Topics: Markov chains, queues. (Problems are taken in parts from the course notes of Prof. Gustavo de Veciana and our textbook by Profs. Srikant and Ying.)

Q. 1 Birth-death chains, Ex 3.5

Q. 2 Wireless channel and queue, Ex 3.9

Q. 3 Find the stationary distribution for the Markov chains with state transition diagrams shown in the figure below.

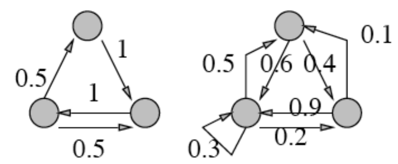

Figure 1: Example Markov chains.

Q. 4 Consider the Markov chain shown in the figure below. Suppose that at time 0 you are in state $X(0) = 0$. Compute the expected time until the next time the Markov Chain is at 0 once again.

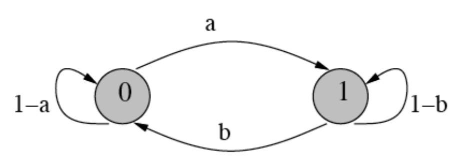

Figure 2: Sample Markov chain.

Q. 5 Let $\xi_1, \xi_2, \ldots$ be i.i.d. with $P(\xi_1 = 1) = P(\xi_1 = -1) = 0.5$. Define

$$\begin{aligned} S_0 &= 0, \\ S_n &= \xi_1 + \ldots + \xi_n, \\ X_n &= \max\{S_m : 0 \le m \le n\}. \end{aligned}$$

Is $\{X_n\}$ a Markv chain?

Q. 6 Let $\xi_1, \xi_2, \ldots$ be i.i.d. uniformly distributed on $\{1, 2, \ldots, N\}$. Show that $X_n = |\{\xi_1, \ldots, \xi_n\}|$ is a Markov chain. Compute its transition matrix. Also compute $E[T_N | X_0 = 0]$ where T_N is the first time when state N is hit. Is the state 0 positive recurrent? Is this Markov chain irreducible?

Note: $X_n = |\{\xi_1, \ldots, \xi_n\}|$ means the cardinality of the set $|\{\xi_1, \ldots, \xi_n\}|$. e.g. $|\{2, 3, 3\}| = |\{2, 3\}| = 2$, $|\{3, 3, 3, 3\}| = 1$.