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CHIZIQLI ALGEBRA

MTH 1234

MAVZU

IKKINCHI VA UCHINCHI TARTIBLI DETERMINANTLAR VA ULARNI HISOBLASH USULLARI

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OLIY MATEMATIKA KAFEDRASI

Ushbu mavzuda siz

1. Determinant nima ekanini;
2. Ikkinchi va uchinchi tartibli determinantlarni;
3. Determinantlarni hisoblash usullarini;
4. Determinantlarni hisoblash formulalari qayerdan kelib chiqqanligini bilib olasiz.

1. Determinantlar

Faqat kvadrat matritsaning determinanti mavjud

Kvadrat matritsa

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

Determinant quyidagi belgilashlardan biri bilan ifodalanadi:

$\det(A)$ yoki $$\Delta=\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Matritsa va determinantning farqi nimada?

Matritsa – jadval,
determinant – shu jadvalning qiymatini aniqlaydigan ifoda

$$A_{1\times1}=(a)$$ 1-tartibli matritsa determinanti $$\Delta=a$$

$$A_{2\times2}=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$ 2-tartibli matritsa determinanti $$\Delta=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$A_{3\times3}=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$ 3-tartibli matritsa determinanti $$\Delta=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

belgilanishlari keltirilgan.

2. Ikkinchi tartibli determinantlar

Jadval shaklida $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ to‘rtta son berilgan bo‘lsa, $\Delta = a_{11}a_{22} - a_{12}a_{21}$ ifodaga **ikkinchi tartibli determinant** deyiladi.

$a_{11}, a_{12}, a_{21}, a_{22}$ - determinantning elementlari

$a_{11}a_{22}$ va $a_{12}a_{21}$ - determinantning hadlari

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

2-tartibli determinantning qiymati asosiy diagonal elementlari ko‘paytmasidan yordamchi diagonal elementlari ko‘paytmasini ayrilganiga teng

1-misol.

$$a)\quad \begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = 3\cdot 4 - 2\cdot 1 = 10$$

$$b)\quad \begin{vmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{vmatrix} = \sin^2 x - (-\cos^2 x) = 1$$

3. Uchinchi tartibli determinantlar

9 ta $\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$ elementdan iborat kvadrat jadval berilgan bo‘lsa,

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{23}a_{32}a_{11}$$

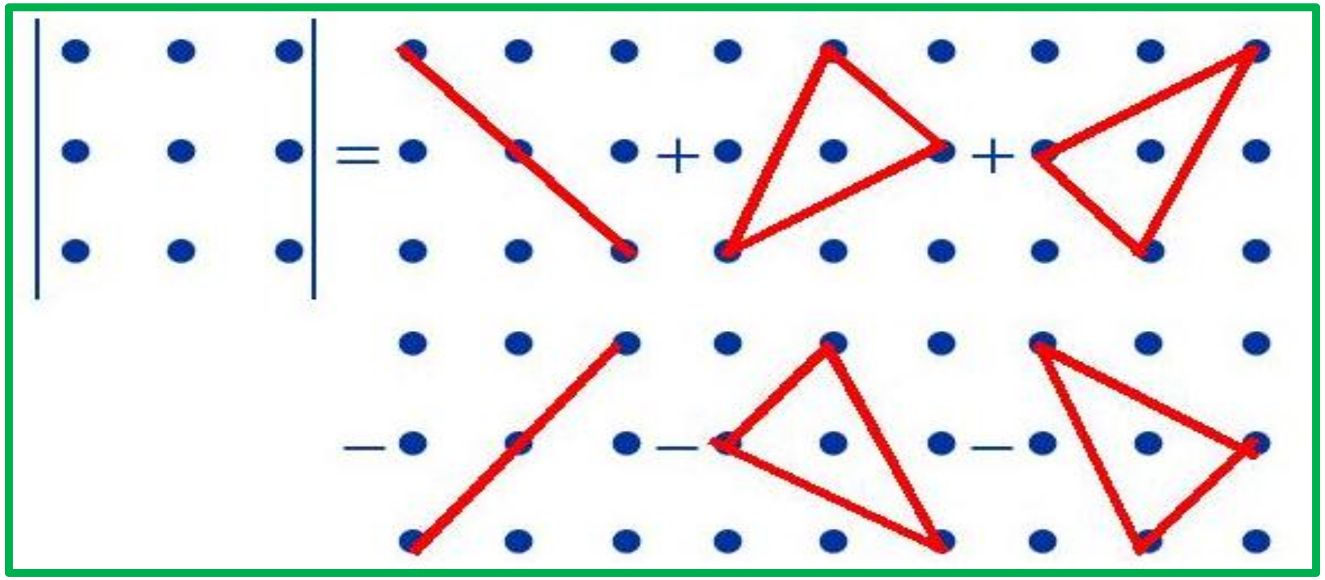
6 ta haddan iborat ifodaga **3-tartibli determinant** deyiladi.

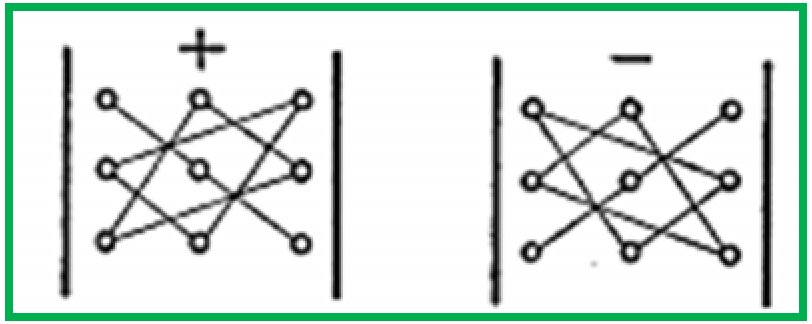
3-tartibli determinantni hisoblashning 3 xil usuli bor:

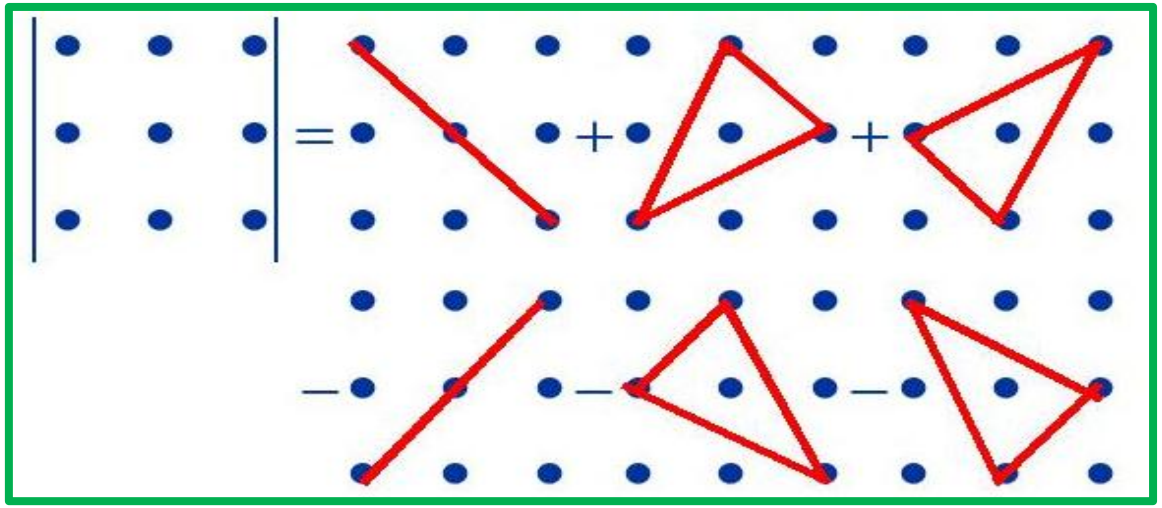
1) uchburchak usuli;
2) Sarryus usuli;
3) Determinant tartibini pasaytirib hisoblash yoki Laplas usuli.

4. Uchinchi tartibli determinantni hisoblashning uchburchak usuli

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{23}a_{32}a_{11}$$







2-misol.

$$\Delta=\begin{vmatrix}3 & -1 & 2\\ 0 & 4 & 5\\ 1 & 6 & 7\end{vmatrix}= \qquad \begin{vmatrix}3 & -1 & 2\\ 0 & 4 & 5\\ 1 & 6 & 7\end{vmatrix} \qquad \begin{vmatrix}3 & -1 & 2\\ 0 & 4 & 5\\ 1 & 6 & 7\end{vmatrix}$$

$$=3\cdot4\cdot7+1\cdot(-1)\cdot5+0\cdot6\cdot2-1\cdot4\cdot2-0\cdot(-1)\cdot7-3\cdot5\cdot6=84-5-8-90=-19$$

5. Uchinchi tartibli determinantni hisoblashning Sarryus usuli

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \left|\begin{array}{ccc|cc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}\right. - \left|\begin{array}{ccc|cc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}\right. =$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

3-misol.

$$\left|\begin{array}{ccc|cc} 1 & 1 & 4 & 1 & 1 \\ -1 & 2 & 3 & -1 & 2 \\ -3 & 2 & 5 & -3 & 2 \end{array}\right. = 1\cdot 2\cdot 5 + 1\cdot 3\cdot(-3) + 4\cdot(-1)\cdot 2 - (-3)\cdot 2\cdot 4 - 2\cdot 3\cdot 1 - 5\cdot(-1)\cdot 1 =$$

$$= 10 - 9 - 8 + 24 - 6 + 5 = 16$$

6. O‘rin almashtirishlar

$1, 2, 3, \ldots, n$ sonlarning biror bir tartibda yozilishiga ***n*-tartibli o‘rin almashtirish** deyiladi.

4-misol. {1, 2, 3} to‘plamning barcha o‘rin alashtirishlarini yozamiz.

$$P_1 = (1, 2, 3)$$
$$P_2 = (2, 3, 1)$$
$$P_3 = (3, 1, 2)$$
$$P_4 = (3, 2, 1)$$
$$P_5 = (2, 1, 3)$$
$$P_6 = (1, 3, 2)$$

O‘rin almashtirishlar soni 6 ta.

7. Inversiyalar

Agar $m > k$ bo‘lib, m soni k sonidan chapda joylashgan bo‘lsa, u holda P o‘rin almashtirishda bu sonlar **inversiya tashkil qiladi** deyiladi.

P o‘rin almashtirishdagi inversiyalarning umumiy soni $invP$ deb belgilanadi.

$invP$ soni juft yoki toq bo‘lganiga qarab P o‘rin almashtirish **juft yoki toq** deyiladi.

5-misol. ***P*=(1, 4, 3, 2)** o‘rin almashtirishda inversiyalar sonini toping.

Yechilishi: 1 dan chapda son yo‘q. Shuning uchun 4 dan boshlaymiz. 4 dan chapda undan katta son yo’q. 3 dan katta bitta son bor, 2 dan katta ikkita son bor. Demak,

$$invP = 0 + 1 + 2 = 3$$

O‘rin almashtirishlar xossalari:

1) $\{1, 2, 3, \ldots, n\}$ to‘plamdagi barcha o‘rin almashtirishlar soni $n!$ ga teng.

2) Juft va toq o‘rin almashtirishlar soni o‘zaro teng, ya’ni har biri $\frac{n!}{2}$ tadan.

3) O‘rin almashtirishda ikkita elementning o‘rni almashtirilsa, uning juft-toqligi o‘zgaradi.

8. O‘rinlashtirishlar

1, 2, 3, … , n sonlar to‘plamini o‘ziga akslantiruvchi, o‘zaro bir qiymatli akslantirish **o‘rinlashtirish** deyiladi:

$$F = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ i_1 & i_2 & i_3 & \dots & i_n \end{pmatrix}$$

O‘rinlashtirishni ikkita o‘rin almashtirish bilan berish mumkin:

$$F = \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}.$$

P_2 juft bo‘lsa, F juft;

P_2 toq bo‘lsa, F toq bo‘ladi.

O‘rinlashtirishning signaturasi

$$h(F) = (-1)^{inv\ P_2}$$

$$h(F) = \begin{cases} 1, \ \text{agar} \ \ invP_2 \ \ \text{juft bo'lsa} \\ -1, \ \text{agar} \ \ invP_2 \ \ \text{toq bo'lsa} \end{cases}$$

6-misol. $F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 2 & 5 \end{pmatrix}$ o‘rinlashtirishning signaturasini toping.

Yechilishi: $invP_2 = 1 + 2 + 2 + 0 = 5$

$$h(F) = (-1)^5 = -1$$

O‘rinlashtirish – determinantga tatbiq qilinadi

Barcha mumkin bo‘lgan $F = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ i_1 & i_2 & i_3 \dots & i_n \end{pmatrix}$ o‘rinlashtirishlarga mos

$h(F) \cdot a_{1i_1} a_{2i_2} a_{3i_3} \dots a_{ni_n}$ ko‘rinishdagi *n*! ta ko‘paytmaning yig‘indisidan iborat songa ***n*-tartibli determinant** deyiladi.

Natija:

1. *n*-tartibli determinant *n*! ta hadning yig‘indisidan iborat bo‘ladi.
2. Yig‘indining har bir hadida, har bir satrdan va har bir ustundan bitta element qatnashadi.
3. Ko‘paytmalarning yarmi o‘z ishorasi bilan, qolgan yarmi qarama-qarshi ishora bilan olinadi.

Eslab qoling! Signatura determinant hadlari oldidagi ishorani hosil qiladi.

Uchinchi tartibli determinantni hisoblash formulasi qayerdan kelib chiqdi?

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{23}a_{32}a_{11}$$

Uchinchi tartibli turli o‘rinlashtirishlar soni 3!=1·2·3=6 ta, determinantda 6 ta had bo‘lishini aniqlab oldik. **Ular qanday hadlar?**

$$S_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \qquad S_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \qquad S_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

Bu o‘rinlashtirishlarda 1-satr indeksdagi 1-nomerni, 2-satr esa 2-nomerni bildiradi. 2-satrdagi o‘rin almashtirishlar inversiyalarini topamiz:

S_1 da P_1=(1,2,3) bo‘lib, ***inv***P_1=0
S_2 da P_2=(2,3,1) bo‘lib, ***inv***P_2=2
S_3 da P_3=(3,1,2) bo‘lib, ***inv***P_3=2

S_1, S_2, S_3 lar juft, ularning signaturalari 1 ga teng.
Bu hadlar “+” ishora bilan olinadi.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{23}a_{32}a_{11}$$

$$S_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \qquad S_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \qquad S_6 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

S_4 da P_4=(3,2,1) bo‘lib, $\boldsymbol{inv}P_1$=1+2=3

S_5 da P_5=(2,1,3) bo‘lib, $\boldsymbol{inv}P_2$=1

S_6 da P_6=(1,3,2) bo‘lib, $\boldsymbol{inv}P_3$=1

S_4, S_5, S_6 lar toq, ularga mos signaturalar -1 ga teng. Shuning uchun bu hadlar “-” ishora bilan olinadi.

7-misol. $a_{13}a_{22}a_{31}a_{46}a_{55}a_{64}$ ko‘paytma biror determinantni aniqlovchi yig‘indining hadlaridan birortasini aniqlaydimi?

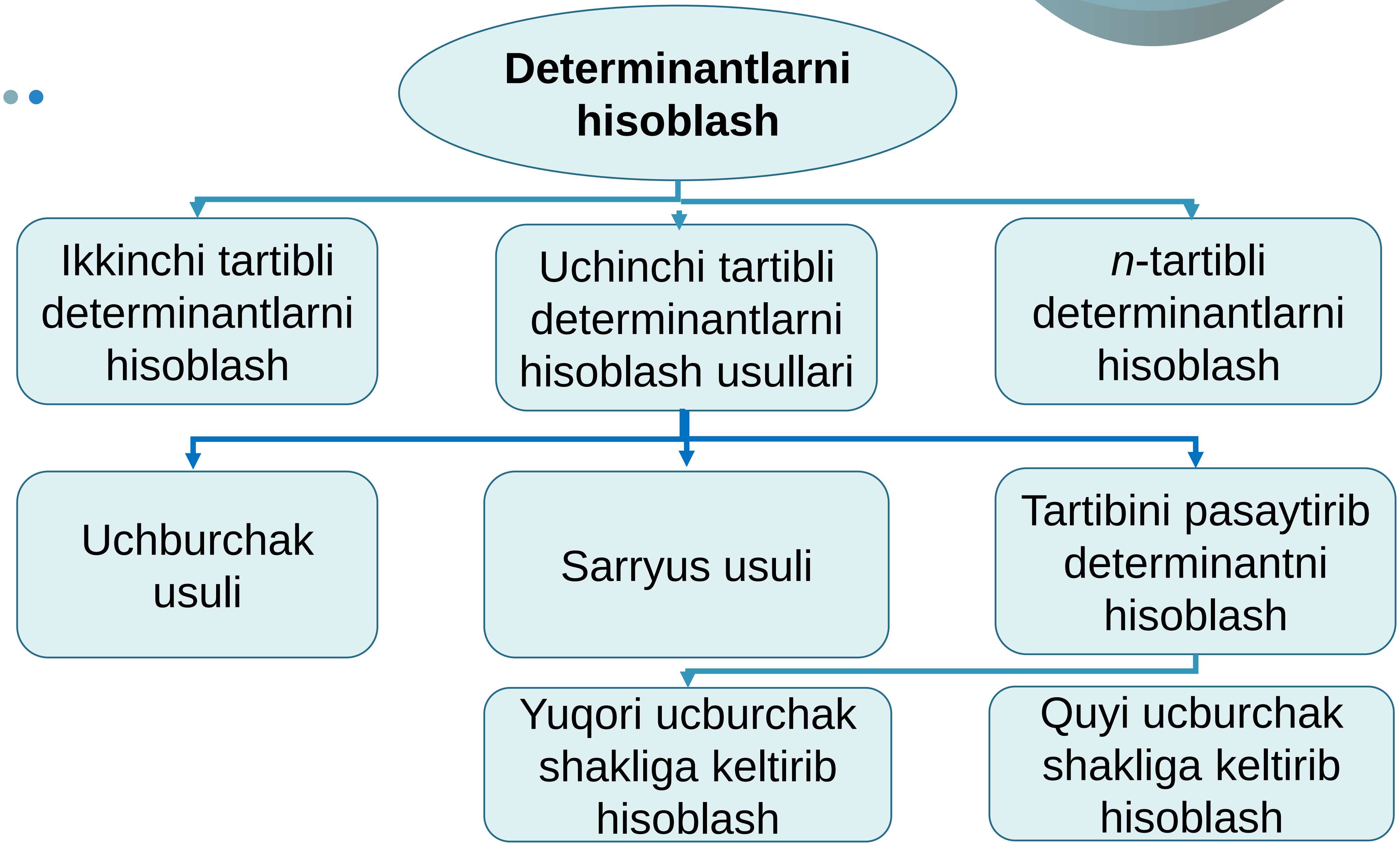
Yechilishi: Determinantning hadi bo‘lishi uchun yig‘indining har bir hadida, har bir satrdan va har bir ustundan bittadan element qatnashishi kerak.

Demak, bu ko‘paytma 6-tartibli determinantning biror hadini ifodalaydi. Buni quyidagi o‘rinlashtirishdan aniqlash mumkin:

$$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 6 & 5 & 4 \end{pmatrix}$$

Bu hadning ishorasini ham topish mumkin.

P=(3, 2, 1, 6, 5, 4) $\quad invP = 1+2+0+1+2=6 \quad h(F)=1$


Determinantlarni hisoblash
Ikkinchi tartibli determinantlarni hisoblash
Uchinchi tartibli determinantlarni hisoblash usullari
n-tartibli determinantlarni hisoblash
Uchburchak usuli
Sarryus usuli
Tartibini pasaytirib determinantni hisoblash
Yuqori ucburchak shakliga keltirib hisoblash
Quyi ucburchak shakliga keltirib hisoblash

O‘z-o‘zini tekshirish uchun savollar:

1. Ikkinchi tartibli determinant deb nimaga aytiladi?
2. Uchinchi tartibli determinantga ta’rif bering.
3. Uchinchi tartibli determinantni hisoblashning qanday usullarini bilasiz?
4. O‘rin almashtirish deb nimaga aytiladi?
5. Inversiya deganda nimani tushundingiz?
6. O‘rin almashtirishlarning qanday xossalari bor?
7. Juft va toq o‘rin almashtirishlar qanday farqlanadi?
8. O‘rinlashtirish deb nimaga aytiladi?
9. Determinantga o‘rinlashtirish asosida ta’rif bering.

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E‘TIBORINGIZ UCHUN RAXMAT!

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