MATH 227 W25 Assignment #2

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Q1 (3)

An astroid is the curve (shown partially completed in red) traced out by a point on the circumference of a smaller circle rolling around the inside of a larger circle with four times the radius. If the larger circle has radius 4, then we can parametrize the astroid by $\vec{r}(t) = (4\cos^3(t), 4\sin^3(t))$ Find the arc length of one quarter of the asteroid (i.e., for $0 \le t \le \frac{\pi}{2}$)

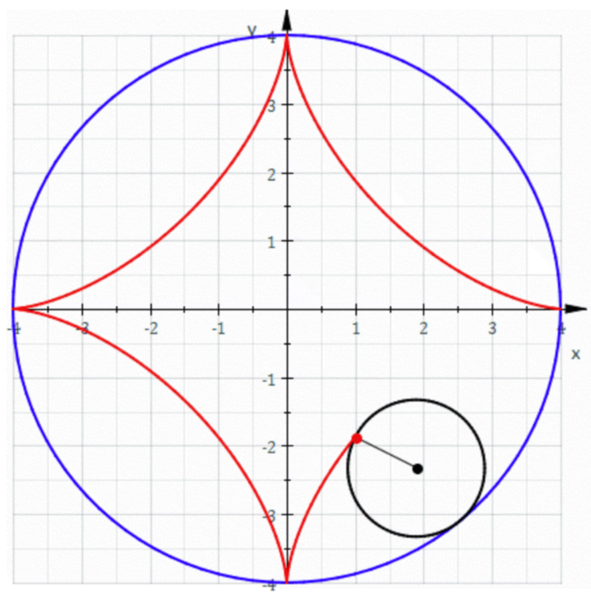

Figure 1: Graph of asteroid being traced

The arclength of a curve is given by the following.

$$L = \int_{t=a}^{b} \sqrt{(x'(t))^2 + (y'(t))^2}\ dt$$

Where x and y are their respecive components of $\vec{r}(t)$.

$$\begin{aligned} x(t) &= 4\cos^3(t) & y(t) &= 4\sin^3(t) \\ x'(t) &= 12\cos^2(t)(-\sin(t)) & y'(t) &= 12\sin^2(t)(\cos(t)) \\ (x'(t))^2 &= 12^2\cos^4(t)(\sin^2(t)) & (y'(t))^2 &= 12^2\sin^4(t)(\cos^2(t)) \end{aligned}$$

$$
\begin{aligned}
\implies \sqrt{(x'(t))^2 + (y'(t))^2} &= \sqrt{12^2 \cos^4(t)\sin^2(t) + 12^2 \sin^4(t)\cos^2(t)} \\
&= \sqrt{12^2 \cos^2(t)\sin^2(t)(\cos^2(t) + \sin^2(t))} \\
&= 12\cos(t)\sin(t) \\
&= 6\sin(2t)
\end{aligned}
$$

So, the arclength can be computed by integrating this function of t according to the bounds of the question.

$$\underline{L} = 6\int_{t=0}^{\frac{\pi}{2}} \sin(2t)\ dt \ = \ 6\left[-\frac{1}{2}\cos(2t)\right]_{t=0}^{\frac{\pi}{2}} = \underline{6}$$

Q2 (6)

A rectangular steel wire frame with length L and height H (as shown) has a lineal mass density (i.e., mass per unit length) given by the following.

$$\rho(x, y) = \rho_0 \left(\frac{2H^2 - y^2}{H^2}\right)$$

Determine the total mass of the wire frame in terms of ρ_0, H, and L.

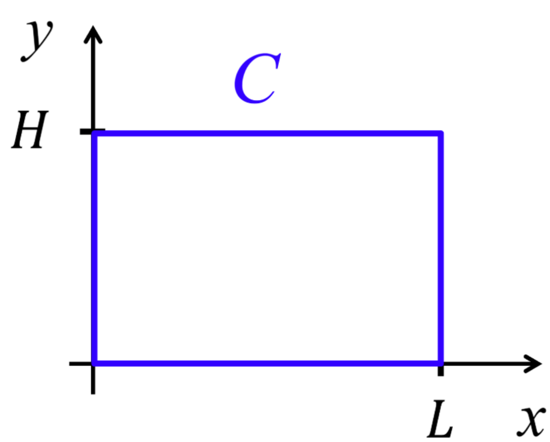

Figure 2: Graph of wire frame