RECURRENCE RELATION PRACTICE

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① $Tn = T(n-1) + n$

$T(n-1) = T(n-2) + n-1$

$T(n-2) = T(n-3) + n-2$

$Tn = T(n-2) + n-1 + n$

$T(n) = T(n-3) + n-2 + n-1 + n$

⋮

$T(n) = T(n-k) + kn - k(k-1)/2$ --- ①

for base case :-

$n-k = 1$

$k = n-1$

subst in ① $\quad T(1) = 1$

$T(n) = T(1) + (n-1)n - (n-1)(n-2)/2$

$\therefore$ Order => n^2 [$O(n^2)$]

② $T(n) = T(n/2) + 1$

$T(n) = T(n/2) + 1$

assuming $n = 2^k$, ie $k = \log n$

$T(2^k) = T\left(\frac{2^k}{2}\right) + 1$

$T(2^k) = T(2^{k-1}) + 1$

$= (T(2^{k-2}) + 1) + 1$

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$= T(2^{k-2}) + 2$

$= [T(2^{k-3}) + 1] + 2$

$= T(2^{k-3}) + 3$

$T(2^k) = T(2^{k-k}) + k$

$= T(2^0) + k = T(1) + k$

if $T(1) = 1$ then,

$T(2^k) = 1 + k$

$T(n) = \log n + 1$

$\therefore \Theta(\log n)$

(3)

$T(n) = 2T(n/2) + n$ —— (1)

$T(n/2) = 2T(n/2^2) + n/2$

$= 2[2T(n/2^2) + n/2] + n$

$= 2^2 T(n/2^2) + 2n$ —— (2)

$T(n/2^2) = 2T(n/2^3) + n/2^2$

$T(n/2) = 2^2\left[2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right] + 2n$

$T(n) = 2^3 T(n/2^3) + 3n$ —— (3)

⋮

k times;

$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn$

let $T\left(\frac{n}{2^k}\right) = T(1)$

$n = 2^k$

$k = \log n$

$T(n) = \cancel{2^k\ T(1) + kn}$

$= n(1) + \underline{n \log n}$

$O(n \log n)$