SOLUTIONS TO ASSIGNMENT 5

Q6. *Solution:* $T = T(x, t)$

$$dT = \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial t}dt$$

$$\left.\frac{dT}{dt}\right)_{particle} = \frac{\partial T}{\partial x}\left.\frac{dx}{dt}\right)_{particle} + \frac{\partial T}{\partial t}$$

or

$$\frac{DT}{Dt} = \left.\frac{dT}{dt}\right)_{particle} = u\frac{\partial T}{\partial x} + \frac{\partial T}{\partial t}$$

For the given data, $u = U = constant$

$$\frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left[T_0 - \alpha e^{-x/L}\sin\left(\frac{2\pi t}{\tau}\right)\right] = \frac{\alpha}{L}e^{-x/L}\sin\left(\frac{2\pi t}{\tau}\right)$$

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial t}\left[T_0 - \alpha e^{-x/L}\sin\left(\frac{2\pi t}{\tau}\right)\right] = -\frac{2\pi}{\tau}\alpha e^{-x/L}\cos\left(\frac{2\pi t}{\tau}\right)$$

substituting,

$$\frac{DT}{Dt} = \left[\frac{U}{L}\sin\left(\frac{2\pi t}{\tau}\right) - \frac{2\pi}{\tau}\cos\left(\frac{2\pi t}{\tau}\right)\right]\alpha e^{-x/L} \quad deg/s$$

Q7. Solution:

Basic equations : $0 = \frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$

$$-\nabla p + \rho\vec{g} = \rho\frac{D\vec{V}}{Dt} \qquad \vec{F} = -\int p\, d\vec{A}$$

(a) For the deformable CV shown

$$0 = \frac{\partial}{\partial t}\int_0^y \rho w x\, dy + \rho u w y = \rho w x\frac{dy}{dt} + \rho u w y$$

But $dy/dt = -V$ and hence $u = \frac{Vx}{y}$

If $y = b_0$ at $t = 0$, then $y = b = b_0 - Vt$ at any time t

$$\therefore\ u = \frac{Vx}{b} \qquad u(t)$$

(b) $a_x = \frac{Du}{Dt} = u\frac{\partial u}{\partial x} + \cancel{v\frac{\partial u}{\partial y}}^{0(1)} + \cancel{w\frac{\partial u}{\partial z}}^{0(1)} + \frac{\partial u}{\partial t}$

Assumptions: (1) $u \neq u(y)$, $w = 0$

$$a_x = \frac{Vx}{b}\left(\frac{V}{b}\right) + \frac{\partial u}{\partial b}\frac{\partial b}{\partial t} = \frac{V^2x}{b^2} + \left(-\frac{Vx}{b^2}\right)(-V) = \frac{2V^2x}{b^2} \qquad a_x$$

(c) From Euler's equation in the x direction with $g_x = 0$

$$\frac{\partial p}{\partial x} = -\rho a_x = -\frac{2\rho V^2 x}{b^2} \qquad \frac{\partial p}{\partial x}$$

(d) $p - p_{atm} = \int_L^x \frac{\partial p}{\partial x}dx = \int_L^x -\frac{2\rho V^2}{b^2}x\,dx = -\frac{\rho V^2x^2}{b^2}\Big]_L^x = \frac{\rho V^2}{b^2}L^2\left[1 - \left(\frac{x}{L}\right)^2\right]$

(e) $F_y = \int_A (p - p_{atm})\,dA = 2\int_0^L \frac{\rho V^2L^2}{b^2}\left[1 - \left(\frac{x}{L}\right)^2\right]w\,dx$

$$= 2\int_0^1 \frac{\rho V^2L^3}{b^2}\left[1 - \left(\frac{x}{L}\right)^2\right]w\,d\left(\frac{x}{L}\right) = \frac{2\rho V^2L^3w}{b^2}\left[\left(\frac{x}{L}\right) - \frac{1}{3}\left(\frac{x}{L}\right)^3\right]_0^1$$

$$F_y = \frac{4\rho V^2L^3w}{3b^2} \quad \text{(upward, since } F_y > 0\text{)} \qquad F_y$$

Q8. Solution:

Basic equation: $\frac{\partial p}{\partial n} = \rho \frac{V^2}{R}$

Assumptions: (1) steady flow (2) frictionless flow
(3) neglect body forces
(4) constant speed along each streamline

At the inlet section, $p = p(y)$

$$\therefore \quad \frac{dp}{dn} = -\frac{dp}{dy} = \rho \frac{V^2}{R} = \rho V^2 \frac{2y}{R_0 L}$$

$$\therefore \quad dp = -\frac{\rho V^2}{R_0 L} 2y\,dy$$

$$p_{L/2} - p_0 = \int_0^{L/2} dp = -\frac{2\rho V^2}{R_0 L}\int_0^{L/2} y\,dy = -\frac{2\rho V^2}{R_0 L}\left.\frac{y^2}{2}\right]_0^{L/2}$$

$$p_{L/2} - p_0 = -\frac{\rho V^2}{R_0 L}\frac{L^2}{4} = -\frac{\rho V L}{4R_0}$$

$$p_{L/2} - p_0 = -1.225\,\frac{kg}{m^3}\times\left(20\,\frac{m}{s}\right)^2\times 0.15\,m \times \frac{1}{4}\times\frac{1}{0.6\,m}\times\frac{N\cdot s^2}{kg\cdot m}$$

$$p_{L/2} - p_0 = -30.6\ N/m^2 \longleftarrow p_{L/2} - p_0$$

Q9.

For 2-D incompressible flow $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, so $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$

$$u = \int \frac{\partial u}{\partial x}dx + f(y) = \int -\frac{\partial v}{\partial y}dx + f(y) = \int -2Ay\,dx + f(y) = -2Axy + f(y)$$

Choose the simplest solution, $f(y) = 0$, so $u = -2Axy$. Hence

$$\vec{V} = -2Axy\,\hat{i} + Ay^2\,\hat{j} = A\left[-2xy\,\hat{i} + y^2\,\hat{j}\right]$$

$$\vec{a}_p = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} = -2Axy\left[-2Ay\,\hat{i}\right] + Ay^2\left[-2Ax\,\hat{i} + 2Ay\,\hat{j}\right]$$

$$\vec{a}_p = 2A^2xy^2\,\hat{i} + 2A^2y^3\,\hat{j} = 2Ay^2\left[x\,\hat{i} + y\,\hat{j}\right]$$

At the point (2,1)

$$\vec{a}_p = 2\times\frac{1}{m\cdot s}\times(1)^2 m^2\left[2m\,\hat{i} + 1m\,\hat{j}\right] = 4\hat{i} + 2\hat{j}\ m/s^2 \longleftarrow \vec{a}_{(2,1)}$$

$$\vec{V} = \frac{1}{m\cdot s}\left[-2(2m)(1m)\,\hat{i} + (1)^2 m^2\,\hat{j}\right] = -4\hat{i} + \hat{j}\ m/s$$

The unit vector tangent to the streamline is

$$\hat{e}_t = \frac{\vec{V}}{|\vec{V}|} = \frac{-4\hat{i} + \hat{j}}{\left[(-4)^2 + (1)^2\right]^{1/2}} = -0.970\,\hat{i} + 0.243\,\hat{j}$$

The unit vector normal to the streamline is

$$\hat{e}_n = \hat{k}\times\hat{e}_t = \hat{k}\times(-0.970\,\hat{i} + 0.243\,\hat{j}) = -0.243\,\hat{i} - 0.970\,\hat{j}$$

The normal component of acceleration is

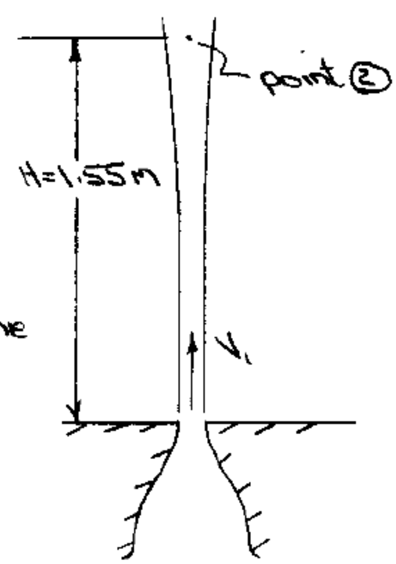
$$a_n = -\frac{V^2}{R} = \vec{a}\cdot\hat{e}_n = (4\hat{i} + 2\hat{j})\cdot(-0.243\,\hat{i} - 0.970\,\hat{j})$$

$$-\frac{V^2}{R} = -2.91\ m/s^2$$

$$R = \frac{V^2}{2.91} = \frac{17\ m^2/s^2}{2.91\ m/s^2} = 5.84\ m \longleftarrow R(1,2)$$

Q10.

Solution: Apply Bernoulli and then y-momentum equation



Basic eq.: $\cancel{\frac{p_1}{\rho}} + \frac{V_1^2}{2} + gz_1 = \cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2} + gz_2$

Assumptions:
(1) steady flow
(2) incompressible flow
(3) frictionless flow
(4) flow along a streamline
(5) $p_1 = p_2 = p_{atm}$

Then

$$V_2 = \left[V_1^2 + 2g(z_1 - z_2)\right]^{1/2}$$

$$V_2 = \left[(6.3)^2 \frac{m^2}{s^2} + 2 \times 9.81 \frac{m}{s^2}(-1.55m)\right]^{1/2}$$

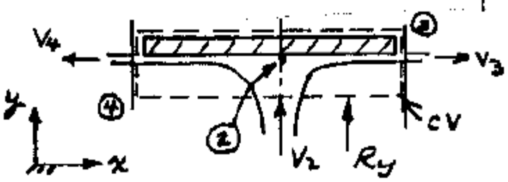
$$V_2 = 3.05 \text{ m/s}$$

By definition, $p_{0_2} = p_2 + \frac{1}{2}\rho V_2^2 = p_{atm} + \frac{1}{2}\rho V_2^2$, so

$$p_{0_2\,gage} = \frac{1}{2} \times 999 \frac{kg}{m^3} \times (3.05)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 4.65 \text{ kPa (g)}$$

Apply y-momentum equation to CV surrounding plate

Basic eq.: $F_{S_y} + \underbrace{\cancel{F_{B_y}}}_{\approx 0(6)} = \underbrace{\cancel{\frac{\partial}{\partial t}}}_{=0(1)} \int_{CV} v \rho \, d\forall + \int_{CS} v \rho \vec{V} \cdot d\vec{A}$



Assumptions:
(6) neglect mass in CV
(7) V_2 enters CV uniformly
(8) $v_3 = v_4 = 0$

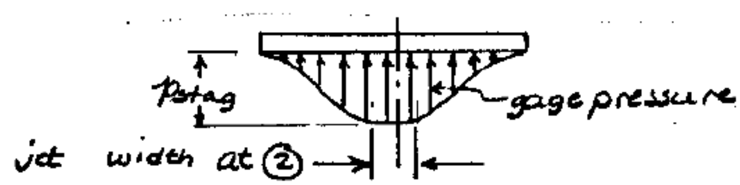
Then

$$R_y = v_2\{-\rho V_1 A_1\} + \underbrace{\cancel{v_3}}_{=0(8)}\{\dot{m}_3\} + \underbrace{\cancel{v_4}}_{=0(8)}\{\dot{m}_4\} = -\rho V_1 A_1 V_2 \quad \text{and}$$

$$K_y = -R_y = \rho V_1 A_1 V_2 = 999 \frac{kg}{m^3} \times 6.3 \frac{m}{s} \times 600 \, mm^2 \times 3.05 \frac{m}{s} \times \frac{m^2}{10^6 \, mm^2} \times \frac{N \cdot s^2}{kg \cdot m}$$

$K_y = 11.5 \text{ N}$ (force up)

The pressure distribution on the plate is as shown.



Q11.

Solution:

Basic equation: $\frac{p_1}{\rho} + \frac{V_1^2}{2} + \cancel{gz_1} = \frac{\overset{0(\text{gage})}{\cancel{p_2}}}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2}$.

Assumptions: (1) steady flow (2) incompressible flow (3) no friction (4) flow along a streamline (5) $\Delta z = 0$

From the Bernoulli equation $p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2)$

From continuity for uniform flow, $V_1A_1 = V_2A_2$

$$\therefore V_2 = V_1 \frac{A_1}{A_2} = V_1 \frac{D^2}{D^2-d^2} = V_1 \frac{1}{1-(d/D)^2} = 7\,\frac{m}{s} \times \frac{1}{1-(0.8)^2} = 19.4\ m/s$$

and

$$p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2) = \frac{1}{2} \times 999\,\frac{kg}{m^3}\left[(19.4)^2 - (7)^2\right]\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 164\ kPa(\text{gage})$$

To determine the force required to hold the plug, apply the x-component of the momentum equation to the CV shown.

$$F_{S_x} + \underset{=0}{\cancel{F_{B_x}}} = \underset{=0(1)}{\cancel{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall}} + \int_{CS} u\rho \vec{V} \cdot d\vec{A}$$

$$p_{1g}A_1 - F = u_1\{-\dot{m}\} + u_2\{\dot{m}\} = \dot{m}(u_2 - u_1) = \rho V_1 A_1 (V_2 - V_1)$$

$$F = p_{1g}A_1 - \rho V_1 A_1 (V_2 - V_1)$$

$$= 164 \times 10^3\,\frac{N}{m^2} \times \frac{\pi}{4}(0.05)^2 m^2 - 999\,\frac{kg}{m^3} \times 7\,\frac{m}{s} \times \frac{\pi}{4}(0.05)^2 m^2 (19.4 - 7)\frac{m}{s} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$F = 322\,N - 170\,N = 152\ N \text{ (in direction shown)}$$

p_1

F