Q1 In a 2-D unsteady flow, the velocity field is given by **V** = (Bt – Ax) **i** + Ay **j** and the temperature filed is given by T = Ctx + Dy, where A, B, C and D are constants (in appropriate units).

(i) Does this represent a possible incompressible flow? If yes, then find the associated stream function ψ. If no, give reasons why the flow is not incompressible. (2 marks)

(ii) Does this represent a possible irrotational flow? If yes, then find the associated velocity potential ϕ. If no, give reasons why the flow is not irrotational. (2 marks)

(iii) Find the acceleration of a fluid particle located at (1, 1) at time t = 1. (4 marks)

(iv) Find the rate of change of temperature of a fluid particle located at (1, 1) at t = 1. (3 marks).

(v) Assume that A = 0 and gravity can be neglected. The gauge pressure at (0,0) is 0. Find the pressure at (1,1). Give justification for the equation used. (4 marks)

$u = Bt - Ax \quad v = Ay \quad T = Ctx + Dy$

a) $\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -A + A = 0 \Rightarrow$ incomp. $u = \frac{\partial \psi}{\partial y} \Rightarrow \psi = (Bt - Ax)y + f(x)$

$-\frac{\partial \psi}{\partial x} = Ay - f'(x) = v_y = Ay \Rightarrow f'(x) = 0 \Rightarrow f(x) = C \rightarrow \psi = (Bt - Ax)y + C$

b) $\frac{\partial u}{\partial y} = 0 \quad \frac{\partial v}{\partial x} = 0 \rightarrow \nabla \times \vec{V} = 0; \frac{\partial \phi}{\partial x} = (Bt - Ax) \Rightarrow \phi = (Bt x - \frac{Ax^2}{2}) + g(y)$

$\frac{\partial \phi}{\partial y} = g'(y) = Ay \Rightarrow g = \frac{Ay^2}{2}$

$\phi = Btx - \frac{Ax^2}{2} + \frac{Ay^2}{2} + c$

c) $a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = B + (Bt - Ax)(-A) + Ay(0) = B - ABt + A^2x$

$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 0 + (Bt - Ax)(0) + AyA = A^2y$

$\vec{a} = (B - AB + A^2)\hat{i} + A^2\hat{j}$ at (1,1) at $t=1$

d) $\frac{dT}{dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = Cx + (Bt - Ax)Ct + Ay(D)$

$= C + BC - AC + AD$ at (1,1) at $t=1$

e) since flow irrotational & incompressible, B.E. can be used (in unsteady form) $\frac{\partial \phi}{\partial t} + \frac{p}{\rho} + \frac{V^2}{2} = \text{const.}$

$\frac{\partial \phi}{\partial t} = Bx \quad \Rightarrow Bx + \frac{p_0}{\rho} + \left(\frac{B^2t^2}{2}\right)\Big|_0 = Bx + \frac{p_0}{\rho} + \left(\frac{B^2t^2}{2}\right)\Big|_1$

$\Rightarrow B(0) + 0 + \frac{B^2t^2}{2} = B(1) + \frac{p_1}{\rho} + \frac{B^2t^2}{2}$

$p_g(1,1) = -\rho B$ or $p(1,1) = p_{atm} - \rho B$

Q2 Consider the potential flow from the superposition of a source located at (x,y) = (-a,0) adjacent to a wall given by the y axis (x=0) and a uniform stream with velocity = Vj.

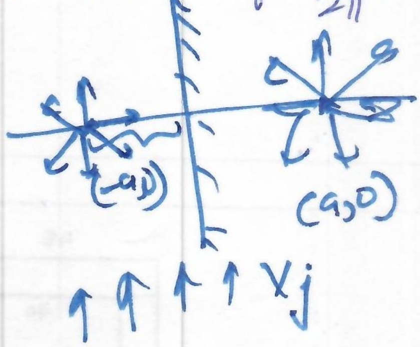
a) Find the stream function, velocity potential and the velocity field for this flow.

b) Also find the pressure distribution on the y axis.

Use either q or $\frac{q}{2\pi}$ or m. or m

q

source at origin: $\psi = \frac{q\theta}{2\pi}$, $\phi = \frac{q}{2\pi} \ln r$



for uniform flow $V\hat{j}$ —

$\phi = Vy$, $\psi = -Vx$

$$\psi = -Vx + \frac{q}{2\pi}\tan^{-1}\frac{y}{x+a} + \frac{q}{2\pi}\tan^{-1}\frac{y}{x-a}$$

$$\phi = Vy + \frac{q}{4\pi}\ln\left[(x+a)^2+y^2\right] + \frac{q}{4\pi}\ln\left[(x-a)^2+y^2\right]$$

$$V_x = \frac{\partial\phi}{\partial x} = \frac{q}{4\pi}\frac{2(x+a)}{\left[(x+a)^2+y^2\right]} + q\frac{2(x-a)}{4\pi\left[(x-a)^2+y^2\right]}$$

$$= \frac{q}{2\pi}\left[\frac{(x+a)}{(x+a)^2+y^2} + \frac{(x-a)}{(x-a)^2+y^2}\right]$$

$$V_y = \frac{\partial\phi}{\partial y} = V + \frac{q}{2\pi}\left[\frac{2y}{(x+a)^2+y^2} + \frac{2y}{(x-a)^2+y^2}\right]$$

On y axis: $(x=0)$ $V_x = \frac{q}{2\pi}\left(\frac{a}{a^2+y^2} - \frac{a}{a^2+y^2}\right) = 0$

$$V_y = V + \frac{q}{2\pi}\left[\frac{2y}{a^2+y^2} + \frac{2y}{a^2+y^2}\right]$$

$$= V + \frac{2yq}{2\pi}\cdot\frac{2}{a^2+y^2} = \frac{4q}{2\pi}\frac{y}{a^2+y^2} + V$$

$$p_\infty + \frac{\rho V^2}{2} = p(y) + \frac{\rho}{2}\left[V + \frac{4q}{2\pi}\frac{y}{a^2+y^2}\right]^2$$

$$p(y) = p_\infty + \frac{\rho V^2}{2} - \frac{\rho}{2}\left(V + \frac{4q}{2\pi}\frac{y}{a^2+y^2}\right)^2$$