Tutorial Sheet – IV (Logic, Propositional functions & quantifiers)

1. (a) Which of the following sentences are propositions:
   - (i) Is this true?
   - (ii) $5 \in \{1,6,7\}$
   - (iii) Answer this question.
   - (iv) 5 + 6 = 12
   - (v) Four is even.

   (b) What is the negation of:
   - (i) Today is Tuesday.
   - (ii) 5 + 1 = 6.
   - (iii) No one wants to buy my house.
   - (iv) Some students have no mobile phone.
   - (v) Every even integer greater than 4 is the sum of two primes.

   (c) Determine the truth value of the following:
   - (i) 6 + 2 =7 or 4 + 4 = 8
   - (ii) 3 +1 = 4 and 5 + 4 = 7
   - (iii) 4 + 3 = 7 and 6 + 2 = 8
   - (iv) If 3 * 5 = 24 then 3 + 5 = 8
   - (v) If 3 * 5 = 15 then 3 + 5 =12

2. Consider the following:

   p: This computer is good.

   q : This computer is cheap.

   Write each of the following statements in symbolic form:
   - (i) This computer is good and cheap.
   - (ii) This computer is not good but cheap.
   - (iii) This computer is costly but good.
   - (iv) This computer is neither good nor cheap.
   - (v) This computer is good or cheap.

3. Consider the following:

   p : you take a course in Discrete Mathematics

   q : you understand logic.

   r : you get an A+ in this course.

   Write in simple sentences the meaning of the following:

   $(i)\ p \vee q$ $\quad (ii)\ q \to r$

   $(iii) \sim p \wedge \sim q.$ $\quad (iv)\ (p \wedge q) \to r$ $\quad (v)\ (p \wedge \sim q) \to \sim r$

4. Construct the truth table for the following:

   $(i)(p \vee \neg q) \wedge p$

   $(ii)\neg(p \vee q) \vee (\neg p \wedge \neg q)$

   $(iii)\ p \wedge (q \vee r)$

   $(iv)\neg p \vee q \to \neg q$

   $(v)\ p \wedge \neg r \leftrightarrow q \vee r$

5. Determine whether the following propositions are tautologies or not.

   $(i)\ p \vee \neg(p \wedge q)$

   $(ii)\neg(p \vee q) \vee [(\neg p) \wedge q] \vee p$

   $(iii)\ (p \wedge q) \to (p \to q)$

   $(iv)[p \wedge (p \to q)] \to q$

   $(v)\ p \wedge (q \wedge r) \leftrightarrow (p \wedge q) \wedge r$

6. Show that the propositions $p \wedge (q \wedge \neg p)$ and $(p \vee q) \wedge (\neg p) \wedge (\neg q)$ are contradiction.

7. Show that the following pairs of propositions are logically equivalent:

   $(i)\ (p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$

   $(ii)\ p \vee (p \wedge q) \equiv p$

   $(iii)\neg(p \vee q) \equiv \neg p \wedge \neg q$

   $(iv)\ p \wedge (\neg q \vee q) \equiv p$

8. State the converse, inverse and contrapositive of the following:
   (i) If today is Easter then tomorrow is Monday
   (ii) If John is a poet then he is poor.
   (iii) If triangle ABC is right angled then $AB^2 + BC^2 = AC^2$
   (iv) If P is a square then P is a rectangle.
   (v) If a triangle is not isosceles then it is not equilateral.
   (vii) If the square of an odd integer is odd then that number is odd.
9. Write the negation of each statement as simply as possible.
   (i) If she works, she will earn money.
   (ii) He swims if and only if the water is warm
   (iii) If it snows, then they do not drive the car.
10. Determine the validity of the following arguments.
   $(i)\ p \to q,\ r \to \sim q \quad \mapsto \quad p \to \sim r$
   $(ii)\ (p \vee \sim q), \sim q \to r,\ q \mapsto\ \sim r$
   $(iii)\ p \to \sim q,\ r \to q,\ r \mapsto\ \sim p$
   (iv) If I study then I will pass in examination.
   If I don't go to cinema, then I will study.
   But I failed in examination.
   …………………………….
   Therefore I went to cinema.

11. Let A = {1, 2, 3, 4, 5}. Determine the truth value of each of the following statements:
   $(i)\ (\exists x \in A)(x+3=10)$ $(ii)\ (\forall x \in A)(x+3<10)$
   $(iii)\ (\exists x \in A)(x+3<5)$ $(iv)\ (\forall x \in A)(x+3 \le 7)$
12. Determine the truth value of each of the following statements where **U =**{1, 2, 3} is the universal set:
   $(i)\ \exists x \forall y\ , x^2 < y+1$ $(ii)\ \forall x \exists y\ , x^2 + y^2 < 12$ $(iii)\ \forall x \forall y\ , x^2 + y^2 < 12$
13. Negate each of the following statements:
   $(i)\ \exists x \forall y\ , p(x,y)$ $(ii)\ \exists x \forall y\ , x^2 + y^2 < a^2$ $(iii)\ \exists y \exists x \forall z\ , x^2 + y^2 - z^2 < a^2$
14. Let *p(x)* denote the sentence "*x+2 > 5*". State whether or not *p(x)* is a propositional function on each of the following sets:
(a) **N**, the set of positive integers (b) **M** = {-1, -2, -3, …} (c) **C**, the set of complex numbers
15. Negate each of the following statements:
   (a) All the students live in the hostels.
   (b) All mathematics majors are male.
   (c) Some students are 18 (years) or older.
16. Let A = {1, 2, 3, …,9, 10}. Consider each of the following sentences. If it is a statement, then determine its truth value. If it is a propositional function, determine its truth set.
   $(a)\ (\forall x \in A)(\exists y \in A)(x+y<14).$ $(c)\ (\forall x \in A)(\forall y \in A)(x+y<14).$
   $(b)\ (\forall y \in A)(x+y<14).$ $(d)\ (\exists y \in A)(x+y<14).$
17. Negate each of the following statements:
   (a) If the teacher is absent, then some students do not complete their homework.
   (b) All the students completed their homework and the teacher is present.
   (c) Some of the students did not complete their homework or the teacher is absent.

18. Find a counterexample for each statement where **U =** {3, 5, 7, 9} is the universal set:
   $(i)\ \ \forall x, x+3 \ge 7$ $(ii)\ \forall x,\ x$ is odd *(iii)* $\forall x,\ x$ is prime. *(iv)* $\forall x,\ |x| = x.$
19. Negate the statement $\exists x \exists y\ (p(x) \wedge \sim q(x))$.