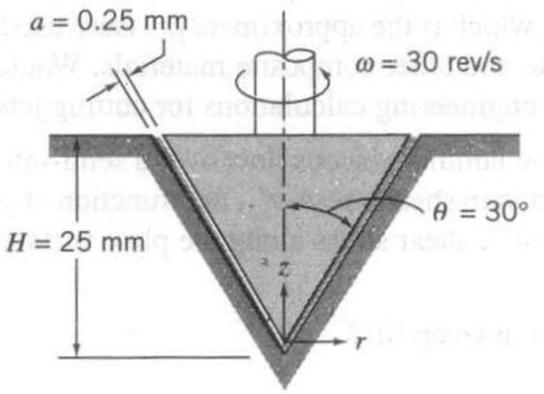
EXERCISE 1

VISCOSITY AND FLOW VISUALIZATION

Viscosity & Flow Visualization

1. A conical pointed shaft turns in a conical bearing. The gap between shaft and the bearing is filled with heavy oil having the viscosity of μ = 0.2 Ns/m^2.

a. Obtain an algebraic expression for the shear stress that acts on the surface of the conical shaft.
b. Calculate the viscous torque that acts on the shaft.



Solution: Assuming Newtonian fluid,

Newton's Law of viscosity for one-dimensional flow:

$$\tau = \mu \frac{dV_t}{dx_n}$$

Assuming linear profile,

$$\tau = \mu \frac{\omega r - 0}{a - 0} = \frac{\mu \omega r}{a}$$

$$\tau(z) =$$

Consider a small element of area

$$dA = 2\pi r\, ds = (\quad)\, dz$$

The viscous torque on the element of area is:

$$dT = r\tau\, dA =$$

Integrating,

$$T = \oint dT = \int_0^H f(z)\, dz =$$

2.The velocity field is given as $\vec{V} = Ax\,\hat{\imath} - Ay\,\hat{\jmath}$ x and y are in meters and A=0.3s^{-1}.
Find:

(a) Equation of the streamline in xy plane
(b) Velocity of the particle at point (2, 8)
(c) Position at t=6 s of the particle located at (2, 8) at t=0 s.
(d) Velocity of the particle found in part (c).
(e) Equation of pathline of particle located at (2, 8) at t=0 s.

Solution:
(a) For streamline we have

$$\frac{dy}{dx} = \frac{v}{u}$$

Substitute values of u and v and integrate to get the equation of streamline.

(b) $\vec{V} = Ax\,\hat{\imath} - Ay\,\hat{\jmath}$. At (2,8) $\vec{V} =$
(c) $\vec{V} = Ax\,\hat{\imath} - Ay\,\hat{\jmath}$

$$u_p = \frac{dx}{dt} =$$

Integrate to get the function of the x coordinate of the particle as a function of time.

$$\Longrightarrow \quad x(t) =$$

$$v_p = \frac{dv}{dt} =$$

Integrate to get the function of the y coordinate of the particle as a function of time.

$$\Longrightarrow \quad y(t) =$$

At $t = 6$ s, $x =$ & $y =$

(d) x & y Coordinates are known. $\Longrightarrow \vec{V} =$
(e) For finding the equations of pathline use the functions $x(t)$& $y(t)$. Eliminate t to get the pathline in x & y .

Viscosity & Flow Visualization

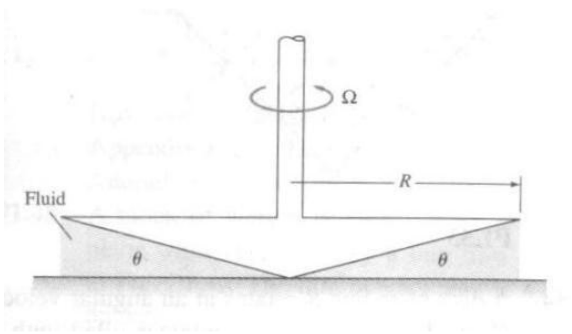
1.The velocity distribution for laminar flow between the parallel plates is given by

$$\frac{u}{u_{max}} = 1 - \left(\frac{2y}{h}\right)^2$$

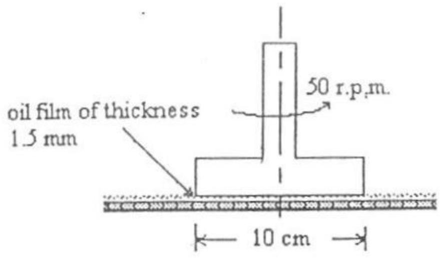
Where h is the distance separating the plates and the origin is placed midway between the plates. Consider a flow of water at 15°C, with $u_{max} = 0.30$ m/s and h = 0.50 mm. Calculate the shear stress on the upper plate and give its direction. Sketch the variation of shear stress across the channel.

2. A female ice skater of mass 60 kg glides on one skate at speed V = 6m/s. Her weight is supported by a thin film of liquid water melted from the ice by the pressure of the skate blade. Assume the blade is L = 300mm long and w = 3 mm wide, and that the water film is h = 0.0016 mm thick. Estimate the deceleration of the skater that results from viscous shear in the water film.

3. The device in Fig. is called a cone-plate viscometer. The angle of the cone is very small and the gap is filled with the test liquid. The torque M to rotate the cone at a rate Ω is measured. Assuming a linear viscosity profile in the fluid film, derive an expression for fluid viscosity μ as function of (M, R, Ω, θ).



4.A flat disc rotates on a table and is separated from it by an oil film. If the torque required to rotate the disc at 50 rpm is 3 gm-cm, find the viscosity of the oil.

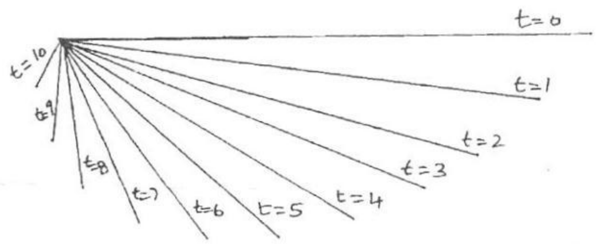


5. In a 2D flow the velocity field is given by $\vec{V} = y\,\hat{\imath} + x\,\hat{\jmath}$.

(a) Find the equation of streamline passing through (0, 0) at t = 5s.
(b) Find the equation of pathline of a particle released from (0, 0) at time t = 0.

6.The figure below shows pathlines of particles released at (0, 0) at a gap of 1 second (0 ≤ t ≤ 10). The time indicated in the figure shows the time at which the particle was released.

(a) Explain giving reasons if this flow is steady or unsteady.
(b) Sketch the figure and on it draw a qualitative pattern of the streakline of particles passing through (0, 0) observed at t = 6s.

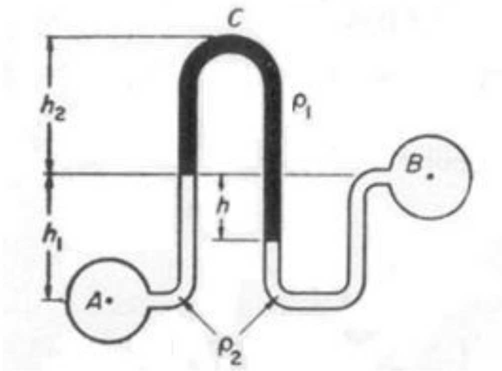


EXERCISE 2

PRESSURE DISTRIBUTION - I

Pressure Distribution in a Fluid

1. Obtain P_a-P_b in terms of the quantities shown:



Solution: Taking pressure difference across each liquid column one by one

$$P_a - P_d =$$

$$P_d - P_c =$$

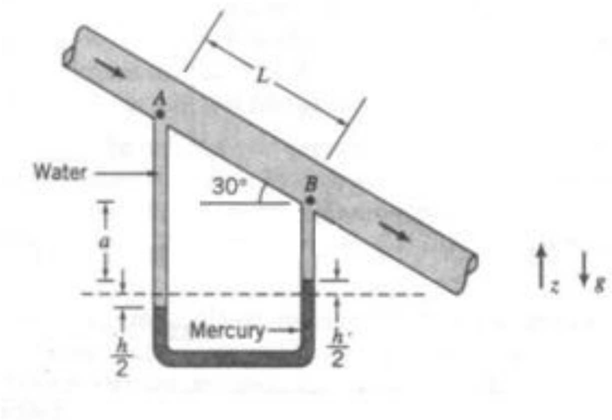
$$P_c - P_e =$$

$$P_e - P_b =$$

Adding all the above equations we get,

$$P_a - P_b =$$

2. Water flows downward along a pipe that is inclined at 30° below the horizontal. The difference in pressure $(P_a - P_b)$ is partly due to gravity and partly due to friction. Derive an expression for the same. Also evaluate for L= 5ft and h = 6 in.



Solution: Calculate the total length between A & C

$$L = \quad + \quad +$$

$$P_a - P_c =$$

Similarly,

$$P_c - P_d =$$

$$P_d - P_b =$$

Adding the above three equations we get,

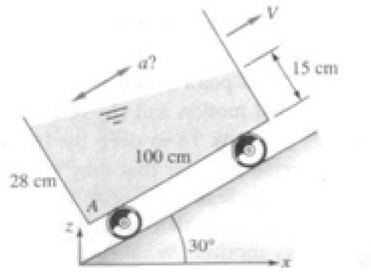
$$P_a - P_b =$$

Substitute values of L, h, g, ρ_w & ρ_m to get,

$$P_a - P_b =$$

3. A tank of liquid is moving with constant acceleration up a 30° inclined pane. Assuming rigid-body motion, compute

a. The value of the acceleration a,
b. Whether the acceleration is up or down, and
c. The gage pressure at point A if the fluid is mercury.



Solution: Assume that the acceleration $\vec{a}$ be up the plane:

$$\vec{a} = (\quad)\hat{\imath} + (\quad)\hat{k}$$

Equation governing pressure distribution in fluid

$$\nabla P = \rho(\vec{g} - \vec{a}) = \left(\frac{\partial P}{\partial x}\right)\hat{\imath} + \left(\frac{\partial P}{\partial z}\right)\hat{k}$$

$$\frac{\partial P}{\partial x} = \qquad\qquad ; \quad \frac{\partial P}{\partial z} =$$

From the component equations,

$$dP = \left(\frac{\partial P}{\partial x}\right)dx + \left(\frac{\partial P}{\partial z}\right)dz$$

At free surface; $dP = 0$

$$\frac{dz}{dx} = -\frac{\left(\frac{\partial P}{\partial x}\right)}{\left(\frac{\partial P}{\partial z}\right)} = 30 - \theta$$

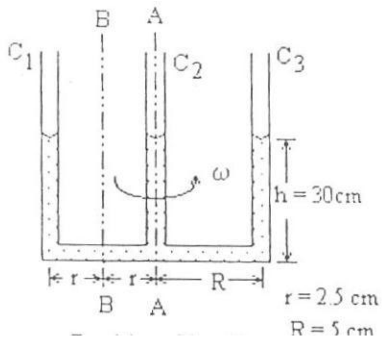
Solve to get the acceleration $\hat{a}$ and its direction:

$$\hat{a} =$$

Gage pressure at A =

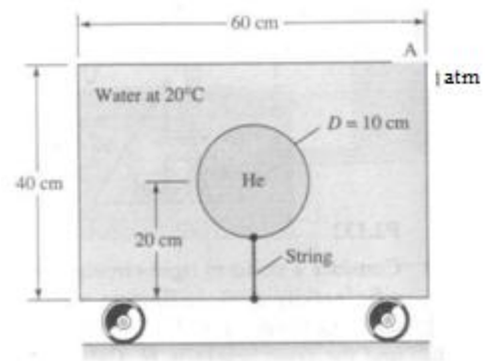
Pressure Distribution in a Fluid

1. The tube arrangement shown rotates about axis AA at 600/π rpm. Determine the levels in column C_1, C_2, C_3in the new position of equilibrium. If the axis is shifted to BB what are the levels? (The tubes are long enough to prevent spillage) The level ‘h’ is when the tube is at rest.



2. A partially full can of soft drink is placed at the outer edge of a child’s merry-go-round, located R =15 m from the axis of rotation. The can diameter and height are D = 65mm and H = 120 mm. The can is half-full of soda, with specific gravity SG = 1.06. Evaluate the slope of the liquid surface in the can if the merry-go-round spins at 0.3 revolutions per second. Calculate the spin rate at which the can would spill, assuming no slippage between the can bottom and the merry-go-round. Would the can most likely spill or slide off the merry-go-round?

3. The tank is Fig. is filled with water and has a vent hole at point A. The tank is 1 m wide into the paper. Inside the tank, a 10 cm balloon, filled with helium at 130 kPa, is tethered centrally by a string. If the tank accelerates to the right at 5 m/s^2 in rigid body motion, at what angle will the balloon lean? Will it lean to the right or left?



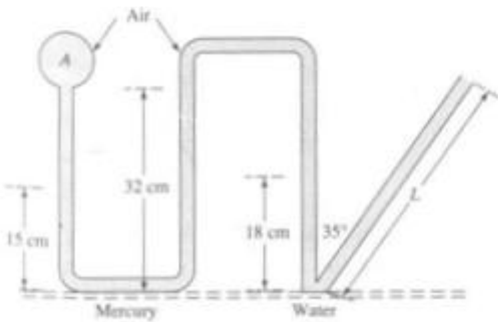
4. A salt water tank is in the shape of a cuboid of dimensions with its depth being 10 m. Due to the presence of salt; the density is not constant but varies with depth as

$\rho = \rho_0 (1 + Kh)$, Where h is the depth in m, $\rho_0 = 1000\ kg/\ m^3$ and $K = 0.03\ m^{-1}$

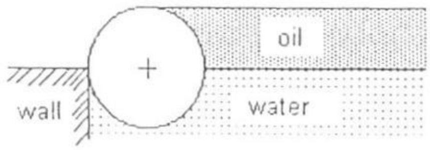
a) Find the pressure distribution in the tank as a function of h
b) Find the force due to salt water on any one of the sides of the tank.

5. The system in the figure shown is open to 1 atm on the right side.

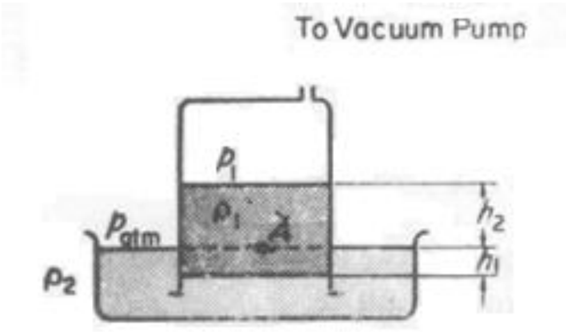
a) If L =120 cm, What is the air pressure in container A
b) If $P_A = 135\ kPa$, what is the length.



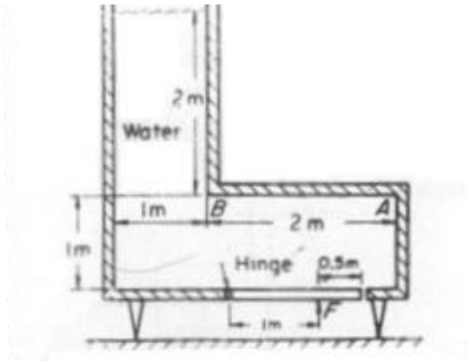
6. A log of 1m diameter supports water and oil (sp. Gravity 0.82) as shown. Find the Weight per unit length of the log if the contact with the wall is frictionless.



7. Obtain the gauge pressure $p_1$in terms of the two densities ρ_1 & ρ_2 and the level differences h_1 & h_2 for the arrangement shown.



8. Compute the force required to support the gate shown at the bottom of the tank. Take the width of the gate to be 1 m.



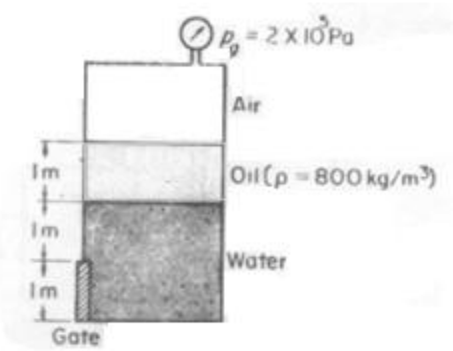
EXERCISE 3

PRESSURE DISTRIBUTION – II

HYDROSTATIC FORCES ON SURFACES

HYDROSTATIC FORCES ON SURFACES

1. Find the pressure at the bottom of the tank. Find the force on the gate if its width is w. $(\rho_{oil} = 800\ kg/m^3)$



Solution: Pressure at the bottom of the tank

$$P_{bottom} = P_{gauge} + P_{hydrostatic}$$

$$P_{bottom} = \quad + \quad +$$

$$P_{bottom} =$$

Non uniform pressure distribution over the gate surface, hence integration is required. Consider an element of length dx at a distance of x from the top of the gate.

$$P_{element} = P_{gauge} + P_{hydrostatic}$$

$$P_{element} = \quad + \quad + \quad +$$

$$A_{element} =$$

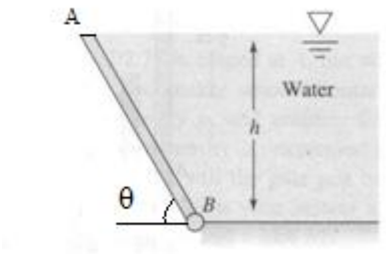
$$\mathrm{dF_{element}} =$$

$$F_{gate} = \int dF_{element}$$

Integrate from $x_1 =$ to $x_2 =$ to get total force.

$$F_{gate} =$$

2. In terms of the given quantities, derive an expression for the force on the gate & the moment it experiences about A. (Take the width of the gate as w and length as L)



Solution: Non uniform pressure distribution over the gate surface, hence integration is required. Consider an element of length dx at a distance of x from A.

$$P_{element} \quad = \quad P_{atm} \quad + \quad P_{hydrostatic}$$

$$P_{element} \quad = \qquad\qquad + $$

$$A_{element} \quad =$$

Force on the element

$$\mathrm{dF_{element}} =$$

$$F_{gate} = \int dF_{element}$$

Integrate from $x_1 =$ to $x_2 =$ to get total force.

$$F_{gate} =$$

Moment on the element about A

$$\mathrm{dM_{element}} =$$

$$M_{gate} = \int dM_{element}$$

Integrate from $x_1 =$ to $x_2 =$ to get total moment about A.

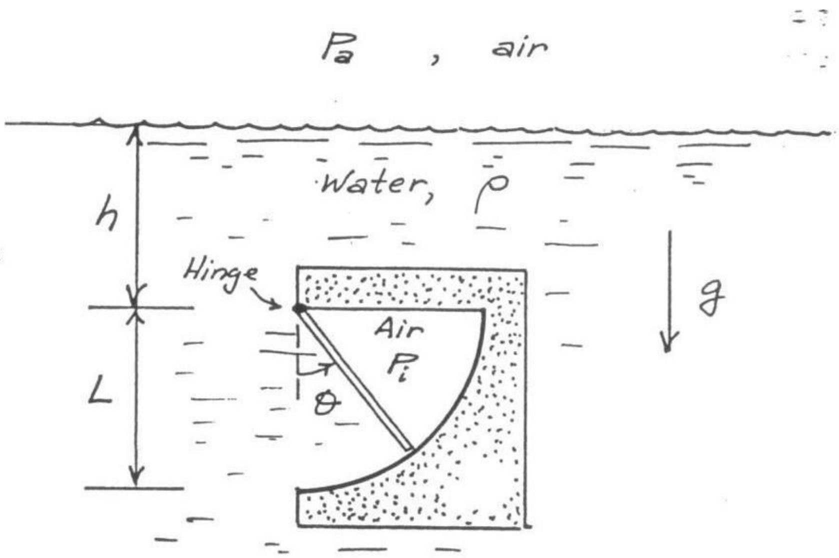
$$M_{gate} =$$

3. The container shown in the sketch in cross-section (it has a length b in the direction perpendicular to the paper) has a massless door of height L which is freely hinged at the top. Initially, the pressure of the air inside the container is atmospheric (p_a), the same as on the outside, and the door hangs vertically ($\theta = 0$).

Suppose now the container is pushed slowly under water (water density = ρ). Assuming that the pressure p_1 of the air is trapped in the container is related to its volumeV_1 by

$$p_i V_i = constant$$

Derive an expression for the depth h of the container at which the door will be turned inward at an angle θ.



Solution:

Force on the door = Force due to water + Force due air trapped in the container

Consider container at depth h and door at an angle θ from the vertical

<u>Force due to water:</u>

Absolute pressure due to water on the door at a distance *l*from the hinge:

$$P(l) =$$

Force on an element dl : $\quad dF_{water} = P(l)b\,dl$

Integrate to get force on door due to water:

$$F_{water} = \int dF_{water} =$$

Force due to trapped air:

Pressure on door due to trapped air: $P_{air,\theta} = \frac{P_i V_i}{V_\theta} =$

Force due to air: $F_{air,\theta} = P_{air,\theta} X\ Area =$

When container is at height h & door has moved angle θ inwards

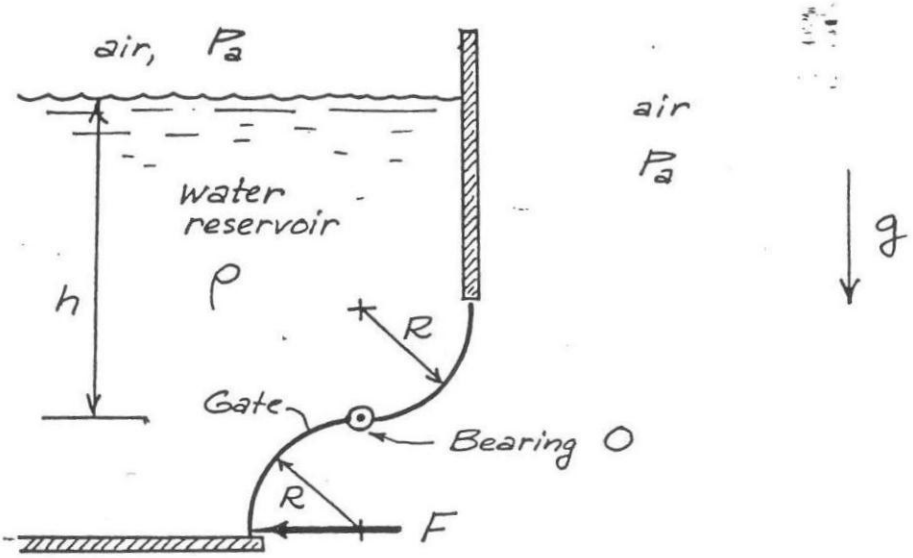
Net force on the door = $F_{water} - F_{air} =$

Solve to get: $h(\theta) =$

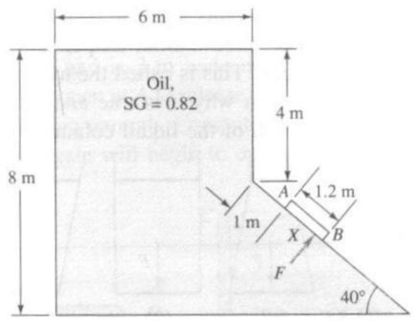
HYDROSTATIC FORCES ON SURFACES

1. Given h, R, g, and the water density ρ, determine the horizontal F force per unit of length normal to the paper required to keep the S-shape gate closed. Neglect friction in bearing O.

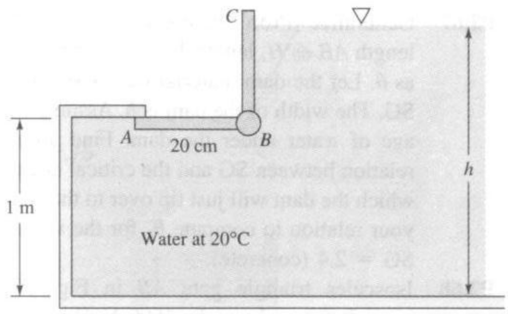
Evaluate the force if h = 3m, R = 1m, and ρ = 1000 kg/m^3



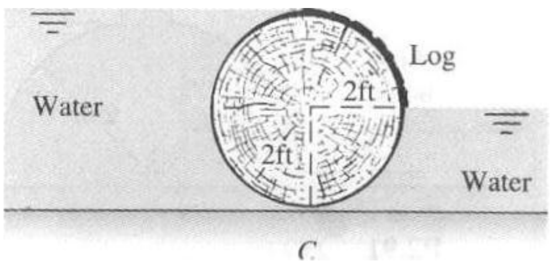
2. Given AB in the figure is 1.2 m long and 0.8 m into the paper. Neglecting the atmospheric pressure, compute the force F on the gate and its centre of pressure position X.



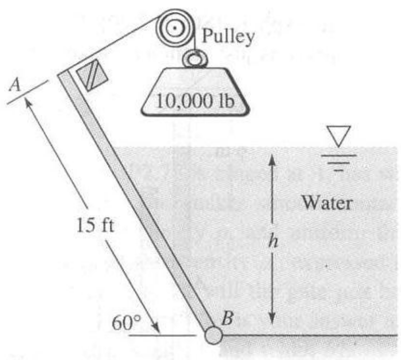
3. Gate ABC in the figure has a fixed hinge line at B and is 2 m wide into the paper. The gate will open at A to release water if the water depth is high enough. Compute the depth h for which the gate will begin to open.



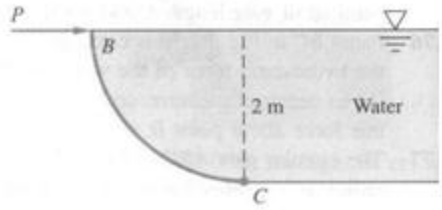
4. The 1mdiameter log (SG=0.80) in the 2m long into the paper and dams water as shown. Compute the net vertical and horizontal reactions at point C.



5. Gate AB in the figure shown is 15 ft long and 8 ft wide into the paper and is hinged at B with a stop at A. The gate is 1-in thick steel, SG = 7.85. Compute the water level h for which the gate will start to fall.



6. The quarter circle BC in figure is hinged at C. Find the horizontal force P required tohold the gate stationary. Neglect of the weight of the gate.



EXERCISE 4

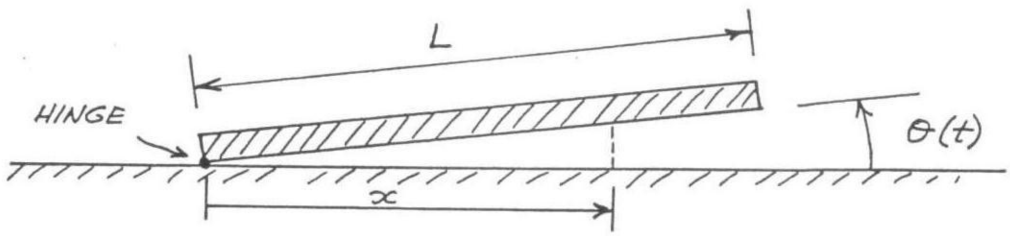
INTEGRAL RELATIONS FOR A CONTROL VOLUME - I

Integral Relations

1. A long, flat plate of breadth L(L being small compared with the length perpendicular to the sketch) is hinged at the left side to a flat wall, and the gap between the plate and wall is filled with an incompressible liquid of density ρ. If the plate is at a small angle θ(t) and is depressed at an angular rate.

$$\omega(t) = -d\theta/dt$$

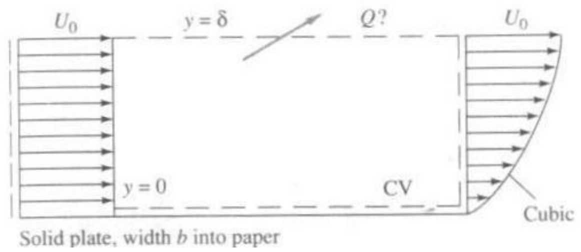
Obtain an expression for the average liquid speed u(x, t) in the x-direction at station x and time t.



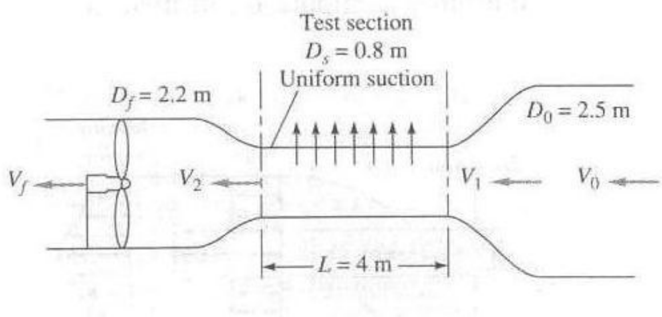
2. An incompressible fluid flows past an impermeable flat plate, as in Fig with a uniform inlet profile $u = U_0$ and a cubic polynomial exit profile

$$u = U_0\left(\frac{3\eta - \eta^3}{2}\right) \qquad where\ \eta = \frac{y}{\delta}$$

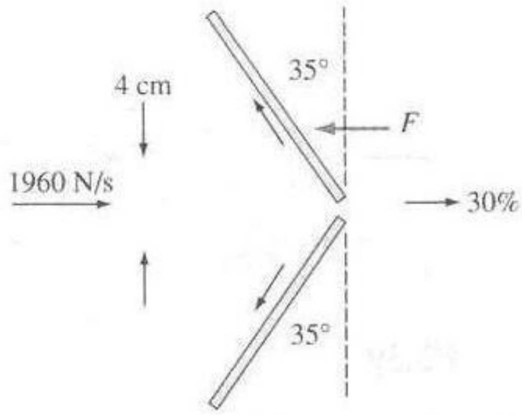
Compute the volume flow Q across the top surface of the control volume.



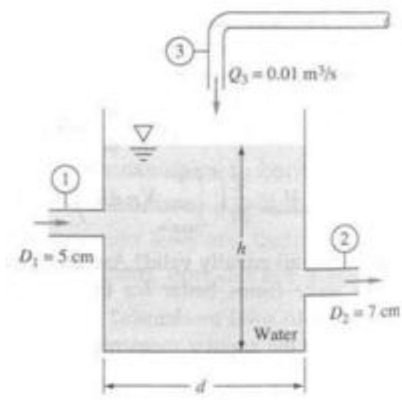
3. In some wind tunnels the test section is performed to suck out fluid and provide a thin viscous boundary layer. The test section wall in Fig contains 1200 holes of 5-mm diameter each per square meter of wall area. The suction velocity through each hole is V_1 = 35 m/s. Assuming incompressible steady flow of air at 20°C, compute a) V_0, b) V_2 and c) V_f, in m/s.



4.A steady two-dimensional water jet, 4 cm thick with a weight flow rate of 1960 N/s, strikes an angled barrier as shown in Fig. Pressure and water velocity are constant everywhere. Thirty percent of the jet passes through the slot. The rest splits symmetrically along the barrier. Calculate the horizontal force F needed to hold the barrier per unit thickness into the paper.



5. The open tank in Fig. contains water at 20°C and is being filled through section 1. Assume incompressible flow. First derive an analytic expression for the water-level change *dh/dt*in terms of arbitrary volume flows (Q_1, Q_2, Q_3) and tank diameter *d*. Then, if the water level *h* is constant, determine the exit velocity V_2 for the given data V_1 = 3 m/s and Q_1 = 0.01 m3/s.



EXERCISE 5

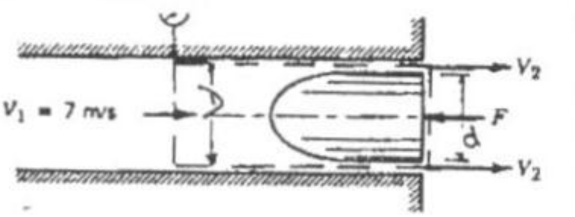
INTEGRAL REALTIONS FOR A CONTROL VOLUME - II

Integral Relations

1.Low speed water flow through a circular tube of diameter, D = 50 mm.Smoothly contoured plug of diameter d = 40 mm, is held in the end of the tube where the water discharges to the atmosphere.

Frictional effects are to be neglected. Velocity profile may be assumed uniform at each section. Find

a. Pressure measured by the gage shown
b. Force required to hold plug



Solution:

Given D = 50 mm, d = 40mm, $V_1 = 7\ m/s$

Applying continuity equation between inlet (1) and outlet (2):

$$V_2 = \frac{A_1 V_1}{A_2} =$$

Frictional effects are neglected.

Applying Bernoulli's equation between 1 and 2, pressure at 1 can be found

BE $\Longrightarrow$

$p_1 =$

<u>Momentum Equation in x direction:</u>

$$\sum F_x = \frac{\partial}{\partial t}\left(\iiint \mathrm{V_x}\,\rho\ d\forall\right) + \oiint \mathrm{V_x}\rho\vec{V}.d\vec{A}$$

For steady flow,

$$\frac{\partial}{\partial t}\left(\iiint \mathrm{V_x}\,\rho\ d\forall\right) =$$

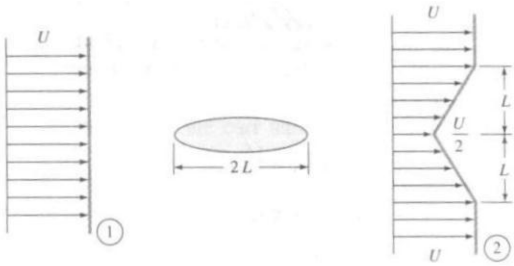
$$\sum F_x =$$

$$\oiint \mathrm{V_x} \rho \vec{V} . d\vec{A} \ =$$

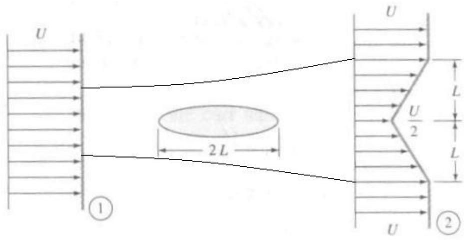
Equate the above equations to get F,

$$F \ =$$

2. When a uniform stream flows past an immersed thick cylinder, a broad low-velocity wake is created downstream, idealized as a V shape in Fig. Pressures p_1&p_2 are approximately equal. If the flow is two-dimensional and incompressible, with width b into the paper, derive a formula for the drag force Fon the cylinder. Rewrite your result in the form of a dimensionless *drag coefficient*based on body length $C_D = F/\rho U^2 bL$.



Solution: The control volume is chosen such that the outlet is V-shape of length 2L as shown in Fig. For such an outlet inlet is so chosen so that no fluid leaves out of the CV from inlet to outlet. In other words, CV is enclosed by streamlines at top and bottom.



Therefore from continuity equation,

$$\dot{m}_{outlet} = \dot{m}_{inlet} =$$

<u>Momentum Equation in x direction:</u>

$$\sum F_x = \frac{\partial}{\partial t}\left(\iiint V_x \rho \, d\forall\right) + \oiint V_x \rho \vec{V}.d\vec{A}$$

For steady flow,

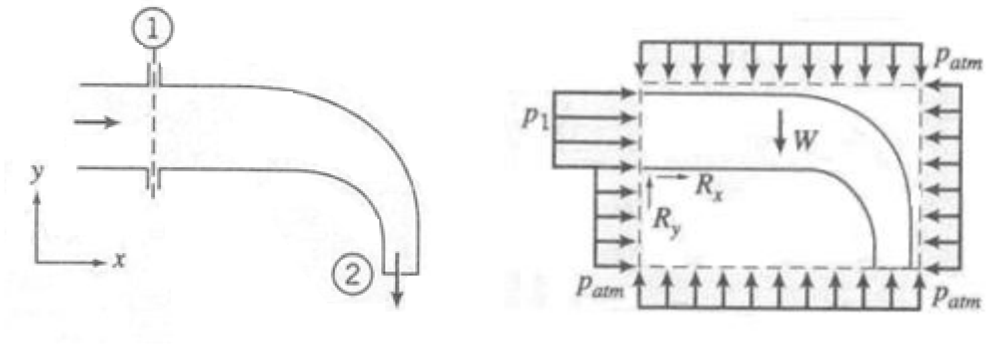
$$\frac{\partial}{\partial t}\left(\iiint V_x \rho \, d\forall\right) =$$

$$\sum F_x = -F_{drag}$$

$$\oiint V_x \rho \vec{V}.d\vec{A} =$$

Equate the above equations to get $F_{drag} =$

3. Water flows steadily through the 90° reducing elbow. At the inlet the absolute pressure is 220 kPa and the cross sectional area is $0.01m^2$. At the outlet the cross sectional area is $0.0025\ m^2$ and the velocity is $16m/s$. Determine the force required to hold the elbow in place.



Solution: $P_1 = 220\ kPa,\ A_1 = 0.01m^2, A_2 = 0.0025\ m^2\ ,\ \vec{V_2} = -16\ \hat{j}$

Assume the force required to hold the elbow be $R = R_x\hat{\imath} + R_y\hat{\jmath}$

To find $\vec{V_1}$ we first use the mass conservation equation.

$$\oint \rho\vec{V}\,.\,d\vec{A} \quad = \qquad\qquad\qquad + \qquad\qquad\qquad = \quad 0$$

$$\vec{V_1} =$$

<u>Momentum Equation in x direction:</u>

$$\sum F_x = \frac{\partial}{\partial t}\left(\iiint \mathrm{V_x}\,\rho\ d\forall\right) + \oiint \mathrm{V_x}\rho\vec{V}.\,d\vec{A}$$

For steady flow,

$$\frac{\partial}{\partial t}\left(\iiint \mathrm{V_x}\,\rho\ d\forall\right) =$$

$$\sum F_x = \qquad\qquad \oiint \mathrm{V_x}\rho\vec{V}.\,d\vec{A} \ =$$

Equate the above equations to get R_x

$$R_x =$$

<u>Momentum Equation in y direction:</u>

$$\sum F_y = \frac{\partial}{\partial t}\left(\iiint \mathrm{V_y}\,\rho\ d\forall\right) + \oiint \mathrm{V_y}\rho\vec{V}.\,d\vec{A}$$

For steady flow,

$$\frac{\partial}{\partial t}\left(\iiint \mathrm{V_y}\, \rho \;\, d\forall\right) =$$

$$\sum F_y =$$

$$\oiint \mathrm{V_y} \rho \vec{V}. d\vec{A} \;\; =$$

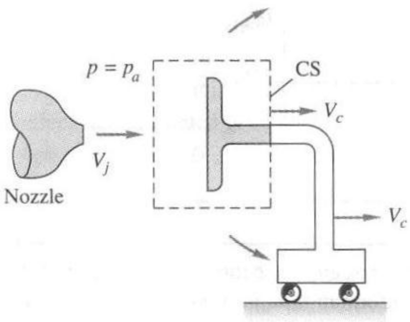
Equate the above equations to get R_y

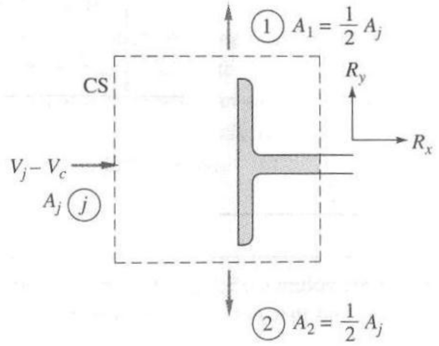
$$R_y =$$

Net force required to hold the elbow =

θ From the vertical =

4. A water jet of velocity V_j impinges normal to a flat plate that moves to the right at velocity V_c. Find the force required to keep the plate moving at constant velocity.





Solution: Assume the force required to hold the elbow be $R = R_x\hat{\imath} + R_y\hat{\jmath}$

To find $\vec{V_1}$ we first use the mass conservation equation.

$$\oint \rho\vec{V}.d\vec{A} \quad = \qquad + \qquad + \qquad = \quad 0$$

$$\vec{V_1} =$$

<u>Momentum Equation in x direction:</u>

$$\sum F_x = \frac{\partial}{\partial t}\left(\iiint \mathrm{V_x}\,\rho\ d\forall\right) + \oiint \mathrm{V_x}\rho\vec{V}.d\vec{A}$$

For steady flow,

$$\frac{\partial}{\partial t}\left(\iiint \mathrm{V_x}\,\rho\ d\forall\right) =$$

$$\sum F_x =$$

$$\oiint \mathrm{V_x}\rho\vec{V}.d\vec{A} \ =$$

Equate the above equations to get R_x

$$R_x =$$

Momentum Equation in y direction:

$$\sum F_y =$$

$$\oiint V_y \rho \vec{V}.d\vec{A} \quad = \qquad\qquad\qquad +$$

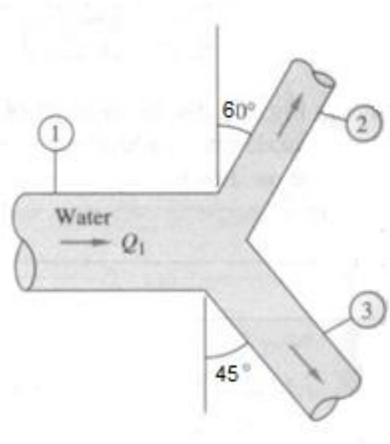
Equate the above equations to get R_y

$$R_y =$$

We can also say by symmetry that $R_y = 0$ (Same flow velocity and area in $\hat{j}$& -$\hat{j}$ directions)

5. Water flows at 0.05 m^3/s through a Y connector shown & splits into two streams having equal velocity. The pressure at point 1 is $2 * 10^5\ Pa$. Find the x & y components of force required to hold the Y connector in place. Neglect the weight of the liquid & connector. The connector is connected to pipes at both inlets & both the exits.

$(A_1 = 0.01m^2, A_2 = 0.002m^2, A_3 = 0.003m^2)$



Solution:

Flow Rate: Q =0.05 m^3/s, $V_1 =$

As both the streams divided equally $\Longrightarrow |\vec{V_2}| = |\vec{V_3}| = V$

To find $\vec{V_2}$&$\vec{V_3}$we first use the mass conservation equation.

$$\oint \rho\vec{V}.d\vec{A} = \qquad + \qquad + \qquad = 0$$

$$V =$$

$$\vec{V_2} = \qquad \hat{\imath} + \qquad \hat{\jmath}$$

$$\vec{V_3} = \qquad \hat{\imath} + \qquad \hat{\jmath}$$

<u>Momentum Equation in x direction:</u>

$$\sum F_x = \frac{\partial}{\partial t}\left(\iiint V_x\, \rho\ d\forall\right) + \oiint V_x\rho\vec{V}.d\vec{A}$$

For steady flow,

$$\frac{\partial}{\partial t}\left(\iiint V_x\, \rho\ d\forall\right) =$$

$$\sum F_x =$$

$$\oiint \mathrm{V_x}\rho\vec{V}.d\vec{A} \quad = \qquad\qquad\qquad + \qquad\qquad\qquad +$$

Equate the above equations to get R_x

$$R_x =$$

<u>Momentum Equation in y direction:</u>

$$\sum F_y = \frac{\partial}{\partial t}\left(\iiint \mathrm{V_y}\ \rho\ \ d\forall\right) \ + \ \oiint \mathrm{V_y}\rho\vec{V}.d\vec{A}$$

For steady flow,

$$\frac{\partial}{\partial t}\left(\iiint \mathrm{V_y}\ \rho\ \ d\forall\right) =$$

$$\sum F_y =$$

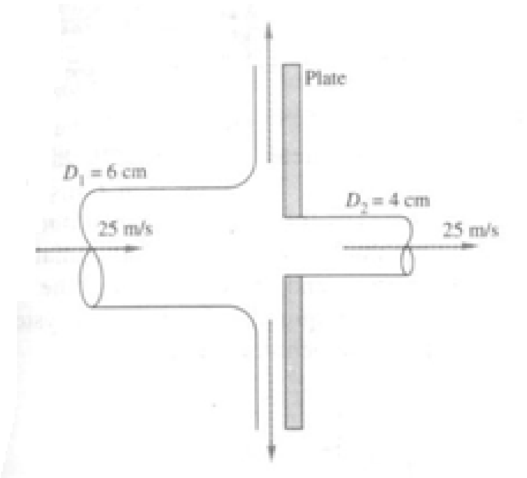
$$\oiint \mathrm{V_y}\rho\vec{V}.d\vec{A} \quad = \qquad\qquad\qquad +$$

Equate the above equations to get R_y

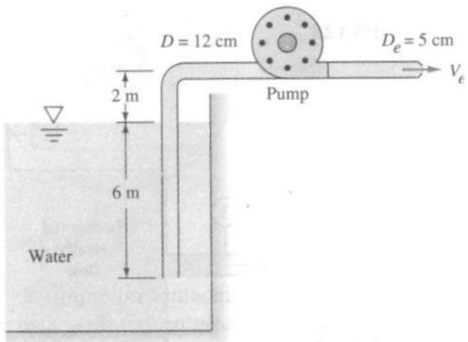
$$R_y =$$

Integral Relations

1. The 6 cm diameter water jet as shown in the figure strikes a plate containing a hole of 4 cm diameter. Part of the jet passes through the hole and part is deflected. Determine the horizontal force required to hold the plate.

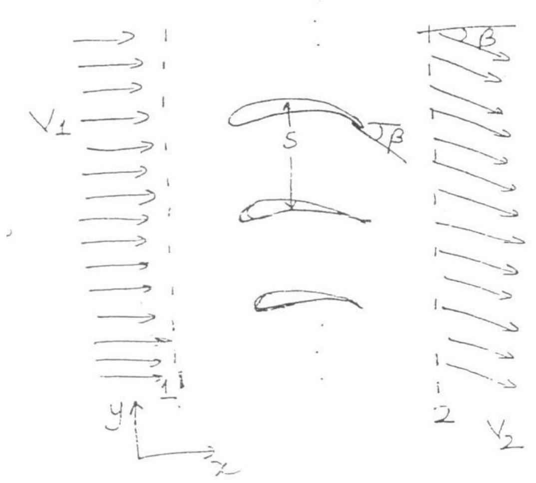


2. When the pump draws 220 m^3/h of water from the reservoir, the total friction head loss is 5m. The flow discharges through a nozzle to the atmosphere. Estimate the pump power in kW delivered to the water.

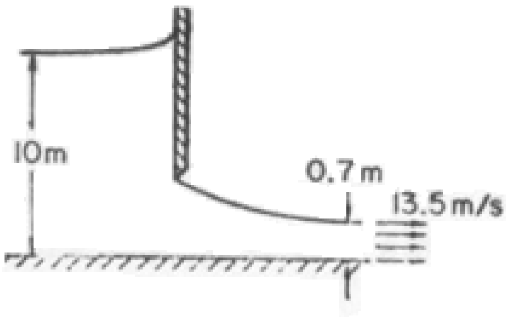


3. Consider flow of water past an infinite set of blades which turn the flow through an angle β as shown in the figure. The blade spacing is S and the width of the blade (in the direction normal to the paper) is W. The effect of gravity can be neglected. In terms of S, W, V_1,β and the density ρ derive expressions for

a) The exit Speed
b) The pressure drop ($p_1 - p_2$)
c) Forces F_x& F_y that act on a single blade

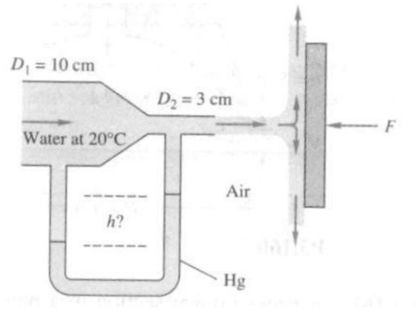


4. The Sluice gate on a dam is raised to allow the flow of water as shown. Estimate the force acting on the gate per unit width. Assume 1-D flow downstream of the gate and the pressure distribution to be hydrostatic far upstream and downstream.



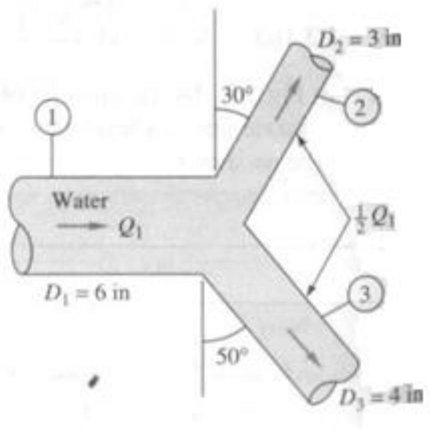
5. Water flows through a circular nozzle, exits into the air as a jet and strikes on a plate as shown. The force required to hold the plate steady is 70 N. Assuming steady, frictionless and 1-D flow estimate

a) The velocities of sections (1) and (2)
b) The mercury manometer reading h.



6. The horizontal wye fitting splits the water flow rate equally. If $Q_1 = 5\ ft^3/s$ and $p_1 = 5\ lbf/in^2$ and losses are neglected, estimate

a) p_2
b) p_3
c) The vector force required to keep the wye in place.



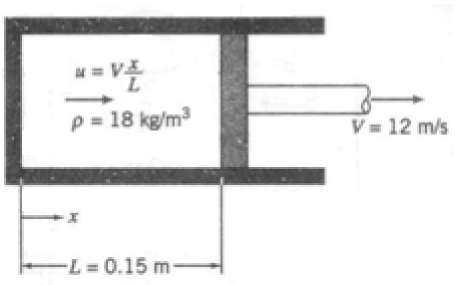
EXERXISE 6

DIFFERENTIAL RELATIONS FOR FLUID FLOW

Differential Relations

1.A gas-filled pneumatic strut in an automobile suspension system behaves like a piston-cylinder apparatus. At one instant when the piston is L = 0.15 m away from the closed end of the cylinder, the gas density is uniform at ρ = 18 kg/m^3 and the piston begins to move away from the closed end at V = 12 m/s. The gas motion is 1-D and proportional to distance from the closed end; it varies linearly from zero at the end to u = V at the piston.

a. Evaluate the rate of change of gas density at this instant.
b. Obtain an expression for the average density as a function of time.



Solution: From law of conservation of mass:

$$\nabla.\rho\vec{V} + \frac{\partial\rho}{\partial t} = 0$$

For 1-D flow CE: $\qquad + \qquad = 0$

Velocity: $\qquad u = \qquad , \quad \frac{\partial u}{\partial x} =$

Gas density is assumed uniform in the volume, $\frac{\partial\rho}{\partial x} =$

$$\frac{\partial\rho}{\partial t} = \frac{d\rho}{dt} = \qquad + \qquad =$$

Put values to get rate of change of density at t=0:

$$\frac{d\rho}{dt}_{t=0} =$$

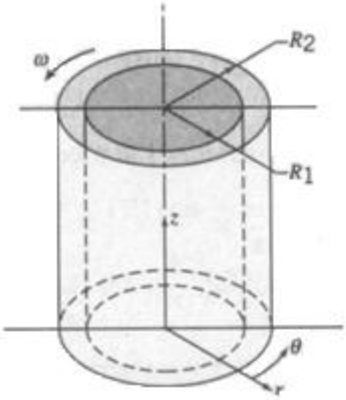
Now, $\qquad L(t) = \qquad +$

Substitute this in rate of change of density equation,

Separate variables and integrate to get $\rho(t)$

2. A viscous liquid fills the annular gap between vertical concentric cylinders. The inner cylinder is stationary, and the outer one rotates at constant speed. The flow is laminar.

a. Simplify the continuity, Navier-Stokes, and tangential shear stress equations to model this flow field.
b. Velocity profile in the annular gap
c. Shear stress distribution in the annular gap.
d. Shear stress at the surface of the inner cylinder.



Solution: Assume

a. Steady and incompressible flow; $\partial/\partial t =$, $\rho =$
b. Axisymmetric flow; $\partial/\partial\theta =$
c. No flow or variation of properties in z-direction; $V_z =$; $\frac{\partial}{\partial z} =$

$\Rightarrow$ $V_r, V_\theta(\quad)$; $p(\quad,\quad)$

C.E. in polar coordinates: + + = 0

$\Rightarrow$ = 0 $(Simplify)$

(From here deduce that $V_r = 0$)

<u>Navier-Stokes Equation</u>

r component: (Simplify)

θ component: (Simplify)

z component: (Simplify)

Shear stress in annular gap:

$$\tau_{\_\_} = \mu(\qquad + \qquad)$$

From the θ component of NS equations, solve for $V_\theta(r)$

$\Longrightarrow$

$\Longrightarrow$

Boundary conditions on V_θ

i. At $r = R_2, \quad V_\theta =$
ii. At $r = R_1, \quad V_\theta =$

Evaluate constants to get V_θ

$\Longrightarrow$

$\Longrightarrow$

Substitute V_θ in shear stress relation to get the distribution:

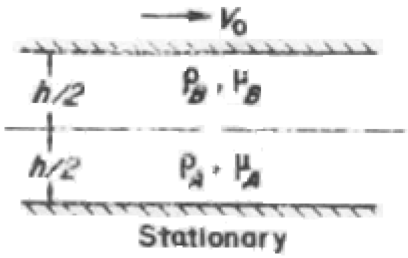
$\Longrightarrow$

$\Longrightarrow$

At the surface of the inner cylinder, $r = R_1$, so

$$\tau_{surface} =$$

3. Consider the steady, laminar, incompressible flow between two large parallel plates as shown. The upper plate moves with velocity V_0 to the right and the lower plate is stationary. The pressure gradient in the flow direction is zero. The lower half of the region between the plates is filled with fluid A, and the upper half is filled with fluid B. Assume the fully developed laminar flow to obtain the velocity profile.



Solution: Assumptions

1. Laminar flow i.e. 2D flow $\Longrightarrow$
2. Steady flow $\Longrightarrow$
3. Incompressible flow $\Longrightarrow$
4. Fully developed flow$\Longrightarrow$

Apply the continuity equation (C.E.) for both the fluids

C.E. For A: $\Longrightarrow$ V_{ya} =

C.E. For B: $\Longrightarrow$ V_{yb} =

Apply boundary conditions to get V_{ya} = &V_{ya} =

<u>Navier Stokes Equations X direction:</u>

$$\rho\left(\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}+V_z\frac{\partial V_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2}\right)$$

<u>For fluid A:</u> (Strike out the zero terms)

Integrate to get the expression for V_{xa}

$\Longrightarrow \quad V_{xa}$ =

<u>For fluid B:</u>

Integrate to get the expression for V_{xb}

$\Longrightarrow \quad V_{xb}$ =

From the boundary conditions simplify the above expressions

1. y = 0 ; V_{xa} =
2. y = h ; V_{xb} =

V_{xa} = &V_{xb} =

Also at y = h/2 we have

1. $V_{xa} = V_{xb}$
2. $\tau_{surface\ at\ A} = \tau_{surface\ at\ B}$

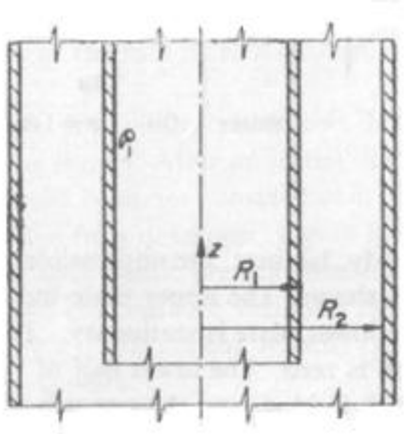
Equation 1 $\Longrightarrow$

Equation 2 $\Longrightarrow$

Solve the above equations to get

V_{xa} = V_{xb} =

4.An incompressible fluid flows radially through a long and thin inner porous cylinder of radiusR_1, across to the outer, concentric porous cylinder of radiusR_2. The non gravitational pressure at R_1 is P_1 and the radial velocity at the inner cylinder isV_1 . For steady, laminar flow between the two cylinders, simplify the continuity equation and the three components of the Navier Stokes equation and then solve them to obtain p in terms of P_1andV_1.



Solution: Assumptions

1. Radial flow $\Longrightarrow$
2. Steady flow $\Longrightarrow$
3. Incompressible flow $\Longrightarrow$
4. At r = R_1 $\Longrightarrow$ Pressure P =
   Velocity V_r =

Apply Continuity Equation for an incompressible flow

C.E.$\Longrightarrow$ V_r $\Longrightarrow$

Eliminate the constant by using velocity at r = R_1.

$$\frac{\partial V_r}{\partial r} =$$

<u>Navier Stokes Equations r direction:</u>
(Strike out the zero terms)

$$\rho\left(\frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} + V_z\frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r}\right)$$
$$= -\frac{\partial p}{\partial r} + \rho g_r + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} + \frac{\partial^2 V_r}{\partial z^2} - \frac{V_r}{r^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right)$$

Simplify to get Equation 1

$$\frac{\partial p}{\partial r} =$$

<u>Navier Stokes Equations z direction:</u>
(Strike out the zero terms)

$$\rho\left(\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_r}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2}\right)$$

Simplify to get a function of pressure in r and z.

$\Longrightarrow$ P(r, z) =
Equation 2

$$\Longrightarrow \quad \frac{\partial p}{\partial r} =$$

Equate equation 1 & 2 to get the final function of p in r &z

$\Longrightarrow$p(r, z) =

Eliminate the constant using the pressure at R_1 $\Longrightarrow$ P =

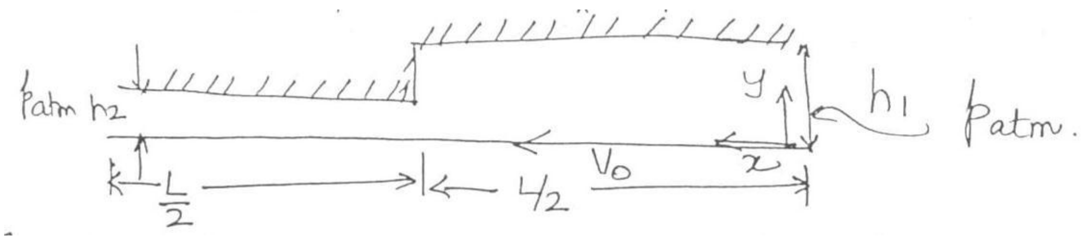
$\Longrightarrow$P(r) =

Differential Relations

1. For an incompressible flow in the $r\theta$ plane, the r component of velocity is given as $V_r = -\Lambda \cos\theta / r^2$. Determine a possible θ component of velocity. How many possible θ components are there?

2. The y component of velocity in a steady, incompressible flow field in the xy plane is $v = -Bxy^3$, where B = 0.2 m^{-3}s^{-1}, and x and y are measured in meters. Find the simplest x component of velocity for this flow field. Find the equation of the streamlines for this flow. Plot the streamlines through (1,4) and (2,4).

3. Idealized flow in a 2D stepped channel
Consider flow of viscous incompressible fluid in a stepped channel, with L >> h_1. The bed of the channel moves with a velocity V_o. Upper Wall is fixed. The aim of the problem is to find the total vertical load which can be carried by the fluid. Neglect gravity and make suitable simplifying assumptions.

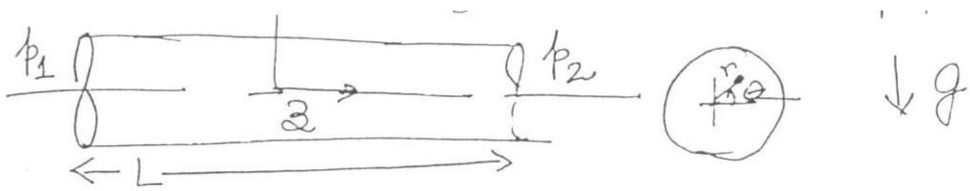


4. Flow in a horizontal circular pipe

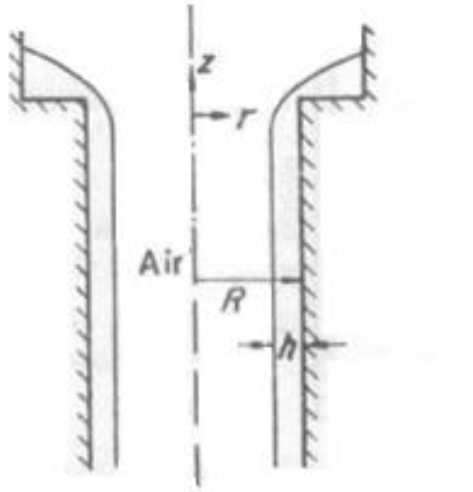
Consider flow of viscous incompressible fluid through a long pipe of Radius R (L>>R) caused by a pressure difference between the two ends of the pipe. Assume axisymmetric velocity field with $V_\theta = 0$. Find

a) The velocity profile of the fluid
b) The volumetric flow rate of the fluid

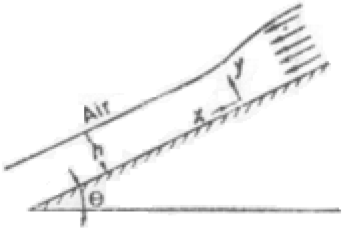
Assume fully developed flow.



5. A wetted wall column is used to measure mass transfer coefficients. A liquid of density ρ and viscosity μ flows down of a tube of radius R shown. After an initial region, the flow becomes fully developed and the thickness of fluid layer is constant at h. Simplify the Navier-Stokes equation and continuity equations to obtain $V_z(r)$ for laminar conditions.



6. A liquid of density ρ and viscosity μ flows laminarly down a wide flat inclined plate as shown. After an initial developing flow region, the depth of the liquid becomes constant at h. Show that the pressure within the fluid in the fully developed region is a function of y alone, and is given by the hydrostatic pressure distribution with g replaced by g$\cos\theta$. Obtain the velocity profile and the flow rate per unit width of the plate.



EXERCISE 7

DIMENSIONAL ANALYSIS AND SIMILARITY

Dimensional Analysis

1.A model propeller 600 mm in diameter is tested in a wind tunnel. Air approaches the propeller at 45 m/s when it rotates at 2000 rpm. The thrust and torque measured under these conditions are 110 N and 10 N-m respectively. It is known that F_t and Tdepends on diameter, D, propeller speed, ω, air speed, V, viscosity, μ and density ρ. A prototype 10 times as large as the model is to be built. At a dynamically similar operating point, the approach air speed is to be 120 m/s. Calculate the speed, thrust, and torque of the prototype propeller under these conditions, neglecting the effect of viscosity but including density.

Solution:

$$F_t = f(D, \omega, V, \mu, \rho)$$

Primary dimensions:

Express each parameter in terms of primary dimensions,

$[F_t] =$; $[D] =$; $[\omega] =$; $[V] =$; $[\mu] =$; $[\rho] =$

Repeating variables:

(Do not choose F_t and μ)

Form Π groups:

$\Pi_1 =$; $\Pi_2 =$; $\Pi_3 =$

Express the Π group containing F_t in terms of other Π groups:

$$\frac{F_t}{(\quad)} = g(\Pi_{\_\_}, \Pi_{\_\_})$$

Neglect the effect of viscosity, therefore,

$$\frac{F_t}{(\quad)} = g(\Pi_x)$$

For dynamic similarity,

$$\Pi_x|_{model} = \Pi_x|_{prototype}$$

$\Longrightarrow$

(From here get the value of$\omega_{prototype}$)

$$\omega_{prototype} =$$

When $\Pi_x|_{model} = \Pi_x|_{prototype}$ then, $g(\Pi_x|_{model}) = g(\Pi_x|_{prototype})$

$\Longrightarrow$

Thus, $F_{t,proto} =$

Similarly solve for torque of the prototype propeller,

2. Your favorite professor likes mountain climbing, so there is always a possibility that the professor may fall into a crevasse in some glacier. If that happened today, and the professor was trapped in a slowly moving glacier, you are curious to know whether the professor would reappear at the downstream drop off the glacier during this academic year. Assuming ice is a Newtonian fluid with the density of glycerine but a million times as viscous, you decide to build a glycerin model and use dimensional analysis and similarity to estimate when the professor would reappear. Assume the real glacier is 15m deep and is on a slope that falls 1.5 m in a horizontal distance of 1850m. Develop the dimensionless parameters and conditions expected to govern dynamic similarity in this problem. If the model professor reappears in the laboratory after 9.6 hours, when should you return to the end of the real glacier to provide help to your favorite professor?

Solution:

$$\vec{V} = f(\rho, g, \mu, D, H, L)$$

Primary dimensions:

Express each parameter in terms of primary dimensions, n=7

$[\vec{V}] =$; $[\rho] =$; $[g] =$; $[\mu] =$; $[D] =$; $[H] =$; $[L] =$

Repeating variables:

(Do not choose $\vec{V}$ and μ)

No. of π groups: n-m = 4

$\pi_1 = \rho^a g^b D^c V = M^0 L^0 T^0$; Solve for a, b and c to get the π group

$$\pi_1 =$$

$\pi_2 = \rho^a g^b D^c \mu = M^0 L^0 T^0$; Solve for a, b and c to get the π group

$\pi_2 =$

$\pi_3 =$ (By inspection)

$\pi_4 =$ (By inspection)

$$\pi_1 = f(\pi_2, \pi_3, \pi_4)$$

For dynamic similarity

$$\Pi_2|_{model} = \Pi_2|_{prototype}$$

Solve the above equation to get

$$\Longrightarrow \frac{D_m}{D_p} = \qquad\qquad (S.G\ ice = 0.92, S.G.gly = 1.26)$$

Also, $f(\Pi_2|_{model}, \Pi_3|_{model}, \Pi_4|_{model}) = f(\Pi_2|_{prototype}, \Pi_3|_{prototype}, \Pi_4|_{prototype})$

$$\Longrightarrow \frac{V_m}{V_p} =$$

$$T = \frac{L}{V} \Longrightarrow \frac{T_p}{T_m} = \frac{L_p}{L_m}\frac{V_m}{V_p}$$

$$T_m = 9.6\ hrs \quad \Longrightarrow \quad T_p =$$

Dimensional Analysis

1. When freewheeling, the angular velocity Ω of a windmill diameter D, the wind velocity V, the air density ρ, the windmill height H as compared to the atmospheric boundary layer height L, and the number of blades N:

$$\Omega = fcn\left(D, V, \rho, \frac{H}{L}, N\right)$$

Viscosity effects are negligible. Find appropriate pi groups for this problem and rewrite the function in dimensionless form.

2. A dam spillway is to be tested by using Froude scaling with a one-thirtieth-scale model. The model flow has an average velocity of 0.6m/s and a volume flow of 0.05m^3/s. What will be the velocity and flow of the prototype? If the measured force on a certain part of the model is 1.5 N, what will be the corresponding force on the prototype?

3. The power P generated by a certain windmill design depends on its diameter D, the air density ρ, the wind velocity V, the rotation rate Ω, and the number of blades n.

a) Write this relationship in dimensionless form. A model windmill, of diameter 50 cm, develops 2.7 kW at sea level when V = 40 m/s and when rotating at 4800 r/min.

b) What power will be developed by a geometrically and dynamically similar prototype, of diameter 5 m, in winds of 12 m/s at 2000 m standard altitude?

c) What is the appropriate rotation rate of the prototype?

4. A simply supported beam of diameter D, length L, and modulus of elasticity E is subjected to a fluid cross flow of velocity V, density ρ, and viscosity μ. Its center deflection δ is assumed to be a function of all these variables.

a) Rewrite this proposed function in dimensionless form.

b) Suppose it is known that δ is independent of μ, inversely proportional to E, and dependent only on ρV^2, not ρ and V separately. Simplify the dimensionless function accordingly.

5.A V-notch weir is a vertical plate with a notch angle ϕ cut at the top and placed in an open channel. The flow rate $\dot{Q}$ through the channel is a function of H, the elevation of the fluid upstream of the notch, acceleration due to gravity, g, the approach velocity of the fluid V_o and the notch angle ϕ.

a) Express this as a functional relationship of non-dimensional numbers (π groups).
b) Simplify the expression in a) if it is found that $\dot{Q} \propto \sqrt{g}\tan\phi$
c) A 1:10 similar model of the weir is used and it is found that in the model, the flow rate is $1m^3/s$. What would be the flow rate in the prototype? Both model and prototype are the same fluid. Assume conditions in b) are satisfied. The approach velocity $V_o\ model$ is half of that for the prototype.

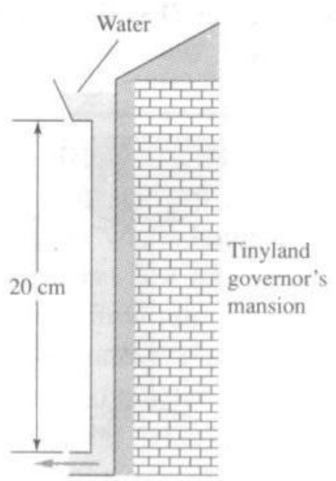
EXERXISE 8

VISCOUS FLOW IN DUCTS

Viscous Flow in Ducts

1. In tinyland, houses are less than a foot high! The rainfall is laminar! The drainpipe is only 2 mm in diameter.

a) When the gutter is full, what is the rate of draining?
b) The gutter is designed for a sudden rainstorm of up to 5mm per hour. For this condition what is the maximum roof area that can be drained successfully?
c) What is Re_d?



Solution: Computing equation:

$$\frac{p_1}{\rho g}+\frac{{V_1}^2}{2g}+z_1=\left(\frac{p_2}{\rho g}+\frac{{V_2}^2}{2g}+z_2\right)+h_f$$

$$p_1-p_2= \quad ; \quad V_1= \quad ; \quad V_2= \quad ; \quad z_1-z_2=$$

$$h_f=$$

For laminar flow,

$$h_f=$$

From here, obtain a quadratic equation in *V.*

Solve for V,

$$V_2=$$

Rate of draining =

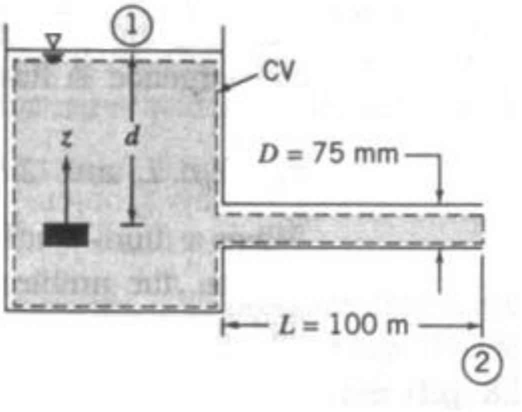
For rainstorm of 5 mm/hour, area of rooftop,

$$A =$$

$$Re_d = \qquad\qquad =$$

Check for laminar flow!!

2. A 100 m length of smooth horizontal pipe is attached to a large reservoir. What depth, d, must be maintained in the reservoir to produce a volume flow rate of 0.0084 m^3/s of water? The inside diameter of the smooth pipe is 75 mm. The inlet is square-edged and water discharges to the atmosphere.



Solution: Computing equation:

$$\frac{p_1}{\rho g} + \frac{{V_1}^2}{2g} + z_1 = \left(\frac{p_2}{\rho g} + \frac{{V_2}^2}{2g} + z_2\right) + h_f + \sum K \frac{V^2}{2g}$$

$$p_1 - p_2 = \quad ; V_1 = \quad ; \quad V_2 = \quad ; \ z_1 - z_2 = \quad ; K_{entrance} =$$

Thus, d =

Assuming water at 20°C, $\rho =$, $\mu =$

$$Re_d = \qquad =$$

For smooth pipes, get f from table,

$$f =$$

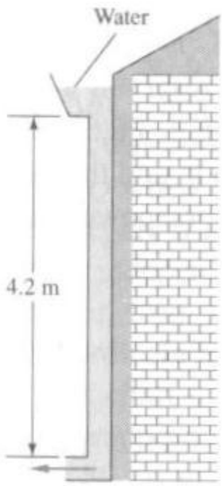
Substitute this value to get d,

$$d =$$

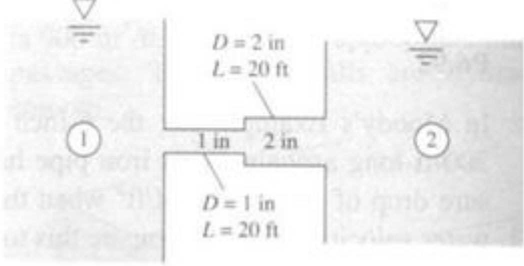
Viscous Flow in Ducts

1. The gutter and smooth drainpipe remove rainwater from the roof if a building. The smooth drainpipe is 7 cm in diameter.

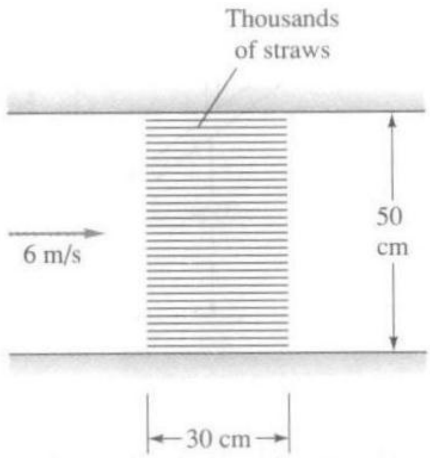
a) When the gutter is full, estimate the rate of draining.
b) The gutter is designed for a sudden rainstorm of up to 5 inches per hour. For this condition, what is the maximum roof area that can be drained successfully?



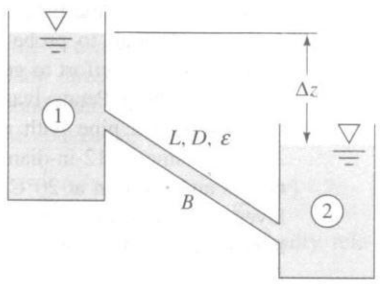
2. The reservoirs are connected by cast iron pipes joined abruptly, with sharp-edged entrances and exit. Including minor losses, estimate the flow of water if the surface of reservoir 1 is 15 m higher than that of reservoir 2.



3. For straightening and smoothing airflow in a 50 cm diameter duct, the duct is packed with a honeycomb of thin straws of length 30 cm and diameter 4 mm. The inlet flow is air at 110 kPa and 20°C, moving at an average velocity of 6 m/s. Estimate the pressure drop across the honeycomb.



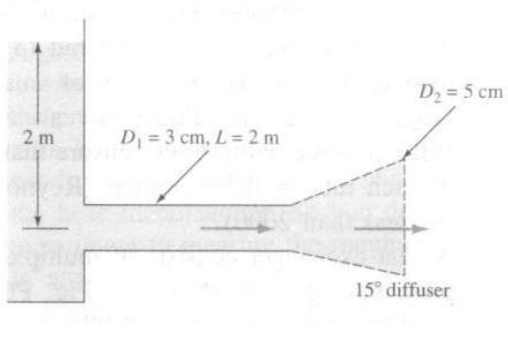
4. The reservoirs in Fig contain water at 20°C. If the pipe is smooth with L=4500 m and d = 4 cm, what will the flow rate in m^3/h be for Δz = 100m?



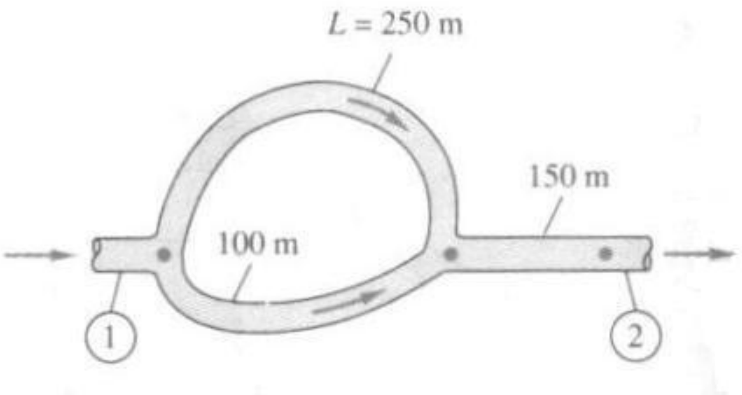
5. Fig shows a smooth inlet pipe, with or without a 15° conical diffuser expanding to a 5-cm diameter exit. The pipe entrance is sharp-edged. Calculate the flow rate

a) without the diffuser

b) with the diffuser



6: For the series-parallel system,all pipes are 8 cm diameter asphalted cast iron. If the total pressure drop p_1-p_2 = 750 kPa, find the resulting flow rate Q m^3/h for water at 20°C. Neglect minor losses.



EXERCISE 9

POTENTIAL FLOW

Potential Flow

1. An incompressible flow field is characterized by stream function $\psi = 3Ax^2y - Ay^3$ where $A = 1m^{-1}s^{-1}$. Show that the flow is irrotational and derive the expression for the velocity potential of the flow.

Solution: For 2 D irrotational flow,

$$\nabla^2\psi = 0$$

Solve the above equation to prove that the flow is irrotational

$$\Rightarrow \nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} =$$

To find the potential function we first find the velocity. Say $\vec{V} = u\,\hat{\imath} + v\,\hat{\jmath}$

$$\Rightarrow \quad u = \frac{\partial\psi}{\partial y} =$$

$$\Rightarrow \quad v = -\frac{\partial\psi}{\partial x} =$$

Now calculate potential function from both the components and compare them

$$\Rightarrow \phi = -\int u\,dx + g(y) =$$

$$\Rightarrow \phi = -\int v\,dy + f(x) =$$

Compare both the values of ϕ to get $g(y)$ and $f(x)$

$$\Rightarrow \quad g(y) = \qquad \& \qquad f(x) =$$

$$\Rightarrow \phi =$$

2. A source and a sink with strengths of equal magnitude, q = $3\pi\ m^2/s$, are placed on the x axis at $x = -a$ & $x = a$, respectively. A uniform flow, with speed $U = 20\ m/s$, in the positive x direction, is added to obtain the flow past a Rankine body. Obtain the stream function, velocity potential, and velocity field for the combined flow. Find the value of $\psi = constant$ on the stagnation streamline. Locate the stagnation points if a = 0.3 m.

Solution: Stream Function

$$\psi = \psi_{source} + \psi_{sink} + \psi_{uniform\ flow}$$

$$\Rightarrow \psi =$$

Potential Function:

$$\phi = \phi_{source} + \phi_{sink} + \phi_{uniform\ flow}$$

$$\Rightarrow \phi =$$

Velocity Function:

$$u = u_{source} + u_{sink} + u_{uniform\ flow}$$

$$\Rightarrow u =$$

$$v = v_{source} + v_{sink} + v_{uniform\ flow}$$

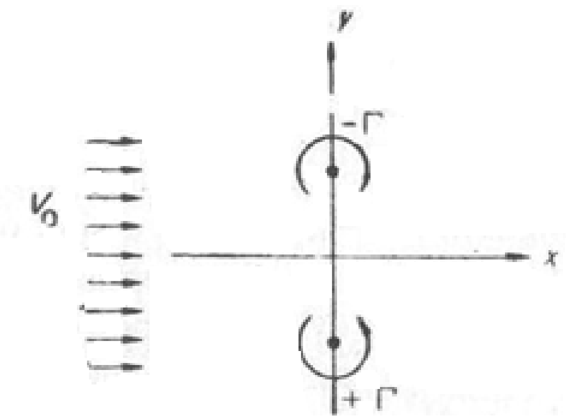
$$\Rightarrow v =$$

$$\Rightarrow \vec{V} = \hat{\imath} + \hat{\jmath}$$

At stagnation point: $\vec{V} = 0$, Find the stagnation point (a = 0.3 m)

Potential Flow

1. Consider the superposition of a free stream and a vortex pair at (0,-a) and (0, +a) of circulations I and –I. Obtain the stream function of this combination and determine the location of stagnation points along the x axis.



2. 2D flow represented by the velocity field $\vec{V} = (Ax - By)\hat{\imath} - (Bx + Ay)\hat{\jmath}$ where $A = 1s^{-2}$& $B = 2\, s^{-1}$, t is in s and coordinates are in metres.

Find:

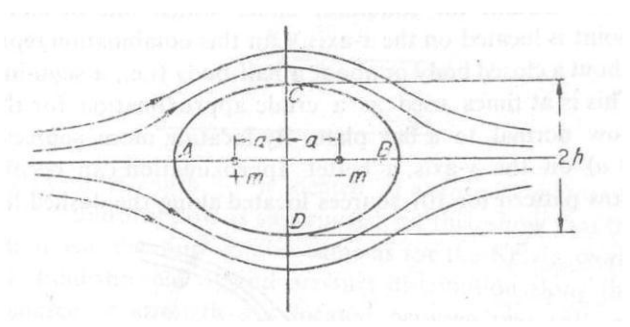
(a) Is this a possible incompressible flow?
(b) Is the flow steady or unsteady?
(c) Show that the flow is irrotational.
(d) Derive an expression for the velocity potential.

3. A flow is described by the stream function $\psi = 4xy$. Locate the point at which the velocity vector has a magnitude of 7 units and makes an angle of 150° with X-axis.

4. For a 2D flow the velocity function is given by the expression $\phi = x^2 - y^2$

(a) Determine velocity components in x and y directions.
(b) Show that the velocity components satisfy the conditions of flow continuity and irrotationality.
(c) Also, determine the stream function.

5. Consider the superposition of a free stream $\vec{V} = V_0\hat{\imath}$ a source of strength m at (-a,0) and a sink of strength m at (a,0). Write the stream function for this combination. Determine the location of stagnation points A and B.



6. Flow field formed b superposition of a uniform flow in +*x*direction (V = 10m/s) and a counterclockwise vortex, with strength $K = 4\pi\ m^2/s$, located at the origin. Find:

(a) ψ, ϕ &$\vec{V}$for the flow field.
(b) Stagnation point(s).