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CHIZIQLI ALGEBRA

MTH 1234

MAVZU

DETERMINANTNING XOSSALARI

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**OLIY MATEMATIKA
KAFEDRASI**

Reja

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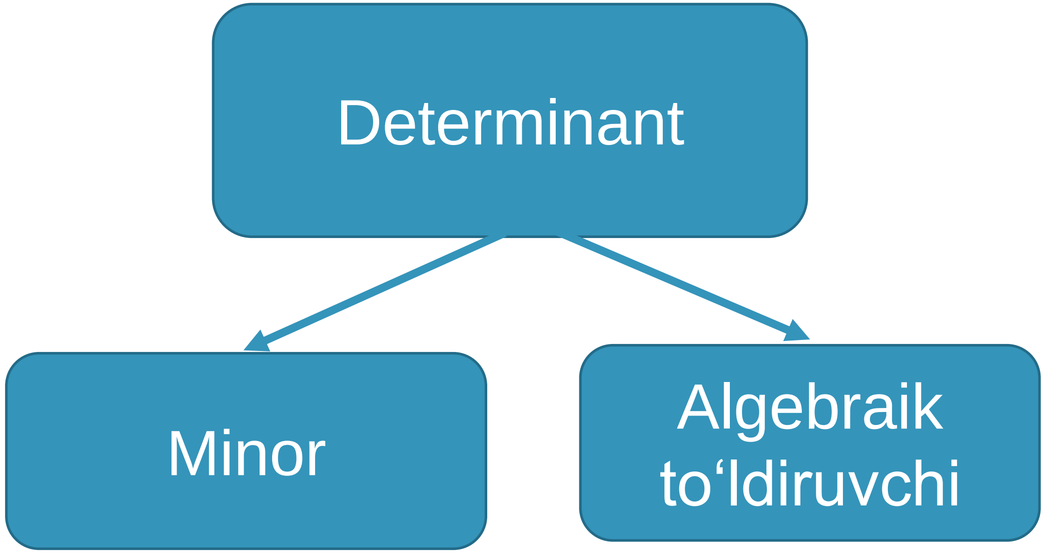

1. Determinantning asosiy xossalari

1^0. Agar determinant biror satri(ustuni)ning barcha elementlari nolga teng bo‘lsa, uning qiymati nolga teng.

$$\begin{vmatrix} 6 & 7 & 3 \\ 0 & 0 & 0 \\ 3 & 4 & 2 \end{vmatrix} = 6\cdot 0\cdot 2 + 0\cdot 4\cdot 3 + 7\cdot 0\cdot 3 - 3\cdot 0\cdot 3 - 6\cdot 4\cdot 0 - 7\cdot 0\cdot 2 = 0.$$

2^0. Diagonal matritsaning determinanti diagonal elementlarining ko‘paytmasiga teng:

$$\det(A) = a_{11}\cdot a_{22}\cdots a_{nn} = \prod_{i=1}^{n} a_{ii}.$$

$$\Delta = \begin{vmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{vmatrix} = 3\cdot 4\cdot 7 = 84$$

3⁰. Yuqori(quyi) uchburchakli matritsaning determinanti uning bosh diagonal elementlari ko'paytmasiga teng:

$$\det(A) = a_{11} \cdot a_{22} \cdots a_{nn} = \prod_{i=1}^{n} a_{ii}.$$

$$\begin{vmatrix} 2 & 3 & 4 \\ 0 & 5 & 9 \\ 0 & 0 & 6 \end{vmatrix} = 2 \cdot 5 \cdot 6 = 60.$$

4⁰. Determinantning biror satri(ustuni) elementlarini $k \neq 0$ songa ko'paytirish determinantni shu songa ko'paytirishga teng kuchlidir:

$$k \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & ka_{13} \\ a_{21} & a_{22} & ka_{23} \\ a_{31} & a_{32} & ka_{33} \end{vmatrix}.$$

5⁰. *n*-tartibli determinant uchun quyidagi tenglik o‘rinli: $\det(kA) = k^n \cdot \det(A)$

$$\begin{vmatrix} 2 & 4 & 6 \\ 12 & 8 & 14 \\ 10 & 6 & 4 \end{vmatrix} = 2^3 \cdot \begin{vmatrix} 1 & 2 & 3 \\ 6 & 4 & 7 \\ 5 & 3 & 2 \end{vmatrix}$$

6⁰. Determinantda ikkita satr (ustun) o‘rinlari almashtirilsa, determinantning ishorasi o‘zgaradi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{12} & a_{11} & a_{13} \\ a_{22} & a_{21} & a_{23} \\ a_{32} & a_{31} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 5 \\ -1 & -2 & 1 \end{vmatrix} = 39 \qquad \begin{vmatrix} 3 & -4 & 2 \\ 5 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix} = -39$$

7⁰. Agar determinant ikkita bir xil satr(ustun)ga ega bo‘lsa, uning qiymati nolga teng.

$$\begin{vmatrix} 5 & 3 & 6 \\ 5 & 3 & 6 \\ 2 & 5 & 3 \end{vmatrix} = 45 + 36 + 150 - 36 - 150 - 45 = 0.$$

8⁰. Agar determinantning biror satr(ustun) elementlariga boshqa satr(ustun)ning mos elementlarini biror songa ko‘paytirib qo‘shilsa, determinantning qiymati o‘zgarmaydi.

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1} + ka_{j1} & a_{i2} + ka_{j2} & \cdots & a_{in} + ka_{jn} \\ \cdots & \cdots & \cdots & \cdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

9⁰. Agar determinant ikkita satri(ustuni)ning mos elementlari proporsional bo‘lsa, uning qiymati nolga teng bo‘ladi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{11} & ka_{12} & ka_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & ka_{11} \\ a_{21} & a_{22} & ka_{21} \\ a_{31} & a_{32} & ka_{31} \end{vmatrix} = 0.$$

10⁰. Agar determinant biror satri(ustuni)ning har bir elementi ikkita qo‘shiluvchi yig‘indisidan iborat bo‘lsa, u holda berilgan determinant ikkita determinant yig‘indisiga teng bo‘ladi.

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1}+b_{i1} & a_{i2}+b_{i2} & \cdots & a_{in}+b_{in} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{i1} & b_{i2} & \cdots & b_{in} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}.$$

11⁰. Transponirlash natijasida determinantning qiymati o‘zgarmaydi.

$$\begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 5 \\ -1 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 3 & -1 \\ -4 & 1 & -2 \\ 3 & 5 & 1 \end{vmatrix} = 39.$$

12⁰. Bir xil tartibli ikkita matritsalar ko‘paytmasining determinanti, bu matritsalar determinantlarining ko‘paytmasiga teng.

$$\det(A \cdot B) = \det(A) \cdot \det(B).$$

2. Determinantning minori

n- tartibli A kvadrat matritsaning $1 \leq k \leq n-1$ tengsizlikni qanoatlantiruvchi ixtiyoriy k ta satri va k ta ustuni kesishgan joyda turgan elementlardan tashkil topgan k-tartibli matritsaning determinanti A matritsa determinantining ***k*-tartibli minori** deyiladi.

$$M_{ij}$$

1-misol. Determinant minorlarini toping:

$$\begin{vmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{vmatrix}$$

1-satr elementlarining minorlari:

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix} \longrightarrow M_{11} = \begin{pmatrix} 2 & 3 \\ 7 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix} \longrightarrow M_{12} = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix} \longrightarrow M_{13} = \begin{pmatrix} -1 & 2 \\ 2 & 7 \end{pmatrix}$$

2-satr elementlarining minorlari:

$$\begin{vmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{vmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix} \longrightarrow M_{21} = \begin{pmatrix} 1 & -2 \\ 7 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix} \longrightarrow M_{22} = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix} \longrightarrow M_{23} = \begin{pmatrix} 1 & 1 \\ 2 & 7 \end{pmatrix}$$

3-satr elementlarining minorlari:

$$\begin{vmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{vmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix} \longrightarrow M_{31} = \begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix} \longrightarrow M_{32} = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix} \longrightarrow M_{33} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$$

3. Algebraik to‘ldiruvchi

a_{ij} elementning **algebraik to‘ldiruvchisi** deb, bu elementning musbat yoki manfiy ishora bilan olingan minoriga aytiladi: $A_{ij} = (-1)^{i+j} M_{ij}$

2-misol. $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ matritsada A_{21} va A_{22} larni toping.

$$M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} \longrightarrow A_{21} = (-1)^{2+1} \cdot M_{21} = -M_{21}$$

$$M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} \longrightarrow A_{22} = (-1)^{2+2} \cdot M_{22} = M_{22}$$

4. Yuqori tartibli determinantlarni hisoblashda tartibini pasaytirish usuli

Laplas teoremasi. Determinantning qiymati uning ixtiyoriy satr (ustun) elementlari bilan shu elementlarga mos algebraik to‘ldiruvchilar ko‘paytmalari yig‘indisiga teng:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{in} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} = a_{i1}A_{i1} + a_{i2}A_{i2} + \dots + a_{in}A_{in} = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} M_{ij}$$

Natija. Determinantning biror satr(ustun) elementlari bilan uning boshqa satri(ustuni) elementlari algebraik to‘ldiruvchilari ko‘paytmalarining yig‘indisi nolga teng.

Laplas formulasi

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\begin{aligned} \Delta &= a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = \\ &= a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = \\ &= a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33} = \\ &= a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31} = \\ &= a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} = \\ &= a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33} \end{aligned}$$

3-misol.

$$\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix} = 2A_{11} + A_{12} + 3A_{13} = 2(-1)^{1+1} \cdot M_{11} + (-1)^{1+2} \cdot M_{12} + 3(-1)^{1+3} \cdot M_{13} =$$

$$= 2 \cdot \begin{vmatrix} 3 & 2 \\ 4 & 3 \end{vmatrix} - \begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix} + 3 \cdot \begin{vmatrix} 5 & 3 \\ 1 & 4 \end{vmatrix} = 2(9-8) - (15-2) + 3(20-3) = 2 - 13 + 51 = 40.$$

4-misol. $\Delta = \begin{vmatrix} 1 & 0 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 3 & 2 & 1 & 5 \\ 1 & 0 & 6 & 2 \end{vmatrix}$ determinantni Laplas formulasi bilan hisoblaymiz.

I usul.

Determinantni eng ko‘p nol element qatnashgan qatorini aniqlaymiz.
Bu yerda 2-ustunda eng ko‘p nol element bo‘lgani uchun, determinantni 2-ustun elementlari bo‘yicha yoyib chiqamiz.

$$\boxed{\Delta = -1\cdot(-1)^{2+2}\begin{vmatrix} 1 & 3 & 2 \\ 3 & 1 & 5 \\ 1 & 6 & 2 \end{vmatrix} + 2\cdot(-1)^{3+2}\begin{vmatrix} 1 & 3 & 2 \\ 2 & 4 & 1 \\ 1 & 6 & 2 \end{vmatrix} = -21.}$$

II usul.

a_{32}= 2 elementni nolga keltirishimiz ham mumkin. Buning uchun 2-satrni 2 ga ko‘paytirib 3-satrga qo‘shamiz. Hosil bo‘lgan determinantni 2-ustun elementlariga nisbatan yoyamiz va hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & 0 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 3 & 2 & 1 & 5 \\ 1 & 0 & 6 & 2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & 0 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 7 & 0 & 9 & 7 \\ 1 & 0 & 6 & 2 \end{vmatrix} = -1 \cdot (-1)^{2+2} \cdot \begin{vmatrix} 1 & 3 & 2 \\ 7 & 9 & 7 \\ 1 & 6 & 2 \end{vmatrix} = -21.$$

5. Yuqori tartibli determinantlarni uchburchak shakliga keltirib hisoblash

Bosh diagonalidan yuqoridagi yoki pastdagi barcha elementlari nollardan iborat bo‘lgan determinant **uchburchak shaklidagi determinant** deyiladi.

Bunday determinantning qiymati bosh diagonali elementlari ko‘paytmasiga teng.

Har qanday determinantni **uchburchak shakliga keltirib hisoblash** mumkin.

5-misol. $$\Delta = \begin{vmatrix} 2 & -4 & 1 & 5 \\ 1 & -3 & 2 & 5 \\ 2 & 2 & 0 & -3 \\ 3 & -1 & 1 & 2 \end{vmatrix}$$ determinantni uchburchak ko‘rinishiga o‘tkazib hisoblaymiz.

Determinantning a_{11} elementi 1 ga teng bo‘lsa, hisoblash oson bo‘ladi. Shuning uchun birinchi va ikkinchi satrlarning o‘rnini almashtiramiz. Bunda determinantning ishorasi qarama-qarshisiga o‘zgaradi.

$$\Delta = -\begin{vmatrix} 1 & -3 & 2 & 5 \\ 2 & -4 & 1 & 5 \\ 2 & 2 & 0 & -3 \\ 3 & -1 & 1 & 2 \end{vmatrix} = -\begin{vmatrix} 1 & -3 & 2 & 5 \\ 0 & 2 & -3 & -5 \\ 0 & 8 & -4 & -13 \\ 0 & 8 & -5 & -13 \end{vmatrix}$$

Hosil bo‘lgan birinchi satrni -2, -2, -3 ga ko‘paytirib, mos ravishda ikkinchi, uchunchi va to‘rtinchi satrlarga qo‘shamiz.

Uchinchi satrini -1 ga ko‘paytirib to‘rtinchi satrlarga qo‘shamiz:

$$-\begin{vmatrix} 1 & -3 & 2 & 5 \\ 0 & 2 & -3 & -5 \\ 0 & 8 & -4 & -13 \\ 0 & 8 & -5 & -13 \end{vmatrix} \qquad \Delta = -\begin{vmatrix} 1 & -3 & 2 & 5 \\ 0 & 2 & -3 & -5 \\ 0 & 8 & -4 & -13 \\ 0 & 0 & -1 & 0 \end{vmatrix}$$

Ikkinchi satrini -4 ga ko‘paytirib, uchinchi satriga qo‘shamiz. So‘ng uchinchi va to‘rtinchi satrlar o‘rinlarini almashtiramiz:

$$\Delta = -\begin{vmatrix} 1 & -3 & 2 & 5 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & 8 & 7 \\ 0 & 0 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 2 & 5 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 8 & 7 \end{vmatrix}$$

Uchinchi satrini 8 ga ko‘paytirib, to‘rtinchi satriga qo‘shamiz :

$$\Delta = \begin{vmatrix} 1 & -3 & 2 & 5 \\ 0 & 2 & -3 & -5 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 7 \end{vmatrix} = 1 \cdot 2 \cdot (-1) \cdot 7 = -14$$

Determinantning qiymati -14 ga teng ekan.

O‘z-o‘zini tekshirish uchun savollar:

1. Ikkinchi tartibli determinantdan nechta minor hosil qilish mumkin?
2. Minor deb nimaga aytiladi?
3. Determinant xossalarini sanab bering.
4. Algebraik to‘ldiruvchi deganda nimani tushunasiz?
5. O‘rab turuvchi minor ta’rifini ayting.
6. Laplas formulasi qanday?
7. Laplas teoremasini ayting.
8. Determinantni hisoblashning qanday usullarini bilasiz?

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