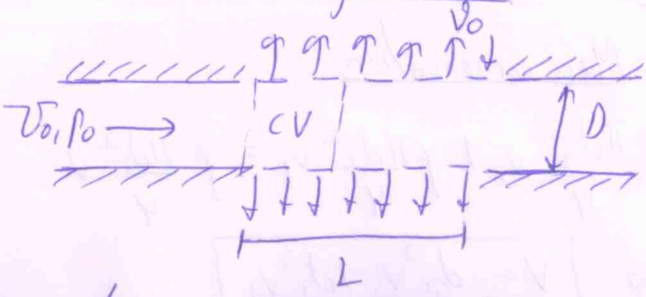
Assignment 4

(Q1)



a) Mass conservation $\Rightarrow -\rho v_0 \frac{\pi D^2}{4} + \rho v_{out} \frac{\pi D^2}{4} + \rho (v_0) \pi D x = 0$

$$V_{out}(x) = \frac{v_0 \frac{\pi D^2}{4} - v_0 \pi D x}{\frac{\pi D^2}{4}} \Rightarrow \boxed{\frac{v_{out}(x)}{v_0} = 1 - \left(\frac{v_0}{v_0}\right)\left(\frac{4x}{D}\right)}$$

b) Force balance $\Rightarrow p_0 \frac{\pi D^2}{4} - p(x)\frac{\pi D^2}{4} = \oiint_{C.S} \rho(\vec{v}\cdot\hat{n}) ds\, v_x$

$$[p_0 - p(x)]\frac{\pi D^2}{4} = -\rho v_0^2 \frac{\pi D^2}{4} + \rho \frac{\pi D^2}{4}[v_{out}(x)]^2$$

$$(p(x) - p_0) = \rho v_0^2 - \rho [v_{out}(x)]^2$$

$$\frac{p(x)-p_0}{\rho v_0^2} = 1 - \left[\frac{v_{out}(x)}{v_0}\right]^2 = 1 - \left[1 + \left(\frac{v_0}{v_0}\right)^2 \frac{16x^2}{D^2} - \frac{8x}{D}\left(\frac{v_0}{v_0}\right)\right]$$

$$\boxed{\frac{p(x)-p_0}{\rho v_0^2} = \frac{8x}{D}\left(\frac{v_0}{v_0}\right) - \frac{16x^2}{D^2}\left(\frac{v_0}{v_0}\right)^2}$$

(Q2) (a) $\sum \vec{F} = \oiint_S \rho(\vec{v}\cdot\hat{n})\vec{v}\, dA$

$$\vec{F_{ext}} = -\rho(v_{jet}^2)A(\hat{i}) + \rho[v_{jet}^2 \cos\theta]A(\hat{i}) + \rho(v_{jet}^2 \sin\theta)A\hat{j}$$

$$F_x(\hat{i}) + F_y(\hat{j}) - Mg(\hat{j}) = -\rho v_{jet}^2 A(\hat{i}) + \rho v_{jet}^2 \cos\theta A(\hat{i}) + \rho A v_{jet}^2 \sin\theta(\hat{j})$$

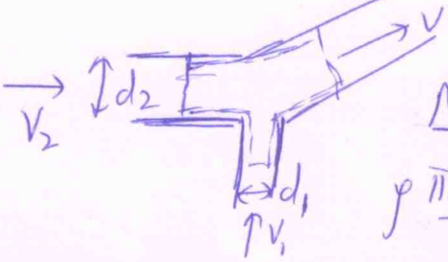
$$\boxed{F_x = \rho v_{jet}^2 A[\cos\theta - 1]\hat{i}}$$

$$\boxed{F_y = Mg + \rho A v_{jet}^2 \sin\theta}$$

(b) $F_x = \rho(v_{jet} - v_{cart})^2 A(\cos\theta - 1)\hat{i}$

(c) $a_x = \dfrac{\rho[v_{jet}^2 - v_{cart})^2 A\,\cancel{(\cos\theta-1)}(1-\cos\theta)}{M}(\hat{i})$

Q3



Mass conservation

$$\rho \frac{\pi d_2^2}{4} v_2 + \rho \frac{\pi d_1^2}{4} v_1 = \rho \frac{\pi d^2}{4} v$$

$$\Rightarrow \boxed{v = \frac{d_2^2 v_2 + d_1^2 v_1}{d^2}}$$

$$\sum F_{ex} = \iint \rho (\vec{v}\cdot\hat{n}) v_x \, dS = 0$$

$$-\rho v_1 \frac{\pi d_1^2}{4} v_1 (\hat{j}) - \rho v_2 \left(\frac{\pi d_2^2}{4}\right) v_2 (\hat{i})$$

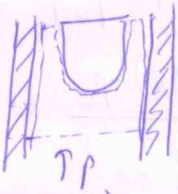
$$+ \rho \frac{\pi d^2}{4} v^2 \cos\theta (\hat{i}) + \rho \frac{\pi d^2}{4} v^2 \sin\theta (\hat{j}) = 0$$

$$\frac{\pi d^2}{4} v^2 \cos\theta = \frac{\pi d_2^2}{4} v_2^2$$

$$\frac{\pi d^2}{4} v^2 \sin\theta = \frac{\pi d_1^2}{4} v_1^2 \Rightarrow \boxed{\tan\theta = \frac{v_1^2 d_1^2}{v_2^2 d_2^2}}$$

Q4

$$\boxed{v_{out} = \frac{Q}{A_{out}}} \Rightarrow \oiint_{C.S} \rho (\vec{v}\cdot\hat{n}) v_y \, dS = \sum F_y$$



$$\Rightarrow \boxed{v_{in} = \frac{Q}{A_{in}}} \Rightarrow \rho Q (v_{out} - v_{in}) = P\left(\frac{\pi D^2}{4}\right) - Mg - P_{atm}\left(\frac{\pi D^2}{4}\right)$$

$$\rho Q \left[\frac{Q}{A_{out}} - \frac{Q}{A_{in}}\right) = \rho Q^2 \left[\frac{A_{in} - A_{out}}{A_{in} A_{out}}\right] = (P - P_{atm}) \frac{\pi D^2}{4} - Mg \quad -(1)$$

Bernoulli $\Rightarrow P + \frac{1}{2}\rho v_{in}^2 = P_{atm} + \frac{1}{2}\rho v_{out}^2$

$$\Rightarrow P - P_{atm} = \frac{\rho}{2}\left[\frac{Q^2}{A_{out}^2} - \frac{Q^2}{A_{in}^2}\right]$$

$$\boxed{P - P_{atm} = \frac{\rho Q^2}{2}\left[\frac{A_{in}^2 - A_{out}^2}{A_{in}^2 A_{out}^2}\right]} \quad -(2)$$

From (1) $\Rightarrow \rho Q^2 \left[\frac{A_{in} - A_{out}}{A_{in}^2 A_{out}}\right] + \frac{Mg}{A_{in}} = (P - P_{atm})$

Substitute (2) in above equation

$$\frac{Mg}{A_{in}} = \frac{\rho Q^2}{2}\left[\frac{A_{in}^2 - A_{out}^2}{A_{in}^2 A_{out}^2}\right] - \rho Q^2 \left[\frac{A_{in} - A_{out}}{A_{in}^2 A_{out}}\right]$$

$$\Rightarrow \frac{Mg}{A_{in}} = \frac{\rho Q^2}{2}\left[\frac{A_{in}^2 - A_{out}^2}{A_{in}^2 A_{out}^2}\right] - \rho Q^2\left[\frac{A_{in}A_{out} - A_{out}^2}{A_{in}^2 A_{out}^2}\right]$$

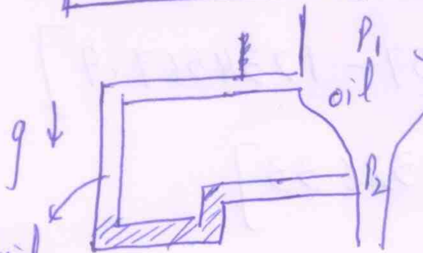
$$\frac{Mg}{\rho Q^2 A_{in}} = \frac{A_{in}^2 - A_{out}^2 - 2A_{in}A_{out} + 2A_{out}^2}{2A_{in}^2 A_{out}^2} = \frac{(A_{in} - A_{out})^2}{2A_{in}^2 A_{out}^2}$$

$$Mg = \frac{\rho Q^2}{2}\left(\frac{(A_{in} - A_{out})^2}{A_{in} A_{out}^2}\right) \Rightarrow \frac{2Mg}{\rho}\frac{A_{in}A_{out}^2}{(A_{in} - A_{out})^2} = Q^2$$

$$\boxed{Q = \sqrt{\frac{2Mg}{\rho}\frac{A_{in}A_{out}^2}{(A_{in} - A_{out})^2}}}$$

$A_{in} = \frac{\pi D^2}{4}$, $A_{out} = \pi\frac{(D^2 - d^2)}{4}$

Q5



fluid static here

fluid dynamic here

(a) $P_1 + \rho_{oil}\, g(H+h) - \rho_{Hg}(h) = P_2$

$$P_1 - P_2 = g\rho_{Hg}(h) - \rho_{oil}\, g(H+h)$$

$$P_1 - P_2 = 13600 \times 10 \times \frac{100}{1000} - 900 \times 10 \times \frac{700}{1000}$$

$$\boxed{P_1 - P_2 = 7300\ Pa}$$

(b) Using bernoulli's theorem & continuity equation :-

$$P_1 + \rho g H + \frac{1}{2}\rho_{oil} v_1^2 = P_2 + \frac{1}{2}\rho_{oil} v_2^2 \Rightarrow v_1 = \frac{Q}{A_1}, v_2 = \frac{Q}{A_2}$$

$$P_1 - P_2 + \rho g H = \frac{\rho_{oil}}{2}\left[\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}\right]$$

$$\Rightarrow Q^2\left[\frac{A_1^2 - A_2^2}{A_1^2 A_2^2}\right] = \frac{2}{\rho_{oil}}\left[P_1 - P_2 + \rho g H\right]$$

$$\boxed{Q = \sqrt{\frac{2}{\rho_{oil}}\frac{A_1^2 A_2^2}{A_1^2 - A_2^2}\left[P_1 - P_2 + \rho g H\right]}}$$

$A_1 = \frac{\pi D_1^2}{4}$, $A_2 = \frac{\pi D_2^2}{4}$

$D_1 = 300mm$, $D_2 = 100mm$

$A_1 = 0.07m^2$, $A_2 = 7.85 \times 10^{-3}$

$$Q = \sqrt{\frac{2}{900}\left[\frac{0.07^2 \cdot (7.85\times10^{-3})^2}{0.07^2 - (7.85\times10^{-3})^2}\right]\left(7300 + 9000 \times \frac{600}{1000}\right)}$$

$$Q = \sqrt{\frac{1}{450}\left(6.24\times10^{-5}\right)\left[12700\right]}$$

$$\underline{Q = 0.041\ m^3/s}$$

Q6 $W_{pump} = \dot{m}\left[\frac{v_1^2}{2} - \frac{v_2^2}{2}\right]$ (flow rate conserved due to mass conservation)

$v_1 = \frac{4Q}{\pi D_1^2}$, $V_2 = \frac{4Q}{\pi D_2^2}$ $\Rightarrow W_{pump} = \rho Q\left[\frac{8Q^2}{\pi^2 D_1^4} - \frac{8Q^2}{\pi^2 D_2^4}\right]$

$W_{pump} = \frac{8\rho Q^3}{\pi^2}\left[\frac{1}{D_1^4} - \frac{1}{D_2^4}\right]$, $Q = 57 m^3/hr$
$= \underline{\underline{0.0158 m^3/s}}$

$$W_{pump} = \frac{8 \times 1000 \times (0.0158)^3}{\pi^2}\left[\frac{1}{(9\times10^{-2})^4} - \frac{1}{(3\times10^{-2})^4}\right]$$

$$= 3.197\times10^{-3}\left[15241.57 - 1234567.9\right]$$

$$= 3.197\times10^{-3}\left[-1219326.33\right]$$

Work by fluid on pump. $= -\underline{\underline{3.898 kW}}$

$W_{pump} = \underline{\underline{3.898 kW}}$

x ——— x ——— x