MUHAMMAD AL-XORAZMIY NOMIDAGI
TOSHKENT AXBOROT TEXNOLOGIYALARI UNIVERSITETI

CHIZIQLI ALGEBRA

MTH 1234

MAVZU

MATRITSALAR VA ULAR USTIDA AMALLAR

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DOTSENT

OLIY MATEMATIKA KAFEDRASI

Matritsalar va ular ustida amallar

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Asosiy tushunchalar

1. Matritsa tushunchasi

Matritsa tushunchasi birinchi marta XIX asrning o‘rtalarida *U. Gamilton*, *A. Keli* va *J. Silvester* asarlarida uchraydi.

Matritsalar nazariyasining asoslarini *K. Weyershtrass* va *G. Frobenius* XIX asrning oxiri va XX asrning boshlarida yaratishgan.

Hozirda matritsalar tabiiy va amaliy jarayonlarning **matematik modellarini qurishda** ishlatiladi

Ko‘pincha İqtisodiy masalalarni chiziqli tenglamalar sistemasiga keltirib yechishda qo‘llaniladi.

1-misol.

$$\begin{array}{cccc} \text{F.I.O} & \text{yoshi} & \text{jinsi} & \text{oylik maoshi} \\ \end{array}$$
$$\begin{pmatrix} 1 & 23 & 1 & 3\,\text{mln} \\ 2 & 47 & 0 & 5.1\,\text{mln} \\ 3 & 60 & 1 & 13\,\text{mln} \\ \dots & \dots & \dots & \dots \\ 100 & 39 & 1 & 20\,\text{mln} \end{pmatrix}$$

100 ta satr va 4 ta ustun

Korxonalar va ularda ishlab chiqariladigan mahsulotlar jadvali bo'lisi mumkin

$$\begin{array}{c|ccccc} & \text{telefon} & \text{TV} & \text{sovutgich} & \text{soat} & \text{gaz plita} \\ \hline \text{Artel} & 1235 & 2304 & 1500 & 3514 & 1000 \\ \text{Samsung} & 2180 & 4710 & 1600 & 5120 & 1101 \\ \text{LG} & 1345 & 1860 & 1450 & 1384 & 1007 \\ \text{Roison} & 1408 & 1543 & 1670 & 3456 & 996 \\ \text{Indesit} & 1654 & 1392 & 1300 & 2015 & 1109 \end{array}$$

5 ta satr va 5 ta ustun

$m \times n$ o‘lchovli matritsa

m ta satrdan va n ustundan iborat $m \cdot n$ ta elementdan tuzilgan to‘g‘ri burchakli jadvalga $m \times n$ **o‘lchovli matritsa** deyiladi.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \qquad A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}. \qquad A = \begin{Vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{Vmatrix}$$

Matritsalar lotin alifbosining bosh harflari bilan belgilanadi: *A*, *B*, *C*, …

Matritsaning o‘lchamlarini ifodalash uchun $\boldsymbol{A_{m\times n}}$ kabi belgilanadi.

Matritsaning *i*-satr, *j*-ustun kesishmasidagi element - a_{ij}.

$$A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

$a_{1n}, a_{2n-1}, \dots, a_{n1}$

yordamchi diagonal

$a_{11}, a_{22}, \dots, a_{nn}$

asosiy diagonal

2. Matritsalarning turlari

$(1 \times n)$ o‘lchovli **satr matritsa**

$$K = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \end{pmatrix}$$

$(m \times 1)$ o‘lchovli **ustun matritsa**

$$L = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}.$$

Barcha elementlari nollardan iborat ixtiyoriy o‘lchovli matritsa – **nol matritsa**

$$\Theta = \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{pmatrix}.$$

Boshlang‘ich element

Nolmas satrning **boshlang‘ich elementi** deb, chapdan hisoblaganda dastlabki noldan farqli elementga aytiladi.

$$K=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12 \\ 0 & -9 & 5 & 9 \end{pmatrix}$$

2-satrning boshlang‘ich elementi k_{24}=12,

3-satrning boshlang‘ich elementi $k_{32} = -9$

Pog‘onasimon matritsa

Agar $A_{m\times n} = \left(a_{ij}\right)$ matritsa quyidagi shartlarni qanoatlantirsa, unga **pog‘onasimon matritsa** deyiladi:

1) nollar satri mavjud bo‘lsa, u barcha nolmas satrlardan pastda joylashgan bo‘lishi kerak;
2) nolmas satrlarning boshlang‘ich elementlari o‘sish tartibida yozilishi kerak, ya’ni $a_{k_1 \mathrm{n}_1}, a_{k_2 \mathrm{n}_2}, \dots, a_{k_r n_\mathrm{r}}$ – boshlang‘ich elementlar, $k_1 < k_2 < \cdots < k_r$ – nolmas satrlar tartibi.

$$K = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12 \\ 0 & -9 & 5 & 9 \end{pmatrix}$$

K – pog‘onasimon emas

$$N = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 5 & 9 \end{pmatrix}$$

N – pog‘onasimon

Teng matritsalar

Agar *A* va *B* matritsalarning o‘lchamlari bir xil va mos elementlari teng bo‘lsa, ularga **teng matritsalar** deyiladi:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \Leftarrow \begin{matrix} a = e & b = f \\ c = g & d = h \end{matrix}$$

$$A = B$$

2-misol. $\begin{pmatrix} 3 & 2 \\ x+y & 1 \end{pmatrix} = \begin{pmatrix} 3 & y \\ 2 & 1 \end{pmatrix}$. x=? y =?

Yechilishi:

Matritsalarning mos elementlarini taqqoslab quyidagi tengliklarni hosil qilamiz:

$$\begin{cases} y = 2 \\ x + y = 2 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 2 \end{cases}$$

3. Matritsalarni qo‘shish va ayirish

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \pm \begin{pmatrix} e & f \\ k & h \end{pmatrix} = \begin{pmatrix} a \pm e & b \pm f \\ c \pm k & d \pm h \end{pmatrix}$$

Bir xil o‘lchamli matritsalarni **qo‘shish va ayirish mumkin** bo‘lib, bu amallar matritsalarning mos elementlari ustida bajariladi.

3-misol.

$$\begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 5 & 0 \\ 6 & 7 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 4 & 1 \\ 6 & 10 & -1 \end{pmatrix}$$

4. Matritsani songa ko‘paytirish

$$\lambda \neq 0$$

$$\lambda \cdot \begin{pmatrix} b & c \\ d & e \end{pmatrix} = \begin{pmatrix} \lambda \cdot b & \lambda \cdot c \\ \lambda \cdot d & \lambda \cdot e \end{pmatrix}$$

Matritsalarni **songa ko‘paytirishda – ** uning **har bir elementi** shu songa ko‘paytiriladi

4-misol.

$$5 \cdot \begin{pmatrix} 1 & 2 \\ 0 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 10 \\ 0 & 10 \\ 5 & 15 \end{pmatrix}$$

Matritsalar ustida bajariladigan amallarning xossalari:

Matritsalarni qo‘shish, ayirish va songa ko‘paytirish amallariga matritsalar ustida **chiziqli amallar** deyiladi.

$$1)\ A+B=B+A;$$
$$2)\ A+(B+C)=(A+B)+C;$$
$$3)\ k(A+B)=kA+kB;$$
$$4)\ k(nA)=(kn)A;$$

$$5)\ (k+n)A=kA+nA;$$
$$6)\ A+\Theta=A;$$
$$7)\ A+(-A)=\Theta;$$
$$8)\ 1\cdot A=A.$$

Chiziqli amallar o‘rin almashtirish, guruhlash va taqsimot qonunlariga bo‘ysunadi.

5. Matritsalarni ko‘paytirish

A matritsaning ustunlari soni *B* matritsaning satrlari soniga teng bo‘lsa, *A* va *B* lar **zanjirlangan matritsalar** deyiladi.

Misol uchun
4x3 va 3x5

$$A \cdot B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 \end{pmatrix}$$

$A_{m\times k} \cdot B_{k\times n}$ **matritsalarning ko‘paytmasi** deb, shunday *C* matritsaga aytiladiki, uning har bir elementi *A* ning *i*-satrini *B* ning *j*-ustunidagi mos elementlariga ko‘paytmalari yig‘indisiga teng:

$$c_{ij} = \sum_{s=1}^{k} a_{is} \cdot b_{is}; i = 1,2,\ldots,m; \; j = 1,2,\ldots,n$$

5-misol. Matritsalarni ko‘paytiring.

$$\begin{pmatrix} 1 & 2 \\ 0 & 8 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 6 & 5 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} 1\cdot6+2\cdot4 & 1\cdot5+2\cdot0 \\ 0\cdot6+8\cdot4 & 0\cdot5+8\cdot0 \\ 1\cdot6+3\cdot4 & 1\cdot5+3\cdot0 \end{pmatrix} = \begin{pmatrix} 14 & 5 \\ 32 & 0 \\ 18 & 5 \end{pmatrix}$$

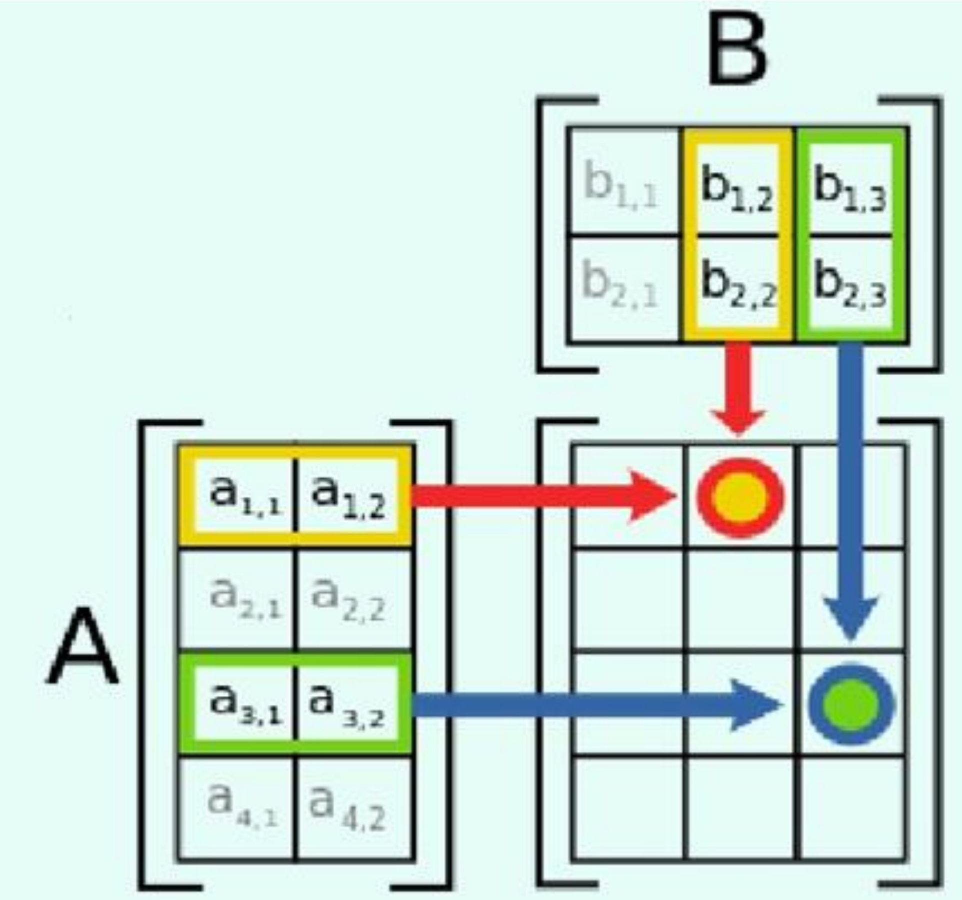
Satrlar ustunlarga ko‘paytiriladi

$$A = \begin{pmatrix} 2 & 3 & 4 \\ 4 & 5 & 2 \\ 9 & 8 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 5 & 8 \\ 1 & 4 \\ 4 & 3 \end{pmatrix}$$

A va *B* matritsalar zanjirlangan, ***AB* ko‘paytma o‘rinli.**

B va *A* matritsalar zanjirlanmagan, ***BA* ko‘paytma mavjud emas.**

Matritsalarni ko‘paytirish sxemasi



$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot \begin{pmatrix} b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a_{11}b+a_{12}c+a_{13}d \\ a_{21}b+a_{22}c+a_{23}d \\ a_{31}b+a_{32}c+a_{33}d \end{pmatrix}$$

$$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} a_1c_1+b_1c_2 \\ a_2c_1+b_2c_2 \end{pmatrix}$$

Matritsalarni ko‘paytirish xossalari

Matritsalarni ko‘paytirish amali kommutativlik xossasiga ega emas: $AB \neq BA$

A va B matritsalar uchun $AB = BA$ munosabat o‘rinli bo‘lsa, ularga **kommutativ matritsalar** deyiladi.

Matritsalarni ko‘paytirishda taqsimot va guruhlash qonunlari o‘rinli:

$$1)\ (kA)B = k(AB) = A(kB);$$

$$2)\ (A+B)C = AC + BC;$$

$$3)\ A(B+C) = AB + AC;$$

$$4)\ A(BC) = (AB)C.$$

6. Matritsani transponirlash

$A_{m\times n} = (a_{ij})$ matritsa $A^T_{n\times m} = (a^T_{ij})$ matritsa bilan **o'zaro transponirlangan** deyiladi, agarda ularning elementlari orasida $a^T_{ij} = a_{ji}$ munosabat o'rinli bo'lsa.

6-misol.

1-satr 1-ustunga aylanadi

$$A = \begin{pmatrix} -1 & 2 & 4 & 0 & 7 \\ 3 & -5 & 2 & 9 & -3 \\ -10 & -8 & -2 & -4 & 11 \end{pmatrix} \qquad A^T = \begin{pmatrix} -1 & 3 & -10 \\ 2 & -5 & -8 \\ 4 & 2 & -2 \\ 0 & 9 & -4 \\ 7 & -3 & 11 \end{pmatrix}$$

2-satr 2-ustunga aylanadi

3-satr 3-ustunga o'tadi

Transponirlangan matritsaning xossalari:

$$1)\left(A^T\right)^T = A,$$

$$2)(kA)^T = kA^T,$$

$$3(A+B)^T = A^T + B^T,$$

$$4(AB)^T = B^T A^T.$$

A kvadrat matritsa uchun $A = A^{\mathrm{T}}$ (yoki $a_{ij} = a_{ji}$) tenglik o‘rinli bo‘lsa, *A* ga **simmetrik matritsa** deyiladi.

7. Kvadrat matritsa

Satrlari va ustunlari soni o‘zaro teng bo‘lgan matritsaga **kvadrat matritsa** deyiladi.

$$A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

Kvadrat matritsada

$i > j$ lar uchun $a_{ij} = 0$ **yuqori uchburchakli matritsa** $A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix}$

$i < j$ lar uchun $a_{ij} = 0$ **quyi uchburchakli matritsa** $A=\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ a_{21} & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$

8. Kvadrat matritsani darajaga oshirish

A kvadrat matritsani $m\,(m>1)$ butun musbat **darajaga ko‘tarish**

$$A^m = \underbrace{A\cdot A\cdot\ldots\cdot A}_{m\ marta}.$$

7-misol:

$$\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 8 & 9 \end{pmatrix}$$

9. Diagonal va birlik matritsalar

Agar $A=(a_{ij})$ kvadrat matritsada $a_{ii} \neq 0$ va $a_{ij}=0, i \neq j$ bo‘lsa, **diagonal matritsa** deyiladi.

$$A=\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix}.$$

Agar diagonal matritsaning barcha diagonal elementlari o‘zaro teng bo‘lsa, unga **skalyar matritsa** deyiladi:

$$A=\begin{pmatrix} a & 0 & \dots & 0 \\ 0 & a & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a \end{pmatrix}.$$

Agar diagonal matritsaning elementlari faqat birlardan iborat bo‘lsa, unga **birlik matritsa** deyiladi:

$$E=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\qquad E=\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

Birlik matritsani ifodalashda ba’zida **Kroneker belgilashi**dan foydalaniladi:

$$\delta_{ij}=\begin{cases}1, & agar \quad i=j \quad \text{bo'lsa},\\ 0, & agar \quad i\neq j \quad \text{bo'lsa}.\end{cases}\qquad E=\begin{pmatrix}1&0&\dots&0\\0&1&\dots&0\\\dots&\dots&\dots&\dots\\0&0&\dots&1\end{pmatrix}=\left(\delta_{ij}\right)$$

E birlik matritsa ixtiyoriy *A* kvadrat matritsa bilan kommutativdir: $AE=EA=A$

O‘z-o‘zini tekshirish uchun savollar:

1. Matritsa deb nimaga aytiladi?
2. Satr matritsa, ustun matritsa deb qanday matritsaga aytiladi?
3. Nol matritsa deb qanday matritsaga aytiladi?
4. Matritsalarni qo‘shish va matritsani songa ko‘paytirish amallari bo‘ysunadigan xossalarni ayting.
5. Matritsa satrlarini mos ustunlari bilan almashtirish amali qanday nomlanadi?
6. O‘zaro zanjirlangan matritsalar qanday ko‘paytiriladi?
7. Matritsalarni ko‘paytirish amali qanday xossalarga bo‘ysunadi?
8. Matritsalarni ko‘paytirish amali o‘rin almashtirish qonuniga bo‘ysunadimi?
9. n-tartibli kvadratik matritsa deb qanday matritsaga aytiladi?
10. Kvadrat matritsaning qanday xususiy ko‘rinishlarini bilasiz?

Adabiyotlar:

1. Gilbert Strang “Introduction to Linear Algebra”, USA, Cambridge press, 5nd edition, 2016. P. 22-30

2. Grewal B.S. “Higher Engineering Mathematics”, Delhi, Khanna publishers, 42nd edition, 2012. P. 26-32

3. Соатов Ё.У. “Олий математика”, Т., Ўқитувчи нашриёти, 1-қисм, 1995. 64-69 бетлар, 3-қисм 15-33 бетлар.

4. Рябушко А.П. и др. “Сборник индивидуальных заданий по высшей математике”, Минск, Высшая школа, 1-часть, 1991. стр. 15-20.

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