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CHIZIQLI ALGEBRA

MTH 1234

MAVZU

CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMASINI YECHISHNING MATRITSA, GAUSS VA GAUSS-JORDAN USULLARI

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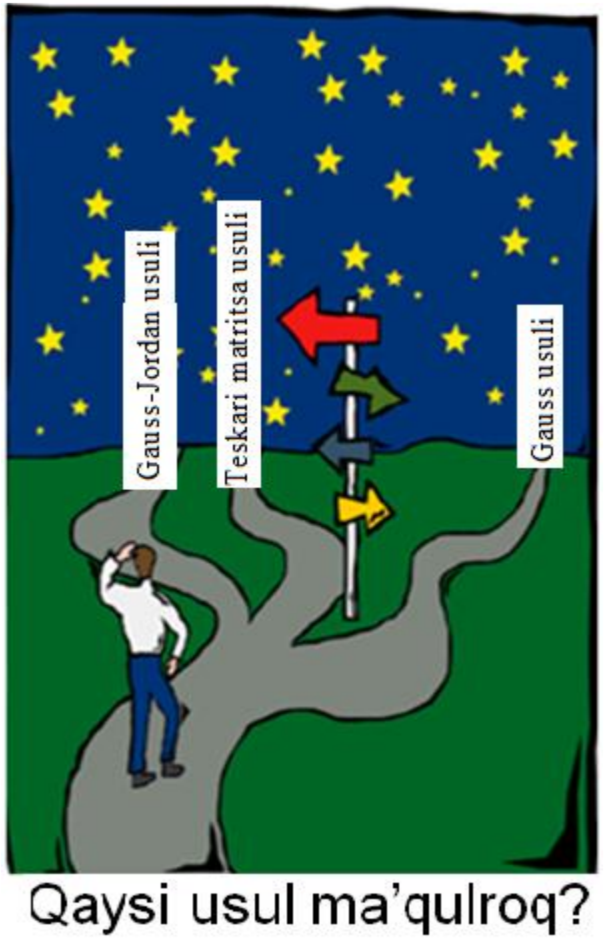
OLIY MATEMATIKA KAFEDRASI



Chiziqli algebraik tenglamalar sistemasini yechish usullari

1. Chiziqli tenglamalar sistemasini teskari matritsa usulida yechish
2. Chiziqli tenglamalar sistemasini Gauss usulida yechish
3. Chiziqli tenglamalar sistemasini Gauss-Jordan usulida yechish

1. Chiziqli tenglamalar sistemasini teskari matritsa usulida yechish

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2, \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n. \end{cases}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

$AX = B$ tenglamalar sistemasining matritsaviy shakli

$$A^{-1} \cdot (A \cdot X) = A^{-1} \cdot B \longrightarrow (A^{-1} \cdot A) \cdot X = A^{-1} \cdot B \longrightarrow E \cdot X = A^{-1} \cdot B$$

$X = A^{-1} \cdot B$ formula *A* matrisasi xosmas, ya’ni $\det|A| \neq 0$ bo‘lganda *n* noma’lumli *n* ta chiziqli tenglamalar sistemasining yechimidan iborat bo‘ladi.

1-misol. $\begin{cases} x_1 - 2x_2 + x_3 = 5 \\ 2x_1 - x_3 = 0 \\ -2x_1 + x_2 + x_3 = -1. \end{cases}$ tenglamalar sistemasini teskari matritsa usulida yechamiz.

$$X = A^{-1} \cdot B$$

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ -2 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 1 & \frac{2}{3} \\ 0 & 1 & 1 \\ \frac{2}{3} & 1 & \frac{4}{3} \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$detA = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ -2 & 1 & 1 \end{vmatrix} = -4 + 2 + 1 + 4 = 3 \neq 0$$

$$\begin{matrix} A_{11} = 1, & A_{21} = 3, & A_{31} = 2 \\ A_{12} = 0 & A_{22} = 3, & A_{32} = 3 \\ A_{13} = 2 & A_{23} = 3, & A_{33} = 4 \end{matrix}$$

$$A^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 3 & 3 \\ 2 & 3 & 4 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$x_1 = 1, \; x_2 = -1, \; x_3 = 2.$$

Sistema matritsasining rangi noma’lumlar sonidan kichik bo‘lsa, yechimni teskari matritsa usulida topish mumkinmi?

2-misol. Chiziqli tenglamalar sistemasini teskari matritsa usulida yechamiz:

$$\begin{cases} x_1 - 2x_2 + 3x_3 - 5x_4 = 2, \\ 2x_1 + x_2 + 4x_3 + x_4 = -3, \\ 3x_1 - 3x_2 + 8x_3 - 2x_4 = -1, \\ 2x_1 - 2x_2 + 5x_3 - 12x_4 = 4 \end{cases}$$

$$A = \begin{pmatrix} 1 & -2 & 3 & -5 \\ 2 & 1 & 4 & 1 \\ 3 & -3 & 8 & -2 \\ 2 & -2 & 5 & -12 \end{pmatrix}, \quad (A|B) = \left(\begin{array}{cccc|c} 1 & -2 & 3 & -5 & 2 \\ 2 & 1 & 4 & 1 & -3 \\ 3 & -3 & 8 & -2 & -1 \\ 2 & -2 & 5 & -12 & 4 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 3 & -5 & 2 \\ 2 & 1 & 4 & 1 & -3 \\ 3 & -3 & 8 & -2 & -1 \\ 2 & -2 & 5 & -12 & 4 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & -2 & 3 & -5 & 2 \\ 0 & 5 & -2 & 11 & -7 \\ 0 & 3 & -1 & 13 & -7 \\ 0 & 2 & -1 & -2 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & -2 & 3 & -5 & 2 \\ 0 & 5 & -2 & 11 & -7 \\ 0 & 0 & 1 & 32 & -14 \\ 0 & 0 & -1 & -32 & 14 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & -2 & 3 & -5 & 2 \\ 0 & 5 & -2 & 11 & -7 \\ 0 & 0 & 1 & 32 & -14 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$r(A) = r(A|B) = 3$$

x_4 – ozod noma’lum, 4-tenglamani tashlab yuboramiz. x_4 ni ozod had tomonga o‘tkazib, hosil bo‘lgan 3 noma’lumli tenglamalar sistemasini teskari matritsa usulida yechamiz:

$$\begin{cases} x_1 - 2x_2 + 3x_3 = 2 + 5x_4, \\ 2x_1 + x_2 + 4x_3 = -3 - x_4, \\ 3x_1 - 3x_2 + 8x_3 = -1 + 2x_4. \end{cases} \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad B = \begin{pmatrix} 2+5x_4 \\ -3-x_4 \\ -1+2x_4 \end{pmatrix} \quad \Delta = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 4 \\ 3 & -3 & 8 \end{vmatrix} = 1$$

Asosiy determinant $\Delta \neq 0$, demak teskari matritsa mavjud. Teskari matritsani Gauss-Jordan usulida elementar almashtirishlar yordamida topamiz:

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 4 & 0 & 1 & 0 \\ 3 & -3 & 8 & 0 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 5 & -2 & -2 & 1 & 0 \\ 0 & 3 & -1 & -3 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 7 & 0 & -8 & 0 & 3 \\ 0 & -1 & 0 & 4 & 1 & -2 \\ 0 & -3 & 1 & 3 & 0 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 20 & 7 & -11 \\ 0 & 1 & 0 & -4 & -1 & 2 \\ 0 & 0 & 1 & -9 & -3 & 5 \end{array}\right),$$

$$A^{-1} = \begin{pmatrix} 20 & 7 & -11 \\ -4 & -1 & 2 \\ -9 & -3 & 5 \end{pmatrix} \qquad X = A^{-1} \cdot B \qquad X = \begin{pmatrix} 20 & 7 & -11 \\ -4 & -1 & 2 \\ -9 & -3 & 5 \end{pmatrix} \begin{pmatrix} 2+5x_4 \\ -3-x_4 \\ -1+2x_4 \end{pmatrix} = \begin{pmatrix} 30+71x_4 \\ -7-15x_4 \\ -14-32x_4 \end{pmatrix}$$

Sistema cheksiz ko‘p yechimga ega: $X = \left(30+71x_4;\, -7-15x_4;\, -14-32x_4;\, x_4\right)^t, \ x_4 \in R$

2. Chiziqli tenglamalar sistemasini Gaussning klassik usulida yechish

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases}$$

Gaussning klassik usulida tenglamalar sistemasini yechish ikki bosqichda amalga oshiriladi:

1) chapdan o‘ngga: sistema yuqori uchburchak ko‘rinishiga keltiriladi.
2) o‘ngdan chapga: noma’lumlar oxirgi tenglamadan boshlab topiladi.

1-bosqich. Sistemani uchburchak ko‘rinishga keltirish uchun $a_{11} \neq 0$ bo‘lishi kerak.

Agar $a_{11} = 0$ bo‘lsa, u holda bu tenglama 1-elementi noldan farqli bo‘lgan $\boldsymbol{i}$ - tenglama bilan almashtiriladi, agar $\boldsymbol{i}$ - tenglamaning 1-elementi a_{i1} =1 bo‘lsa, juda ma’qul.

$$\begin{cases} x_1 + \dfrac{a_{12}}{a_{11}} x_2 + \dfrac{a_{13}}{a_{11}} x_3 + \ldots + \dfrac{a_{1n}}{a_{11}} x_n = \dfrac{b_1}{a_{11}} \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n = b_m \end{cases} \Bigg| \cdot(-a_{21}), (-a_{31}), \ldots$$

$$\begin{cases} x_1 + \dfrac{a_{12}}{a_{11}} x_2 + \dfrac{a_{13}}{a_{11}} x_3 + \ldots + \dfrac{a_{1n}}{a_{11}} x_n = \dfrac{b_1}{a_{11}} \\ a_{22}^{(1)}x_2 + a_{23}^{(1)}x_3 + \ldots + a_{2n}^{(1)}x_n = b_2^{(1)} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m2}^{(1)}x_2 + a_{m3}^{(1)}x_3 + \ldots + a_{mn}^{(1)}x_n = b_m^{(1)} \end{cases}$$

$$\begin{cases} x_1 + \dfrac{a_{12}}{a_{11}} x_2 + \dfrac{a_{13}}{a_{11}} x_3 + \ldots + \dfrac{a_{1n}}{a_{11}} x_n = \dfrac{b_1}{a_{11}} \\ a_{22}^{(1)}x_2 + a_{23}^{(1)}x_3 + \ldots + a_{2n}^{(1)}x_n = b_2^{(1)} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{mn}^{(m-1)}x_n = b_m^{(m-1)} \end{cases}$$

2-bosqich. Oxirgi tenglamadan x_n noma’lum-ning qiymati aniqlanadi.

$$x_n = \frac{b_m^{(m-1)}}{a_{mn}^{(m-1)}}$$

Undan keyin x_{n-1}, x_{n-2}, … , x_2 va x_1 topiladi.

Tenglamalar sistemasini yechishning Gauss usuli **noma’lumlarni ketma-ket yo‘qotish usuli** deb ham ataladi.

2-misol. $\begin{cases} 2x_2 - x_3 = -7, \\ x_1 + x_2 + 3x_3 = 2, \\ -3x_1 + 2x_2 + 2x_3 = -10. \end{cases}$ tenglamalar sistemasini Gaussning klassik usulida yeching.

1-tenglamani 2-tenglama bilan almashtiramiz va noma’lumlarni ketma-ket yo‘qotishni boshlaymiz;

$$\begin{cases} x_1 + x_2 + 3x_3 = 2, \\ 2x_2 - x_3 = -7, \\ -3x_1 + 2x_2 + 2x_3 = -10 \end{cases} \longrightarrow \begin{cases} x_1 + x_2 + 3x_3 = 2, \\ 2x_2 - x_3 = -7, \\ 5x_2 + 11x_3 = -4. \end{cases} \longrightarrow \begin{cases} x_1 + x_2 + 3x_3 = 2, \\ x_2 - \frac{1}{2}x_3 = -\frac{7}{2}, \\ 5x_2 + 11x_3 = -4. \end{cases} \longrightarrow$$

$$\begin{cases} x_1 + x_2 + 3x_3 = 2, \\ x_2 - \frac{1}{2}x_3 = -\frac{7}{2}, \\ \frac{27}{2}x_3 = \frac{27}{2}. \end{cases} \longrightarrow \begin{cases} x_1 + x_2 + 3x_3 = 2 \\ x_2 - \frac{1}{2}x_3 = -\frac{7}{2} \\ x_3 = 1 \end{cases} \longrightarrow \begin{cases} x_1 = 2 \\ x_2 = -3 \\ x_3 = 1 \end{cases}$$

Chiziqli tenglamalar sistemasini Gaussning modifikatsiyalangan usulida yechish

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & a_{m3} & \ldots & a_{mn} \end{pmatrix}$$

$$A/B = \left(\begin{array}{cccc|c} a_{11} & a_{12} & \ldots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ldots & a_{2n} & b_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} & b_m \end{array}\right)$$

Matritsada **elementar almashtirishlar**:

a) Nollardan iborat satrni o‘chirish;

b) Ikkita parallel satrni o‘rnini almashtirish;

c) Bitta satrning barcha elementlarini biror songa ko‘paytirib, boshqa satrning mos elementlariga qo‘shish;

d) Satrning barcha elementlarini noldan farqli bir xil songa ko‘paytirish.

Bu usulda tenglamalar yozib o‘tirilmaydi, faqat koeffitsiyentlar bilan ish ko‘riladi.

2-misol. Chiziqli tenglamalar sistemasini Gaussning modifikatsiyalangan usulida yeching:

$$\begin{cases} x_1 - x_2 + 3x_3 + 2x_4 = 11; \\ 4x_1 + 6x_2 - x_3 = -1; \\ 3x_1 + 2x_2 + 2x_3 - x_4 = 3; \\ 5x_1 - x_2 + 2x_3 + x_4 = 2. \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 11 \\ 4 & 6 & -1 & 0 & -1 \\ 3 & 2 & 2 & -1 & 3 \\ 5 & -1 & 2 & 1 & 2 \end{array}\right) \quad (-4)\ (-3)\ (-5)$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 11 \\ 0 & 10 & -13 & -8 & -45 \\ 0 & 5 & -7 & -7 & -30 \\ 0 & 4 & -13 & -9 & -53 \end{array}\right) \begin{array}{l} \\ (-1)\ (-2) \\ 2 \\ 5 \end{array}$$

$$3)\ \left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 11 \\ 0 & 10 & -13 & -8 & -45 \\ 0 & 0 & -1 & -6 & -15 \\ 0 & 0 & -39 & -29 & -175 \end{array}\right) \begin{array}{l} \\ \\ \cdot(-1) \\ \cdot(-1) \end{array}$$

$$4)\ \left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 11 \\ 0 & 10 & -13 & -8 & -45 \\ 0 & 0 & 1 & 6 & 15 \\ 0 & 0 & 39 & 29 & 175 \end{array}\right) \begin{array}{l} \\ \\ \cdot(-39) \\ \\ \end{array}$$

$$5)\ \left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 11 \\ 0 & 10 & -13 & -8 & -45 \\ 0 & 0 & 1 & 6 & 15 \\ 0 & 0 & 0 & 205 & 410 \end{array}\right)$$

$$5)\ \left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 11 \\ 0 & 10 & -13 & -8 & -45 \\ 0 & 0 & 1 & 6 & 15 \\ 0 & 0 & 0 & 205 & 410 \end{array}\right)$$

5-bosqichni qayta ko‘chirib yozdik. Endi matritsani tenglama shaklida yozib olamiz:

$$\begin{cases} x_1 - x_2 + 3x_3 + 2x_4 = 11; \\ 10x_2 - 13x_3 - 8x_4 = -45; \\ x_3 + 6x_4 = 15; \\ 205x_4 = 410. \end{cases}$$

$$\begin{cases} x_4 = \dfrac{410}{205} = 2 \\ x_3 = 15 - 6x_4 = 3 \\ x_2 = \dfrac{-45 + 13x_3 + 8x_4}{10} = \dfrac{-45 + 39 + 16}{10} = 1 \\ x_1 = 11 + x_2 - 3x_3 - 2x_4 = 11 + 1 - 9 - 4 = -1 \end{cases}$$

$$\begin{cases} x_1 = -1 \\ x_2 = 1 \\ x_3 = 3 \\ x_4 = 2 \end{cases}$$

Yechimning to‘g‘riligi berilgan tenglamaga qo‘yib, tekshirib ko‘riladi:

$$\begin{cases} x_1 - x_2 + 3x_3 + 2x_4 = 11; \\ 4x_1 + 6x_2 - x_3 = -1; \\ 3x_1 + 2x_2 + 2x_3 - x_4 = 3; \\ 5x_1 - x_2 + 2x_3 + x_4 = 2. \end{cases}$$

3. Chiziqli tenglamalar sistemasini Gauss-Jordan usulida yechish

Tenglamalar sistemasini Gauss – Jordan usulida yechishning (Gauss usulining Jordan modifikatsiyasi) mazmun-mohiyati quyidagicha:

1) Berilgan sistemaning kengaytirilgan (*A/B)* matritsasi quriladi.

2) Sistemaning teng kuchliligini saqlovchi elementar almashtirishlar yordamida, kengaytirilgan matritsaning chap qismida birlik matritsa hosil qilinadi.

3) Birlik matritsadan o‘ngda hosil bo‘lgan ustun yechimlar ustuni bo‘ladi.

$$\left(A|B\right) \sim \left(E|X^*\right)$$

3-misol. $\begin{cases} x_1 + x_2 + 2x_3 + 3x_4 = 1, \\ 3x_1 - x_2 - x_3 - 2x_4 = -4, \\ 2x_1 + 3x_2 - x_3 - x_4 = -6, \\ x_1 + 2x_2 + 3x_3 - x_4 = -4. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yechamiz.

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 3 & -1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 & -6 \\ 1 & 2 & 3 & -1 & -4 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 0 & 4 & 7 & 11 & 7 \\ 0 & -1 & 5 & 7 & 8 \\ 0 & 1 & 1 & -4 & -5 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & -4 & -5 \\ 0 & -1 & 5 & 7 & 8 \\ 0 & 4 & 7 & 11 & 7 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 7 & 6 \\ 0 & 1 & 1 & -4 & -5 \\ 0 & 0 & 6 & 3 & 3 \\ 0 & 0 & 3 & 27 & 27 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 7 & 6 \\ 0 & 1 & 1 & -4 & -5 \\ 0 & 0 & 1 & 9 & 9 \\ 0 & 0 & 2 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 0 & -13 & -14 \\ 0 & 0 & 1 & 9 & 9 \\ 0 & 0 & 0 & -17 & -17 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 0 & -13 & -14 \\ 0 & 0 & 1 & 9 & 9 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \begin{cases} x_1 = -1 \\ x_2 = -1 \\ x_3 = 0 \\ x_4 = 1 \end{cases}$$

$$rank(A) = rank(A/B) = k = n$$

4-misol. $\begin{cases} x_1 - x_2 + 2x_3 - x_4 = 4, \\ x_1 + x_2 + x_3 + x_4 = 10, \\ 7x_1 + 2x_2 + 8x_3 - 6x_4 = 44, \\ 5x_1 + 2x_2 + 5x_3 - 6x_4 = 30. \end{cases}$ tenglamalar sistemasini Gauss-Jordan usulida yechamiz.

Sistemada x_1, x_2, x_3 noma'lumlar oldidagi koeffitsiyentlarni nolga aylantiramiz:

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 4 \\ 1 & 1 & 1 & 1 & 10 \\ 7 & 2 & 8 & -6 & 44 \\ 5 & 2 & 5 & -6 & 30 \end{array}\right) \cdot (-1), (-7), (-5) \Rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 4 \\ 0 & 2 & -1 & 2 & 6 \\ 0 & 9 & -6 & 1 & 16 \\ 0 & 7 & -5 & -1 & 10 \end{array}\right) \cdot (-4{,}5), (-3.5) \Rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 4 \\ 0 & 2 & -1 & 2 & 6 \\ 0 & 0 & -1{,}5 & -8 & -11 \\ 0 & 0 & -1{,}5 & -8 & -11 \end{array}\right) (-2)$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 4 \\ 0 & 2 & -1 & 2 & 6 \\ 0 & 0 & 3 & 16 & 22 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$rank(A) = rank(A/B) = k < n$$

$$\Rightarrow\left(\begin{array}{cccc|c}1 & -1 & 2 & -1 & 4\\ 0 & 2 & -1 & 2 & 6\\ 0 & 0 & 3 & 16 & 22\\ 0 & 0 & 0 & 0 & 0\end{array}\right)$$

x_4 – ozod noma’lum ekan

$$\begin{cases} x_1 - x_2 + 2x_3 - x_4 = 4, \\ 2x_2 - x_3 + 2x_4 = 6, \\ 3x_3 + 16x_4 = 22 \end{cases} \Rightarrow \begin{cases} x_1 = 8x_4 - 34/3 \\ x_2 = -(11x_4 + 2)/3 \\ x_3 = -(16x_4 - 22)/3 \end{cases} \Rightarrow$$

Sistemada cheksiz ko‘p yechimga ega:

$$\left(8x_4 - \frac{34}{3}; -\frac{11x_4 + 2}{3}; -\frac{16x_4 - 22}{3}; x_4\right), \quad x_4 \in R.$$

O‘z-o‘zini tekshirish uchun savollar:

1. Chiziqli tenglamalar sistemasini yechishning teskari matritsa usuli qanday?
2. Chiziqli tenglamalar sistemasi matritsaviy shaklda qanday yoziladi?
3. Chiziqli tenglamalar sistemasini yechishda teskari matritsa usulining afzallik va noqulaylik jihatlari nimalardan iborat?
4. Chiziqli tenglamalar sistemasini yechishning Gauss usuli qanday?
5. Chiziqli tenglamalar sistemasi Gaussning klassik usulida qanday yechiladi?
6. Chiziqli tenglamalar sistemasi ustida elementar almashtirishlar deganda nimani tushunasiz?
7. Chiziqli tenglamalar sistemasining barcha yechimlarini topish o’rniga uning umumiy yechimini qurish yetarlimi?
8. Chiziqli tenglamalar sistemasini yechish Gauss usulining Jordan modifikatsiyasi mazmun-mohiyatini so’zlab bering va sxemasini yozing?

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E'TIBORINGIZ UCHUN RAHMAT!

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