APL106

Minor 1

Time: 60+5 minutes
+ 10 minutes uploading

11-11-2020

Name

Entry No.

Please answer all the questions. All bold letters indicate vector quantities. Standard symbols have their usual meanings.

If required use the data as follows: a) $\rho_{air} = 1.25$ kg/m^3, b)) $\rho_{water} = 1000$ kg/m^3, c) $g = 9.81$ m/s^2.

Total 7 pages in Question paper including this page. 1 question on each page from page 2 to page 7.

Maximum Marks

Q1.	9 marks
Q2.	6 marks
Q3.	12 marks
Q4.	6 marks
Q5.	12 marks
Q6.	15 marks
Total	60 marks

All parts of the same question must be answered together.

Please sign the after writing the following statement:

I hereby declare that I have not taken any help from anyone in solving this examination.

Mehaan Jain

(Signature)

ISHAANI JAIN
2019ME10257

Ans) $V = 4\hat{i}$ $\quad 0 \le t \le 4\,s$

$V = 4\hat{i} + 4\hat{j}$ $\quad 4 \le t \le 8\,s$

$V = 4\hat{j}$ $\quad 8 \le t \le 12\,s.$

(a) $\frac{dy}{dx} = 0 \Rightarrow y = C$

$y(2) = 0 \Rightarrow C = 0.$

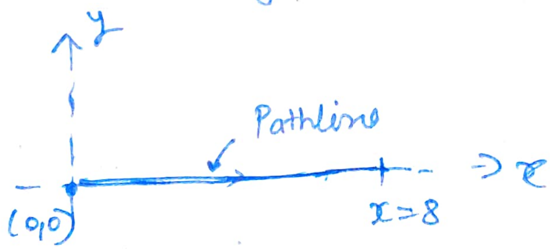
$\Rightarrow y = 0$ (x-axis).

(b) $\frac{dx}{dt} = 4 \Rightarrow \int_0^x dx = \int_0^2 4\,dt$

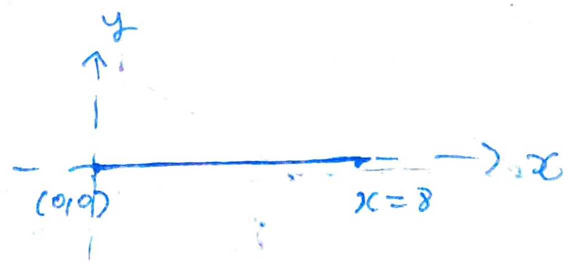
$x - 0 = 4(2-0) \Rightarrow x = 8$

(a) $\frac{dy}{dt} = 0 \Rightarrow y = \text{constant} = K$

$y(0) = 0 \Rightarrow y = 0$



(c) streakline



$4 \le t \le 8$

(d) $\frac{dy}{dx} = 1 \Rightarrow y = x + C$ $\quad$ (0, 16) is satisfied by this

$\Rightarrow \underline{y = x - 16}.$

at $t = 8$ sec

$y = 4 \times 8 - 16$

$= \underline{16}$

b/w $8 \le t \le 12$

$\frac{dy}{dx} = \lim_{V_x \to 0} \frac{4}{V_x} \to \infty$

$\Rightarrow \underline{x = 32}$

ISHAAN JAIN

2019ME10257

(d) Streamline passing through (0,0) at $t = 10\,sec$

$\Rightarrow \quad x = 0 \quad \because \quad \frac{dy}{dx} = \lim_{V_x \to 0} \frac{4}{0} \to \infty$

$\Rightarrow$ Streamline is $\parallel$ to y axis

(e) Pathline of particle at (0,0) at $t = 0$, observed at $t = 10$.

$0 \le t \le 4$: $y = 0$ $\Rightarrow$ at $t = 4$: particle is at $x = 16$, $y = 0$ $(16, 0)$.

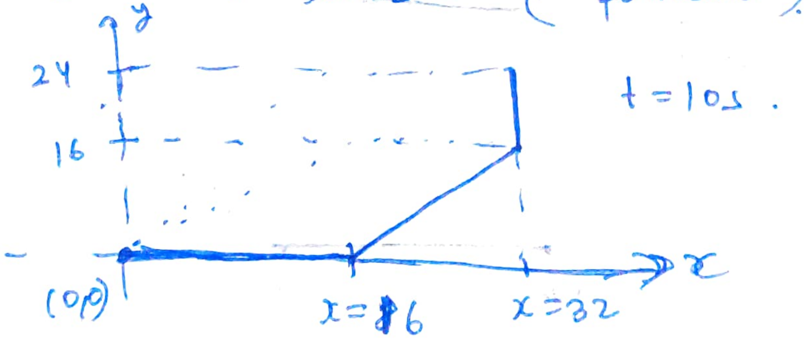
$4 \le t \le 8$ $\quad \frac{dy}{dx} = 1 \Rightarrow y = x + C$ $\quad$ (16,0) is satisfied by this

$\Rightarrow y = x - 16$

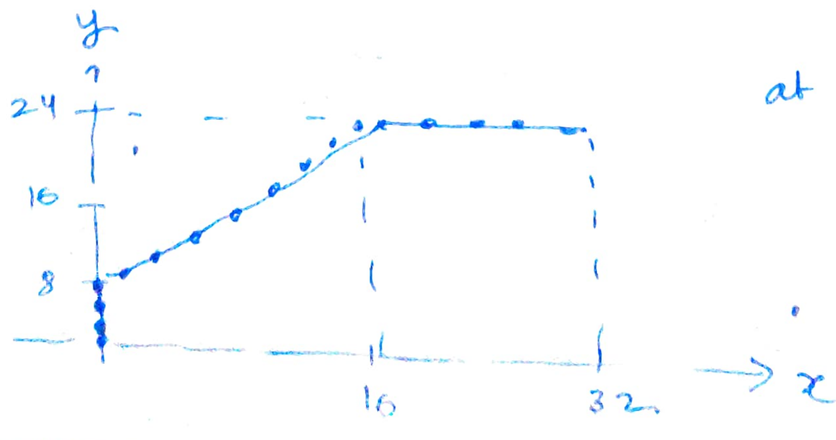
at $t = 8$ sec. $\quad y = 32 - 16 = 16$

$\Rightarrow (32, 16)$.

$8 \le t \le 12$: $x = 32$ (pathline) $\quad$ at $t = 10$ $(32, 24)$.



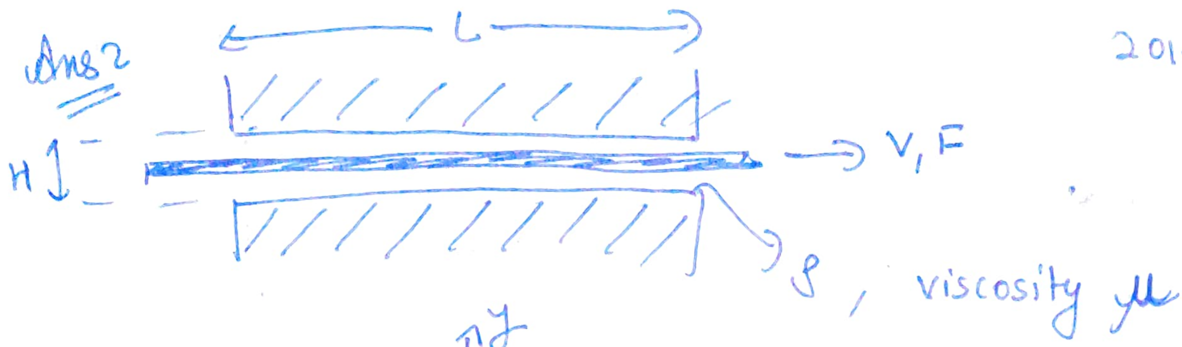
(f) Streakline



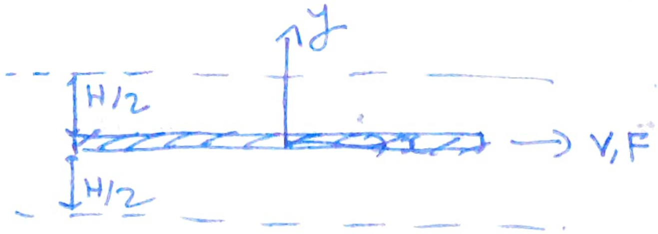
at $t = 10$.

ISHAAN JAIN
2019ME10251

Ans 2



(a)



$$\tau_{yx} = \mu \frac{dV_x}{dy}$$

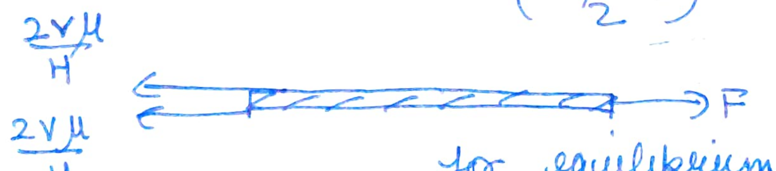
for H/2 part above tape.

$$\tau_{yx} = \mu \frac{(0-V)}{(H/2 - 0)} = -\frac{V\mu 2}{H}$$

Also,

for H/2 part below tape.

$$\tau_{yx} = \mu \frac{(0-V)}{(-H/2 - 0)} = \frac{2\mu V}{H}$$



for equilibrium

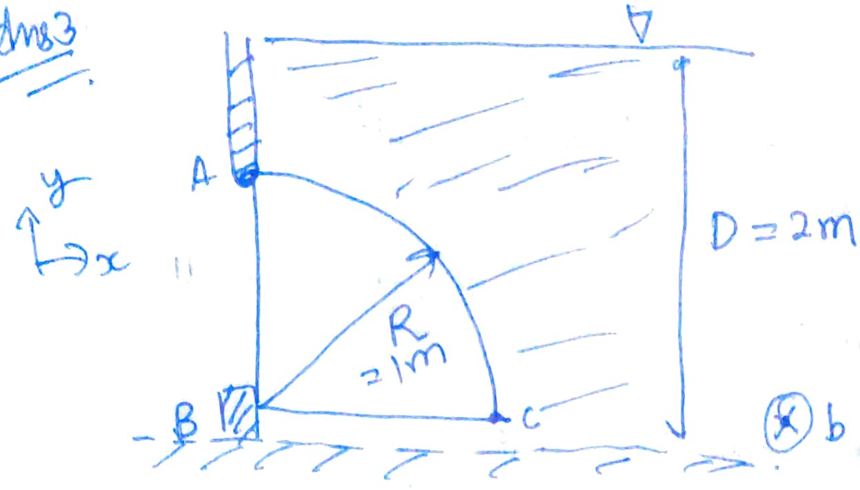
$$F = \frac{4V\mu}{H}\hat{i} \text{ N.}$$

(b) Power $= \vec{F}\cdot\vec{V} = \left(\frac{4V\mu}{H}\right)\hat{i} \cdot \left(V\hat{i}\right)$

$$= \frac{4V^2\mu}{H} \text{ J/s.}$$

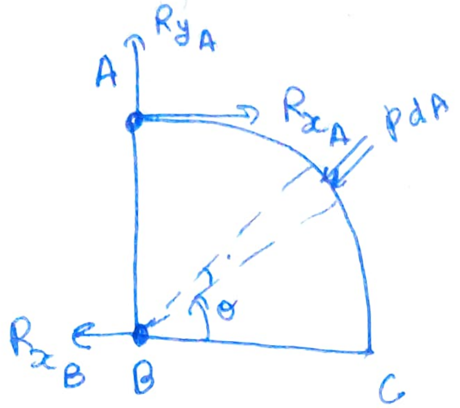
ISHAAN JAIN
2019ME10251

Ans 3.



Neglect weight of gate.

b, width into plane = 1m



F_x : horizontal force due to pressure.

$$F_x = \rho_w g \left(D - \frac{R}{2}\right)(Rb).(-\hat{i})$$

$$F_y = \rho_w g \left(DRb - \frac{\pi}{4}R^2 b\right)\hat{j}$$

for equilibrium

$\Sigma F_x = 0 \Rightarrow R_{x_A} = R_{x_B} + |F_x|$.

Also

Moment About B = 0, ~~pressure~~

$\Rightarrow$ pressure forces passes through B
Hence their contribution = 0.

$\Rightarrow$ R_{y_B} passes through B, Hence 0.

$\Rightarrow R_{x_A} \times R = 0 \Rightarrow R_{x_A} = 0$.

$$\Rightarrow R_{x_B} = -|F_x| = -\rho_w g \left(D - \frac{R}{2}\right)(Rb)$$

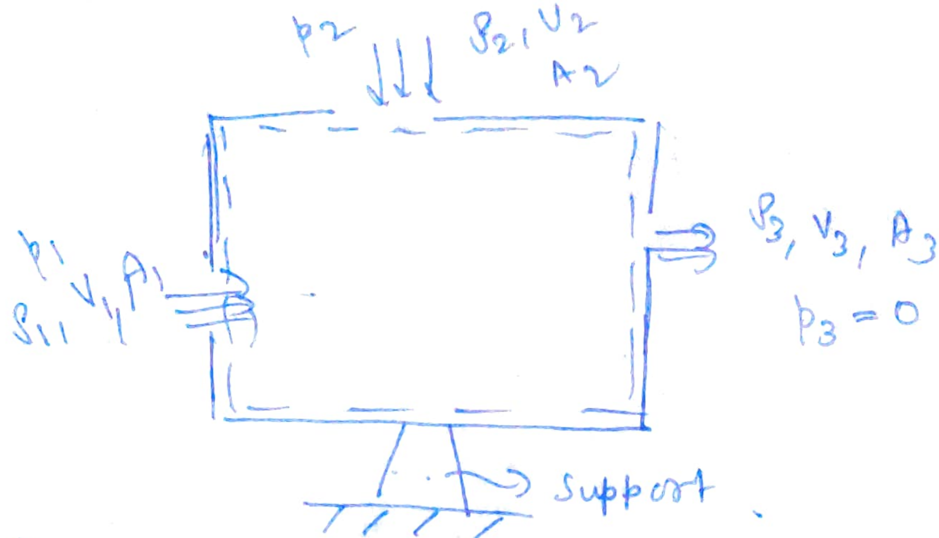
$$= -1000 \times 9.81 \left(\frac{3}{2}\right)(1 \times 1)$$

(Force on ABC) $= -14715$ N

$\therefore \vec{R_{x_B}} = 14715\,\hat{i}$ N $\Rightarrow$ Force on Hinge B $= -\vec{R_{x_B}} = -14715\,\hat{i}$ N

Ans 5

ISHAAN JAIN
2019ME10251



(i)

By conservation of mass of control volume,

$$-\rho_1 A_1 V_1 - \rho_2 A_2 V_2 + \rho_3 A_3 V_3 = 0.$$

$$\Rightarrow \rho_3 A_3 V_3 = \rho_1 A_1 V_1 + \rho_2 A_2 V_2.$$

By volume continuity equation.

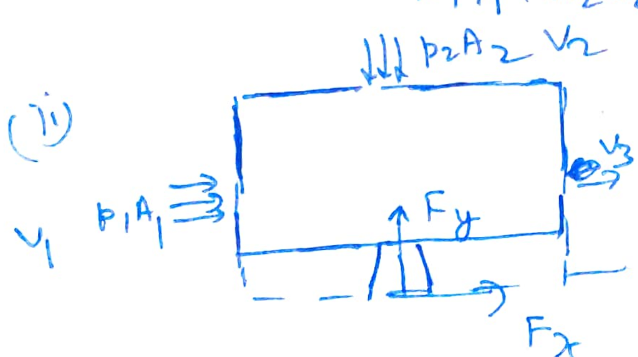
$$-A_1 V_1 - A_2 V_2 + A_3 V_3 = 0$$

$$\Rightarrow V_3 = \frac{A_1 V_1 + A_2 V_2}{A_3} \quad -(1)$$

$$\Rightarrow \rho_3 = \frac{1}{A_3 V_3}\left[\rho_1 A_1 V_1 + \rho_2 A_2 V_2\right]$$

$$\rho_3 = \frac{\left[\rho_1 A_1 V_1 + \rho_2 A_2 V_2\right]}{A_1 V_1 + A_2 V_2} \quad -(2).$$

(ii)



Applying moment equat"

$$F_x + p_1 A_1 = 0 - V_1^2 \rho_1 A_1 + \rho_3 V_3^2 A_3.$$

$$F_x = (\rho_3 V_3^2 A_3 - \rho_1 V_1^2 A_1 - p_1 A_1)\hat{i}$$

$-F_x$ on support.

$$F_y - p_2 A_2 = -\rho_2 V_2^2 A_2.$$

$$F_y = (p_2 A_2 - \rho_2 V_2^2 A_2)\hat{j}$$

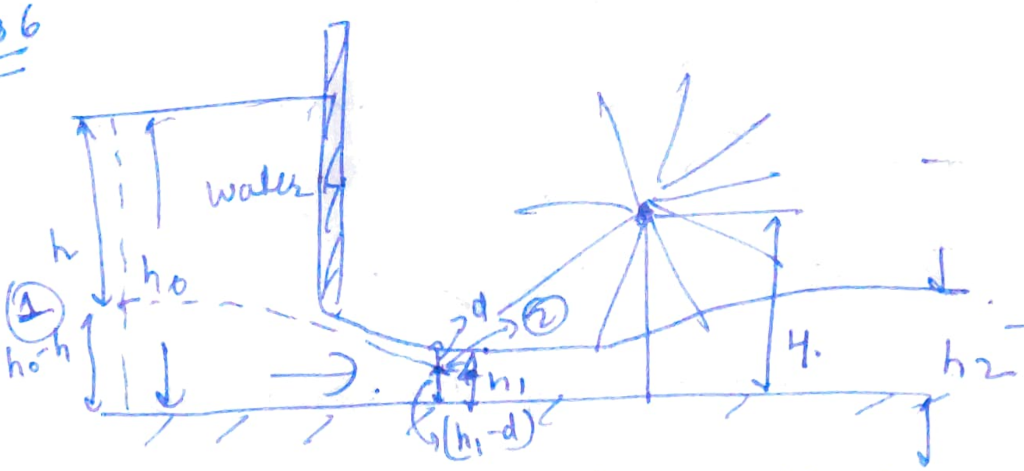
ISHAAN JAIN
2019ME10251

$\Rightarrow$ Force on support : $-F_x\hat{i} - F_y\hat{j}$.

(iii) No, Applying BE b/w 1 & 3 is not a reasonable approximation as there will be losses due to mixing.

ISHAAN JAIN
2014ME10251

Ans 6



$h_1 = 1m$
$h_2 = 2m.$
$h_0 = 5m$
$W = 2m$

Continuity equation. $V_1 H_1 = V_2 H_2$

$\Rightarrow V_0 h_0 = V_1 h_1$

B.E. (1 → 2) (as given

(a) for 1

$$\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g}$$

$$= \frac{\rho g h}{\rho g} + (h_0 - h) + \frac{V_0^2}{2g}$$

$$= h_0 + \frac{V_1^2}{2g}$$

for 2.

$$\frac{p_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

$$= h_1 + \frac{V_1^2}{2g}$$

$$\Rightarrow h_0 + \frac{V_0^2}{2g} = h_1 + \frac{V_1^2}{2g}$$

and $V_0 h_0 = V_1 h_1.$

$$V_0 = \frac{V_1 h_1}{h_0} = V_1 \frac{1}{5} = \frac{V_1}{5}$$

$$\Rightarrow 5 + \frac{V_1^2}{50g} = 1 + \frac{V_1^2}{2g}.$$

$$4 = \frac{V_1^2}{g}\left(\frac{1}{2} - \frac{1}{50}\right)$$

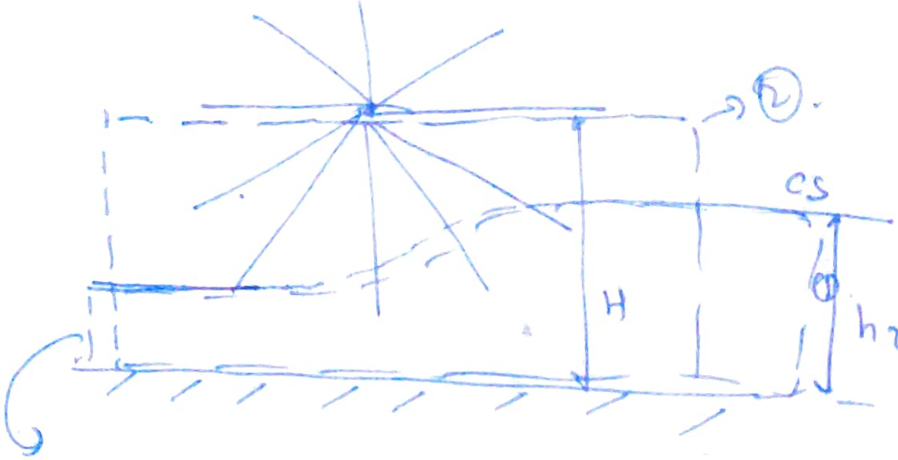
$$= \frac{V_1^2}{g}\left(\frac{\cancel{24}^{12}}{\cancel{50}_{25}}\right) = \frac{\cancel{12}^{3} V_1^2}{25g} = \cancel{4}$$

$$V_1^2 = \frac{25g}{3} \Rightarrow |V_1| = \sqrt{\frac{25g}{3}}\ m/s$$

$$\Rightarrow \vec{V_1} = 9.041\ m/s\ \hat{i}.$$

ISHAAN JAIN
2019ME10251

(b)



① Applying continuity equation on this CV,

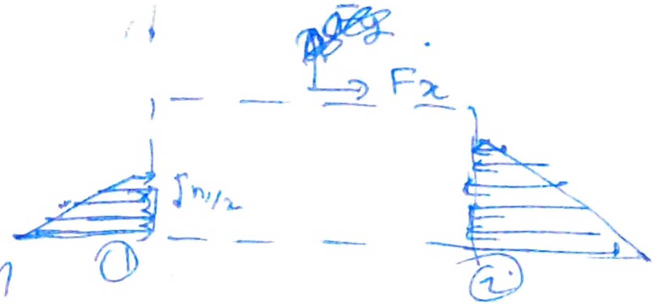
$$V_1 h_1 W = V_2 h_2 W$$

$$\Rightarrow V_1 h_1 = V_2 h_2$$

$$\Rightarrow V_2 = \frac{V_1 h_1}{h_2} = (9.041)\frac{1}{2}\hat{i} \text{ m/s}$$

$$\vec{V_2} = 4.5205\hat{i} \text{ m/s}$$

(c) Taking ② as CV;



Applying Momentum equation

$$\sum F_x = \frac{\partial}{\partial t}\iiint \cancel{V_x \rho d\forall} + \oint_{CS} V_x(\rho\vec{V}\cdot\hat{n})dA.$$

Assuming steady flow

$$= \oint_{CS} V_x \rho(\vec{V}\cdot\hat{n})dA = \iint_{①} V_x\rho(\vec{V}\cdot\hat{n})dA + \iint_{②} V_x\rho(\vec{V}\cdot\hat{n})dA$$

$$F_x + \rho g\frac{h_1^2}{2}w - \rho g\frac{h_2^2}{2}w = -V_1\rho A_1 V_1 + \rho A_2 V_2^2.$$

$$F_x = (\rho h_2 w V_2^2 - \rho V_1^2 w h_1 + \rho g\frac{h_2^2}{2}w - \rho g\frac{h_1^2}{2}w)\hat{i}.$$

$$= -52.309\hat{i} \text{ KN}$$

$\therefore$ Force on paddle wheel $= -\vec{F_x} = \underline{52.309\hat{i} \text{ KN}}$

HELP SHEET

ISHAAN JAIN
2019ME10251

1. $\tau_{nt} = \mu \dfrac{dV_t}{dx_n}$

2. For Streamline:
$$\frac{dy}{dx} = \frac{V_y(x,y,t)}{V_x(x,y,t)}$$

3. Pathline
$$\frac{dx}{dt} = u(x,y,t)$$
$$\frac{dy}{dt} = v(x,y,t)$$

4. Force due to pressure $= -\iint p\, dA\hat{n}$

5. Basic equation of fluid statics : $\rho\vec{a} = -\nabla p + \rho\vec{g}$

6. Buoyancy : upward force $= \rho g \forall \rightarrow$ submerged portion

7. $\dfrac{\partial \beta}{\partial t}$: for steady flow (depends on frame of reference)

8. Volume flow rate $(\dot{Q}) = \oiint_{CS} (\vec{V}\cdot\hat{n})\, dA$

9. Mass flow rate $(\dot{m}) = \oiint_{S} \rho(\vec{V}\cdot\hat{n})\, dA$

10. Reynold's Transport Theorem :
$$\frac{dB}{dt} \text{ or } \frac{DB}{Dt} = \frac{\partial}{\partial t}\left(\iiint \beta\rho\, d\forall\right) + \oiint_{CS} \beta\rho(\vec{V}\cdot\hat{n})\, dA$$
$\hookrightarrow$ relative velocity to CV.

11. Law of Conservation of Mass.
$$0 = \frac{\partial}{\partial t}\iiint \rho\, d\forall + \oiint_{CS} \rho(\vec{V}\cdot\hat{n})\, dA$$

12. Volume continuity eqn.
$$\frac{d\forall}{dt} = \frac{\partial}{\partial t}\iiint d\forall + \oiint_{CS} (\vec{V}\cdot\hat{n})\, dA$$

13. Momentum equation.
$$\Sigma F_{ext} = \frac{\partial}{\partial t}\iiint \vec{V}\rho\, d\forall + \oiint_{CS} \vec{V}\cdot\rho(\vec{V}\cdot\hat{n})\, dA$$

14. Energy equation for CV.
$$\frac{dE}{dt} = \frac{\partial}{\partial t}\iiint e\rho\, d\forall + \oiint_{CS} e(\rho\vec{V}\cdot\hat{n})\, dA = \underbrace{\frac{dQ}{dt}}_{(\dot{Q})} - \underbrace{\frac{dW}{dt}}_{(\dot{W})}$$

$\dot{W}_s < 0$ (pump)
$\dot{W}_s > 0$ (turbine)

$$\dot{W}_p = \oiint_{CS} \frac{p}{\rho}(\rho\vec{V}\cdot\hat{n})\, dA \qquad \dot{W}_v = -\oiint_{CS} (\vec{\tau}\cdot\vec{V})\, dA$$

ISHAAN JAIN
2019ME10XXX

$$\frac{\dot{Q}}{\dot{m}} - \frac{\dot{W}_s}{\dot{m}} - \frac{\dot{W}_v}{\dot{m}} = -\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \left(h_2 + \frac{V_2^2}{2} + gz_2\right)$$

h: specific enthalpy $= \left(\hat{u} + \frac{p}{\rho}\right)$

$\frac{p}{\rho g}$: pressure head $\qquad \frac{V^2}{2g}$: velocity head

$\frac{p}{\rho g} + z$: piezometric head.

15 Bernoulli's Equation: $\left(\frac{p}{\rho} + \frac{V^2}{2} + gz\right)$ along streamline is constant.

16. C_d : coefficient of discharge $= \dfrac{\dot{Q}_{\text{venturi/nozzle/orifice (actual)}}}{\dot{Q}_{\text{ideal (obtained from BE)}}}$

$C_{d_{\text{venturi}}} > C_{d_{\text{Nozzle}}} > C_{d_{\text{orifice}}}$

17. Average velocity : $\dot{Q}/A$

18. Pitot tube : $p_{\text{stagnation}} = p_{\text{static}} + \underbrace{\frac{1}{2}\rho V^2}_{\text{dynamic pressure}}$

19. efficiency : $\eta_{\text{pump}} = \dfrac{E_{\text{fluid}}}{E_{\text{input}}} \qquad \eta_{\text{turbine}} = \dfrac{E_{\text{output}}}{E_{\text{fluid}}}$

20. $\vec{a}_{P|XYZ} = \vec{a}_{P/xyz} + \left\{ \vec{a}_{A|XYZ} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + \dot{\vec{\omega}}\times\vec{r} + \underbrace{2\vec{\omega}\times\vec{V}_{P|xyz}}_{\text{CORIOLIS ACCELERATN}} \right\}$

21. RTT for accelerating CV

$$\iiint \vec{a}_{P|xyz}\,\rho\, d\forall = \frac{\partial}{\partial t}\iiint \vec{V}\rho\, d\forall + \oiint_{CS} \vec{V}(\rho\vec{V}\cdot\hat{n})\, dA.$$

$\underline{\underline{\text{LHS}}} = \underbrace{\iiint a_{P|XYZ}\,\rho\, d\forall}_{\substack{\| \\ m\vec{a}_{c|XYZ} \\ \text{COM}}} - \underbrace{\iiint \left\{ \vec{a}_{A|XYZ} + \vec{\omega}\times\vec{\omega}\times\vec{r} + \dot{\vec{\omega}}\times\vec{r} + 2\vec{\omega}\times\vec{r} \right\}\rho\, d\forall}_{\substack{\| \\ \textcircled{a}}}$

$\iiint \vec{a}_{A|XYZ}\,\rho\, d\forall = m\vec{a}_A \qquad \iiint \vec{\omega}\times\vec{\omega}\times\vec{r}_{AP}\,\rho\, d\forall = \vec{\omega}\times\vec{\omega}\times m\vec{r}_c$

$\iiint \dot{\vec{\omega}}\times\vec{r}\rho\, d\forall = \dot{\vec{\omega}}\times m\vec{r}_c \qquad 2\vec{\omega}\times\iiint \vec{V}_{P|xyz}\,\rho\, d\forall$