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MTH 1234

CHIZIQLI ALGEBRA

MAVZU

CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMASINI TAQRIBIY YECHISH USULLARI

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OLIY MATEMATIKA KAFEDRASI

Chiziqli algebraik tenglamalar sistemasini yechish usullari

- **To‘g‘ridan-to‘g‘ri yechish usuli**

  (aniq yechish)
  - Kramer usuli
  - Gauss usuli
  - Teskari matritsa usuli
- **Iteratsion yechish usuli**

  (taqribiy yechish)
  - Oddiy iteratsiya
  - Zeydel usuli

Chiziqli algebraik tenglamalar sistemasi

n- tartibli chiziqli algebraik tenglamalar sistemasi quyidagi ko‘rinishlarda beriladi:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases} \qquad (1)$$

$$A\bar{x} = \bar{b} \qquad (2)$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & \cdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \qquad \bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \bar{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Taqribiy yechish usullarini qachon qo‘llash mumkin?

Chiziqli tenglamalar sistemalarida tenglamalar soni $10<n<200$ bo‘lsa, yechimni EHM da topish uchun **iteratsion usullardan** foydalaniladi.

Iteratsion usullarda hisoblash avvalida izlanayotgan yechimga qandaydir boshlang‘ich yaqinlashish berilishi talab qilinadi.

Iteratsion jarayonning yaqinlashish tezligi va shartlari sistemaning *A* matritsasi xossalariga va boshlang‘ich yaqinlashishning tanlanishiga bog‘liq.

$$\begin{cases} x_1 = \dfrac{b_1}{a_{11}} - \dfrac{a_{12}}{a_{11}} x_2 - \ldots - \dfrac{a_{1n}}{a_{11}} x_n \\ x_2 = \dfrac{b_2}{a_{22}} - \dfrac{a_{21}}{a_{22}} x_1 - \ldots - \dfrac{a_{2n}}{a_{22}} x_n \\ \qquad \ldots \\ x_n = \dfrac{b_n}{a_{nn}} - \dfrac{a_{n1}}{a_{nn}} x_1 - \ldots - \dfrac{a_{n,n-1}}{a_{nn}} x_{n-1} \end{cases}$$

Bu usulni qo‘llash uchun

1) $a_{ii} \neq 0$

2) $\max\limits_{j} \sum\limits_{i=1; i\neq j}^{n} \left| \dfrac{a_{ij}}{a_{ii}} \right| < 1$

shartlar bajarilishi kerak

Oddiy iteratsiya usuli

(1) yoki (2) sistemalarga iteratsiya usulini qo‘llash uchun quyidagi ko‘rinishga o‘tkazish zarur:

$$\bar{x} = G\bar{x} + \bar{f} \qquad (3)$$

Shundan so‘ng iteratsiya jarayoni rekurrent formulalar yordamida bajariladi:

$$\bar{x}^{(k+1)} = G\bar{x}^{(k)} + \bar{f}, \quad k = 0, 1, 2, \dots . \qquad (3^*)$$

G matritsa va $\bar{f}$ vektor (1) sistemani shakl almashtirish natijasida hosil qilinadi. (3*) yaqinlashuvchi bo‘lishi uchun $|\lambda_i(G)| < 1$ shart bajarilishi zarur va yetarlidir, bunda $\lambda_i(G)$ – G matritsaning barcha xos qiymatlari.

$||G||<1$ yoki $|\lambda_i(G)|<\forall||G||$ shartlardan biri bajarilganda ham (3*) yaqinlashuvchi bo‘ladi. ||...|| - **matritsa normasi**.

Normani aniqlash uchun quyidagi shart tekshirib ko‘riladi:

$$\|G\| = \max_{1\le i\le n} \sum_{j=1}^{n} |g_{ij}| \quad \text{yoki} \quad \|G\| = \max_{1\le j\le n} \sum_{i=1}^{n} |g_{ij}| \qquad (4)$$

bunda $G = \{g_{ij}\}_1^n$ $g_{ij} = -a_{ij} / a_{ii}\,;$

Agar *A* matritsa diagonal xususiyatiga ega bo‘lsa, ya’ni

$$|a_{ii}| > \sum_{i,j=1; i\ne j}^{n} |a_{ij}|, \qquad A = \{a_{ij}\}_1^n \qquad (5)$$

shart bajarilsa, yaqinlashish kafolatlangan bo‘ladi.

Agar (4) yoki (5) shart bajarilsa, iteratsiya usuli ixtiyoriy $\bar{x}^{(0)}$ boshlang‘ich taqribiy qiymat bo‘yicha yaqinlashadi.

A matritsaga ega (2) tenglamalar sistemasini (3) ko‘rinishga keltirish yoki (4) va (5) yaqinlashish shartlarini ta’minlash maqsadida shakl almashtirish bo‘yicha yondoshuvlar ko‘p.

Masalan, $\bar{x} = G\bar{x} + \bar{f}$ (3) ni quyidagi usulda hosil qilish mumkin:

1) $A = B + C$, $\det B \neq 0$ bo‘lsin. U holda

$A\bar{x} = \bar{b}$ $(B+C)\bar{x} = \bar{b}$ $B\bar{x} = -C\bar{x} + \bar{b}$

$$B^{-1}B\bar{x} = -B^{-1}C\bar{x} + B^{-1}\bar{b}$$

$$\bar{x} = -B^{-1}C\bar{x} + B^{-1}\bar{b}$$

$-B^{-1}C = G$ va $B^{-1}\bar{b} = \bar{f}$ deb olib, $\bar{x} = G\bar{x} + \bar{f}$

(4) va (5) yaqinlashish shartlariga ko‘ra $A = B + C$ ixtiyoriy bo‘la olmaydi.

2) Agar A matritsa $|a_{ii}| > \sum_{i,j=1; i\neq j}^{n} |a_{ij}|, \qquad A = \{a_{ij}\}_1^n$ (5) shartni qanoatlantirsa, u holda B matritsa sifatida pastki uchburchak matritsani olish mumkin:

$$B = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ & \cdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \qquad a_{ii} \neq 0.$$

3) Yoki $A\bar{x} = \bar{b}$

$$A\bar{x} - \bar{b} = 0$$

$$\bar{x} + (A\bar{x} - \bar{b}) = \bar{x}$$

$$\bar{x} = \bar{x} + \lambda(A\bar{x} - \bar{b}) = \bar{x} + \lambda A\bar{x} - \lambda\bar{b} = (E + \lambda A)\bar{x} - \lambda\bar{b} = G\bar{x} + \bar{f}$$

λ parametrni shunday tanlash kerakki, $\|G\| = \|E + \lambda A\| < 1$ shart bajarilsin.

Agar (5) o‘rinli bo‘lsa, u holda almashtirishni (3) ga keltirish oson bo‘ladi, ya’ni (1) sistemaning har bir *i*- tenglamasini quyidagi rekurrent formulalar bo‘yicha x_i ga nisbatan yechiladi:

$$x_i^k = -\frac{1}{a_{ii}}\left[\sum_{j=1;\, j\neq i}^{n} a_{ij}\, x_j^{k-1} - b_i\right] = \sum_{j=1}^{n} g_{ij}\, x_j^{k-1} + f_i\,;$$

$$g_{ij} = -\,a_{ij} / a_{ii}\,; \qquad g_{ii} = 0;\ f_i = b_i / a_{ii}\,, \qquad (5^*)$$

ya’ni $G = \{g_{ij}\}_1^n$

Agar *A* matritsada diagonallik xususiyati bo‘lmasa, teng kuchlilikni saqlagan holda qandaydir chiziqli almashtirishlar bajarib, diagonallik xususiyatini hosil qilish kerak.

1-misol. Uch no’malumli chiziqli tenglamalar sistemasini yeching:

$$\begin{cases} 2x_1 - 1{,}8x_2 + 0{,}4x_3 = 1; & (I) \\ 3x_1 + 2x_2 - 1{,}1x_3 = 0; & (II) \\ x_1 - x_2 + 7{,}3x_3 = 0; & (III) \end{cases}$$

Yechilishi:

$$|a_{ii}| > \sum_{i,j=1;i\neq j}^{n} |a_{ij}|$$

1-tenglamani α ga, 2-tenglamani β ga ko‘paytirib, qo‘shamiz:

$$(2\alpha + 3\beta)\, x_1 + (-1{,}8\alpha + 2\beta)\, x_2 + (0{,}4\alpha - 1{,}1\beta)x_3 = \alpha .$$

$\alpha = \beta = 5$ deb olib, $25x_1 + x_2 - 3{,}5x_3 = 5$

2-tenglamani δ ga, 2-tenglamani -γ ga ko‘paytirib, qo‘shamiz:

$$(3\delta - 2\gamma)\, x_1 + (2\delta + 1{,}8\gamma)\, x_2 + (-1{,}1\delta - 0{,}4\gamma)x_3 = -\gamma .$$

$\delta = 2$, $\gamma = 3$ deb olib, $0x_1 + 9{,}4x_2 - 3{,}4x_3 = -3$ tenglamani hosil qilamiz rekurrent formulalarni hosil qilamiz.

$$\begin{cases} 25x_1 + x_2 - 3{,}5x_3 = 5; \\ 9{,}4x_2 - 3{,}4x_3 = -3; \\ x_1 - x_2 + 7{,}3x_3 = 0. \end{cases}$$

$$\begin{cases} x_1 + 0{,}04x_2 - 0{,}14x_3 = 0{,}2; \\ x_2 - 0{,}36x_3 = -0{,}32; \\ 0{,}14x_1 - 0{,}14x_2 + x_3 = 0. \end{cases}$$

$$\begin{cases} x_1 = -0{,}04x_2 + 0{,}14x_3 + 0{,}2; \\ x_2 = 0{,}36x_3 - 0{,}32; \\ x_3 = -0{,}14x_1 + 0{,}14x_2. \end{cases}$$

Ozod hadlar ustunidagi sonlarni $\overline{x}^{(0)}$ yechim sifatida olamiz.

$\overline{x}^{(0)}$ = (0,2; –0,32; 0)^T

$$\begin{aligned} x_1^{(k+1)} &= -0{,}04x_2^{(k)} + 0{,}14x_3^{(k)} + 0{,}2; \\ x_2^{(k+1)} &= 0{,}36x_3^{(k)} - 0{,}32; \\ x_3^{(k+1)} &= -0{,}14x_1^{(k)} + 0{,}14x_2^{(k)}. \end{aligned}$$

k = 0, 1, 2, ...

$$\left| \overline{x}^{(k+1)} - \overline{x}^{(k)} \right| < \varepsilon$$

2-misol. Oddiy iteratsiya usulini qo‘llab, ε=0,001 aniqlik bilan tenglamalar sistemasini yeching:

$$\begin{cases} 0{,}32x_1 - 0{,}05x_2 + 0{,}11x_3 - 0{,}08x_4 = -2{,}15; \\ 0{,}11x_1 + 0{,}16x_2 - 0{,}28x_3 - 0{,}06x_4 = 0{,}83; \\ 0{,}08x_1 - 0{,}15x_2 \qquad + 0{,}12x_4 = -1{,}16; \\ -0{,}21x_1 + 0{,}13x_2 - 0{,}27x_3 \qquad = -0{,}44. \end{cases}$$

Yechilishi: $\boxed{\overline{x}^{(k+1)} = G\,\overline{x}^{(k)} + \overline{f}}$

$$\begin{cases} x_1 = \quad 0{,}32x_1 - 0{,}05x_2 + 0{,}11x_3 - 0{,}08x_4 + 2{,}15; \\ x_2 = \quad 0{,}11x_1 + 0{,}16x_2 - 0{,}28x_3 - 0{,}06x_4 - 0{,}83; \\ x_3 = \quad 0{,}08x_1 - 0{,}15x_2 \qquad + 0{,}12x_4 + 1{,}16; \\ x_4 = -0{,}21x_1 + 0{,}13x_2 - 0{,}27x_3 \qquad + 0{,}44. \end{cases}$$

$$\| \overline{x}^* - \overline{x}^{(k)} \| \le \frac{\|G\|^{k+1}}{1-\|G\|} \cdot \|\overline{f}\| \le 0{,}001. \qquad \|\overline{f}\| = 2{,}15$$

Diagonallik xususiyati mavjud: $\|G\| = \max_{1\le i\le 4} \sum_{j=1}^{4} |g_{ij}| = \max\{0{,}56; 0{,}61; 0{,}35; 0{,}61\} = 0{,}61 < 1$

Dastlabki yaqinlashish sifatida ozod hadlar ustunini olamiz:

$$\begin{cases} x_1 = \ \ 0{,}32x_1 - 0{,}05x_2 + 0{,}11x_3 - 0{,}08x_4 + 2{,}15; \\ x_2 = \ \ 0{,}11x_1 + 0{,}16x_2 - 0{,}28x_3 - 0{,}06x_4 - 0{,}83; \\ x_3 = \ \ 0{,}08x_1 - 0{,}15x_2 + 0{,}12x_4 + 1{,}16; \\ x_4 = -0{,}21x_1 + 0{,}13x_2 - 0{,}27x_3 + 0{,}44. \end{cases}$$

$$\bar{x}^{(0)} = (2{,}15; -0{,}83; 1{,}16; 0{,}44)^T$$

$$x_1^{(1)} = \ \ 0{,}32 \cdot 2{,}15 + 0{,}05 \cdot 0{,}83 + 0{,}11 \cdot 1{,}16 - 0{,}08 \cdot 0{,}44 + 2{,}15 = 2{,}9719;$$

$$x_2^{(1)} = \ \ 0{,}11 \cdot 2{,}15 - 0{,}16 \cdot 0{,}83 - 0{,}28 \cdot 1{,}16 - 0{,}06 \cdot 0{,}44 - 0{,}83 = -1{,}0775;$$

$$x_3^{(1)} = \ \ 0{,}08 \cdot 2{,}15 + 0{,}15 \cdot 0{,}83 + 0{,}12 \cdot 0{,}44 + 1{,}16 = 1{,}5093;$$

$$x_4^{(1)} = -0{,}21 \cdot 2{,}15 - 0{,}13 \cdot 0{,}83 - 0{,}27 \cdot 1{,}16 + 0{,}44 = -0{,}4326.$$

$$\bar{x}^{(1)} = (2{,}9719;\ -1{,}0775;\ 1{,}5093;\ -0{,}4326)$$

$$\left\| \ \bar{x}^* - \bar{x}^{(k)} \right\| \le 0{,}001$$

Hisoblashni davom etib, natijalarni jadvalga kiritamiz:

k	x_1	x_2	x_3	x_4
0	2,15	–0,83	1,16	0,44
1	2,9719	–1,0775	1,5093	–0,4326
2	3,3555	–1,0721	1,5075	–0,7317
3	3,5017	–1,0106	1,5015	–0,8111
4	3,5511	–0,9277	1,4944	–0,8321
5	3,5637	–0,9563	1,4834	–0,8298
6	3,5678	–0,9566	1,4890	–0,8332
7	3,5760	–0,9575	1,4889	–0,8356
8	3,5709	–0,9573	1,4890	–0,8362
9	3,5712	–0,9571	1,4889	–0,8364
10	3,5713	–0,9570	1,4890	–0,8364

$$\left| \bar{x}^{(k+1)} - \bar{x}^{(k)} \right| < \varepsilon$$

$x_1 = 3,5713$

$x_2 = -0,957$

$x_3 = 1,489$

$x_4 = -0,8364$

Zeydel usuli

Zeydel usuli oddiy iteratsiyaning modifikatsiyalangan usuli bo‘lib,

$$\bar{x} = G\,\bar{x} + \bar{f}$$

$$\begin{aligned}
x_1^{(k+1)} &= g_{11}x_1^{k} + \ldots + g_{1n}x_n^{k} + f_1;\\
x_2^{(k+1)} &= g_{21}x_1^{(k+1)} + \ldots + g_{2n}x_n^{(k)} + f_2;\\
x_3^{(k+1)} &= g_{31}x_1^{(k+1)} + \ldots + g_{31}x_n^{(k)} + f_3;\\
&\ldots\\
x_n^{(k+1)} &= g_{n1}x_1^{(k+1)} + \ldots + g_{nn}x_n^{(k)} + f_n.
\end{aligned}$$

3-misol. Sistemani yeching:

$$\begin{cases} 10x_1 + x_2 - 3x_3 - 2x_4 + x_5 = 6 \\ -x_1 + 25x_2 + x_3 - 5x_4 - 2x_5 = 11 \\ 2x_1 + x_2 - 20x_3 + 2x_4 - 3x_5 = -19 \\ x_2 - x_3 + 10x_4 - 5x_5 = 10 \\ x_1 + 2x_2 + x_3 + 2x_4 - 20x_5 = -32 \end{cases}$$

Yechilishi: $a_{ii} \neq 0$

Sistemadagi tenglamalarni mos ravishda 10, 25,-20,10,-20 larga bo'lib chiqamiz.

$$\begin{cases} 10x_1 + x_2 - 3x_3 - 2x_4 + x_5 = 6 \quad /10 \\ -x_1 + 25x_2 + x_3 - 5x_4 - 2x_5 = 11 \quad /25 \\ 2x_1 + x_2 - 20x_3 + 2x_4 - 3x_5 = -19 \quad /-20 \\ x_2 - x_3 + 10x_4 - 5x_5 = 10 \quad /10 \\ x_1 + 2x_2 + x_3 + 2x_4 - 20x_5 = -32 \quad /-20 \end{cases}$$

$$\begin{cases} x_1 + 0{,}1x_2 - 0{,}3x_3 - 0{,}2x_4 + 0{,}1x_5 = 0{,}6 \\ -0{,}04x_1 + x_2 + 0{,}04x_3 - 0{,}2x_4 - 0{,}08x_5 = 0{,}44 \\ 0{,}1x_1 + 0{,}05x_2 - x_3 + 0{,}1x_4 - 0{,}15x_5 = -0.95 \\ -0{,}1x_2 - 0{,}1x_3 + x_4 - 0{,}5x_5 = 1 \\ 0{,}05x_1 + 0{,}1x_2 + 0.05x_3 + 0{,}1x_4 - x_5 = -1{,}6 \end{cases}$$

$$
\begin{cases}
x_1 = 0,6 - 0,1x_2 + 0,3x_3 + 0,2x_4 - 0,1x_5 \\
x_2 = 0,44 + 0,04x_1 - 0,04x_3 + 0,2x_4 + 0,08x_5 \\
x_3 = 0,95 + 0,1x_1 + 0,05x_2 + 0,1x_4 - 0,15x_5 \\
x_4 = 1 - 0,1x_2 + 0,1x_3 + 0,5x_5 \\
x_5 = 1,6 + 0,05x_1 + 0,1x_2 + 0.05x_3 + 0,1x_4
\end{cases}
$$

$$
\max_i \sum_{j=1; i \neq j}^{n} \left| \frac{a_{ij}}{a_{ii}} \right| < 1
$$

$$
a_{ii} \neq 0
$$

$$
\begin{cases}
0,1 + 0,3 + 0,2 + 0,1 = 0,7 < 1 \\
0,04 + 0,04 + 0,2 + 0,08 = 0,36 < 1 \\
0,1 + 0,05 + 0,1 + 0,15 = 0,4 < 1 \\
0,1 + 0,1 + 0,5 = 0,7 < 1 \\
0,05 + 0,1 + 0,05 + 0,1 = 0,3 < 1
\end{cases}
$$

shartlar bajariladi

Dastlabki yaqinlashish sifatida ozod hadlar ustuni olinadi:

$$\overline{x}^{(0)} = (0,6;\ 0,44;\ 0,95;\ 1;\ 1,6)$$

$$\overline{x}^{(0)} = (x_1^{(0)} = 0,6; x_2^{(0)} = 0,44; x_3^{(0)} = 0,95; x_4^{(0)} = 1; x_5^{(0)} = 1,6)$$

$$\begin{cases} x_1^{(1)} = 0,6 - 0,1x_2^{(0)} + 0,3x_3^{(0)} + 0,2x_4^{(0)} - 0,1x_5^{(0)} \\ x_2^{(1)} = 0,44 + 0,04x_1^{(0)} - 0,04x_3^{(0)} + 0,2x_4^{(0)} + 0,08x_5^{(0)} \\ x_3^{(1)} = 0,95 + 0,1x_1^{(0)} + 0,05x_2^{(0)} + 0,1x_4^{(0)} - 0,15x_5^{(0)} \\ x_4^{(1)} = 1 - 0,1x_2^{(0)} + 0,1x_3^{(0)} + 0,5x_5^{(0)} \\ x_5^{(1)} = 1,6 + 0,05x_1^{(0)} + 0,1x_2^{(0)} + 0.05x_3^{(0)} + 0,1x_4^{(0)} \end{cases}$$

$$\begin{cases} x_1^{(1)} = 0,6 - 0,1 \cdot 0,44 + 0,3 \cdot 0,95 + 0,2 \cdot 1 - 0,1 \cdot 1,6 = 0,881 \\ x_2^{(1)} = 0,44 + 0,04 \cdot 0,6 - 0,04 \cdot 0,95 + 0,2 \cdot 1 + 0,08 \cdot 1,6 = 0,754 \\ x_3^{(1)} = 0,95 + 0,1 \cdot 0,6 + 0,05 \cdot 0,44 + 0,1 \cdot 1 - 0,15 \cdot 1,6 = 0,892 \\ x_4^{(1)} = 1 - 0,1 \cdot 0,44 + 0,1 \cdot 0,95 + 0,5 \cdot 1,6 = 1,851 \\ x_5^{(1)} = 1,6 + 0,05 \cdot 0,6 + 0,1 \cdot 0,44 + 0.05 \cdot 0,95 + 0,1 \cdot 1 = 1,72 \end{cases}$$

$$\overline{x}^{(1)} = (0,881;\ 0,754;\ 0,892;\ 1,851;\ 1,72)$$

Kerakli aniqlikkacha davom qilinadi:

$$\left\| \overline{x}^{*} - \overline{x}^{(k+1)} \right\| \le \varepsilon$$

Hisoblashni davom etib, natijalarni jadvalga kiritamiz:

k	x_1	x_2	x_3	x_4	x_5
0	0,6	0,44	0,95	1	1,6
1	0,881	0,754	0,892	1,851	1,72
2	0,9884	0,9482	1,0029	1,9147	1,9859
3	0,9904	0,9814	0,9908	1,9939	1,9854
4	0,99944	0,99753	0,99768	1,99364	1,99897
5	0,99839	0,99865	0,99929	1,99954	1,99970
6	0,99986	0,99989	0,99977	1,99976	1,99960
7	0,999934	0,999920	1,000018	1,999788	1,999947
8	0,999974	0,999951	0,999976	2,000042	1,999978

Yechim: $x = (1;\ 1;\ 1;\ 2;\ 2)$

O‘z-o‘zini tekshirish uchun savollar:

1. Chiziqli algebraik tenglamalar sistemasini yechish usullarini aytib bering.
2. Chiziqli algebraik tenglamalar sistemasini taqribiy yechish usullari qachon qo‘llaniladi?
3. Matritsa normasi qanday aniqlanadi?
4. Chiziqli algebraik tenglamalar sistemasini taqribiy yechishning oddiy iteratsiya usulini tushuntiring.
5. Chiziqli algebraik tenglamalar sistemasini taqribiy yechishning Zeydel usuli qanday amalga oshiriladi?
6. Iteratsion jarayon yaqinlashishining zaruriy va yetarli shartlarini bilasizmi?
7. Chiziqli algebraik tenglamalar sistemasini yechishning eng kichik kvadratlar usulini bilasizmi?

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E’TIBORINGIZ UCHUN RAHMAT!

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