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Design and Optimization of 3D Radial Slot Grain Configuration

Ali Kamran, Liang Guozhu*

School of Astronautics, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

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Abstract

Upper stage solid rocket motors (SRMS) for launch vehicles require a highly efficient propulsion system. Grain design proves to be vital in terms of minimizing inert mass by adopting a high volumetric efficiency with minimum possible sliver. In this article, a methodology has been presented for designing three-dimensional (3D) grain configuration of radial slot for upper stage solid rocket motors. The design process involves parametric modeling of the geometry in computer aided design (CAD) software through dynamic variables that define the complex configuration. Grain burn back is achieved by making new surfaces at each web increment and calculating geometrical properties at each step. Geometrical calculations are based on volume and change-in-volume calculations. Equilibrium pressure method is used to calculate the internal ballistics. Genetic algorithm (GA) has been used as the optimizer because of its robustness and efficient capacity to explore the design space for global optimum solution and eliminate the requirement of an initial guess. Average thrust maximization under design constraints is the objective function.

Keywords: solid rocket motors; 3D grains; radial slot configuration; internal ballistics; computer aided design; heuristic optimization; genetic algorithm

1. Introduction

Grain design is to evolve burning surface area and develop the relationship with web burnt. Grain design proves to be vital in terms of minimizing inert mass by adopting a high volumetric efficiency with minimum possible sliver. Three-dimensional (3D) grains are complex in shape; hence their design methodology is also complicated. Different methods have been used to calculate the geometrical properties of grain burn back analysis [1-2]. Analytical methods, though accurate but limited to specific geometries, have been used scarcely for 3D grain configurations.

The most prominent analytical method is the generalized coordinate grain calculation method which uses basic geometrical shapes to define the initial grain void[3-5]. This method has long been used in industry for grain design, though it is complex and may have small errors. The calculation step size for burn back analysis could prove to be critical and leads to oscillation in the burning area calculations. Ref.[6] presented an improved approach for removing pulsating errors in grain design due to the web and axial increments. Refined numerical approach still encounters considerable errors. In these conventional methods, the accuracy of solution largely depends upon the web and axial increment chosen for volume calculation, and will indeed require certain approximation to limit computational time.

Ref.[7] generated carpet plots for a large amount of data for star grain configurations. It presented optimization for geometrical parameters of star grain while leaving number of star points and varying other geometrical parameters. The approach has severe limitations for the large number of design variables. Ref.[8] moved one step further and applied pattern search technique to the design and optimization of 3D grain configuration. The approach has limited applicability in modern era as solution quality is heavily dependent on starting solution. The approach has a tendency to fall prey to local optima similar to any gradient descent/ascent method and has extreme sensitivity to the starting solution.

Ref.[9] presented design and optimization for finocyl grain using generalize coordinate method. Ref.[10] presented a hybrid optimization technique for finocyl grain configuration using the same method.

The above discussion necessitates the requirement of adopting heuristic optimization technique not only to avoid local optima but also to eliminate the requirement of starting point. Introducing computer aided design (CAD) to the process will improve the accuracy of calculated geometrical properties.

CAD based programs are available in industry and

*Corresponding author. Tel.: +86-10-82339944.
E-mail address: lgz@buaa.edu.cn

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have proved to be tremendously useful for the design process of solid rocket motor (SRM). Two softwares, PIBAL [11] and ELEA [12], use CAD modeling for 2D and 3D grains design of SRM. The former uses a simplified ballistic model and the latter one can give a point to point burning rate taking account of local gas dynamics.

The methodology adopted in this work is CAD modeling of the propellant grain. This approach creates a parametric model with dynamic variables to define the grain geometry. Surface offset simulates grain burning regression and evaluates subsequent volume at each step.

Upper stage SRM of launch vehicles requires highly efficient propulsion system. An infinite number of possibilities exist, therefore, the need arises for intelligent optimization approach which can control the design domains and configure an optimum design within set design limits and constraints.

3D radial slot geometry is extremely complex. It has 24 independent design variables that need to be optimized to attain the best possible solution. The large number of design variables complicates the optimization process. The present study employs genetic algorithm (GA) as the optimizer because of its robustness and efficient capacity to explore the design space for global optimum solution and eliminate the requirement of an initial guess. The aim is to find the optimal configuration while adhering to performance objectives and design constraints.

2. Geometric Modeling and Regression

The grain geometry is based on CAD software that has the capability of handling parametric modeling. Grain is modeled in parts to provide ease and ensure lesser chances of surface creation failure. A simple variable input is sufficient to create the geometry. CAD software is linked to MATLAB via Visual Basic. MATLAB sends variable array to CAD software enabling automatic creation of the grain geometry. CAD software evaluates the geometrical properties and sends to MATLAB for further calculations. Fig.1 presents the flowchart of the design process.

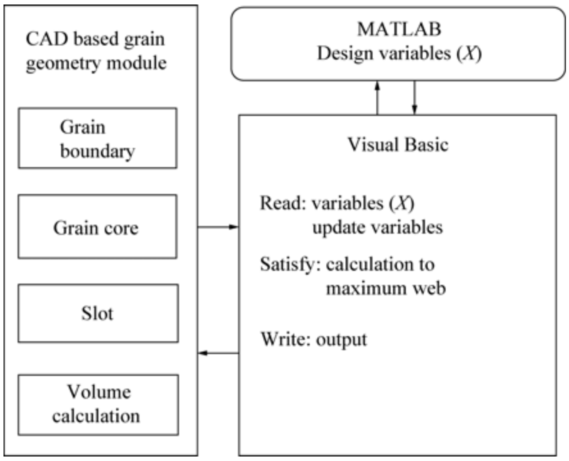

Fig.1 Grain design process.

Fig.2 shows a detailed description of the grain modeling. The following steps explain the construction of grain configuration:

(1) Front and rear opening radii for chamber case, motor length, ellipsoid ratio, and diameter are the input parameters required to create the grain external boundary (see Fig.2(a)).

(2) To construct the bore, front-end web along with different dimensions are the input variables to be provided (see Fig.2(b)). The rear end can have large cylindrical cavity provision for nozzle submergence.

(3) The input requirements to create slot are slot thickness, web above slot and axial distance from certain references (see Figs.2(c) and (d)).

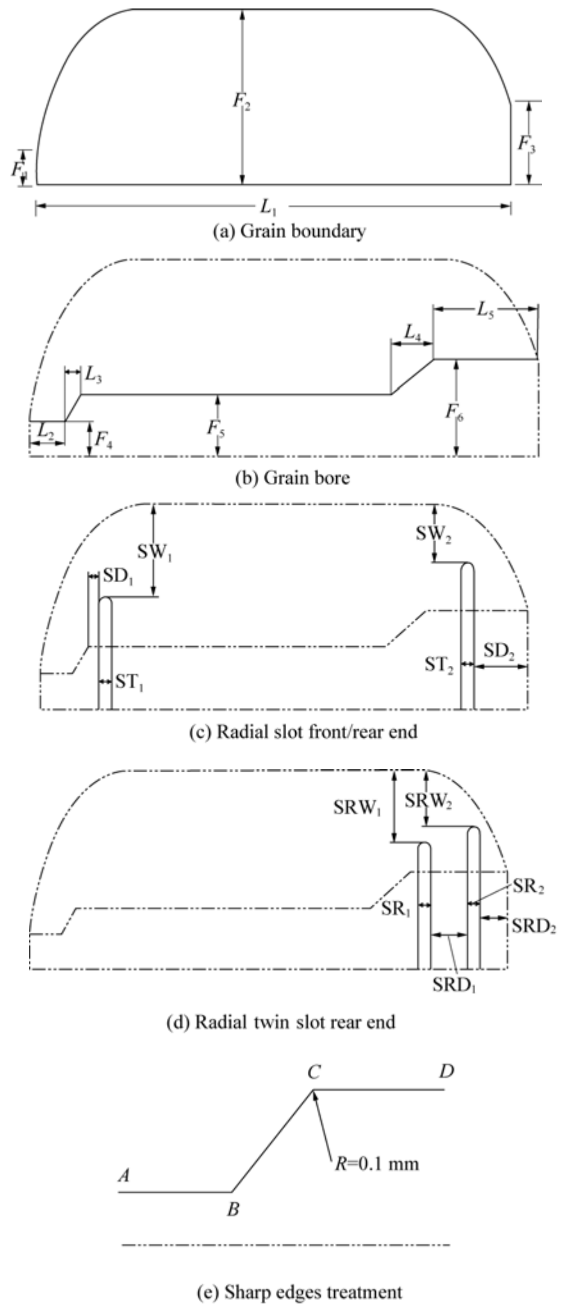

Fig.2 Grain modeling process.

(4) In case a slot is not required the slot web is increased to bore radius (see Figs.2(c) and (d)).

(5) Two configurations can be designed: front/ rear slot configuration (see Fig.2(c)) and twin slot at the rear end (see Fig.2(d)).

(6) Sharp corners are filleted to account for new surfaces that are created during burning as shown in Fig.2(e). Lines *AB* and *BC* are connected using CAD function “Connect”, so that they remain connected during offsetting operation. Lines *BC* and *CD* are connected through a small fillet of radius 0.1 mm in the initial geometry. Offsetting process involves increasing the fillet radius by a value equal to web increment.

Table 1 lists a description of 24 independent design variables for complex grain geometry.

Table 1 Design variables for grain geometry

Variable	Description
L_1	Grain length
L_2	Front end web
L_3	Front cone length
L_4	Rear cone length
L_5	Rear cylinder length
F_1	Motor front opening
F_2	Grain radius
F_3	Motor rear opening
F_4	Grain front opening
F_5	Bore radius
F_6	Rear cylinder radius
ST_1	Front slot width
ST_2	Rear slot width
SD_1	Front slot distance
SD_2	Rear slot distance
SW_1	Front slot web
SW_2	Rear slot web
SR_1	Slot width 1
SR_2	Slot width 2
SRD_1	Slot distance 1
SRD_2	Slot distance 2
SRW_1	Slot web 1
SRW_2	Slot web 2
$m_{1,2}$	Ellipsoid ratio

CAD software performs the following steps for constructing the parametric geometric model after defining the variables for grain configuration:

(1) Grain boundary is solid and constructed by revolve protrusion with no burning surface.

(2) Grain bore is constructed by revolve surface and all surfaces burning.

(3) Boolean function is used to subtract the solid within grain bore.

(4) Similar operation is performed for radial slots and all surfaces burning.

(5) Surface offset function available in CAD software is used to simulate burning, by offsetting the surface by a web increment equal and orthogonal in all directions.

(6) Boolean function is used at each web increment to subtract the solid within grain bore and slots to calculate new volume.

(7) Offsetting and boolean operations are repeated till the web is completely burnt.

Model verification is performed by calculating star grain burning area with the present method and analytical method. Star grain analytical expressions are adopted from Ref.[13]. Fig.3 shows the comparison of burning area between the two methods. Modeling presented in this article shows excellent performance compared with analytical method.

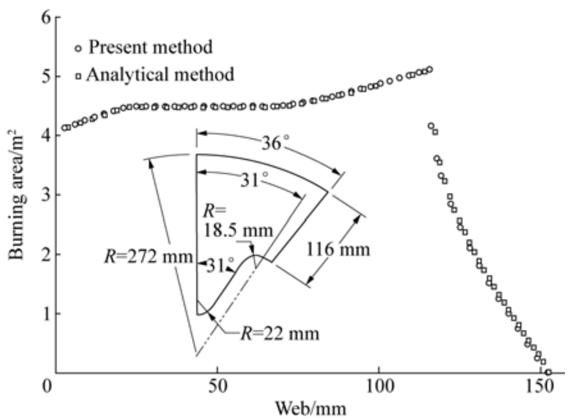

Fig.3 Burning area comparison for model verification.

The grain regression is achieved by equal web increment in all directions. The selection of web increment is critical to grain regression. At each step new grain geometry is created automatically thereafter volume at each web increment is calculated. A decreasing trend is obtained for volume of the grain.

Burning surface area is calculated by

$$A_{bk}=\frac{V_{k+1}-V_k}{w_{k+1}-w_k} \tag{1}$$

where k is the web step, V the volume of propellant, and w the web.

Propellant mass is calculated by

$$m_p=\rho_p V_k \tag{2}$$

where ρ_p is the propellant density.

3. Performance Prediction and Optimization Model

The SRM performance is calculated using simplified ballistic model. Steady state chamber pressure p_c is calculated by equating mass generated in chamber to mass ejected through nozzle throat [14-16].

$$p_c=(\rho_p a c^* K)^{1/(1-n)} \tag{3}$$

where $K=A_b/A_t$, A_t is the area of throat, a the burn rate coefficient, n the pressure sensitivity index, and c^* the characteristic velocity.

Thrust is determined by

$$F=C_F p_c A_t \tag{4}$$

Thrust coefficient is given by

$$C_F = \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}\left[1-\left(\frac{p_e}{p_c}\right)^{(\gamma-1)/\gamma}\right]} + \frac{p_e - p_{amb}}{p_c}\varepsilon \tag{5}$$

where γ is the specific heat ratio, p_e nozzle exit pressure, p_{amb} ambient pressure and ε nozzle area ratio.

Requirements have been given for fixed length and outer diameter of the grain while remaining within constraints of burning time, propellant mass and nozzle parameters. Maximization of average thrust $F_{av,max}(X)$ is the design objective, where X is given as

$$X = f(F_1, F_2, F_3, F_4, F_5, F_6, ST_1, ST_2, SD_1, SD_2, SW_1, SW_2, L_1, L_2, L_3, L_4, L_5, SR_1, SR_2, SRW_1, SRW_2, SRD_1, SRD_2)$$

subject to constraints

$$C_j(X) \le 0 \qquad (j = 1, 2, \cdots, n)$$

Bound for all variables is provided for efficient search in design space:

$$\begin{cases} \text{Lower bound} = \min(X_i) \\ \text{Upper bound} = \max(X_i) \end{cases} (i = 1, 2, \cdots, 23)$$

4. Optimization Method

GA can handle both discrete and continuous variables, making them well suited to major design problems. GA is capable of examining historical data from previous design and attempts to look for patterns in the input parameters which produce favorable output. GA uses neither sensitivity derivatives nor a reasonable starting solution and yet proves to be a powerful optimization tool.

GA employs three operators to propagate its population from one generation to another (a population of 30 members for 20 generations is found sufficient in the present study). The first operator is the "Selection" operator that mimics the principle of "Survival of the Fittest". Stochastic uniform option is used for selection. The second operator is the "Crossover" operator, which mimics mating in biological populations. The crossover operator propagates features of good surviving designs from the current population into the future population, which will have better fitness value on average. Thirty percent of the population is used for matting on a single point basis. The last operator is "Mutation", which promotes diversity in population characteristics. The mutation operator allows for global search of the design space and prevents the algorithm from getting trapped in local minima. A uniform mutation strategy is used with approximately a quarter of the population. Details on GA can be found in Refs. [17]-[20].

The optimization algorithm has been tested on widely stated benchmark functions[21]. The algorithm proves robust enough for engineering application.

Fig.4 presents the flowchart of GA.

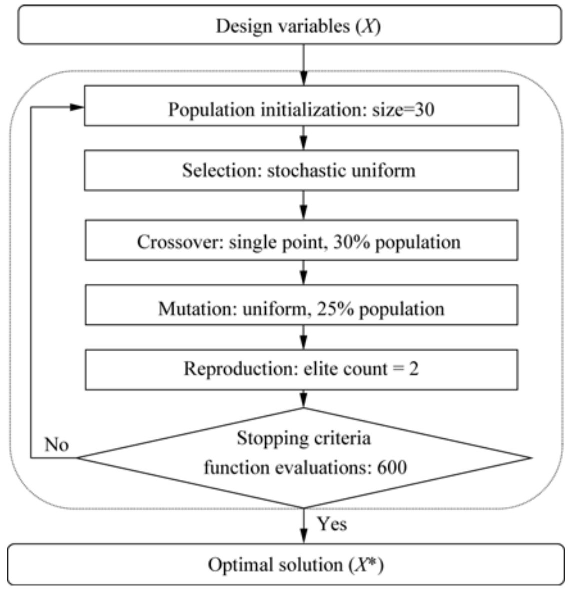

Fig.4 Flowchart of genetic algorithm.

Pseudo-code of the optimization is listed as follows:

Optimization routine

Initialize

- Set population size
- Set total number of generation
- Set stopping criteria

While (stopping criteria Not achieved)

- Create public-board to store information
- Generate population (random)

For i = 1 to total generations

For j = 1 to population size

Call Visual Basic

Arrange **Input** data for **CAD**

Call CAD

For k = 1 to web

(a) Make grain geometry

(b) Calculate physical properties

(c) Write **Output** data

End

End

Evaluate constraints

Evaluate fitness

CALL Crossover

Check crossover rate

Create new off-springs

CALL Mutation
Mutate prescribed amount of individuals (random)

Send information to public-board

End
End

5. Optimization Results

Hydroxy terminated polybutadine (HTPB) based propellant is selected for the grain configuration. Table 2 lists propellant and nozzle parameters used in ballistic analysis, in which D_t is the throat diameter, AP represents ammonium per chlorate, and Al represents aluminum.

Front/ rear radial slot configuration is chosen as case study as shown in Fig.2(c). Table 3 presents the design constraints for grain configuration, in which t_b is burning duration.

The design variables and respective bounds for thirteen variables in the optimization model are shown in Table 4.

Table 2 Propellant and nozzle parameters

Parameter	Value
D_t/mm	160
ε	16
c^*/(m·s^{-1})	1 550
ρ_p/(kg·m^{-3})	1 750
n	0.34
a/(mm·s^{-1}·Pa^{-n})	0.031 1
Propellant	HTPB/AP/Al

Table 3 Design constraints for configuration

Variable	Value
L_1/mm	2 395
F_2/mm	700
t_b/s	74±3
p_{max}/bar	< 65
m_p/kg	5 000±100
ε	16

Table 4 Bound for design variables

Variable	Lower bound	Upper bound
F_4/mm	80	120
F_5/mm	220	280
F_6/mm	330	400
ST_1/mm	25	50
ST_2/mm	25	50
SD_1/mm	100	200
SD_2/mm	80	200
SW_1/mm	150	280
SW_2/mm	150	250
L_2/mm	70	130
L_3/mm	80	120
L_4/mm	80	120
L_5/mm	150	250

Table 5 shows the optimum dimensions obtained from GA.

Table 6 depicts the ballistic performance achieved.

Fig.5 shows the optimum grain configuration and burning regression at different web steps.

Table 5 Optimum design variables

Variable	Optimum value
F_4/mm	96.5
F_5/mm	266.4
F_6/mm	352
ST_1/mm	28.6
ST_2/mm	36.6
SD_1/mm	160.8
SD_2/mm	122.5
SW_1/mm	268.5
SW_2/mm	196
L_2/mm	83.7
L_3/mm	98.3
L_4/mm	96
L_5/mm	188

Table 6 Ballistic performance

Parameter	Optimum value
F_{av}/kN	176.6
m_p/kg	4 937
t_b/s	74.5
p_{max}/bar	61.6

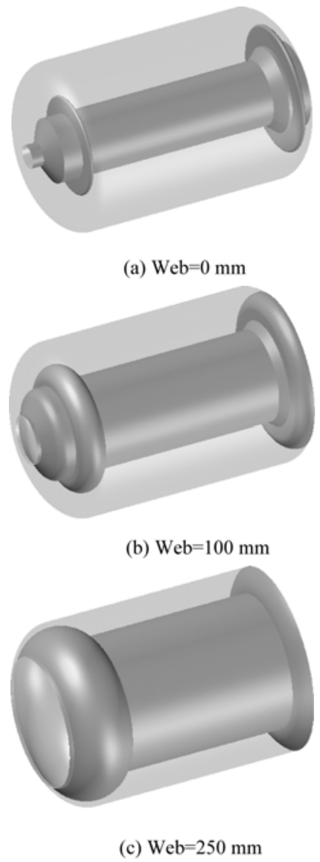

Fig.5 Grain configuration and burning regression.

Fig.6 shows the burning area and volume with respect to web burnt. Fig.7 depicts pressure and thrust time history.

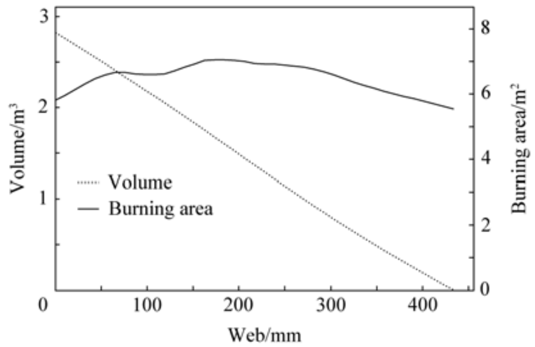

Fig.6　Volume/ burning area vs web trace.

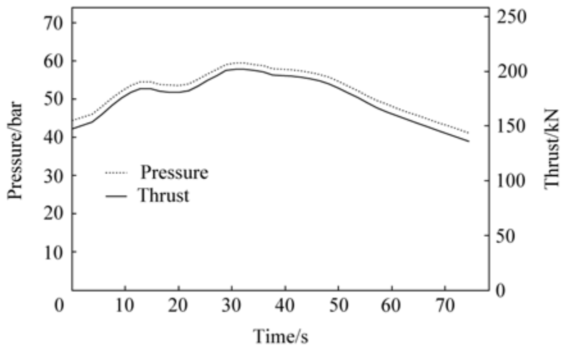

Fig.7　Pressure/ thrust vs time trace.

Results reveal that the optimum grain configuration achieved with the proposed approach has provided promising results. The average thrust achieved is 176 kN, which satisfies all strict constraints.

6. Conclusions

This research effort presents an automated approach for the design and optimization of 3D radial slot configurations. This approach integrates CAD software and optimization module, and based on geometrical data, ballistic performance is evaluated.

CAD model allows different entities of the grain, to be modeled separately, which not only prevents surface creation failures but also allows for future modification of the model. Similar complex grain geometries can be created by using simple input parameters and then optimized. The use of GA eliminates the problem of suitable initial guess. This approach attains optimized design variables, adheres to design constraints and proves a noteworthy increase in capability of searching optimal solutions. A maximum of 600 function evaluation is enough to converge to a global optimum.

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Biographies:

Ali Kamran Born in 1975 at Karak, Pakistan, he received his B.E. mechanical degree in 1999 from University of Engineering and Technology (UET) Peshawar, Pakistan. He received his M.S. degree in solid rocket propulsion from Beijing University of Aeronautics and Astronautics (BUAA), China in 2004. Currently he is a Ph.D. candidate in the same university. His research interest includes design and optimization of space propulsion systems.
E-mail:alklsl@yahoo.com

Liang Guozhu　Born in 1966, he is a professor in department of Space Propulsion, School of Astronautics, Beijing University of Aeronautics and Astronautics. His research interests include propulsion theory and engineering of aeronautics and astronautics. His current research field is design and simulation of solid rocket motor and liquid rocket engine.
E-mail: lgz@buaa.edu.cn