Tutorial Sheet – II (Relations)

1. Let A= {1, 3, 5}, B={x, y}, C = { α, β } Find $A \times B \times C$.
2. Let A = {a, b, c, d, e, f} and B = {beef, dad, ace, cab} and let R be the relation from A to B where (x, y)$\in$ R if x is a letter in the word y. Find the matrix M which represents R.
3. Let A= {2, 3, 4, 5} the relations R and S on A defined by
   R={(2,2),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5),(5,3)}
   S={(2,3),(2,5),(3,4),(3,5),(4,2),(4,3),(4,5),(5,2),(5,5)}
   Find the matrices of the above relations. Use the matrices to find out the following compositions of the relations R & S:
   $(i)\ R \circ S \qquad (ii)\ R \circ R \qquad (iii)\ S \circ R$
4. List the ordered pairs in the relation R from A={0,1,2,3,4} to B={0,1,2,3}where (a, b)$\in R$ iff
   (i) a = b (ii) a + b = 3 (iii) GCD(a, b) = 1
   Draw the Matrix of relation R in each case.
5. Let $A = \{1, 2, 3, 4\}$, $B = \{4, 5, 6\}$ and $C = \{5, 6, 7, 8\}$. Define relation $R : A \to B$: xRy if $x + y = 2k$; $S : B \to C$: xSy if $y - x = 1$. Determine M_R, M_S and $M_{R \circ S}$.
6. On the set of real numbers, R & S are two relations defined as
   $R = \{(a,b):\ a \le 2b\}$ & $S = \{(b,c):\ b \le 3c\}$. Describe $S \circ R$ and $R \circ S$. Does $(1,5) \in S \circ R$.
7. R is a relation on the set $A = \{1, 2, 3, 4, 5\}$ such that xRy if $|x - y| \le 1$ then draw the digraph of R.
8. Give an example of a relation which is
   (i) neither symmetric nor antisymmetric
   (ii) both symmetric & antisymmetric
   (iii) symmetric but not antisymmetric
   (iv) antisymmetric but not symmetric
   (v) neither reflexive nor irreflexive

9. Which of the properties are followed by following relations:

$$(i)\quad M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (ii)\quad M_S = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (iii)\quad M_T = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

   (Reflexive, Irreflexive, Symmetric, Asymmetric, Antisymmetric and Transitive)
10. From question no. 9, obtain $M_{R^{-1}}, M_{\overline{R}}, M_{R \cup S}$ and $M_{R \cap S}$. Is $M_{R^{-1}} = M_{\overline{R}}$?
11. Determine whether the relation R, which is given as follows; is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive:
    (i) Empty relation on $A = \{1, 2, 3\}$ i.e. $R = \emptyset$
    (ii) Universal relation on $A = \{1, 2, 3\}$ i.e. $R = A \times A$
    (iii) Relation of $\subseteq$
    (iv) Relation of $\le$
    (v) Relation of $<$
    (vi) Relation of $=$
    (vii) Relation of divisibility i.e. / on the set of Natural numbers.
    (viii) Relation of divisibility i.e. / on the set of Integers.
    (ix) Relation of perpendicular lines i.e. $\perp$
    (x) Relation of parallel lines i.e. $\|$

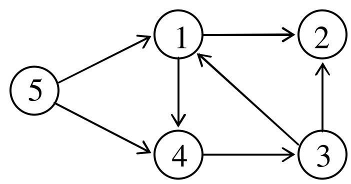
12. Consider the following relations on {1, 2, 3, 4}

$$R_1 = \{(1,2)\}$$
$$R_2 = \{(2,2), (2,3), (3,2)\}$$
$$R_3 = \{(1,2), (4,2), (4,1), (3,2), (3,1), (3,4)\}$$
$$R_4 = \{(1,1), (1,2), (2,1), (2,2), (4,3), (3,1), (3,3)\}$$
$$R_5 = \{(1,1), (1,4), (1,2), (4,1), (4,4), (3,3), (3,2), (2,1), (2,2)\}$$
$$R_6 = \{(1,1), (1,3), (1,2), (1,4), (3,3), (3,2), (3,4), (2,2), (2,4), (4,4)\}$$

Which of these relations are reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive?

13. Explain the following:
    i) If R is reflexive $\Rightarrow R \cap R^{-1}$ is non empty.
    ii) If R & S are reflexive, symmetric and transitive $\Rightarrow R \cap S$ is also…
    iii) If R & S are transitive $\Rightarrow R \cup S$ need not be transitive.

14. Obtain the closure (Reflexive, symmetric and transitive) of the following relation:



15. What will be the reflexive and symmetric closures of the relation $R = \{(a,b):\ a < b\}$ on the set of integers?

16. Find the zero-one matrix of the transitive closure of the relation R where

$$M_R = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix}.$$

17. In the digraph (given in Q. no. 14) which of the following are paths: 3, 1, 2; 5, 1, 3, 2; 5, 4, 3, 1; 5, 1, 4, 3, 1; 1, 4, 3, 1? What are the lengths of those that are paths? Which of the paths in this list are circuits?

18. Show that the relation "is congruent modulo 3" on the set of integers is an equivalence relation. Find also all equivalence classes.

19. Show that on a set, an equivalence relation always induces a partition.

20. R is a relation defined on $Z \times Z$ by $(a,b)R(c,d)$ if $a + d = b + c$. Show that R is an equivalence relation. Find the equivalence class of $(1,5)$ and $(2,6)$.

21. Show that the "less than or equal" relation $(\leq)$ is a partial ordering on the set of integers.

22. Show that the inclusion relation $\subseteq$ is a partial ordering on the power set of a set A.

23. Draw the Hasse diagram representing the partial ordering $\{(a,b): a \leq b\}$ on $\{1,3,5,7\}$.

24. Draw the Hasse diagram representing the partial ordering $\{(a,b): a \text{ divides } b\}$ on $\{1,2,3,4,6,9,15\}$.

25. Draw the Hasse diagram for the partial ordering $\{(A,B): A \subseteq B\}$ on the power set $P(S)$ where $S = \{1,2,3\}$.

<u>Remark</u>: We denote composition of relations S and R by RoS. If you wish to denote composition of S and R by SoR then must mention about it explicitly.