Tutorial Sheet – VII

1. Let the binary operation $*$ be defined on the set $\{a,b,c\}$ by means of the composite table

$*$	a	b	c
a	b	a	c
b	a	c	b
c	c	b	a

Is $*$ associative?

2. In a group show that
   i. cancellation law holds
   ii. the equations $a*x=b$ and $x*a=b$ have unique solutions
   iii. identity and inverse are unique
   iv. $(a*b)^{-1}=b^{-1}*a^{-1}$
   v. in the composition table of a group, an element can appear only once in each row and each column
   vi. if $x*x=e\ \forall x\in$ group, then it is abelian.
3. Show that arbitrary intersection of subgroups is a subgroup but arbitrary union of subgroups need not be a subgroup.
4. If a group G has four elements, show that it must be an abelian.
5. If R is the additive group of real numbers and R_+ the multiplicative group of positive real numbers, prove that the mapping $f:R\rightarrow R_+$ defined by $f(x)=e^x\ \forall x\in R$ is an homomorphism of R onto R_+. Is it isomorphism? Explain.
6. Test $(Z,+,\times),(Q,+,\times),(R,+,\times)$ and $(C,+,\times)$ for rings, integral domains and fields.
7. Let R be the ring and $M_n(R)$ be the set of all $n\times n$ matrices with elements from R. With the usual operation of addition and multiplication show that $M_n(R)$ is a ring. Is it a commutative ring? Is it a ring with unity? Does it have zero divisors?
8. In a ring show that
   i. $0.a=a.0=0$
   ii. $a.(-b)=(-a).b=-a.b$
   iii. $(-a).(-b)=a.b$
9. Let R be a ring. Show that the set of all polynomials with coefficients from R form a ring with respect to addition and multiplication of polynomials.
10. Prove that the set $z(\sqrt{2})$ of all real numbers of the form $a+b\sqrt{2}$, with a and b as integers is an integral domain with respect to ordinary addition and multiplication. Is it a field?
11. Every integral domain is not a field but every finite integral domain is a field. Give the examples in the support of above statement.