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CHIZIQLI ALGEBRA

MAVZU

XOS VEKTORLARI BAZIS TASHKIL QILUVCHI CHIZIQLI OPERATORLAR

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OLIY MATEMATIKA KAFEDRASI

XOS VEKTORLARI BAZIS TASHKIL QILUVCHI CHIZIQLI OPERATORLAR

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1. Sodda chiziqli operatorlar

R^n fazodagi eng sodda chiziqli operatorlar shunday operatorlarki, ular *n* ta chiziqli erkli vektorga ega bo‘ladi.

Haqiqatan, $T: R^n \to R^n$ operator chiziqli erkli $\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\}$ vektorlarga ega bo‘lgan operator bo‘lsin. Shu vektorlarni bazis uchun qabul qilamiz.
U holda ***T*** operatorni quyidagicha tasvirlash mumkin:

$$\left.\begin{array}{l} T(\vec{e}_1) = \lambda_1 \vec{e}_1, \\ T(\vec{e}_2) = \lambda_2 \vec{e}_2, \\ \dots\dots\dots\dots\dots\dots \\ T(\vec{e}_n) = \lambda_n \vec{e}_n, \end{array}\right\}$$

Bunda $\lambda_1, \lambda_2, \ldots, \lambda_n$ - operatorning xos sonlari;

$\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\}$ - operatorning xos vektorlari bo‘ladi.

Bundan $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n$ xos vektorlar tashkil qilgan bazisda ***T*** operatorning matritsasi eng sodda bo‘lib, u diagonal ko‘rinishda bo‘ladi:

$$A = \begin{bmatrix} \lambda_1 & 0 & \ldots & 0 \\ 0 & \lambda_2 & \ldots & 0 \\ . & . & ... & . \\ 0 & 0 & \ldots & \lambda_n \end{bmatrix}$$

2. Bazis tashkil qiluvchi xos vektorlar

1-teorema. Agar $\boldsymbol{R^n}$ fazoda ***T*** chiziqli operatorning xos qiymatlari $\lambda_1, \lambda_2, ..., \lambda_s$ ($s \le n$) haqiqiy sonlar to‘plamiga tegishli juft-jufti bilan har xil sonlar bo‘lsa, bu xos qiymatlarga mos keluvchi $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_s$ xos vektorlar chiziqli erkli bo‘ladi.

Xususan, ($s = n$) bo‘lsa, $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n$ xos vektorlar $\boldsymbol{R^n}$ da bazis tashkil qiladi.

Teoremaning isboti matematik induksiya metodi asosida olib boriladi.

Siz teorema isbotini Raxмatov R.R. va Adizov A.A. larning “Chiziqli fazo va chiziqli operatorlar” (TATU, Toshkent, 2019), o‘quv uslubiy qo‘llanmasidan ko‘rishingiz mumkin.

Bazis tashkil qiluvchi xos vektorlarni topish

1- misol. $T:R^3 \to R^3$ chiziqli operatorning $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ bazisdagi matritsasi berilgan. *T* operatorning xos sonlari, xos vektorlarini va (agar mumkin bo‘lsa) operatorning matritsasi diagonal ko‘rinishni oladigan bazisini toping:

$$A = \begin{bmatrix} -1 & 3 & -1 \\ -3 & 5 & -1 \\ -3 & 3 & 1 \end{bmatrix}$$

Yechilishi: *T* operatorning xarakteristik ko‘phadini tuzib olamiz:

$$\det[A-\lambda E] = \begin{vmatrix} -1-\lambda & 3 & -1 \\ -3 & 5-\lambda & -1 \\ -3 & 3 & 1-\lambda \end{vmatrix} = (1+\lambda)(5-\lambda)(\lambda-1)+9+9-3(5-\lambda)+9(1-\lambda)-3(1+\lambda) =$$

$$= (\lambda^2-1)(5-\lambda)-9(\lambda-1) = (\lambda-1)(5\lambda-\lambda^2+5-\lambda-9) = -(\lambda-1)(\lambda^2-4\lambda+4) = (1-\lambda)(\lambda-2)^2$$

T operatorning xarakteristik sonlari 3 ta ekan: $\lambda=1,\ \lambda_2=\lambda_3=2$

$$\det[A-\lambda E]=\begin{vmatrix} -1-\lambda & 3 & -1 \\ -3 & 5-\lambda & -1 \\ -3 & 3 & 1-\lambda \end{vmatrix}=(1-\lambda)(\lambda-2)^2$$

Operatorning xos soni:

$$\lambda_1=1$$

Operatorning **xos vektorini** topish uchun quyidagi sistemani yechamiz:

$$q_1=x_1\overrightarrow{e_1}+x_2\overrightarrow{e_2}+x_3\overrightarrow{e_3}$$

$$\begin{cases} -2x_1+3x_2-x_3=0 \\ -3x_1+4x_2-x_3=0 \\ -3x_1+3x_2=0 \end{cases} \Rightarrow \begin{cases} x_1=x_3 \\ x_2=x_3 \\ x_1=x_2 \end{cases} \Rightarrow \begin{cases} x_1=1 \\ x_2=1 \\ x_3=1 \end{cases}$$

Xos vektor: $q_1=\overrightarrow{e_1}+\overrightarrow{e_2}+\overrightarrow{e_3}$

Operatorning $\lambda_2=\lambda_3=2$ xos sonlariga mos keladigan xos vektorni topamiz:

$$\begin{cases} -3x_1+3x_2-x_3=0 \\ -3x_1+3x_2-x_3=0 \\ -3x_1+3x_2-x_3=0 \end{cases} \Rightarrow \begin{cases} x_1=x_2 \\ x_3=0 \end{cases}$$

Xos vektorlar: $\vec{q}_2=\vec{e}_1-\vec{e}_2$

$\vec{q}_3=\vec{e}_3$

Bu topilgan xos vektorlarning chiziqli erkli ekanini tekshirish kerak:

$$\begin{cases} \vec{q}_1 = (1,1,1) \\ \vec{q}_2 = (1,-1,0) \\ \vec{q}_3 = (0,0,1) \end{cases} \qquad \begin{cases} \vec{q}_1 = \vec{e}_1 + \vec{e}_2 + \vec{e}_3 \\ \vec{q}_2 = \vec{e}_1 - \vec{e}_2 \\ \vec{q}_3 = \vec{e}_3 \end{cases} \qquad \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1-1 = -2 \neq 0$$

Shu sababli $\overrightarrow{q_1}, \overrightarrow{q_2}, \overrightarrow{q_3}$ vektorlar bazis tashkil qiladi.

T operatorning eng sodda diagonal matritsasini tuzamiz:

$$\begin{aligned} \lambda_1 &= 1 \\ \lambda_2 &= 2 \\ \lambda_3 &= 2 \end{aligned} \qquad B = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \Rightarrow B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

3. Chiziqli operator matritsasining xos soni va xos vektori

2- misol. Quyidagi matritsaning xos soni va xos vektorini aniqlang: $A=\begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix}$

Yechilishi: Matritsaning noma’lum xos vektorini belgilab olamiz: $\bar{u}=\begin{pmatrix} x \\ y \end{pmatrix}$

Matritsaviy tenglamasini yozamiz: $A\bar{u}=\lambda\bar{u}$

$$\begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix}\cdot\begin{pmatrix} x \\ y \end{pmatrix}=\lambda\cdot\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -x-6y \\ 2x+6y \end{pmatrix}=\begin{pmatrix} \lambda x \\ \lambda y \end{pmatrix} \longrightarrow \begin{cases} -x-6y=\lambda x \\ 2x+6y=\lambda y \end{cases}$$

$$\begin{cases} -x-6y-\lambda x=0 \\ 2x+6y-\lambda y=0 \end{cases} \longrightarrow \begin{cases} (-1-\lambda)x-6y=0 \\ 2x+(6-\lambda)y=0 \end{cases} \qquad \bar{u}=\begin{pmatrix} x \\ y \end{pmatrix}\neq\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Xos vektor nolga teng bo‘lmaydi, notrivial bo‘lishi kerak, shuning uchun $x=0, y=0$ yechimga mos kelmaydi.

Bundan, chiziqli tenglamalar bir-biriga bog‘liq va sistemaning matritsasi determinanti nolga teng: $\begin{vmatrix} -1-\lambda & -6 \\ 2 & 6-\lambda \end{vmatrix} = 0$

Bu *A* matritsaning **xarakteristik tenglamasi** deyiladi, uning ildizlari berilgan matritsaning xos soni hisoblanadi.

$$(-1-\lambda)(6-\lambda)+12=0$$

$$\lambda^2-5\lambda+6=0$$

$$\lambda_1=2,\quad \lambda_2=3$$

Endi xos sonlarga mos xos vektorlarni aniqlaymiz: $\lambda_1=2$ bo’lganda

$$\begin{cases} (-1-\lambda)x-6y=0 \\ 2x+(6-\lambda)y=0 \end{cases} \qquad \begin{cases} -3x-6y=0 \\ 2x+4y=0 \end{cases} \Rightarrow x=-2y$$

“x” ning o‘rniga qiymat berib, cheksiz ko‘p xos vektorlarni hosil qilamiz. Vektorning “x” koordinatasi **musbat, butun** va **minimal** bo‘lishi, “y” esa kasr son bo‘lmasligi kerak: 1-xos vektor: $\begin{cases} x=2 \\ y=-1 \end{cases}$ $\quad \overline{u}_1=\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\lambda_2 = 3$ bo‘lganda $\begin{vmatrix} -1-\lambda & -6 \\ 2 & 6-\lambda \end{vmatrix} = 0$ $\begin{cases} -4x - 6y = 0 \\ 2x + 3y = 0 \end{cases}$ $\begin{cases} x = 3 \\ y = -2 \end{cases}$

Ikkinchi xos vektor: $\bar{u}_2 = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Natija: xos sonlar: $\lambda_1 = 2,\ \lambda_2 = 3$

xos vektorlar: $\bar{u}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix},\ \bar{u}_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

4. *T* operatorning matritsasi diagonal ko‘rinishida bo‘ladigan bazisni topish

3- misol. $T: C^2 \to C^2$ operator $\overrightarrow{e_1}, \overrightarrow{e_2}$ bazis vektorlarni $\begin{matrix} T(\vec{e}_1) = \vec{e}_1 + i\vec{e}_2 \\ T(\vec{e}_2) = i\vec{e}_1 + \vec{e}_2 \end{matrix}$ vektorga o‘tkazuvchi operator bo‘lsin.

T operatorning matritsasi diagonal ko‘rinishda bo‘ladigan bazisni toping.

Yechilishi: $\overrightarrow{e_1}, \overrightarrow{e_2}$ bazisda *T* ning matritsasi quyidagicha bo‘ladi: $A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$

Bunda *T* operatorning xarakteristik ko ‘phadini yozib, xos sonlarini topamiz:

$$\det[A - \lambda E] = \begin{vmatrix} 1-\lambda & i \\ i & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - i^2 = (1-\lambda-i)(1-\lambda+i)$$

$$\lambda_1 = 1 - i \quad \text{va} \quad \lambda_2 = 1 + i$$

Bu xos sonlarga mos keladigan xos vektorlarni topamiz: $\begin{matrix} \vec{q}_1 = x_1\vec{e}_1 + x_2\vec{e}_2 \\ \vec{q}_2 = y_1\vec{e}_1 + y_2\vec{e}_2 \end{matrix}$

Xos vektorlar quyidagi tenglamalar sistemasidan topiladi:

$$\begin{cases} x_1 + ix_2 = (1-i)x_1 \\ x_1 + ix_2 = (1+i)x_1 \\ ix_1 + x_2 = (1-i)x_2 \\ ix_1 + x_2 = (1+i)x_2 \end{cases} \Rightarrow \begin{cases} ix_1 + ix_2 = 0 \\ -ix_1 + ix_2 = 0 \\ ix_1 + ix_2 = 0 \\ ix_1 - ix_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = -x_2 \\ x_1 = x_2 \end{cases} \Rightarrow \begin{cases} \vec{q}_1 = \vec{e}_1 - \vec{e}_2 \\ \vec{q}_2 = \vec{e}_1 + \vec{e}_2 \end{cases}$$

Bazis vektorning diagonal matritsasi:

$$B = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \Rightarrow \begin{matrix} \lambda_1 = 1-i \\ \lambda_2 = 1+i \end{matrix} \Rightarrow B = \begin{pmatrix} 1-i & 0 \\ 0 & 1+i \end{pmatrix}$$

O‘z-o‘zini tekshirish uchun savollar:

1. Chiziqli fazoning chiziqli almashtirishi yoki operatori deb nimaga aytiladi?
2. Xos vektorlari bazis tashkil qiluvchi chiziqli operator deb qanday operatorga aytiladi?
3. Chiziqli operatorning xos vektorlari bazis tashkil qilishining yetarli sharti?
4. L^n fazoda bir bazisdan ikkinchi bazisga o‘tish matritsasi qanday tuziladi?
5. Chiziqli operator ustida bajariladigan qanday amallarni bilasiz?
6. Chiziqli operatorning xos vektori va xos qiymati deb nimaga aytiladi?
7. Xos vektorlarning qanday xossalarini bilasiz?
8. Qanday bazisda matritsa diagonal ko‘rinishga ega bo‘ladi?

Adabiyotlar:

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E'TIBORINGIZ UCHUN RAHMAT!

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