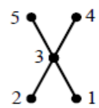
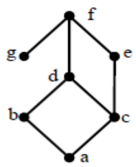
Tutorial Sheet – VIII
(Ordered Sets & Lattices)

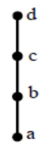
1. Find maximal, minimal, greatest(Last/Unit) element and least(First/Zero) element in the following posets:
   (i) $(A, \leq)$ where $A = \{x : \quad 0 \leq x \leq 1, \quad x \in R\}$
   (ii) The posets depicted in Hasse diagrams: a1-a4
2. Find upper bound, lower bound, least upper bound(LUB) and greatest lower bound (GLB) of subsets $\{J, F, B\}$ and $\{F, G, H\}$ in the poset given in the Fig. b.1.
3. Construct the $\wedge$ & $\vee$ table and find for which pair LUB and GLB do not exist in Fig:c1-c3.



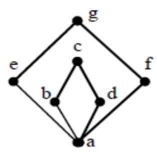
(a.1)



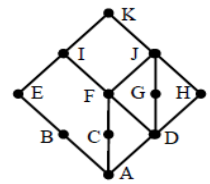
(a.2)



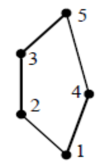
(a.3)



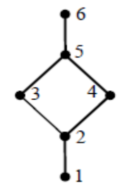
(a.4)



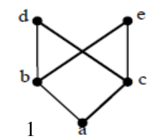
(b.1)



(c.1)

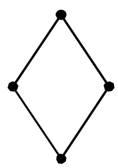


(c.2)

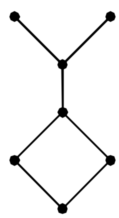


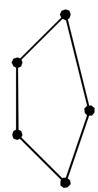
(c.3)

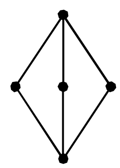
4. Construct a diagram of a poset having four elements, two maximal, two minimal but no greatest and no least element.
5. Which of the following posets are lattice?

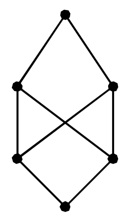

(d.1)

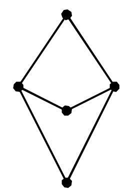

(d.2)

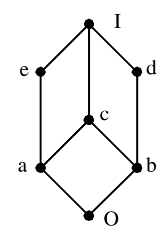

(d.3)


(d.4)


(d.5)


(d.6)


(d.7)



(e.1)

6. Let L be a lattice. Then for every a and b in L show that:
   (i) $a \vee b = b$ iff $a \leq b$
   (ii) $a \wedge b = a$ iff $a \leq b$
   (iii) $a \wedge b = a$ iff $a \vee b = b$
7. Let S be a set and $P(S)$ the power set of S. Let $\subseteq$ be a partial order relation on $P(S)$. Show that $P(S)$ is a lattice. Interpret LUB and GLB.
8. Let n be a positive integer and D_n the set of all positive divisors of n. Show that D_n is a lattice under the relation of divisibility. Interpret LUB and GLB. Draw the Hasse diagrams for $n = 20$ and $n = 30$.
9. Consider the lattice L in the Fig. e.1:
   (i) Which nonzero elements are join irreducible?
   (ii) Which elements are atoms?
   (iii) Which of the following are sublattices of L:
   $L_1 = \{O, a, b, I\}$ $\qquad$ $L_2 = \{O, a, d, I\}$
   $L_3 = \{a, c, e, I\}$ $\qquad$ $L_4 = \{O, c, e, I\}$
   (iv) Is L distributive?
   (v) Find the complements, if they exists, for the elements, a, b and c.
   (vi) Is L a complemented lattice?