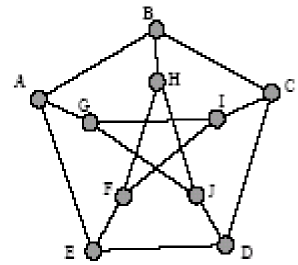
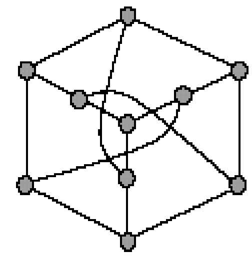
Tutorial Sheet – V (Graphs part 1)

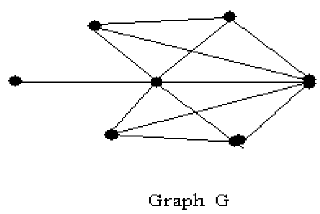
1. By suitably lettering the vertices, prove that the following two graphs are isomorphic:

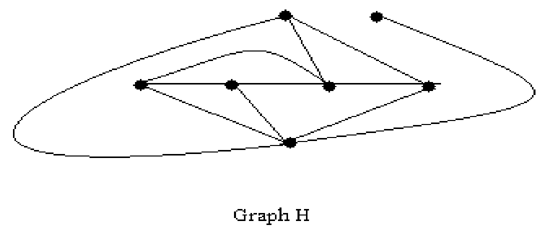
Graph 9.1 and 9.2





2. Are the following graphs isomorphic? Justify your answer.





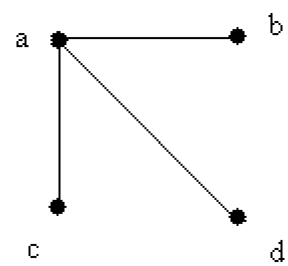
Graph 10.1 Graph 10.2

3. Draw the following graphs:
   $(i)\, K_7 \quad (ii)\, K_{1,8} \quad (iii)\, K_{4,4} \quad (iv)\, C_7 \quad (v)\, W_7 \quad (vi)\; Q_3 \quad (vii)\, Q_4$.

4. How many vertices and how many edges do these graphs have?
   $(i)\, K_n \quad (ii)\, K_{m,n} \quad (iii)\, C_n \quad (iv)\, W_n \quad (v)\; Q_n$.

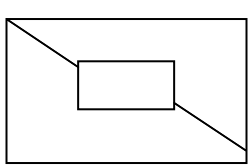
5. Draw all subgraphs of this graph:

Graph 13

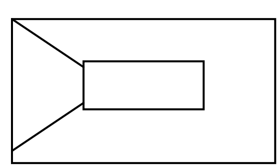


6. Show that the following graphs are not isomorphic:

Graph 15a

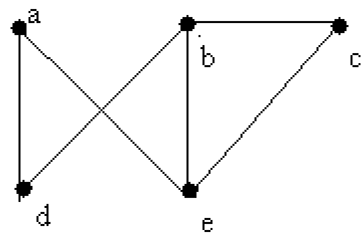


Graph 15b



7. Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?
(a) a,e,b,c,b (b) a,e,a,d,b,c,a (c) e,b,a,d,b,e (d) c, b, d, a, e, c

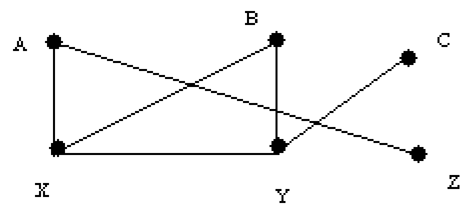
Graph 16



8. Let G be a graph. Determine whether or not each of the following sequences of edges forms a path:
   a. [{A, X}, {X,B}, {C, Y}, {Y, X}]
   b. [{A, X}, {X,Y}, {Y, Z}, {Z, A}]
   c. [{X, B},{B, Y}, {Y, C}]
   d. [{B, Y},{Y, C}, {C,Y}]

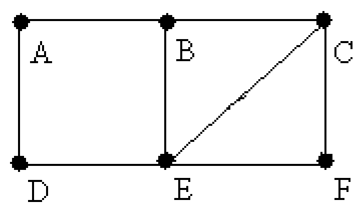
   Find all possible cycle in the graph.

Graph 17

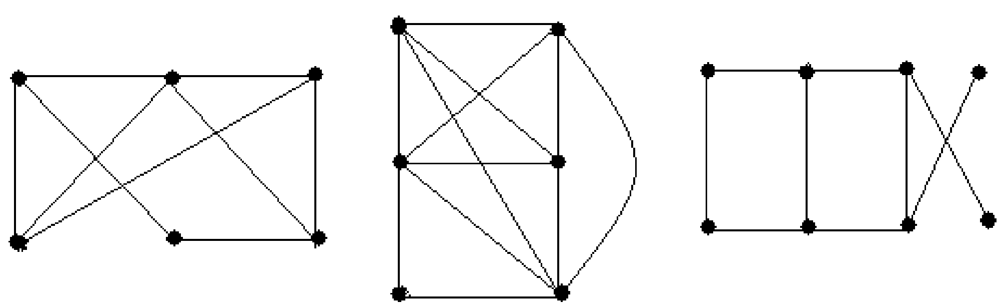


9. For the graph given above, find (a) all simple paths from A to C (b) d (A, C).
10. Consider the graph 19. Find : (a) all simple paths from A to F; (b) all trails from A to F; (c) d (A, F); the distance from A to F (d) diam (G); the diameter of G; (e) all cycles which include vertex A; (f) all cycles in G.

Graph 19



11. Identify which of the graphs are planar. If not planar then draw subgraph which is homeomorphic to $K_{3,3}$ or K_5.



12. Consider the graph G where V(G) = { A, B, C, D, E} and E (G) = {(A,D),(B,C),(C,E), (D,B),(D,D),(D,E),(E,A)}
   (a) Express G by its adjacency table
   (b) Does G have any loops or parallel edges?
   (c) Find all simple paths from D to E
   (d) Find all cycles in G.
   (e) Find the number of subgraphs of G with vertices C, D, E.
   (f) Find the H of G generated by C, D, E.

13. Draw the multigraph G corresponding to each of the following adjacency matrices:

$$\begin{pmatrix} 0 & 2 & 0 & 1 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 2 & 0 & 2 & 2 \end{pmatrix}$$

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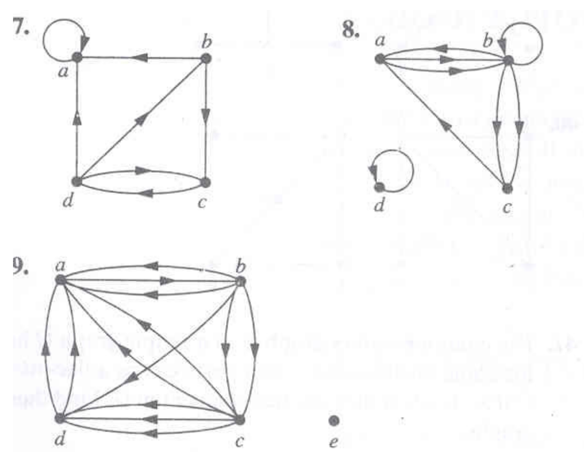
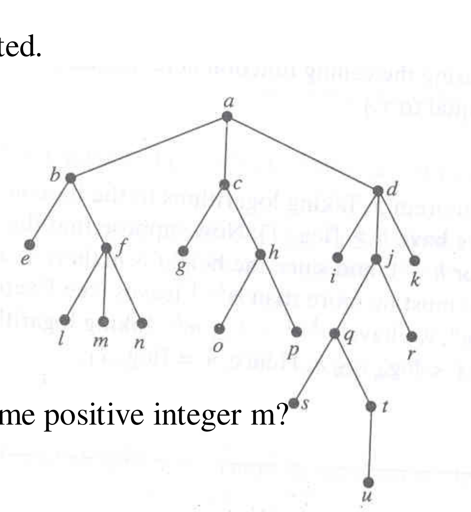
14. Discuss with examples: Euler path and graph, Hamiltonian path and graph
15. Prove Euler's formula. Discuss chromatic number and four color theorem.
16. Use Welch Powell Algorithm to color the graph given in the Exercise: 8.21 (Schaum's Series)

Tutorial Sheet – VI (Graphs part 2)

1. For each of the given graph, determine the sum of the in-degrees and the sum of the out-degrees of the vertices directly. Show that they are both equal to the number of edges in the graph.



2. Construct the underlying undirected graph for the above directed graphs.
3. Question No. 9.5, 9.23, 9.24 (Schaum's Series)
4. Let V = {2,3,4,9,15} and E = { (x, y): x < y and x is relatively prime to y } $\subseteq V \times V$. (Note that E is a relation on V)
   (a) Draw the directed graph G(V, E).
   (b) Is G strongly connected? Is G weakly connected? Is G unilaterally connected?
5. Answer these questions about the rooted tree illustrated.
   Which vertex is the root?
   (a) Which vertices are internal?
   (b) Which vertices are leaves?
   (c) Which vertices are children of j?
   (d) Which vertex is the parent of h?
   (e) Which are siblings of o?
   (f) Which vertices are ancestors of m?
   (g) Which vertices are descendants of b?



6. Is the rooted tree in Q. No. 5 a full m-ary tree for some positive integer m?

7. The following addresses are in random order:

2.1.1, 3.1, 2.1, 1, 2.2,.1.2, 0, 3.2, 2.2, 1.1, 2, 3.1.1, 2.2.1, 3, 2.2.1.1

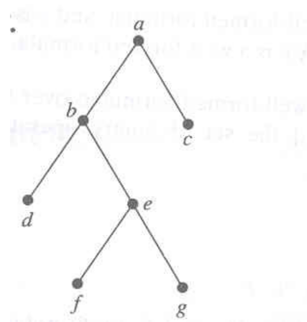
(a) Place the addresses in lexicographical order.

(b) Draw the corresponding rooted tree?

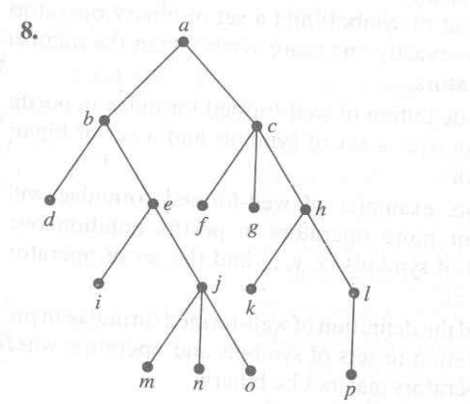
(c) Identify the path p from the root 0 to each of the following vertices, and find the level number of the vertex: (i) 2.2.1.1 (ii) 3.1 (iii) 2.1.1

(d) Find the leaves of T.

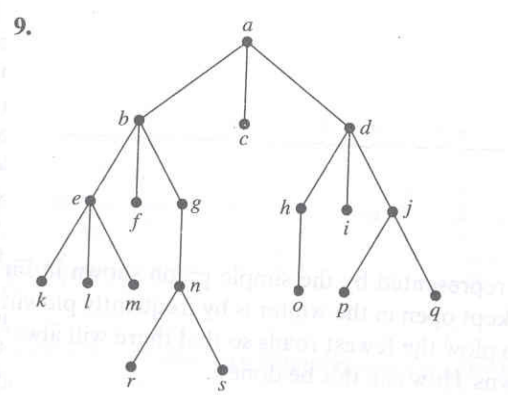
8. Question No. 9.10 (Schaum's Series)
9. Question No. 9.33 (Schaum's Series)
10. What is the value of the prefix expression $+ - * 2 \quad 3 \quad 5 \ / \uparrow 2 \ 3 \ 4$? Ans: 3.
11. Represent the expression (x + xy) + (x/y) and x + ((xy + x)/ y) using an ordered rooted tree.
12. Represent $(A \cap B) - (A \cup (B - A))$ using an ordered rooted tree.
13. Write the expressions of Q. No. 11 and Q. No. 12 in prefix form.
14. Question No. 9.34, 9.35 (Schaum's Series)
15. Consider the digraph G of Question No. 9.34, determine how many paths of length 3 exist in g and which vertices are connected by a path of length 3.
16. Draw the subtree of the tree (in Q. No. 5) that is rooted at
    (a) a (b) c (c) e.
17. Give an example of a complete bipartite graph $K_{m,n}$, m and n are positive integers, which is a tree.
18. Example 10.1 (Schaum's Series)
19. Consider the following trees



Tree 1



Tree 2



Tree 3

(a) Determine the order in which a preorder traversal visits the vertices of the above
rooted trees.

(b) Determine the order in which an inorder traversal visits the vertices of the above
rooted trees.

(c) Determine the order in which a postorder traversal visits the vertices of the above
rooted trees.