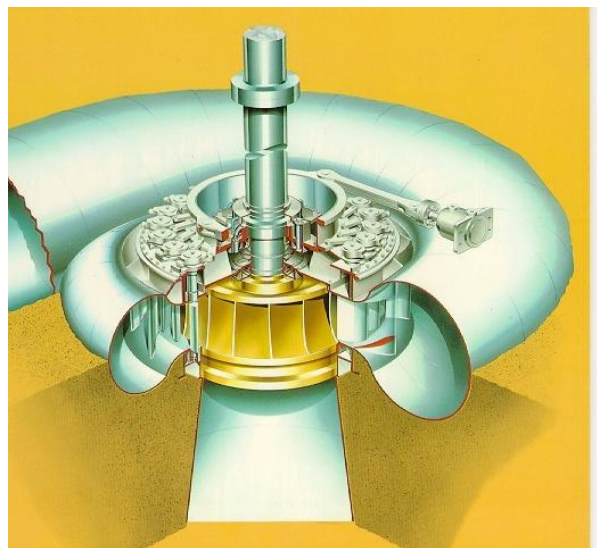
for  $u_2 = u_1$  we can write:

$$P_R = \rho Q u_1 (c_{u1} - c_{u2})$$

$$\Delta W = u w_1 (1 - k \cos \beta_2) = u (c_1 - u) (1 - k \cos \beta_2)$$

# Reaction turbines

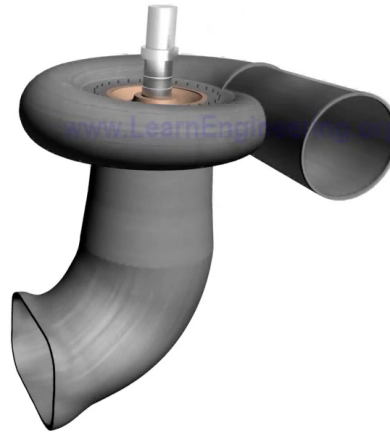
Francis – physical installation



# Reaction turbines

Francis turbine – video

## FRANCIS TURBINE

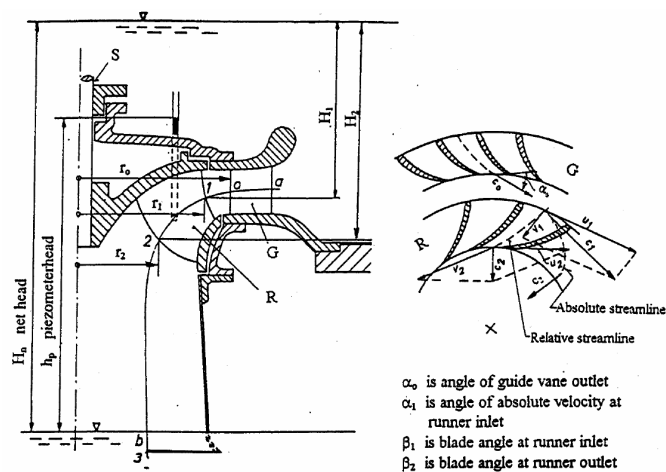


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Figure: <https://youtu.be/3BCiFeykRzo>

# Reaction turbines

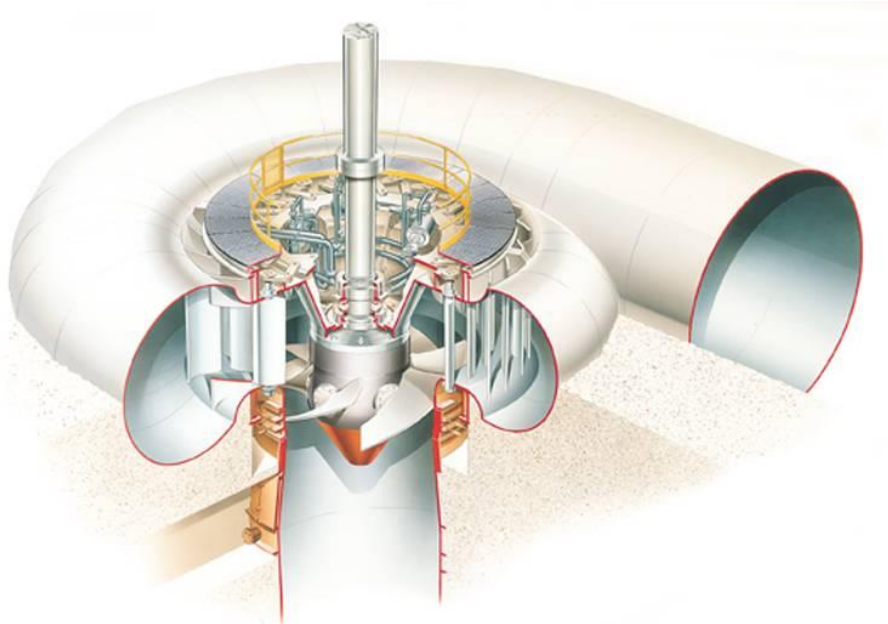
Francis – flow diagram



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# Reaction turbines

Kaplan – physical installation



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# Reaction turbines

Kaplan – video

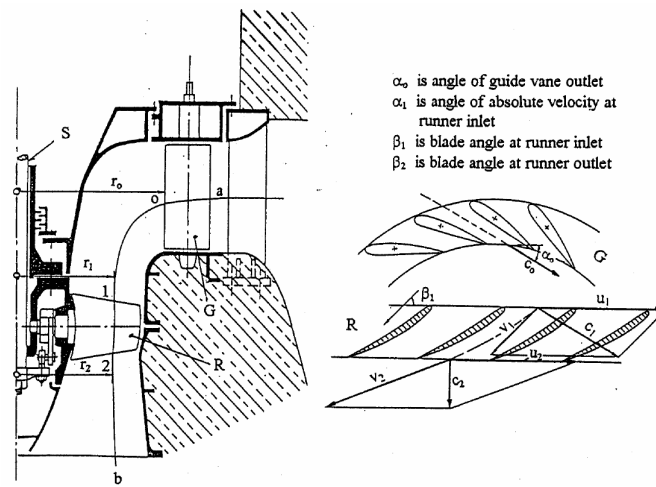


Figure: <https://youtu.be/0p03UTgpnDU>



# Reaction turbines

## Kaplan – flow diagram



# Reaction turbines

## Kaplan – power calculation

### Definition (Rotor Power)

$$P_R = \rho Q(u_1 c_{u1} - u_2 c_{u2})$$

for  $u_2 = u_1$  we can write:

$$P_R = \rho Q u_1 (c_{u1} - c_{u2})$$

# Hydraulic efficiency

## Part 1

### Definition (Total available power)

$$P_n = \rho Q g H_n \quad (3)$$

### Definition (Power of the runner)

$$P_R = \rho Q (u_1 c_{u1} - u_2 c_{u2}) \quad (4)$$

### Definition (Hydraulic efficiency)

$$\eta_h = \frac{P_R}{P_n} = \frac{1}{g H_n} (u_1 c_{u1} - u_2 c_{u2}) \quad (5)$$

# Hydraulic efficiency

## Part 2

### Definition (Total sum of losses)

$$h_L = \frac{1}{2} (\zeta_1 c_1^2 + \zeta_2 v_2^2 + (1 + \zeta_3) c_3^2 + E_I^2) \quad (6)$$

Rearranging (5) gives the

### Definition (Main turbine equation)

$$\eta_h H_n = \frac{1}{g} (u_1 c_{u1} - u_2 c_{u2}) \quad (7)$$

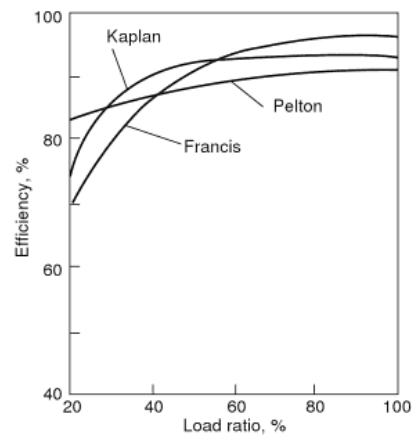
Substituting (6) into (7) then gives:

### Definition (Total sum of energy transfer)

$$H_n = (1 + \zeta_1) \frac{c_1^2}{2g} - \frac{c_2^2}{2g} + (1 + \zeta_2) \frac{v_2^2}{2g} - \frac{v_1^2}{2g} + \frac{u_1^2}{2g} - \frac{u_2^2}{2g} + (1 + \zeta_3) \frac{c_3^2}{2g} + \frac{E_I^2}{2g} \quad (8)$$

# Turbine efficiencies

Francis, Kaplan, Pelton



Variation of hydraulic efficiency for various types of turbine over a range of loading, at constant speed and constant head.