ISH PHANDA
102016098
3CS12

Q-1. Given Random sample: $(x_1, \ldots, x_n)$

$$L(Q_1, Q_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Taking ln on both sides

$$\ln(L(Q_1, Q_2)) = \sum_{i=1}^{n} \left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 - \frac{1}{2}(\ln 2\pi\sigma^2)\right)$$

To find MLE, diff. log likelihood w.r.t θ_1, θ_2.

$\Rightarrow$ for θ_1,

$$\frac{\delta}{} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma^2}\right) = 0$$

$\delta\theta_1$

$$\Rightarrow \sum_{i=1} x_i - n\mu = 0$$

$$\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i \rightarrow \text{Mean}$$

$\Rightarrow$ for θ_2,

$$\frac{\delta}{\delta\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n}\left(-\frac{(x_i-\theta_1)^2}{2\theta_2^2} + \frac{1}{2\theta_2}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n}\left(\frac{(x_i-\theta_1)^2}{\theta_2}\right) = \frac{n}{\theta_2} = 0$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

$$\Rightarrow \theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

$\rightarrow$ Variance

Q-2. Find MLE of θ for a binomial distb. $B(m, \theta)$, where m is a +ve integer.

$$L(\theta) = \prod_{i=1}^{n} {}^{m}C_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking ln,

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left(\ln({}^{m}C_{x_i}) + x_i \ln\theta + (m-x_i)\ln(1-\theta) \right)$$

$$\frac{\delta}{\delta\theta} \ln(L(\theta)) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

Solve for θ

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\theta}$$

$$\theta \sum_{i=1}^{n} x_i = m \sum_{i=1}^{n} \theta$$

$$\theta = \frac{1}{m} \sum_{i=1}^{n} x_i$$

MLE of θ is a sample mean of observation.