Q. 1 Simulate a discrete-time Geo/Geo/1 queue, i.e., a discrete time queue with Bernoulli arrivals and services. Play with it by changing different the parameters. In particular, study the following scenarios.

- Pick $\mu = 0.9$. Simulate for $\lambda = 0.1, 0.2, \ldots 0.9, 1$. By simulating the queue for T time slots (large $T \gg 1000$), compute $\frac{1}{T} \sum_{t=1}^{T} Q(t)$ in each case, where $Q(t)$ is the number of jobs in the system. Plot against λ.

- You learned in class about the $\pi P = \pi$. Compare the above quantity with the expectation under the PMF π.

- Let D_i be the sojourn time of the ith job/packet. (Sojourn time is the time from arrival till service completion.) Compute $\frac{1}{N} \sum_{i=1}^{N} D_i$ for large N for all λ in the above list. Plot against λ.

- Plot the ratio of the two quantities computed in the previous parts against λ. (This relates to an interesting theorem in queuing theory called Little's theorem. Check that out in the textbook.)