Assignment on pipe flows and losses in fittings

6.21 In Tinyland, houses are less than a foot high! The rainfall is laminar! The drainpipe in Fig. P6.21 is only 2 mm in diameter. (a) When the gutter is full, what is the rate of draining? (b) The gutter is designed for a sudden rainstorm of up to 5 mm per hour. For this condition, what is the maximum roof area that can be drained successfully? (c) What is Red?

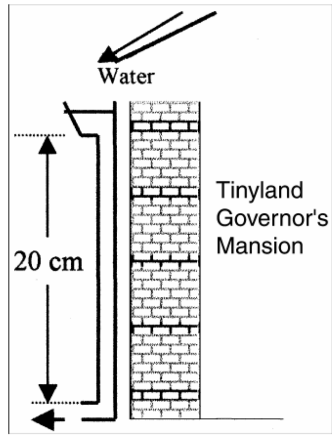

Fig. P6.21

Solution: If the velocity at the gutter surface is neglected, the energy equation reduces to

$$\Delta z = \frac{V^2}{2g} + h_f, \quad \text{where } h_{f,\text{laminar}} = \frac{32\mu LV}{\rho g d^2}$$

For water, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. (a) With Δz known, this is a quadratic equation for the pipe velocity V:

$$0.2\ m = \frac{V^2}{2(9.81\ m/s^2)} + \frac{32(0.001\ kg/m{\cdot}s)(0.2\ m)V}{(998\ kg/m^3)(9.81\ m/s^2)(0.002\ m)^2},$$

$$\text{or:} \quad 0.051V^2 + 0.1634V - 0.2 = 0, \quad \textit{Solve for } V = 0.945\ \frac{m}{s},$$

$$Q = \frac{\pi}{4}(0.002\ m)^2\left(0.945\ \frac{m}{s}\right) = 2.97E\text{–}6\ \frac{m^3}{s} = \mathbf{0.0107}\ \frac{\boldsymbol{m^3}}{\boldsymbol{h}} \quad \textit{Ans.}\ \text{(a)}$$

(b) The roof area needed for maximum rainfall is 0.0107 m^3/h ÷ 0.005 m/h = **2.14 m^2**. *Ans.* (b)
(c) The Reynolds number of the gutter is Red = (998)(0.945)(0.002)/(0.001) = **1890** *laminar*. *Ans.* (c)

6.28 For straightening and smoothing an airflow in a 50-cm-diameter duct, the duct is packed with a "honeycomb" of thin straws of length 30 cm and diameter 4 mm, as in Fig. P6.28. The inlet flow is air at 110 kPa and 20°C, moving at an average velocity of 6 m/s. Estimate the pressure drop across the honeycomb.

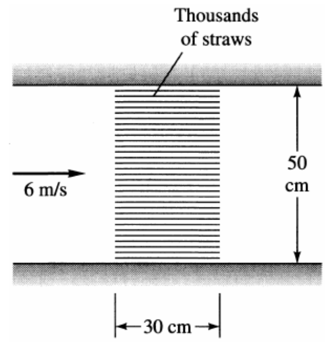

Fig. P6.28

Solution: For air at 20°C, take $\mu \approx$ 1.8E−5 kg/m·s and ρ = 1.31 kg/m^3. There would be approximately 12000 straws, but each one would see the average velocity of 6 m/s. Thus

$$\Delta p_{laminar} = \frac{32\mu LV}{d^2} = \frac{32(1.8E{-}5)(0.3)(6.0)}{(0.004)^2} \approx \mathbf{65\ Pa} \quad \textit{Ans.}$$

Check Re = $\rho Vd/\mu$ = (1.31)(6.0)(0.004)/(1.8E−5) ≈ 1750 OK, laminar flow.

6.47 The gutter and smooth drainpipe in Fig. P6.47 remove rainwater from the roof of a building. The smooth drainpipe is 7 cm in diameter. (a) When the gutter is full, estimate the rate of draining. (b) The gutter is designed for a sudden rainstorm of up to 5 inches per hour. For this condition, what is the maximum roof area that can be drained successfully?

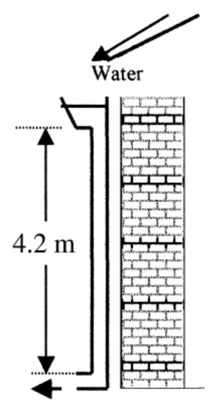

Fig. P6.47

Solution: If the velocity at the gutter surface is neglected, the energy equation reduces to

$$\Delta z = \frac{V^2}{2g} + h_f, \quad h_f = f\frac{L}{d}\frac{V^2}{2g}, \quad \text{solve } V^2 = \frac{2g\Delta z}{1 + fL/d} = \frac{2(9.81)(4.2)}{1 + f(4.2/0.07)}$$

For water, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Guess $f \approx$ 0.02 to obtain the velocity estimate V ≈ 6 m/s above. Then Red ≈ $\rho Vd/\mu$ ≈ (998)(6)(0.07)/(0.001) ≈ 428,000 (turbulent). Then, for a smooth pipe, $f \approx$ 0.0135, and V is changed slightly to 6.74 m/s. After convergence, we obtain

$$V = 6.77\text{ m/s}, \quad Q = V(\pi/4)(0.07)^2 = \mathbf{0.026\ m^3/s} \quad \textit{Ans.}\text{ (a)}$$

A rainfall of 5 in/h = (5/12 ft/h)(0.3048 m/ft)/(3600 s/h) = 0.0000353 m/s. The required roof area is

$$A_{roof} = Q_{drain}/V_{rain} = (0.026\text{ m}^3/\text{s})/0.0000353\text{ m/s} \approx \mathbf{740\ m^2} \quad \textit{Ans.}\text{ (b)}$$

6.55 The reservoirs in Fig. P6.55 contain water at 20°C. If the pipe is smooth with L = 4500 m and d = 4 cm, what will the flow rate in m^3/h be for Δz = 100 m?

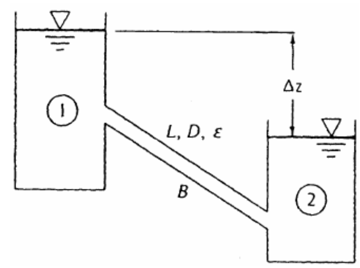

Fig. P6.55

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. The energy equation from surface 1 to surface 2 gives

$$p_1 = p_2 \quad \text{and} \quad V_1 = V_2,$$
$$\text{thus} \quad h_f = z_1 - z_2 = 100 \text{ m}$$

$$\text{Then } 100 \text{ m} = f\left(\frac{4500}{0.04}\right)\frac{V^2}{2(9.81)}, \quad \text{or} \quad fV^2 \approx 0.01744$$

Iterate with an initial guess of f ≈ 0.02, calculating V and Re and improving the guess:

$$V \approx \left(\frac{0.01744}{0.02}\right)^{1/2} \approx 0.934 \ \frac{\text{m}}{\text{s}}, \quad \text{Re} \approx \frac{998(0.934)(0.04)}{0.001} \approx 37300, \quad f_{\text{smooth}} \approx 0.0224$$

$$V_{\text{better}} \approx \left(\frac{0.01744}{0.0224}\right)^{1/2} \approx 0.883 \ \frac{\text{m}}{\text{s}}, \quad \text{Re}_{\text{better}} \approx 35300, \quad f_{\text{better}} \approx 0.0226, \text{ etc......}$$

This process converges to

$$f = 0.0227, \ \text{Re} = 35000, \ V = 0.877 \text{ m/s}, \ Q \approx 0.0011 \text{ m}^3/\text{s} \approx \mathbf{4.0\ m^3/h}. \quad \textit{Ans.}$$

P6.99 In Sec. 6.11 it was mentioned that Roman aqueduct customers obtained extra water by attaching a diffuser to their pipe exits. Fig. P6.99 shows a simulation: a smooth inlet pipe, with and without a 15° diffuser expanding to a 5-cm-diameter exit. The pipe entrance is sharp-edged.

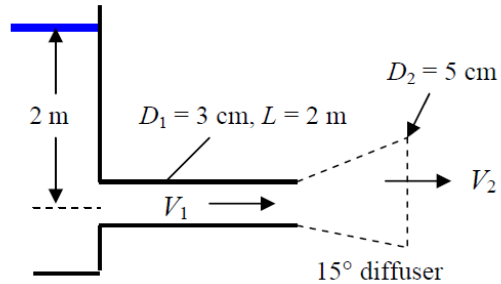

Fig. P6.99

Calculate the flow rate (*a*) without, and (*b*) with the diffuser.

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m-s. The energy equation between the aqueduct surface and the pipe exit yields

$$z_{surf} = z_2 + \frac{V_2^2}{2g} + h_f + \Sigma h_m = z_2 + \frac{V_2^2}{2g} + \frac{V_1^2}{2g}\left(f\frac{L}{D_1} + K_{entrance} + K_{diffuser}\right)$$

(*a*) Without the diffuser, $K_{diff} = 0$, and $V_1 = V_2$. For a sharp edge, take $K_{ent} = 0.5$. We obtain

$$2m = \frac{V_1^2}{2g}\left(1 + f\frac{2m}{0.03m} + 0.5\right), \quad with \quad f = fcn(\mathrm{Re} = \rho V_1 D_1/\mu)$$

$$Solve:\ \mathrm{Re} = 115{,}000\ ;\ f = 0.0175\ ;\ V_1 = 4.48\,m/s\ ;\ Q_{without} = 0.00271\,m^3/s \quad Ans.(a)$$

(*b*) *With* the diffuser, from Fig. 6.23, for $D_1/D_2 = 3/5 = 0.6$ and $2\theta = 15°$, read $K_{diffuser} \approx 0.2$. From one-dimensional continuity, $V_2 = V_1(3/5)^2 = 0.36V_1$. The energy equation becomes

$$2m = \frac{(0.36V_1)^2}{2g} + \frac{V_1^2}{2g}(1 + f\frac{2m}{0.03m} + 0.5 + 0.2)$$

$$Solve: \ \mathrm{Re} = 134{,}000 \ ; \ f = 0.0169 \ ; \ V_1 = 3.84\,m/s \ ; \ Q_{with} = 0.00316\,m^3/s \quad Ans.(b)$$

Adding the diffuser increases the flow rate by **17%**. [NOTE: Don't know if the Romans did this, but a well-rounded entrance, $K_{ent} = 0.05$, would increase the flow rate by another 15%.]

6.116 For the series-parallel system of Fig. P6.116, all pipes are 8-cm-diameter asphalted cast iron. If the total pressure drop $p1 - p2 = 750$ kPa, find the resulting flow rate Q m³/h for water at 20°C. Neglect minor losses.

Solution: For water at 20°C, take $\rho = 998$ kg/m³ and $\mu = 0.001$ kg/m·s. For asphalted cast iron, $\varepsilon \approx 0.12$ mm, hence $\varepsilon/d = 0.12/80 \approx 0.0015$ for all three pipes. The head loss is the same through AC and BC:

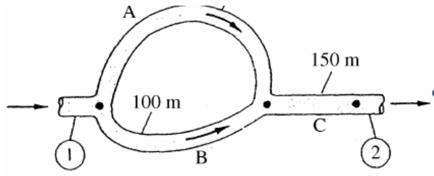

Fig. P6.116

$$\frac{\Delta p}{\rho g} = h_{fA} + h_{fC} = h_{fB} + h_{fC} = \left(f\frac{L}{d}\frac{V^2}{2g}\right)_A + \left(f\frac{L}{d}\frac{V^2}{2g}\right)_C = \left(f\frac{L}{d}\frac{V^2}{2g}\right)_B + \left(f\frac{L}{d}\frac{V^2}{2g}\right)_C$$

Since d is the same, $V_A + V_B = V_C$ and f_A, f_B, f_C are found from the Moody chart. Cancel g and introduce the given data:

$$\frac{750000}{998} = f_A\frac{250}{0.08}\frac{V_A^2}{2} + f_C\frac{150}{0.08}\frac{V_C^2}{2} = f_B\frac{100}{0.08}\frac{V_B^2}{2} + f_C\frac{150}{0.08}\frac{V_C^2}{2}, \quad V_A + V_B = V_C$$

Guess $f_{rough} \approx 0.022$ and solve laboriously: $V_A \approx 2.09\ \frac{m}{s}$, $V_B \approx 3.31\ \frac{m}{s}$, $V_C \approx 5.40\ \frac{m}{s}$

Now compute $Re_A \approx 167000$, $f_A \approx 0.0230$, $Re_B \approx 264000$, $f_B \approx 0.0226$, $Re_C \approx 431000$, and $f_C \approx 0.0222$. Repeat the head loss iteration and we converge: $V_A \approx 2.06$ m/s, $V_B \approx 3.29$ m/s, $V_C \approx 5.35$ m/s, $Q = (\pi/4)(0.08)^2(5.35) \approx \mathbf{0.0269\ m^3/s}$. *Ans.*