Solution Problem 1:

Variables

A_1, A_2, A_3, A_4 : Amounts of constituents 1, 2, 3 and 4 in A.

B_1, B_2, B_3, B_4 : Amount of constituents 1,2,3,4 in B

C_1, C_2, C_3, C_4 : Amount of constituents 1,2,3,4 in C.

Maximize

$$(A_1+A_2+A_3+A_4)(15.50) + (B_1+B_2+B_3+B_4)(8.50)$$
$$+((C_1, C_2, C_3, C_4)(13.50) - (A_1+B_1+C_1)(9.0)$$
$$-(A_2+B_2+C_2)(9.0) - (A_3+B_3+C_3)(3.0)$$
$$-(A_4+B_4+C_4)(7.0).$$

Subject to

$(A_1+B_1+C_1) \leq 5000$

$(A_2+B_2+C_2) \leq 3000$

$(A_3+B_3+C_3) \leq 3000$

$(A_4+B_4+C_4) \leq 2000$

Bounds:

$A_1, A_2, A_3, A_4,$
$B_1, B_2, B_3, B_4,$
C_1, C_2, C_3, C_4 All must be ≥ 0.

Specifications: Linear Constraints:

$$\frac{A_1}{A_1+A_2+A_3+A_4} \leq 0.3 \Rightarrow 0.7A_1 - 0.3A_2 - 0.3A_3 - 0.3A_4 \leq 0$$

$$\frac{A_2}{A_1+A_2+A_3+A_4} \geq 0.4 \Rightarrow -0.4A_1 + 0.6A_2 - 0.4A_3 - 0.4A_4 \geq 0$$

$$\frac{A_3}{A_1+A_2+A_3+A_4} \leq 0.5 \Rightarrow -0.5A_1 - 0.5A_2 + 0.5A_3 - 0.5A_4 \leq 0$$

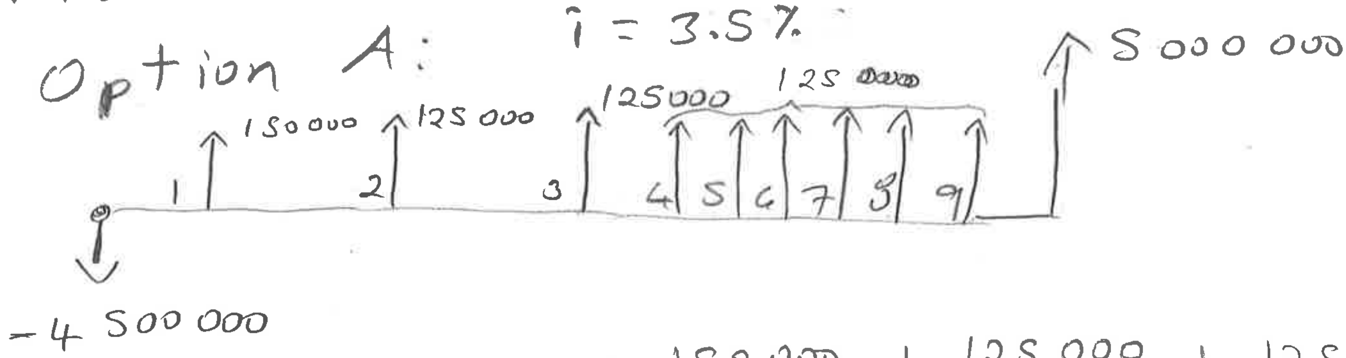
$$\frac{B_1}{B_1+B_2+B_3+B_4} \leq 0.5 \Rightarrow 0.5B_1 - 0.5B_2 - 0.5B_3 - 0.5B_4 \leq 0$$

$$\frac{B_2}{B_1+B_2+B_3+B_4} \geq 0.1 \Rightarrow -0.1B_1 + 0.9B_2 - 0.1B_3 - 0.1B_4 \geq 0$$

$$\frac{C_1}{C_1+C_2+C_3+C_4} \geq 0.7 \Rightarrow 0.3C_1 - 0.7C_2 - 0.7C_3 - 0.7C_4 \geq 0$$

Problem 2: Solution

Option A: $i = 3.5\%$



$$NPV = -4\,500\,000 + \frac{150\,000}{1.035} + \frac{125\,000}{1.035^2} + \frac{125\,000}{1.035^3} + \frac{125\,000}{1.035^4} + \frac{125\,000}{1.035^5} + \frac{125\,000}{1.035^6} + \frac{125\,000}{1.035^7} + \frac{125\,000}{1.035^8} + \frac{125\,000}{1.035^9} + \frac{5\,000\,000}{1.035^{10}} = 19709.47$$

Option B:

$NPV = 0$, since $i = 3.5\%$, the interest is re-invested, and all the capital plus interest are collected at the end of the ten years.

Option A is better according with this criteria.

Problem 3:

Max xyz

Subject to:

$$x+y+z=120$$
$$x>0$$
$$y>0$$
$$z>0$$

Using Lagrange:

$$L(x,y,z) = xyz - \lambda(x+y+z-120)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial x} &= yz-\lambda = 0 \\ \frac{\partial L}{\partial y} &= xz-\lambda = 0 \\ \frac{\partial L}{\partial z} &= xy-\lambda = 0 \end{aligned}\right\} \begin{aligned} &\lambda = yz = xz = xy \\ &\Rightarrow yz = xz \Rightarrow y = x \\ &\quad xz = xy \Rightarrow z = y \\ &\Rightarrow x = y = z. \end{aligned}$$

$$\frac{\partial L}{\partial \lambda} = x+y+z-120 = 0$$

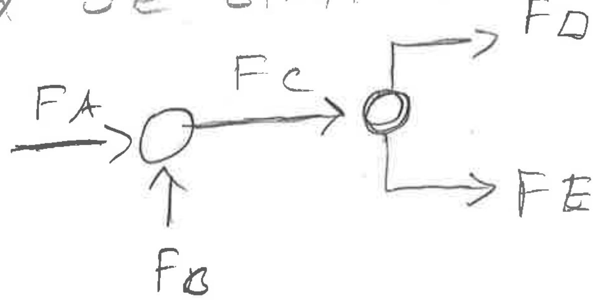
since $y=x$ and $z=x$:

$$3x-120=0$$
$$x=40$$
$$y=40$$
$$z=40$$

$xyz = 64000$

Problem 4: Solution

There are different solutions depending on the variables selected, but the procedure should be similar:



Selecting to measure FA, FB, Fc and FD, it is possible to do data reconcilation on FA, FB and FC, because they are redundant:

$$\min_{\hat{F}_A, \hat{F}_B, \hat{F}_C} \frac{(F_A-\hat{F}_A)^2}{\sigma^2} + \frac{(F_B-\hat{F}_B)^2}{\sigma^2} + \frac{(F_C-\hat{F}_C)^2}{\sigma^2}.$$

subject to:

$$\hat{F}_A + \hat{F}_B - \hat{F}_C = 0$$

Notice that FD is not included because is not redundant.

Using Lagrange multipliers:

$$L(\hat{F}_A, \hat{F}_B, \hat{F}_C, \lambda) = (F_A-\hat{F}_A)^2 + (F_B-\hat{F}_B)^2 + (F_C-\hat{F}_C)^2 + 2\lambda(\hat{F}_A + \hat{F}_B - \hat{F}_C)$$

Notice that since the varian is the same for all the sensors, it is not necessary to include it on the procedure. Also, using "2λ" instead of "λ" simplifies the operations.

$$\frac{\partial L}{\partial \hat{F}_A} = -2(F_A - \hat{F}_A) + 2\lambda = 0 \qquad \text{Eq. 1}$$

$$\frac{\partial L}{\partial \hat{F}_B} = -2(F_B - \hat{F}_B) + 2\lambda = 0 \qquad \text{Eq. 2}$$

$$\frac{\partial L}{\partial \hat{F}_C} = -2(F_C - \hat{F}_C) - 2\lambda = 0 \qquad \text{Eq. 3}$$

$$\frac{\partial L}{\partial \lambda} = 2(\hat{F}_A + \hat{F}_B - \hat{F}_C) = 0 \qquad \text{Eq. 4}$$

From Eq. 1 and Eq. 3 $\Rightarrow \lambda = (F_A - \hat{F}_A) = -(F_C - \hat{F}_C)$.

$$\Rightarrow \hat{F}_A = F_A + F_C + \hat{F}_C \qquad \text{Eq. 5}$$

From Eq. 2 and Eq. 3 $\Rightarrow \lambda = (F_B - \hat{F}_B) = -(F_C - \hat{F}_C)$

$$\Rightarrow \hat{F}_B = F_B + F_C - \hat{F}_C \qquad \text{Eq. 6}$$

Substituting Eq. 5 and Eq. 6 in Eq. 4 :

$$F_A + F_C - \hat{F}_C + F_B + F_C - \hat{F}_C - \hat{F}_C = 0$$

$$-3\hat{F}_C + F_A + F_B + 2F_C = 0$$

$$\boxed{\hat{F}_C = \frac{F_A + F_B + 2F_C}{3}} \qquad \text{Eq. 7}$$

Substituting Eq. 7 in Eq. 5

$$\hat{F}_A = F_A + F_C - \frac{(F_A + F_B + 2F_C)}{3}$$

$$\Rightarrow \boxed{\hat{F}_A = \frac{2F_A - F_B + F_C}{3}} \qquad \text{Eq. 8}$$

Substituting Eq. 7 in Eq. 6:

$$\hat{F}_B = F_B + F_C - \frac{(F_A + F_B + 2F_C)}{3}$$

$$\Rightarrow \boxed{\hat{F}_B = \frac{2F_B - F_A + F_C}{3}} \quad \text{Eq. 9.}$$

$\hat{F}_E$ can be calculated using the estimated value for $\hat{F}_C$ and the measured value for F_D:

$$\boxed{\hat{F}_E = \hat{F}_C - F_D} \quad \text{Eq. 10.}$$

Problem 5. Solution

Variables:

B_1: Select project 1. Binary
B_2: select project 2. Binary
B_3: select project 3. Binary
B_4: select project 4. Binary
B_5: select project 5. Binary
B_6: select project 6. Binary
B_7: select project 7. Binary

Can only take values "0" or "1".

Maximize: $150\,000 B_1 + 130\,000 B_2 + 50\,000 B_3 + 20\,000 B_4 + 200\,000 B_5 + 5\,000 B_6 + 75\,000 B_7$

Subject to:

$500\,000 B_1 + 200\,000 B_2 + 250\,000 B_3 + 150\,000 B_4 + 200\,000 B_5 + 1000 B_6 + 170\,000 B_7 \leq 850\,000$

$100\,000 B_1 + 200\,000 B_2 + 50\,000 B_3 + 150\,000 B_5 + 50\,000 B_7 \leq 250\,000$

$3000 B_1 + 2000 B_2 + 2000 B_3 + 500 B_4 + 5000 B_5 + 100 B_6 + 500 B_7 \leq 10000$

Managament policies constraints:

$B_1 + B_2 \leq 1$

$B_2 - B_4 \geq 0$

$B_3 - B_7 = 0$

$B_6 - B_5 \geq 0$