MUHAMMAD AL-XORAZMIY NOMIDAGI
TOSHKENT AXBOROT TEXNOLOGIYALARI UNIVERSITETI

MTH 1234

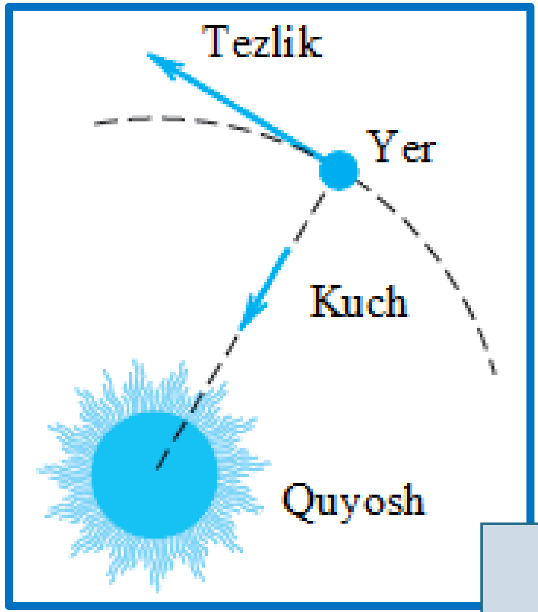
CHIZIQLI ALGEBRA

MAVZU

VEKTORLAR NAZARIYASI ELEMENTLARI

SADADDINOVA SANOBAR SABIROVNA,
DOTSENT

OLIY MATEMATIKA KAFEDRASI



Vektorlar nazariyasi elementlari

Tekislikda vektorlar

Fazoda vektorlar

Vektorlar ustida chiziqli amallar

Vektorlarni qo‘shish

Vektorlarni ayirish

Vektorni skalyar songa ko‘paytirish

Uchburchak qoidasi

Parallelogramm qoidasi

Vektorlarni skalyar ko‘paytirish

Vektorlarni vektor ko‘paytirish

Vektolarni aralash ko‘paytirish

1. Vektorlar nazariyasiga kirish

Vektor atamasini siz umumiy o‘rta maktabning geometriya fanida ko‘rgansiz.

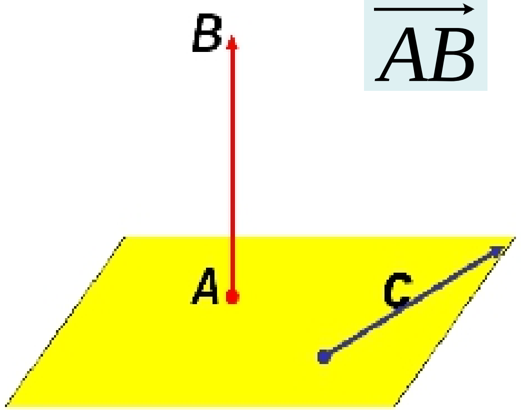
Fizikadan **tezlik, tezlanish, kuch** kabi yo‘nalishga ega kattaliklarni vektor kattalik ekanini,

yo‘l, massa, yuza, hajm – skalyar kattalik ekanini bilib olgansiz.

Biror narsaga ta’sir qilayotgan qandaydir kuchni olib qaraylik. **Kuch qanday miqdorda va qaysi yo‘nalishda ta’sir qilishi juda muhim.**

Tortishish kuchi, itarish kuchi, ishqalanish kuchi borki, ular har xil yo‘nalishlarda ta’sir qiladi.

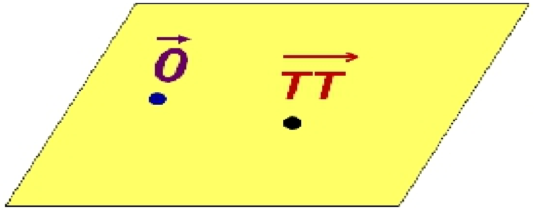
2. Tekislikda va fazoda vektorlar



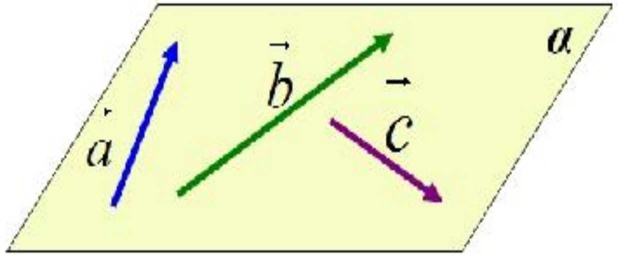
$\overrightarrow{AB}$ Yo‘nalgan kesma yoki nuqtalarning tartiblangan juftligiga **vektor** deyiladi; 1-nuqta vektorning **boshi**, 2-nuqta uning **oxiri** bo‘ladi.

$\vec{a}, \vec{b}, \vec{c}, \ldots$
$\mathbf{a}, \mathbf{b}, \mathbf{c}, \ldots$ vektorlar lotin alifbosining kichik harflari tepasida strelka bilan yoki bosma harflar bilan belgilanadi.

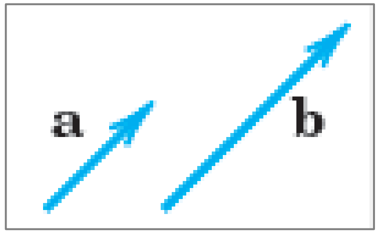
$|\vec{a}|$ Vektorning **uzunligi** deb, uning boshi va oxiri orasidagi masofaga aytiladi.



Boshi va oxiri bir nuqtada bo‘lgan vektor **nol vektor;**

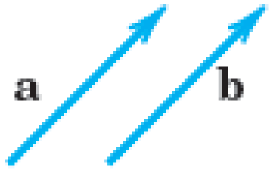


Bir to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotuvchi vektorlar **kollinear vektorlar;**

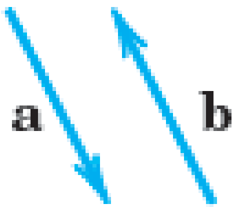


Bir tekislikda yoki parallel tekisliklarda yotuvchi vektorlar **komplanar vektorlar** deyiladi.

Teng va qarama-qarshi vektorlar

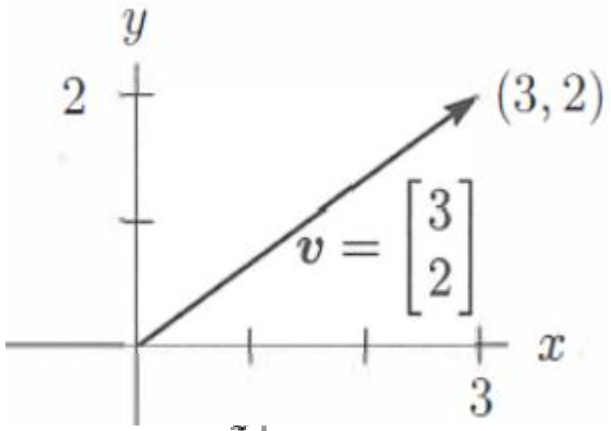


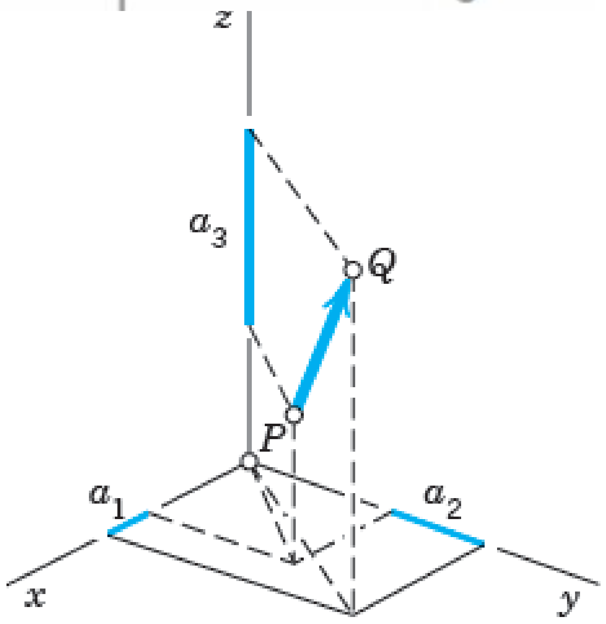
Uzunliklari teng, kollinear va yo‘nalishlari bir xil bo‘lgan vektorlar **teng vektorlar** deyiladi: $\vec{a} = \vec{b}$



Uzunliklari teng, kollinear va yo‘nalishlari har xil bo‘lgan vektorlar **qarama-qarshi vektorlar** deyiladi: $\vec{a} = -\vec{b}$

Vektorning koordinatalari





Vektorning koordinatalari deb, uning koordinata o‘qlaridagi proeksiyalariga aytiladi.

$$Oxy \text{ tekislikda } \quad A(x_1, y_1),\ B(x_2, y_2) \quad \Rightarrow \quad \vec{AB} = \{x_2 - x_1, y_2 - y_1\}$$

$$\vec{BA} = \{x_1 - x_2, y_1 - y_2\}$$

$$\vec{AB} = -\vec{BA}$$

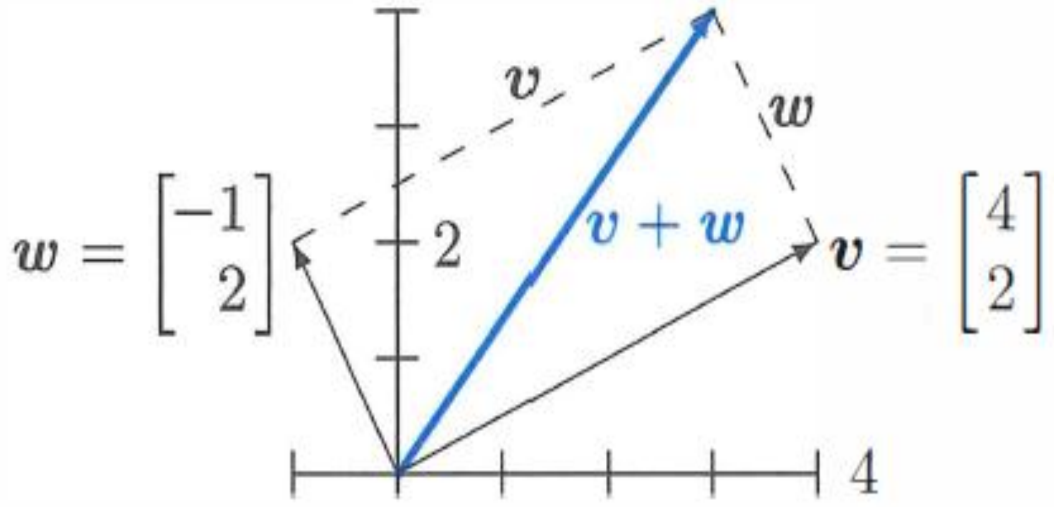
$$Oxyz \text{ fazoda } \quad P(x_1, y_1, z_1),\ Q(x_2, y_2, z_2)$$

$$\vec{PQ} = \{x_2 - x_1, y_2 - y_1, z_2 - z_1\}$$

$$x_2 - x_1 = a_1, \qquad y_2 - y_1 = a_2, \qquad z_2 - z_1 = a_3$$

$$\Rightarrow \quad \vec{PQ} = \{a_1, a_2, a_3\}$$

Vektorlarni qo‘shish



$$v + w = \begin{bmatrix} 4 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$$

$$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$$

$$\mathbf{a} + \mathbf{0} = \mathbf{0} + \mathbf{a} = \mathbf{a}$$

$$\mathbf{a} + (-\mathbf{a}) = \mathbf{0}.$$

Oxy tekislikda

$$\vec{a} = \{x_1, y_1\},\ \vec{b} = \{x_2, y_2\}$$

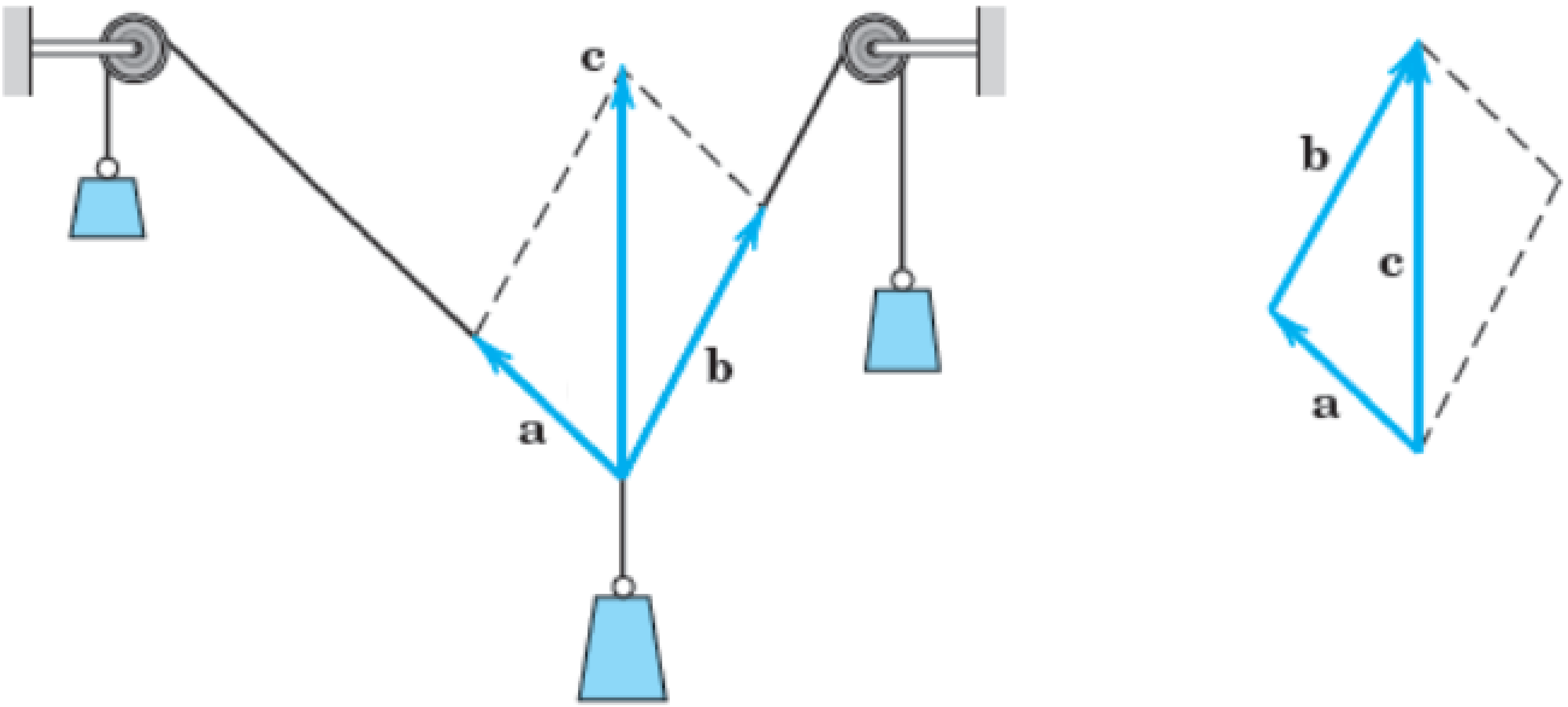
$$\vec{a} + \vec{b} = \vec{c} = \{x_1 + x_2, y_1 + y_2\}$$

$Oxyz$ fazoda

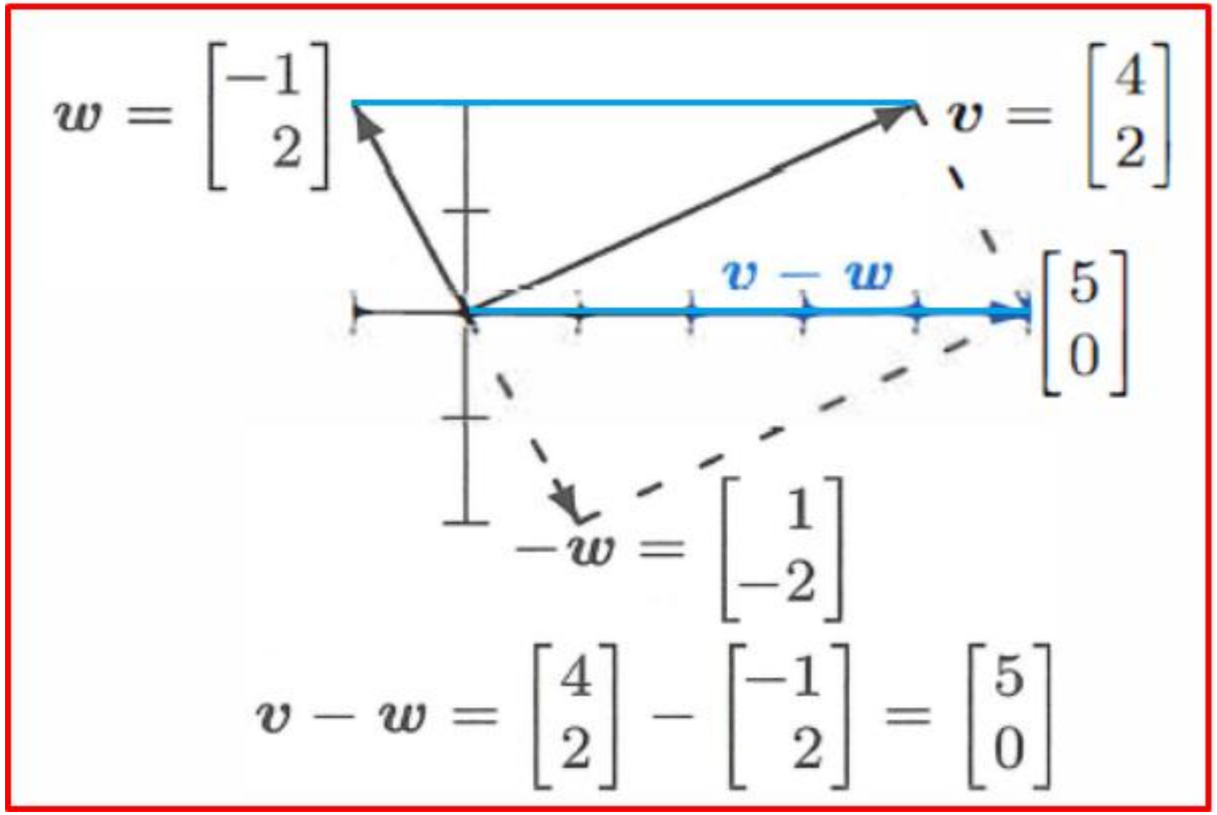
$$\vec{a} = \{x_1, y_1, z_1\},\ \vec{b} = \{x_2, y_2, z_2\}$$

$$\vec{a} + \vec{b} = \vec{c} = \{x_1 + x_2, y_1 + y_2, z_1 + z_2\}$$

Mexanikada kuchlarni qo‘shish



Vektorlarni ayirish



Oxy tekislikda

$$\vec{a} = \{x_1, y_1\},\ \vec{b} = \{x_2, y_2\}$$

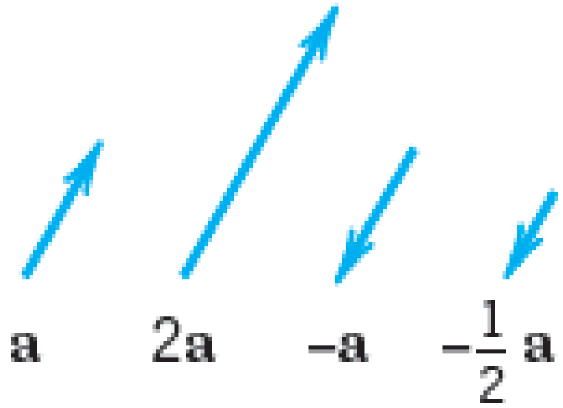
$$\vec{a} - \vec{b} = \vec{c} = \{x_1 - x_2, y_1 - y_2\}$$

$Oxyz$ fazoda

$$\vec{a} = \{x_1, y_1, z_1\},\ \vec{b} = \{x_2, y_2, z_2\}$$

$$\vec{a} - \vec{b} = \vec{c} = \{x_1 - x_2, y_1 - y_2, z_1 - z_2\}$$

Vektorni skalyar songa ko‘paytirish



Oxy tekislikda

$$\lambda \neq 0,\ \vec{a} = \{x, y\} \ \Rightarrow \ \lambda\vec{a} = \{\lambda x, \lambda y\}$$

$Oxyz$ fazoda

$$\lambda \neq 0 \text{ va } \vec{a} = \{x, y, z\} \ \Rightarrow \ \lambda\vec{a} = \{\lambda x, \lambda y, \lambda z\}$$

$$c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$$

$$(c + k)\mathbf{a} = c\mathbf{a} + k\mathbf{a}$$

$$c(k\mathbf{a}) = (ck)\mathbf{a}$$

$$1\mathbf{a} = \mathbf{a}.$$

$$0\mathbf{a} = \mathbf{0}$$

$$(-1)\mathbf{a} = -\mathbf{a}.$$

Bazis vektorlar

n ta $\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n$ vektor va n ta $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ son berilgan bo‘lsin, vektorlar sistemasining chiziqli kombinatsiyasini tuzamiz:

$$\alpha_1 \cdot \vec{a}_1 + \alpha_2 \cdot \vec{a}_2 + \alpha_3 \cdot \vec{a}_3 + \ldots + \alpha_n \cdot \vec{a}_n$$

Agar $\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n$ vektorlar sistemasi uchun kamida bittasi noldan farqli n ta $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ son mavjud bo‘lsa-ki, ular uchun vektorlar sistemasining chiziqli kombinatsiyasi nolga teng, ya’ni $\alpha_1 \cdot \vec{a}_1 + \alpha_2 \cdot \vec{a}_2 + \alpha_3 \cdot \vec{a}_3 + \ldots + \alpha_n \cdot \vec{a}_n = 0$ **(1)** bo‘lsa, bunday vektorlar sistemasiga **chiziqli bog‘liq vektorlar sistemasi** deyiladi.

Aks holda vektorlar **chiziqli erkli** deyiladi, ular uchun (1) tenglik faqat $\alpha_1 = \alpha_2 = \alpha_3 = \ldots = \alpha_n = 0$ bo‘lgandagina o‘rinli bo‘ladi.

Agar vektorlar chiziqli bog‘liq bo‘lsa, (1) tenglikdagi biror vektorni boshqalari orqali ifodalash mumkin.

Fazoning o‘lchovi

Ixtiyoriy $\vec{a}$ vektorni *n* ta chiziqli erkli $\vec{e}_1, \vec{e}_2, \vec{e}_3, \ldots, \vec{e}_n$ vektorlarning chiziqli kombinasiyasi ko‘rinishida ifodalash mumkin bo‘lsa, shu *n* ta vektor fazoning **bazisi** deyiladi.

Bazisni hosil qiladigan vektorlar soni **fazoning o‘lchami,** bazisga kiruvchi vektorlar **bazis vektorlar** deyiladi.

1) **To‘g‘ri chiziq 1 o‘lchovli fazo**, chunki to‘g‘ri chiziqda istalgan $\vec{e}$ vektor bazis hosil qiladi, qolgan vektorlar shu bazis vektor orqali ifodalanadi:

$$\vec{a} = \alpha \cdot \vec{e}\ , \qquad \alpha \neq 0\ .$$

2) **Tekislik 2 o‘lchovli fazo**, chunki tekislikda kollinear bo‘lmagan istalgan ikkita $\vec{e}_1$ va $\vec{e}_2$ vektorlar chiziqli erkli bo‘lib, bazis hosil qiladi, qolgan vektorlarni esa ular orqali ifodalash mumkin:

$$\vec{a} = \alpha\vec{e}_1 + \beta\vec{e}_2\ , \quad \left(\alpha^2 + \beta^2 \neq 0\right).$$

3) **3 o‘lchovli fazoda** $\vec{e}_1, \vec{e}_2$ va $\vec{e}_3$ vektorlar bazis tashkil qiladi:

$$\vec{a} = \alpha\vec{e}_1 + \beta\vec{e}_2 + \gamma\vec{e}_2\ , \quad \left(\alpha^2 + \beta^2 + \gamma^2 \neq 0\right).$$

Bazis vektorlarning uzunliklari har xil bo‘ladi.

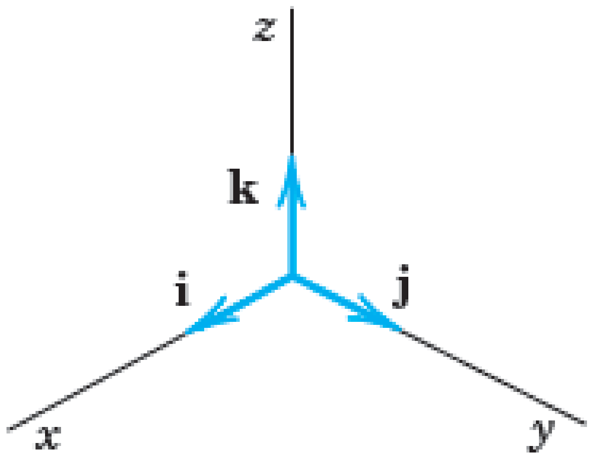
Ortonormal vektorlar

Biz amaliyotda bazis vektorlarning uzunliklarini bir birlikka olib kelamiz.

Agar ikki vektor orasidagi burchak π/2 ga teng bo‘lsa, ular **ortogonal vektorlar** deyiladi.

Ortogonal vektorlarning uzunliklari 1 ga teng bo‘lsa, **ortonormal vektorlar** yoki **ortlar** deyiladi.

Ortonormal vektorlarni asos qilib, Dekart koordinata sistemasini kiritamiz.

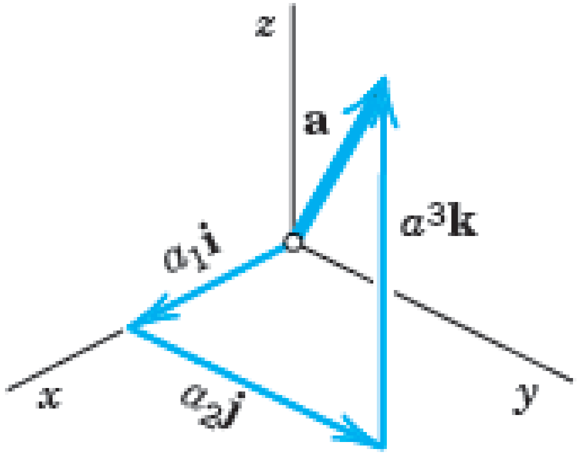


Oxy tekislikda

$$\vec{a}=\{x,y\} \Rightarrow \vec{a}=x\vec{i}+y\vec{j}$$

$Oxyz$ fazoda

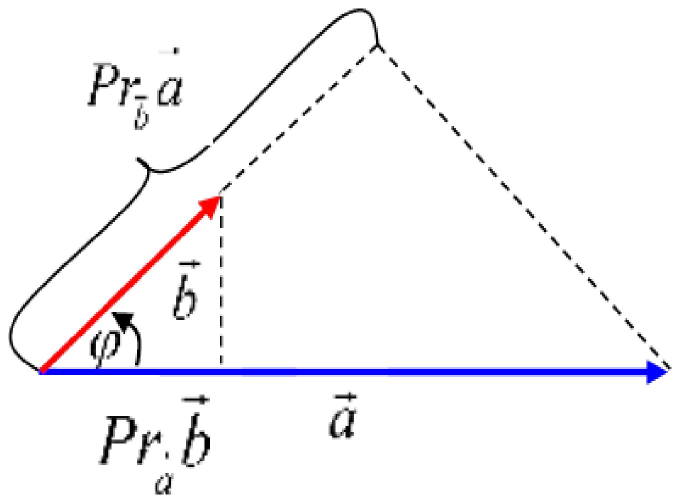
$$\vec{a}=\{a_1,a_2,a_3\} \Rightarrow \vec{a}=a_1\vec{i}+a_2\vec{j}+a_3\vec{k}$$



Vektorlarning skalyar ko‘paytmasi

$\vec{a}$ va $\vec{b}$ **vektorlarning skalyar ko‘paytmasi** deb, bu vektorlar uzunliklarini ular orasidagi burchak kosinusiga ko‘paytmasiga teng bo‘lgan songa aytiladi:

$$\left(\vec{a},\vec{b}\right)=\vec{a}\cdot\vec{b}=\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cdot cos\varphi$$



Agar skalyar ko‘paytmada

$$\frac{\mathrm{Pr}_{\vec{a}}\,\vec{b}}{\left|\vec{b}\right|}=\cos\varphi \;\Rightarrow\; \mathrm{Pr}_{\vec{a}}\,\vec{b}=\left|\vec{b}\right|\cos\varphi \;\Rightarrow\; \vec{a}\cdot\vec{b}=\left|\vec{a}\right|\mathrm{Pr}_{\vec{a}}\,\vec{b}$$

$$\vec{a}\cdot\vec{b}=\left|\vec{a}\right|\cdot Pr_{\vec{a}}\,\vec{b}=\left|\vec{b}\right|\cdot Pr_{\vec{b}}\,\vec{a}$$

Ikki vektorning skalyar ko‘paytmasi ulardan birining uzunligi bilan ikkinchisining shu vektor yo‘nalishidagi proeksiysi ko‘paytmasiga teng.

$$\vec{a}=\{a_1,a_2,a_3\} \text{ va } \vec{b}=\{b_1,b_2,b_3\} \;\Rightarrow\; \vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3$$

Vektorlar skalyar ko‘paytmasining xossalari

1-teorema. Agar $\vec{a}\cdot\vec{b}=0$ bo‘lsa, u holda $\vec{a}$ va $\vec{b}$ vektorlar ortogonal bo‘ladi.

2-teorema. Har qanday vektorning o‘ziga skalyar ko‘paytmasi bu vektorning uzunligi kvadratiga teng: $\vec{a}\cdot\vec{a}=\left|\vec{a}\right|^2$

3-teorema. Skalyar ko‘paytma o‘rin almashtirish qonuniga bo‘ysunadi:

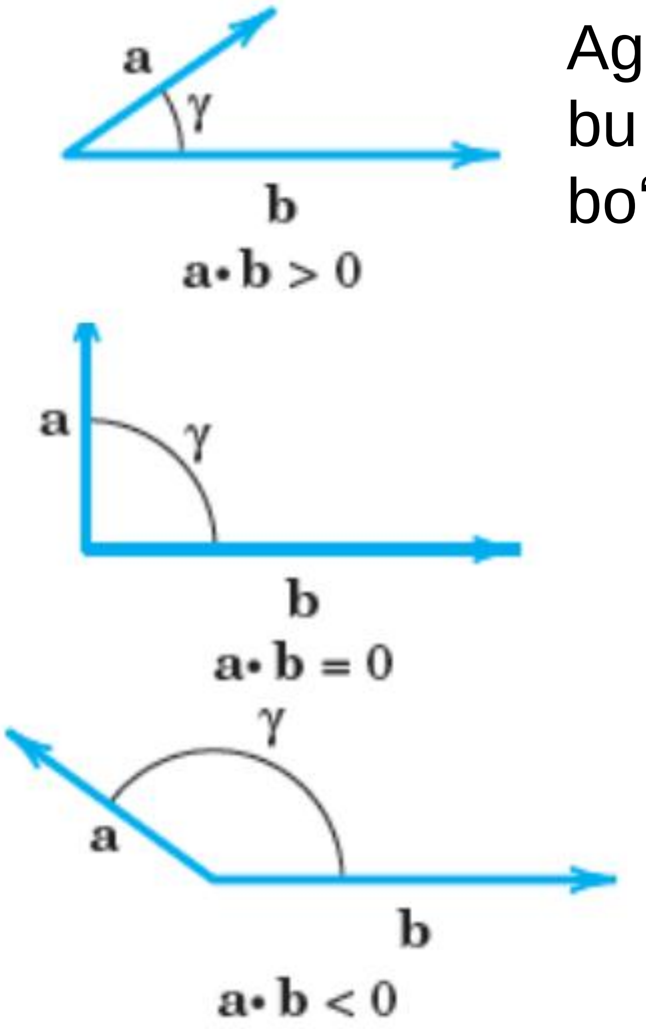
$$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$$

4-teorema. Skalyar ko‘paytma skalyar ko‘paytuvchiga nisbatan gruppalash qonuniga bo‘ysunadi: $\left(\lambda\cdot\vec{a}\right)\cdot\vec{b}=\vec{a}\cdot\left(\lambda\cdot\vec{b}\right)=\lambda\cdot\left(\vec{a}\cdot\vec{b}\right)$

5-teorema. Skalyar ko‘paytma qo‘shishga nisbatan taqsimot qonuniga bo‘ysunadi:

$$\left(\vec{a}+\vec{b}\right)\cdot\vec{c}=\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}$$

Skalyar ko‘paytmaning manbai mexanikadir



Agar jismga o‘zgarmas $\vec{a}$ kuch ta’sir qilayotgan bo‘lsa va bu kuch ta’sirida jism $\vec{b}$ yo‘nalishda harakat qilayotgan bo‘lsa, u holda kuchning bajargan ishi:

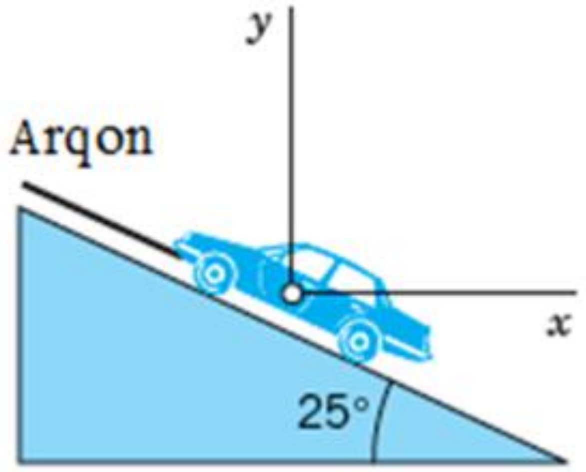
$$A = \left|\vec{a}\right| \cdot \left|\vec{b}\right| \cdot \cos\varphi$$

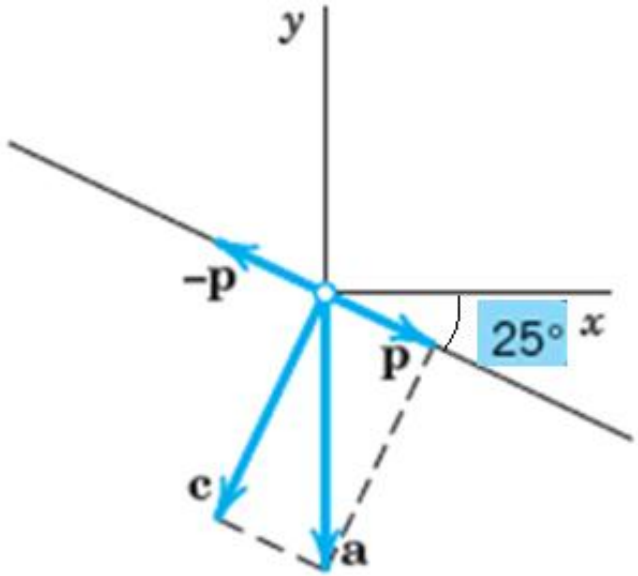
Agar $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak o‘tkir burchak bo‘lsa, bajarilgan ish musbat.

Agar $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak 90^0 bo‘lsa, bajarilgan ish nolga teng.

Agar $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak o‘tmas burchak bo‘lsa, bajarilgan ish manfiy bo‘ladi.

Ma’lum yo‘nalishda ta’sir qiluvchi kuch





1-masala. Agar gorizontal yo‘nalishga 25^0 burchak ostida massasi 2268 kg bo‘lgan avtomobilni arqon bilan tortmoqchi bo‘lsak, arqonga ta’sir qiluvchi kuch nimaga teng bo‘ladi?

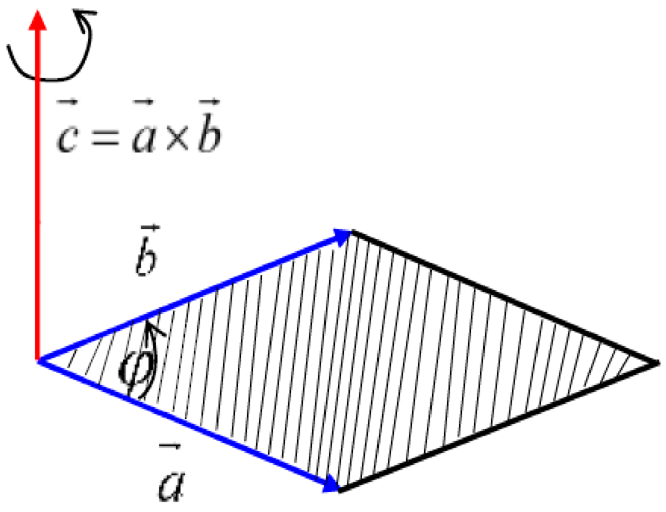
Yechilishi: Voqeani koordinata sistemasida tasvirlaymiz. Avtomobilning og‘irlik kuchi *OY* o‘qiga teskari yo‘nalgani uchun manfiy bo‘ladi:

$$\vec{a} = -2268 \ \text{kg} \qquad \vec{a} = \vec{c} + \vec{p}$$

$\vec{c}$ avtomobilning qiyalikka ko‘rsatayotgan bosim kuchi, bu bizni hozir qiziqtirmaydi. Arqonga ta’sir qiluvchi kuch arqon bo‘ylab yo‘nalgan vektor – taranglik kuchi:

$$|\vec{p}| = |\vec{a}|\cos(90^0 - \varphi) = 2268 \cdot \cos 65 = 958{,}5 \ \text{kg}$$

Vektorlarning vektor ko‘paytmasi

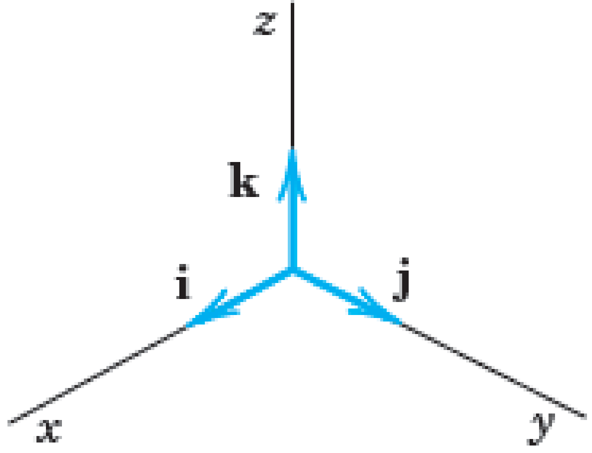


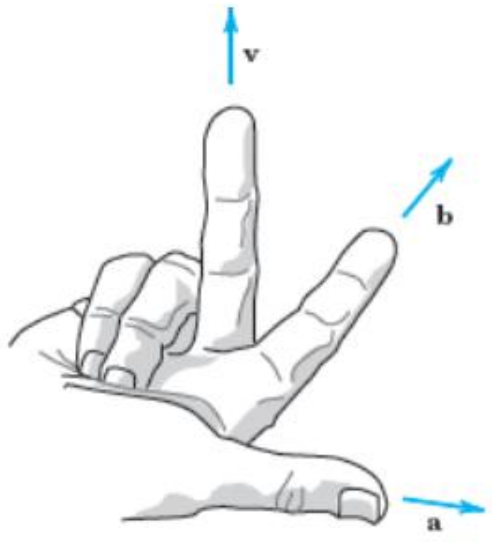
$\vec{a}$ va $\vec{b}$ vektorlarning **vektor ko‘paytmasi** deb, quyidagi shartlarni qanoatlantiradigan $\vec{c}$ vektorga aytiladi:

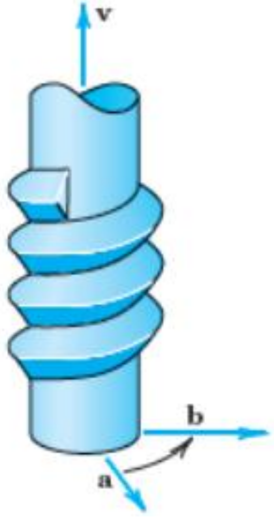
1) $\vec{c}$ vektor $\vec{a}$ va $\vec{b}$ vektorlarga perpendikulyar;

2) $\left|\vec{c}\right|=\left|\vec{a}\right|\cdot\left|\vec{b}\right|\cdot\sin\left(\vec{a},\vec{b}\right);$

3) $\vec{a}\,,\vec{b},\vec{c}$ vektorlarning tartiblangan uchligi o‘ng uchlikni tashkil etadi.







O‘ng qo‘l qoidasi

Vektor ko‘paytmaning xossalari

1^0. Vektor ko‘paytmada ko‘paytuvchilar o‘rnini almashtirilsa, uning ishorasi o‘zgaradi:

$$\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$$

2^0. Vektor ko‘paytma skalyar ko‘paytuvchiga nisbatan gruppalash qonuniga bo‘ysunadi:

$$\left(\lambda\cdot\vec{a}\right)\times\vec{b} = \vec{a}\times\left(\lambda\cdot\vec{b}\right) = \lambda\cdot\left(\vec{a}\times\vec{b}\right)$$

3^0. $\vec{a}$ va $\vec{b}$ vektorlar yig‘indisi bilan $\vec{c}$ vektorning vektor ko‘paytmasi taqsimot qonuniga bo‘ysunadi:

$$\left(\vec{a}+\vec{b}\right)\times\vec{c} = \vec{a}\times\vec{c}+\vec{b}\times\vec{c}$$

Vektor ko‘paytmaning tatbiqlari

$$\vec{a}\{a_x, a_y, a_z\} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\vec{b}\{b_x, b_y, b_z\} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$$

$$\vec{i}\times\vec{i}=0\,,\quad \vec{i}\times\vec{j}=\vec{k}\,,\quad \vec{i}\times\vec{k}=-\vec{j}\,,$$
$$\vec{j}\times\vec{j}=0,\quad \vec{j}\times\vec{k}=\vec{i}\,,\quad \vec{j}\times\vec{i}=-\vec{k}\,,$$
$$\vec{k}\times\vec{k}=0\,,\quad \vec{k}\times\vec{i}=\vec{j}\,,\quad \vec{k}\times\vec{j}=-\vec{i}\,.$$

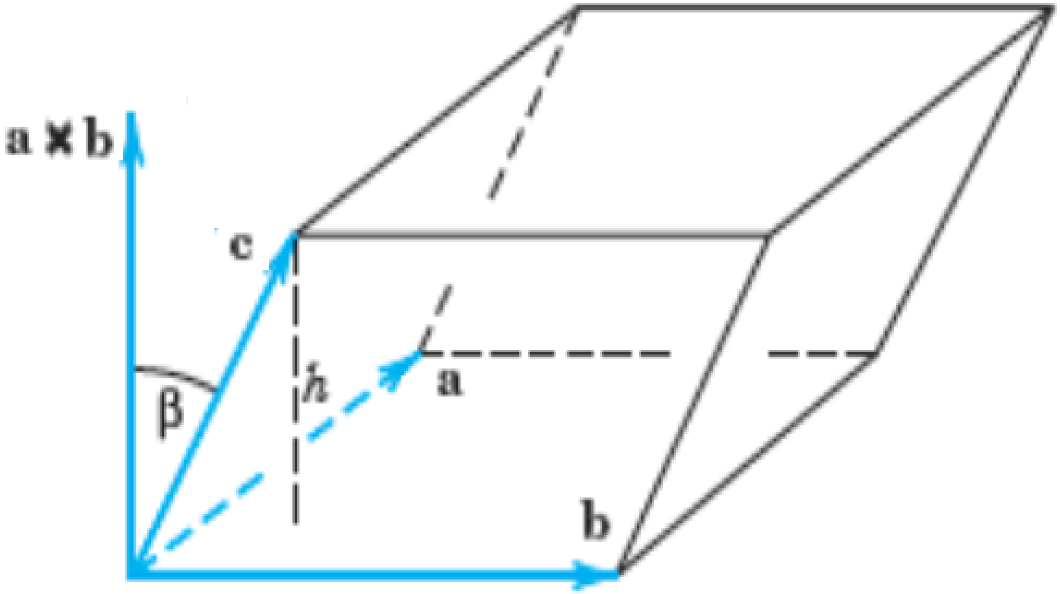
1) $\vec{a}$ va $\vec{b}$ vektorlar kollinear bo‘lishi uchun $\vec{a}\times\vec{b}=0$ bo‘lishi zarur va yetarlidit.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

2) $\vec{a}$ va $\vec{b}$ vektorlarga uchburchak yasalgan bo‘lsin. U holda bu uchburchakning yuzi:

$$S = \frac{1}{2}\left|\vec{a}\times\vec{b}\right| = \frac{1}{2}\operatorname{mod}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_k \\ b_x & b_y & b_z \end{vmatrix}$$

Vektorlarning aralash ko‘paytmasi



$\vec{a}, \vec{b}, \vec{c}$ vektorlar tartiblangan uchligining **aralash ko‘paytmasi** deb, $\vec{a} \times \vec{b}$ vektor bilan $\vec{c}$ vektorning skalyar ko‘paytmasiga teng songa aytiladi: $\left(\vec{a} \times \vec{b}\right) \cdot \vec{c}$

$$V_{parallelepiped} = \left|\left(\vec{a} \times \vec{b}\right) \cdot \vec{c}\right|$$

$$\vec{a} = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$$

$$\vec{b} = x_2\vec{i} + y_2\vec{j} + z_2\vec{k}$$

$$\vec{c} = x_3\vec{i} + y_3\vec{j} + z_3\vec{k}$$

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = x_3 \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} + y_3 \begin{vmatrix} z_1 & x_1 \\ z_2 & x_2 \end{vmatrix} + z_3 \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}.$$

Aralash ko‘paytmaning tatbiqlari

$$\vec{a} = \{x_1, y_1, z_1\}, \quad \vec{b} = \{x_2, y_2, z_2\}, \quad \vec{c} = \{x_3, y_3, z_3\}$$

vektorlar komplanar bo‘lishi uchun

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$$

tenglik o‘rinli bo‘lishi zarur va yetarlidir.

O‘z-o‘zini tekshirish uchun savollar:

1. Tekislikda va fazoda vektor deb nimaga aytiladi?
2. Vektorning matritsaviy ko‘rinishi qanday?
3. Vektorlar ustida chiziqli amallar deganda qanday amallar tushuniladi?
4. Vektorlar ustida bajariladigan chiziqli amallar xossalarini sanab o‘ting?
5. Vektorlarning skalyar ko‘paytmasi deb nimaga aytiladi?
6. Arifmetik vektor uzunligi deb nimaga aytiladi?
7. Vektorlarning uzunligi bo‘ysunadigan qanday xossalarni bilasiz?
8. Vektorlarning skalyar ko‘paytmasi qanday xossalarga bo‘ysunadi?
9. Vektorlarning vektor va aralash ko‘paytmasi deb nimaga aytiladi?

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E’TIBORINGIZ UCHUN RAHMAT!

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