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Jaypee University of Information Technology, Waknaghat

Test-2 Examination, April 2019

B.Tech (ECE/CSE/IT) - Semester IV

Course Title: Probability Theory and Random Processes   Max. Marks: 25 marks
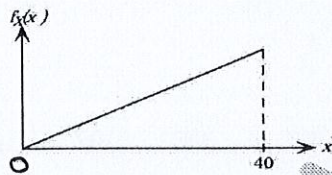Course Code: 10B11MA411
Course Credits: 4   Max. Time: 90 minutes

Note: Answer all the questions. Sketch diagrams and define random variables along with range where applicable. Scientific calculators are allowed.

1. Suppose you just purchased a digital music player and have put 8 tracks on it. After listening to them you decide that you like 2 of the songs. With the random feature on your player, each of the 8 songs is played once in random order. Let $L_1 = A$ liked-song and $L_2 = $ Another liked-song.   (3 Marks) [CO-1]

   (a) Find the probability that among the first two songs played you like both of them.

   (b) Find the probability that among the first two songs played you like neither of them.

2. (a) Consider the probability density function of **X** as shown in figure: (2 Marks) [CO-2]

   Compute the density function $f_X(x)$.

   (b) Consider the joint probability density function of **X** and **Y**:   (4 Marks) [CO-2]

   $$f(x,\ y) = \begin{cases} \frac{1}{1600}, & 0 \le x \le 40 \text{ and } 0 \le y \le 2x \\ 0, & \text{otherwise} \end{cases}$$

   Find the $\mathbb{P}(Y > X)$.

3. Consider the following independent random variables **X** and **Y** with the respective PDFs:

   $$f_X(x) = \begin{cases} \frac{1}{2}, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases} \qquad f_Y(y) = \begin{cases} 1, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

   Find the PDF of the sum of **X** and **Y**.   (4 Marks) [CO-2]

4. An internet search engine looks for a certain keyword in a sequence of independent web sites. It is believed that 20% of the sites contain this keyword.   (4 Marks) [CO-3]

   (a) Let **X** be the number of websites visited until the first keyword is found. Write down the distribution of **X** with *range*. What is the expected value of **X**?

   (b) Out of the first 10 websites, let **Y** be the number of sites that contain the keyword. Write down the distribution of **Y** with *range*. What is the expected value of **Y**?
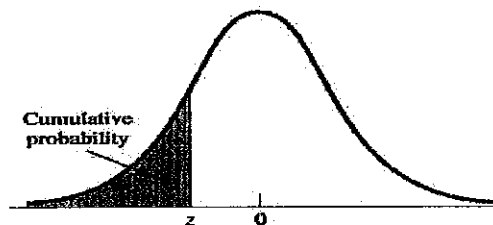
5. Suppose that calls are received at a 24-hour suicide hotline according to a Poisson process with an average of 35 calls per week. (4 Marks) [CO-3]

   (a) Find the probability that there are less than 3 calls in a day.

   (b) Find the probability that the time between two successive calls is less than 4 hours.

6. *Mensa* is an international society whose membership is limited to individuals having **IQ** score **X** above the 98th percentile of the general population; that is, *Mensa* accepts people whose **IQs** put them in the top 2% of the population. It is well-known that the average **IQ** for the general population is 100, the standard deviation is 16, and the distribution itself is normal. (4 Marks) [CO-3]

   (a) Find probability that a randomly selected person has an **IQ** between 105 and 110.

   (b) What is the *lowest* IQ that will qualify a person to belong to *Mensa*?

(Standard) Normal probability table to compute $\mathbb{P}(Z \leq z)$:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641