Problem 1 -

a)

$$\min_{x_1, x_2, \dots x_6} f(x) = x_1 + 5x_2 + 2x_3 + 1.5x_4 + 2x_5 + 1.5x_6$$

Subject to:

$$110x_1 + 205x_2 + 160x_3 + 160x_4 + 420x_5 + 260x_6 \geq 2000$$

$$4x_1 + 32x_2 + 13x_3 + 8x_4 + 4x_5 + 14x_6 \geq 55$$

$$2x_1 + 12x_2 + 54x_3 + 285x_4 + 22x_5 + 80x_6 \geq 800$$

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0,\ x_6 \geq 0.$$

b) Excel:

x_1 =	1	F(x) =	Expression
x_2 =	1	Const 1 =	Expression
x_3 =	1	Const 2 =	Expression
x_4 =	1	Const 3 =	Expression
x_5 =	1		
x_6 =	1		

Call Solver:
- Define goal cell
- Choose minimize
- Define variables cells
- Define constraints
- Choose positive for variables

→ Solve.

Matlab:

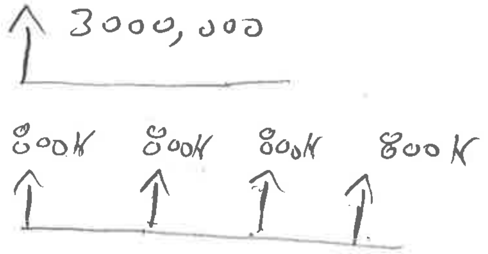
```
f = [1 5 2 1.5 2 1.5];
A = [-110 -205 -160 -160 -420 -260
     -4   -32  -13  -8   -4   -14
     -2   -12  -54  -285 -22  -80];
b = [-2000 -55 -800]';
x = linprog(f, A, b).
```

c) The solution with real numbers should give a lower (or equal) cost than the solution with integer variables, since integers are a subset of the real numbers.

Problem 2:

Option A

Option B



$i = 5\%$

$$NPV_B = 800K + \frac{800K}{1.05} + \frac{800K}{1.05^2} + \frac{800K}{1.05^3} = 2978600$$

Option A is better.

Problem 3:

a)

$$\min_{\hat{F}_A, \hat{F}_B, \hat{F}_C} f(\hat{F}_A, \hat{F}_B, \hat{F}_C) = \frac{(\hat{F}_A - F_A)^2}{\sigma^2} + \frac{(\hat{F}_B - F_B)^2}{\sigma^2} + \frac{(\hat{F}_C - F_C)^2}{\sigma^2}$$

Subject to: $\hat{F}_A - \hat{F}_B - \hat{F}_C = 0$

b) Since the variance is the same for all sensors, we can use: minimize $f = (\hat{F}_A - F_A)^2 + (\hat{F}_B - F_B)^2 + (\hat{F}_C - F_C)^2$

Subject to: $\hat{F}_A - \hat{F}_B - \hat{F}_C = 0$

Using Lagrange:

$$\min_{\hat{F}_A, \hat{F}_B, \hat{F}_C, \lambda} L = (\hat{F}_A - F_A)^2 + (\hat{F}_B - F_B)^2 + (\hat{F}_C - F_C)^2 + \lambda(\hat{F}_A - \hat{F}_B - \hat{F}_C)$$

$$\Rightarrow \frac{\partial L}{\partial \hat{F}_A} = 2(\hat{F}_A - F_A) + \lambda = 0 \quad (1)$$

$$\frac{\partial L}{\partial \hat{F}_B} = 2(\hat{F}_B - F_B) - \lambda = 0 \quad (2)$$

$$\frac{\partial L}{\partial \hat{F}_C} = 2(\hat{F}_C - F_C) - \lambda = 0 \quad (3)$$

$$\frac{\partial L}{\partial \lambda} = \hat{F}_A - \hat{F}_B - \hat{F}_C = 0 \quad (4)$$

From (1), (2) and (3) we can notice that:

$$\lambda = -2(\hat{F}_A - F_A) = 2(\hat{F}_B - F_B) = 2(\hat{F}_C - F_C)$$

$$\text{so: } -(\hat{F}_A - F_A) = (\hat{F}_B - F_B) = (\hat{F}_C - F_C)$$

Using this together with (4):

$$\hat{F}_A - \underbrace{[-(\hat{F}_A - F_A) + F_B]}_{\hat{F}_B} - \underbrace{[-(\hat{F}_A - F_A) + F_C]}_{\hat{F}_C} = 0$$

$$\Rightarrow \hat{F}_A + \hat{F}_A - F_A - F_B + \hat{F}_A - F_A - F_C = 0$$

$$3 \cdot \hat{F}_A = 2F_A + F_B + F_C \Rightarrow \hat{F}_A = \frac{2F_A + F_B + F_C}{3}$$

$$\Rightarrow \hat{F}_A = \frac{[2(120.7) + 110.2 + 8.5]}{3} = \underline{119.56}$$

$$\hat{F}_B = -(\hat{F}_A - F_A) + F_B = (120 - 119.56) + 110.2 = \underline{\underline{110.63}}$$

$$\hat{F}_C = -(\hat{F}_A - F_A) + F_C = (120 - 119.56) + 8.5 = \underline{\underline{8.93}}$$

Problem 4:

a)

$$\min_{b_1,b_2,b_3,b_4} S(b_1,b_2,b_3,b_4) = \left[0.34 - b_1/\left(1-e^{-(b_2+0b_3)b_4}\right)\right]^2/0.0004$$
$$+ \left[0.44 - b_1/\left(1-e^{-(b_2+0.005b_3)b_4}\right)\right]^2/0.0004$$
$$+ \left[0.50 - b_1/\left(1-e^{-(b_2+0.008b_3)b_4}\right)\right]^2/0.0004$$
$$+ \left[0.59 - b_1/\left(1-e^{-(b_2+0.012b_3)b_4}\right)\right]^2/0.0002$$
$$+ \left[0.72 - b_1/\left(1-e^{-(b_2+0.015b_3)b_4}\right)\right]^2/0.0002$$
$$+ \left[0.93 - b_1/\left(1-e^{-(b_2+0.02b_3)b_4}\right)\right]^2/0.0002$$
$$+ \left[0.99 - b_1/\left(1-e^{-(b_2+0.02b_3)b_4}\right)\right]^2/0.0002.$$

b) Excel:

- Objective function

$b_1 =$		$S =$	
$b_2 =$			
$b_3 =$			
$b_4 =$			

Call Solver.
choose Objective cell
Choose minimize
Choose variable cells
Solve.

- The solution can be wrong because can be local minimas.
- We should at least plot the function with the parameters found, and the experimental data, and check the fit. We can also start from different initial approximation for the parameters and call the solver, choosing the solution with the minimal sum of square errors.

Problem 4: continuation

Matlab:

Define the function:

```
function S = f(b)

S = (0.34 - b(1)/(1 - exp(-b(2) + 0 b(3)))^b(4))^2/0.0004 + ...

etc. etc.
```

- Define initial values for b_1, b_2, b_3, b_4:

```
b = [1 1 1 1];
```

- Call fminunc:

```
[x, fval] = fminunc(@f, b);
```

- Same answers than for the Excel case:
  The solution can be a local minimum.

6

Problem 5.-

a) Non-Dominated set:

$\boxed{A, C, D, F, G, H}$

b) He has to prioritize the criteria additionally, for example, if being close to his parents city is more important than saving money, option C is very attractive..

If saving money is more important, option F is very attractive.

Note:

other fundamented answers are accepted. The key is that the student needs to consider other criteria to make his decision.-