

EXP 3 : CONSTRAINT SATISFACTION PROBLEM
CRYPTARITHMETIC PROBLEM

Problem Formulation

Given an expression where two words add to give a third word, assign some unique digits to each letter where some letter cannot be assigned to different digit.

Display possible mappings to each of the letters S, E, N, D, M, O, R and Y.

Initial State

- D, E, Y, N, R, O, S, M = ?
- $C_1 = ?$ $C_2 = ?$ $C_3 = ?$

C_1, C_2, C_3 stands for carry variable resp.

Goal State

The digits to the letters must be assigned in such that sum is satisfied.

Various possible goal state can be achieved.

One of them is :

D = 7 E = 5 Y = 2 N = 6 R = 8 O = 0 S = 9 M = 1

$C_1 = 1$ $C_2 = 1$ $C_3 = 0$

Problem Solving

- Starting from left hand side, the terms are S & M. Assign a digit which could give a satisfactory result. let's assign $S=9$
 $M=1$

$$\begin{array}{r} S \\ + M \\ \hline MO \end{array} \longrightarrow \begin{array}{r} 9 \\ + 1 \\ \hline 10 \end{array}$$

we get O as 0

- next we take terms E & O to get N as result.
considering $E=5$

$$\begin{array}{r} E \\ + O \\ \hline N \end{array} \xrightarrow{X} \begin{array}{r} 5 \\ + 0 \\ \hline 5 \end{array}$$

{ This is not possible
 \therefore E & N cannot
be assigned to
same digit }

assume $C_2 = 1$

$$\begin{array}{r} C_2 \text{ (carry)} \\ + E \\ + O \\ \hline N \end{array} \begin{array}{r} \textcircled{1} \\ 5 \\ 0 \\ \hline 6 \end{array}$$

we get $N=6$

- further adding next two terms N and R.

$$\begin{array}{r} N \\ + R \\ \hline E \end{array} \xrightarrow{X} \begin{array}{r} 6 \\ + 8 \\ \hline 14 \end{array}$$

{ as E is already
assigned 5 }

assume $C_3 = 1$

$$\begin{array}{r} N \\ + R \\ \hline E \end{array} \longrightarrow \begin{array}{r} \textcircled{1} \\ 6 \\ + 8 \\ \hline 15 \end{array}$$

we get $R=8$

on adding last two terms, 1 carry must be produced

$$\begin{array}{r} D \\ + E \\ \hline Y \end{array} \rightarrow \begin{array}{r} 7 \\ + 5 \\ \hline 12 \end{array}$$

- keeping all constraints in mind, the final state is :

$G(0)$	$G(1)$	$G(1)$		
$S(9)$	$E(5)$	$N(6)$	$D(7)$	
$M(1)$	$O(0)$	$R(8)$	$E(5)$	
<hr/>				
$M(1)$	$O(0)$	$N(6)$	$E(5)$	$Y(2)$

$S = 9$	$D = 7$	$R = 8$
$E = 5$	$M = 1$	$Y = 2$
$N = 6$	$O = 0$	

Algorithm :

- Start
- Accept an expression 'SEND + MORE = MONEY'
- Extract the words SEND, MORE & MONEY.
- Permute for different combination of values for S, E, N, D, M, O, R, Y.
- Check if sum of left value, SEND + MORE is equal to right sum, MONEY or not.
- If the sum value matches print the mapping
- Continue for other permutations as well.
- Stop.