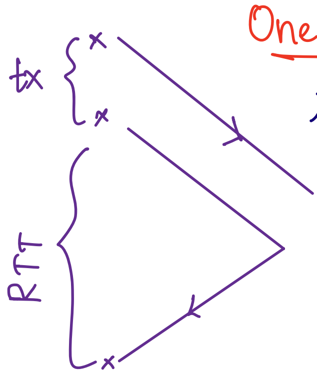
PROBLEM SET - 2

Qn 1:

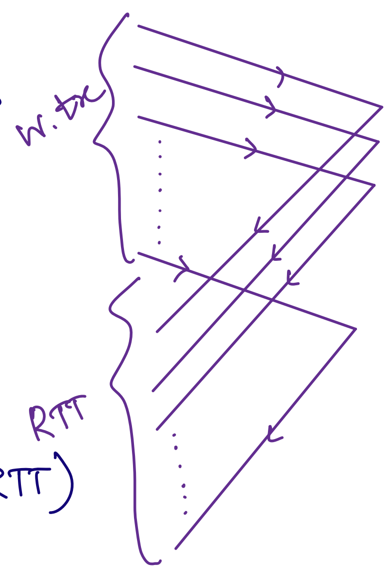


One by one : To send a packet successfully, 'T' trials are needed ; $T \sim Geo(1-q)$

$$\mathbb{E}[\text{time to send a packet successfully}] = \mathbb{E}[T(tx + RTT)] = \frac{tx + RTT}{1-q}$$

$$\mathbb{E}[\#\text{ of successful packet tx / unit time}] = \frac{1-q}{tx + RTT}$$

Go-back-W :



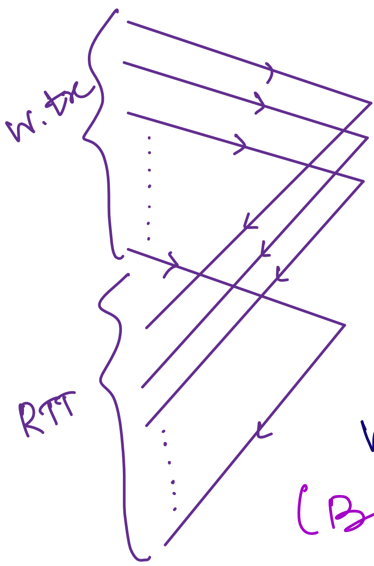
In 'W.tx + RTT' time, # of packets tx successfully be 'X'.

$$X = \begin{cases} W & \text{w.p } (1-q)^W \\ 0 & \text{w.p } [1-(1-q)^W] \end{cases}$$

$$\mathbb{E}[\#\text{succ. packet tx per } (Wtx + RTT) \text{ time}] = W(1-q)^W$$

$$\mathbb{E}[\#\text{succ. packet tx per unit time}] = \frac{W(1-q)^W}{Wtx + RTT}$$

Selective Repeat: This is similar to Go-back-W except that $X \sim \text{Binomial}(W, (1-q))$



$$\mathbb{E}\left[\#\text{ succ. packet tx / unit time}\right] = \frac{W(1-q)}{Wtx + RTT}$$

Here throughput increases as W increases, so NO optimum.

(But in practice, $\frac{RTT}{tx}$ is an upper bound on W). Note that all calculations fail – check $W.tx \approx RTT$ or $W.tx > RTT$.

For Go-back-W, ignore $tx (\simeq 0)$, then

$$\max_{W} W(1-q)^W \Rightarrow \frac{d}{dW} W(1-q)^W = 0$$

(treat W as real no.)

$$W(1-q)^W = W \exp(-KW) \text{ where } K = \ln\frac{1}{1-q}$$

$$\Rightarrow \frac{d}{dW} W(1-q)^W = \exp(-KW) - KW\exp(-KW)$$

$$\Rightarrow W^* = \frac{1}{K} = \frac{1}{\ln\frac{1}{1-q}}$$

$$\ln\frac{1}{1-q} = -\ln(1-q) = -\left(-q - \frac{q^2}{2} - \dots\right) \simeq q \quad \text{(for small } q\text{)}$$

$\Rightarrow W^* \simeq 1/q$ {Comparing with TCP Reno, $W_\infty \simeq \frac{1}{\sqrt{q}}$ square root law

Here it is $1/q$. This is price paid for not knowing 'q'.}

Qn-2. This question constructs a hypothetical scenario to illustrates different aspects of TCP Reno.

↳ case I: ssthreshold = 32 or 64 (i.e) ssthresh < 100

(Note: ssthreshold is generally 32 but is free parameter and can be chosen by OS).

Case II: ssthreshold > 100

In case I, after ss phase (slowstart), TCP gets into CA phase.

RTT = 80ms, The line can transmit atmost

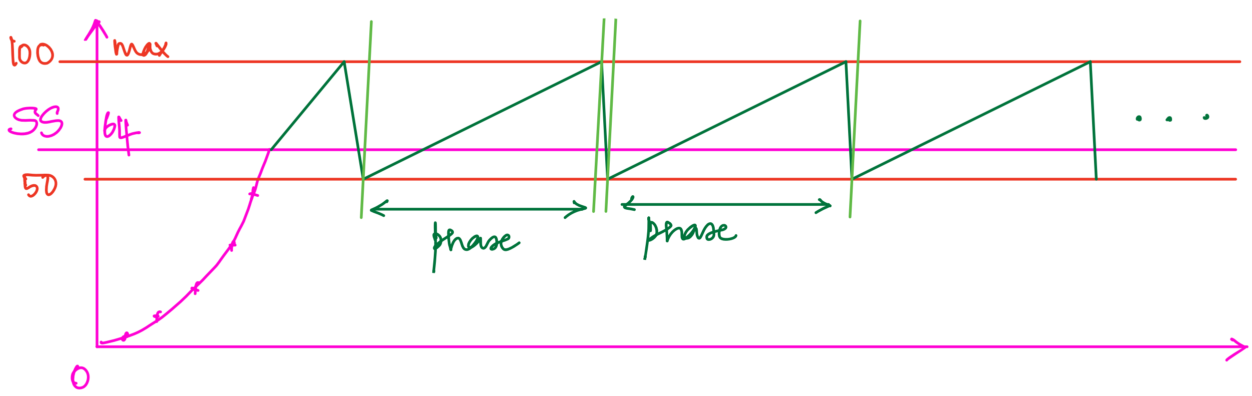
$$\frac{15\text{ Mbps} \times 80\text{ms}}{1500 \times 8} \text{ packets/RTT} = 100 \text{ packets/RTT}.$$

So, as long as W < 100, ACKs will keep coming and W will increase by 1 per 'W' Acks.

But, if W ≥ 100 there will be no ACK and

$W \rightarrow \frac{W}{2}$ and start CA again.

So max window size $W_{max} = 100$.



For average window size, we ignore SS. By symmetry, overall average window size
= average window size over "one phase"

$$\Rightarrow \frac{50+51+\dots+100}{51} = \frac{150\times 25+75}{51} = 75 \text{ packets/RTT}$$

$$\Rightarrow \text{average threshold} = \frac{75\times 1500\times 8}{80\times 10^{-3}} \text{ bps}$$

$$= 11250 \text{ kbps}$$

packets txd. in SS:

RTT #	1	2	3	4	5	6	7
# packets	1	2	4	8	16	32	64

$$\Rightarrow 1+2+4+8+16+32+64 = 127.$$

(or 63 if SS threshold is 32)

For case II, never gets out of SS threshold, everytime W reaches from 64 to 128, there is no Ack and it gets back to SS. Hence the # of packets in SS $\to \infty$ as SS is perpetual.

$$\text{Average window size} = \frac{1}{8}(1+2+2^2+\dots+2^7)$$

$$= 255/8 \text{ packets per RTT.}$$

$$\text{Average threshold} = \frac{255 \times 1500 \times 8}{8 \times 80 \times 10^{-3}} \text{ bps}$$

$$= 4781.25 \text{ kbps}$$

Qn 3:

$$[dw]_+ = \underbrace{\frac{1}{\underbrace{W(t)}_{\text{per ACK } W(t) \text{ increases by } \frac{1}{W(t)}}}} \cdot \overbrace{\frac{W(t-RTT)}{RTT}}^{\#\text{ACKs per unit time at } `t'} \cdot dt \, . \, \underbrace{(1-q(t))^{\frac{W(t-RTT)}{RTT}.dt}}_{\simeq 1 \text{ since } (1-\delta)^{\epsilon} \simeq 1 \text{ for small } \delta \,\&\, \epsilon}$$

$$[dw]_- = -\underbrace{(W(t)-1)}_{\text{as } W(t) \text{ reduces to } 1 \text{ so decrease by } (W(t)-1)} \cdot \overbrace{\frac{W(t-RTT)}{RTT}}^{\text{as above}} \cdot dt \cdot \underbrace{q(t)}_{\text{due to approx. of Union bound (refer class notes).}}$$

Combining, $$\frac{dW(t)}{dt} = \frac{W(t-RTT)}{W(t) \cdot RTT} - \frac{(W(t)-1)W(t-RTT) \cdot q(t)}{RTT}$$

$$\left.\frac{dW(t)}{dt}\right|_{t=\infty} = 0 \Rightarrow \frac{W(\infty)}{W(\infty) \cdot RTT} - \frac{(W(\infty)-1)W(\infty)q(\infty)}{RTT} = 0$$

$$\Rightarrow W^2(\infty)q(\infty) - W(\infty)q(\infty) - 1 = 0.$$

$$W(\infty) = \frac{+q(\infty) \pm \sqrt{q^2(\infty) + 4q(\infty)(1)}}{2q(\infty)} \simeq \frac{1}{2} \pm \frac{1}{\sqrt{q(\infty)}}$$