Digital Image Processing ASSIGNMENT : 1

Ans 1) The equation of the surface for biquadratic interpolation using 9 points (neighbours) is given as :-

$$v(x,y) = \sum_{i=0}^{2}\sum_{j=0}^{2} a_{ij}\, x^i y^j$$

$$v(x,y) = a_{00} + a_{01}y + a_{10}x + a_{02}y^2 + a_{20}x^2 + a_{21}x^2y + a_{12}xy^2 + a_{22}x^2y^2$$

For getting the equation of the surface, we need 9 equations since there are 9 unknowns.

So, we take 9 neighbours of the current point from input.

The equation then can be written as :-

$$v_1(x_1, y_1) = a_{00} + a_{01}y_1 + \dots + a_{22}x_1^2y_1^2$$

$$v_2(x_2\, y_2) = a_{00} + a_{01}y_2 + \dots + a_{22}x_2^2y_2^2$$

$$\vdots$$

$$v_9(x_9, y_9) = a_{00} + a_{01}y_9 + \dots + a_{22}x_9^2y_9^2$$

$v_1, v_2 \dots v_9$ are the pixel values at $(x_1, y_1) \dots (x_9 y_9)$.

So, the above 9 equations can be re-written as :-

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & x_1^2 & y_1^2 & x_1y_1 & x_1^2y_1 & x_1y_1^2 & x_1^2y_1^2 \\ 1 & x_2 & y_2 & x_2^2 & y_2^2 & x_2y_2 & x_2^2y_2 & x_2y_2^2 & x_2^2y_2^2 \\ 1 & x_3 & y_3 & x_3^2 & y_3^2 & x_3y_3 & x_3^2y_3 & x_3y_3^2 & x_3^2y_3^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_9 & y_9 & x_9^2 & y_9^2 & x_9y_9 & x_9^2y_9 & x_9y_9^2 & x_9^2y_9^2 \end{bmatrix} \begin{bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{20} \\ a_{02} \\ a_{11} \\ a_{21} \\ a_{12} \\ a_{22} \end{bmatrix}$$

$$V = XA$$

$$\text{So} \quad A = X^{-1}V$$

using this expression, we can find A, that is the matrix of coefficients.

Ans 2) Given 2×2 image as $\begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}$

Interpolation factor $c = 1.5$

$\therefore$ Dimensions of output image $= \underline{3 \times 3}$

Mappings of the output coordinate to input coordinate :

output	Input
0,0 $\longrightarrow$	0,0
1,1 $\longrightarrow$	$2/3, 2/3$ (0.666, 0.666)
2,2 $\longrightarrow$	$4/3, 4/3$ (1.333, 1.333)

maps to

$$\begin{bmatrix} 5 & 8.3 & 13.3 \\ 8.3 & 13.8 & . \\ & . & . \end{bmatrix}$$

Output Matrix Image

For bilinear interpolation on (1,1) :-

We first find what point does (1,1) map to in the input.

$$x_{in}, y_{in} = \frac{1}{1.5}, \frac{1}{1.5} = 0.666, 0.666$$

Now we find 4 neighbours of x_{in}, y_{in} in input image

they come out to be : (0,0) (0,1) (1,0) and (1,1)

Using $V = XA$ [V : pixel value & X : coordinate]

where $V(x,y) = ax + by + cxy + d$

We can write :

$$\begin{bmatrix} 5 \\ 10 \\ 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & +1 & 0 \\ 1 & -1 & -1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix}$$

$\therefore$ $a = b = c = d = 5$

$\therefore$ Pixel value at output [1][1] $= 5(0.666) + 5(0.666) + 5(0.666)^2 + 5$

$= \underline{13.88} \approx 13.9$