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Influence of plane-parallel finishing trajectories to the roughness obtained by milling of spherical surfaces

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Abstract

The purpose of the present paper is to determine a mathematical model in order to evaluate theoretically the roughness of a spherical surface finished with a torodial end mill. The parameters that influence the theoretical value of the roughness are: (i) technological parameters (cutting speed, feed per tooth, radial feed) or (ii) geometrical (the geometry of the semi-product, the geometry of the cutting tool). The calculus to determine the roughness value is a difficult task taking into consideration the complexity of the mathematical relations; the paper presents a graphic method using a CAD application (AutoCAD) and a programming environment (AutoLISP).

Keywords: Roughness; milling; toric end-mill; mold; surface.

1. The Premises of the Mathematical Model

The optical block of a motor vehicle is commonly-known as the headlight; it serves to light up the road in conditions of low visibility and it comes under road lighting regulations. The main components of the optical block are (Figure 1): (i) cover; (ii) reflector; (iii) the transparent optical part.

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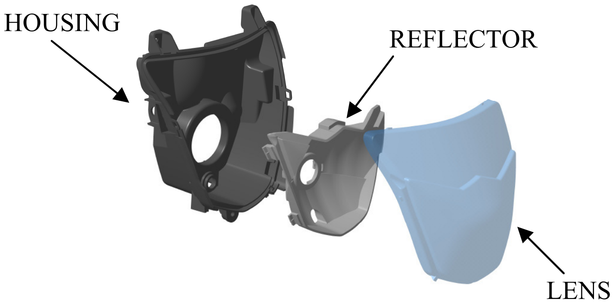

Fig. 1. The main components of the optical block [Tri.o.m. S.p.a.]

In addition to these, there is also the light source, the vertical control system of the light spot, the wire harness and the necessary contacts, fasteners. The three main components are manufactured by plastic injection.The aesthetical and functional requirements for the headlights affect the complexity of the injection molds proper to these components. The emergence of CNC processing machines and of the computer aided design and manufacturing applications revolutionized the engineering of plastic injection molds, this NC-CAD-CAM trio giving free hand to the auto designers. The quality of lighting is decided by the quality level of the mold cores surfaces, though despite the technological progress, the manual polishing perfection is still to be achieved. The present paper deals with aspects regarding the quality of the mold cores and offers technological recommendations, taking into account cost and productivity issues.

The active interior surface of the reflector and implicitly the forming surface of the injection mold's core is a sculptural surface that cannot be expressed by mathematical relations. In order to define equations for determining the theoretical roughness of a mill-generated surface, it is required an approximation of this forming surface with a pitch surface (plane, cylinder, cone, sphere, torus), respectively with a spherical surface (Figure 2). The spherical section defines the portion between two parallel planes intercepting a sphere.

The mathematical approach accounts using a 3 axis processing centre, setting vertically the milling tool axis. The accounted finishing paths are plane-parallel and radial. The following index presents the inputs in the mathematical evaluation of the theoretical micro asperities' heights:

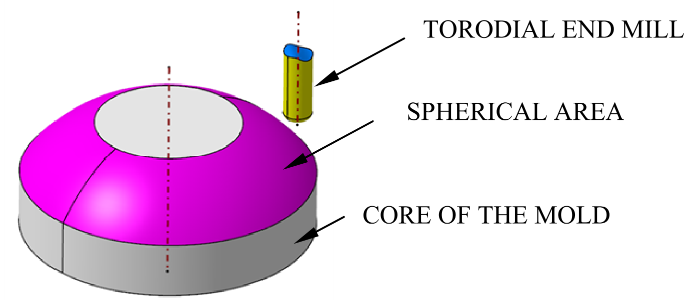

Fig. 2. Theoretical surface for roughness prediction

Nomenclature

H	theoretical height of the irregularities

X	main approach angle
χ’	secondary approach angle
ω	angular speed
V_c	cutting speed
fz	feed per tooth
ap	radial feed
Z	teeth number
D	diameter of the milling tool
R_2	corner radius of the milling tool
R_s	radius of workpiece
θ	position parameter
t	incremental parameter

2. The Mathematical Model

Consider a reference system $X_0Y_0Z_0$ attached to the core of the mold and a reference system $X_1Y_1Z_1$ attached to the torodial end mill. The torodial end mill is revolving around its axis with a rotative speed n and shifts on trajectories in the form of circular contour lines, sweeping the entire forming surface (Figure 3). The torodial end mill can move on a contour line by cut-up milling or cut-down milling.

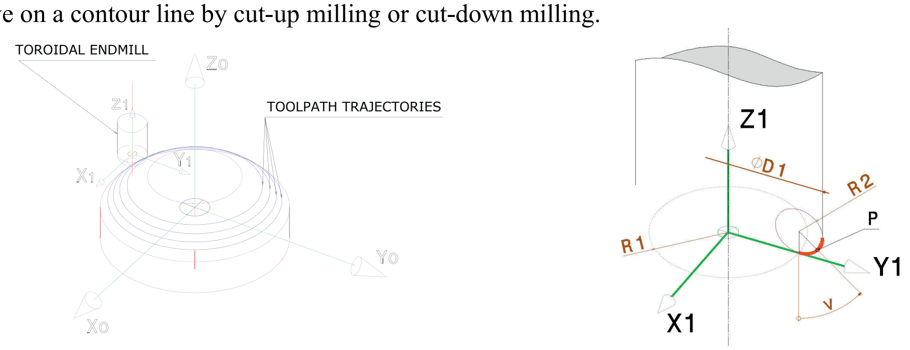

Fig. 3. Case of plane-parallel paths

Both Z axes –of the cutting tool and of the workpiece – are positioned in such way to have the same direction as the Z axis of the CNC machine and its positive sense. As the workpiece is of revolution, the direction and the sense of the X and Y axes are taken randomly.

The bit edge that generates the torus of the cutting tool’s active area, is defined by the parameters R1 and R2 and υ (Figure 3). The equation of the bit edge, of the circular curve described by point P in figure 3 can be written in a parametric form, using the following expression:

$$\begin{cases} X_1 = 0 \\ Y_1 = R_1 + R_2 \sin v \\ Z_1 = R_2 \sin v \end{cases} \tag{1}$$

where υ passes through the range $v \in \left[0, \frac{\pi}{2}\right]$.

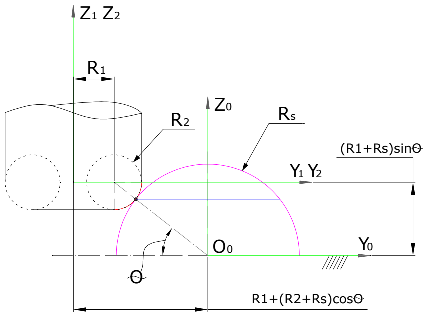

Fig. 4. Positioning the horizontal plane

The directory path on which the cutting tool is moving is a circular trajectory obtained by intersecting the spherical area with the horizontal plane –positioned by the parameter θ (Figure 4). Considering the tangency of the bit edge to the spherical area between the position angle υ of the point P and the angle θ that defines the position of the horizontal plane, we define the relation:

$$\theta = \frac{\pi}{2} - \upsilon \tag{2}$$

As such, the vector that defines the position of point P is written as:

$$\boldsymbol{r}_1 = \begin{vmatrix} 0 \\ R_1 + R_2 \cos\theta \\ R_2 \cos\theta \\ 1 \end{vmatrix} \tag{3}$$

According to figure 5, the cutting tool executes a roto-translation movement along the circular directory path (starting from the initial moment t0) and a point P on the bit edge will describe a curled epicycloid.

The theoretical roughness of this mill-generated surface is the distance between the directory path and the interior intersection point of the curled epicycloid. More bit edges generate more curled epicycloids that intersect themselves, influencing directly the roughness value, meaning that more teeth the mill cutter has, higher the quality of the surface will be.

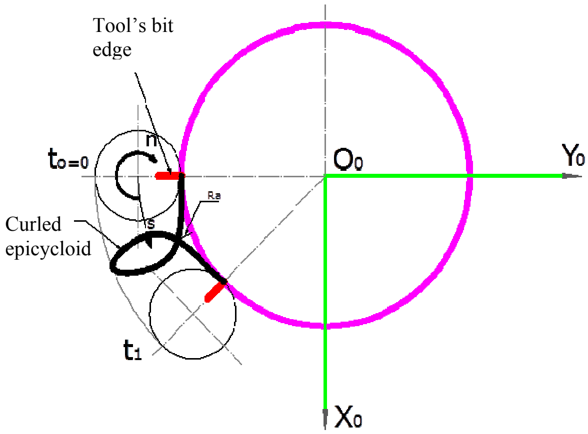

Fig. 5. Curled epicycloid generated by the trajectory of a point of the bit edge

Next, a proposition for a mathematical model to determine the equations for these curled epicycloid curves and to determine the theoretical roughness of the mill-generated surface, depending on the technological parameters, on the cutting tool's parameters and on the geometry of the finished product. As a mathematical instrument, it was used the coordinate transformation method.

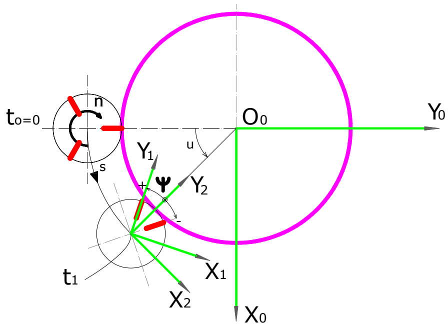

Fig. 6. Defining the secondary system

In order to write easier the curves' equations, it was used a secondary reference system, which executes a translation motion along the equidistant circle to the directory path, and a rotary motion chose in such a manner that the Y axis is oriented towards the origin of the base system attached to the workpiece (Figure 6). Angle ψ forms between the two reference systems, the secondary system and the one attached to the milling tool. The angle can have a positive value corresponding to cut-up milling or a negative value corresponding to cut-down milling.

As such, the position vector of point P on the bit edge – in connection to the reference system X0Y0Z0 attached to the workpiece – is described with the following relation:

$$\boldsymbol{r}_0 = \boldsymbol{M}_{02} \cdot \boldsymbol{M}_{21} \cdot \boldsymbol{r}_1 \tag{4}$$

The transition matrix from the mobile system to the secondary one takes the following form:

$$\boldsymbol{M}_{21} = \begin{vmatrix} \cos\psi & \sin\psi & 0 & 0 \\ -\sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{5}$$

The transition matrix from the secondary system to the base reference system takes the following form:

$$\boldsymbol{M}_{02} = \begin{vmatrix} \cos u & -\sin u & 0 & (R_1 + (R_2 + R_s)\cos\theta)\sin u \\ \sin u & \cos u & 0 & -(R_1 + (R_2 + R_s)\cos\theta)\cos u \\ 0 & 0 & 1 & (R_2 + R_s)\sin\theta \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{6}$$

The following expressions are obtained after the calculus:

$$\begin{cases} x_0 = -x_1\cos\psi\sin u - x_1\sin\psi\cos u + (R_1 + (R_2 + R_s)\cos\theta)\sin u \\ y_0 = x_1\cos\psi\cos u - x_1\sin\psi\sin u - (R_1 + (R_2 + R_s)\cos\theta)\cos u \\ z_0 = (R_2 + R_s)\sin\theta \end{cases} \tag{7}$$

Using trigonometric relations, we obtain the expression for the curled epicycloids curve:

$$\begin{cases} x_0 = -x_1 \sin(u+\psi) + (R_1 + (R_2 + R_s)\cos\theta)\sin u \\ y_0 = x_1 \cos(u+\psi) - (R_1 + (R_2 + R_s)\cos\theta)\cos u \\ z_0 = (R_2 + R_s)\sin\theta \end{cases} \tag{8}$$

The final expression must contain the parameters defined at the beginning, such as cutting speed (V_c), feed per tooth, the cutting tool's number of teeth, the diameter of the milling tool, corner radius and workpiece radius. θ is the position parameter for the horizontal plane and time *t* is the incremental parameter. The rotative speed depends on the cutting speed and on the effective diameter (defined by the position of point P in Figure 4), according to the relation:

$$n = \frac{1000 * V_c}{2 * (R_1 + R_2 \sin v) * \pi} \qquad \textit{[rot/min]} \tag{9}$$

By defining the angular speed we obtain:

$$\omega = 2\pi n = \frac{1000 * V_c}{(R_1 + R_2 \sin v)} \qquad \textit{[rad/min]} \tag{10}$$

Or

$$\omega = \frac{1000 * V_c}{60 * (R_1 + R_2 \sin v)} \qquad \textit{[rot/sec]} \tag{11}$$

The tool's feed along the directory curve becomes:

$$V_f = f_z * Z * \frac{1000 * V_c}{2 * 60 * (R_1 + R_2 \sin v) * \pi} \qquad \textit{[mm/sec]} \tag{12}$$

The angle positioning the bit edge is described by the following relation:

$$\psi = \omega * t = \omega * \frac{180}{\pi} * t \qquad \textit{[deg]} \tag{13}$$

Introducing into the relation we obtain:

$$\psi = \pm \frac{3000 * V_c * t}{\pi * (R_1 + R_2 \sin v)} \qquad \textit{[deg]} \tag{14}$$

The covered distance is expressed as:

$$s = t * V_f = u * (R_1 + (R_2 + R_s)\cos\theta) \quad \textit{[mm]} \tag{15}$$

Expressing the angle *u* in degrees we obtain:

$$u = \frac{3000 * V_c * f_z * z * t}{2 * \pi^2 * (R_1 + R_2 \sin v) * (R_1 + (R_2 + R_s)\cos\theta)} \qquad \textit{[deg]} \tag{16}$$

Introducing into the relation:

$$\theta = \frac{\pi}{2} - v \tag{17}$$

We obtain the expressions for the parameters u and ψ:

$$u = \frac{3000 * V_c * f_z * z * t}{2 * \pi^2 * (R_1 + R_2 \cos\theta) * (R_1 + (R_2 + R_s)\cos\theta)} \quad [deg] \tag{18}$$

$$\psi = \pm \frac{3000 * V_c * t}{\pi * (R_1 + R_2 \cos\theta)} \quad [deg] \tag{19}$$

Introducing these parameters in relation no. 8 we obtain the requested parametric equations – the curled epicycloids described by the bit edge during the processing of the spherical area.

3. Evaluation of the mathematical results

The determination of the intersection points for the curled epicycloids curves according to the relations above is an extremely difficult problem, with complicate relations. A more approachable solution is to use graphic methods. This kind of curves can be easily generated with the help of a CAD environment and of a programming language. This vectorial construction can load up any necessary information, including the required roughness.

Next, a calculus application for plotting the curled epicycloids curves, generated by finishing a spherical area with plane-parallel trajectories. As CAD environment we used AutoCAD and as a programming language AutoLISP.

Considering the next inputs:

- Cutting speed Vc = 180 [m/min]
- Feed per tooth fz = 0.05 [mm/rot]
- Teeth number Z = 2
- Diameter of the milling tool D = 6 [mm]
- Corner radius of the tool R2 = 2 [mm]
- Radius of workpiece Rs = 50 [mm]

The LISP's routine plots two curled epicycloid curves (Figure 7) displayed red and green, proper to the curves described by the two bit edges of the tool. These curves are generated in the plane positioned by the θ parameter on a circle (displayed blue).

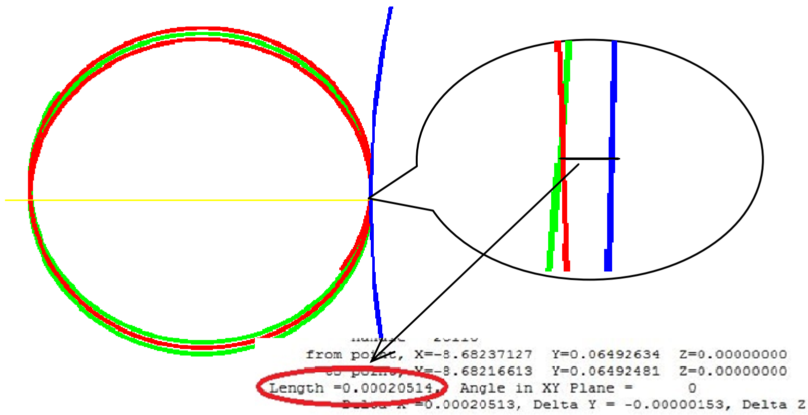

Fig. 7. Determination of the theoretical roughness

Taking into consideration the plane positioned by the θ parameter and containing the tool's trajectory (Figure 6), we obtain the following values for roughness:

Table 1. Determined values for the theoretical roughness

Angle θ	Length of the measured segment	Ra [µm]
0	0,00009948	0,09948
10	0,00010054	0,10054
20	0,00010368	0,10368
30	0,00011283	0,11283
40	0,00011743	0,11743
50	0,00012959	0,12959
60	0,00015075	0,15075
70	0,00017089	0,17089
80	0,00020514	0,20514

4. Conclusions

The numerical and theoretical study presented above reached the following conclusions:

- The active interior surface of a reflector is a sculptural surface that cannot be expressed by mathematical relations;
- Creating a mathematical model to determine the theoretical roughness obtained by milling sculptural surfaces requires an approximation for this surface, using a pitch surface through a spherical section;
- The calculus to determine the value of roughness represents a difficult problem due to the complexity of the mathematical relations. The recommendation is for the use of graphic methods, a CAD environment along with a programming language (in this case AutoCAD and AutoLISP);
- The curves described by the intersection point between the bit edge and the workpiece's spherical area are curled epicycloids;
- Plane-parallel finishing trajectories generate a surface with a theoretical variable roughness, better as the surface is tilted.

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