Ans 1

a) B.C. $y=0 \quad V_x=0, V_y=0$

$y=b \quad \mu_o \frac{dV_x}{dy}\Big|_o = \mu_\omega \frac{dV_x}{dy}\Big|_\omega$

$\frac{dV_x}{dy}\Big|_o = \frac{\mu_\omega}{\mu_o} \frac{dV_x}{dy}\Big|_\omega \approx 0$

At $x = x, y=b, \; p_g = \rho_\omega g (d-x)$

b) $\frac{\partial \vec{V}}{\partial x}=0, \frac{\partial \vec{V}}{\partial z}=0 \quad \frac{\partial \vec{V}}{\partial t}=0 \quad \vec{V}=\vec{V}(y)$

C.E. $\cancel{\frac{\partial V_x}{\partial x}} + \frac{\partial V_y}{\partial y} = 0 \quad \Rightarrow V_y = \text{const.}$

B.C. $\Rightarrow V_y = 0$

N.S. y dir. $0 = -\frac{\partial p}{\partial y} + 0 + 0 \Rightarrow \frac{\partial p}{\partial y} = 0$

$p = f(x)$ Use B.C. $\rho_\omega g(d-x) = f(x)$

$\frac{dp}{dx} = f'(x) = -\rho_\omega g$

N.S. x: $0 = -\frac{\partial p}{\partial x} - \rho_o g + \mu_o \frac{d^2 V_x}{dy^2}$

$\frac{d^2 V_x}{dy^2} = \frac{1}{\mu_o}(-\rho_\omega g + \rho_o g) = \frac{g}{\mu_o}(\rho_o - \rho_\omega)$

$V_x = \frac{g}{\mu_o}(\rho_o - \rho_\omega)\frac{y^2}{2} + C_1 y + C_2$

B.C. $\Rightarrow V_{xb} = \frac{g}{\mu_o}(\rho_\omega - \rho_o)\left(by - \frac{y^2}{2}\right)$

$Q = \int_0^b V_x \, dy = \frac{b^3}{3}\frac{g}{\mu_o}(\rho_\omega - \rho_o)$

~~Ans 3~~ **Ans. 2**

a) $Z=0$, $V_r=0$, $V_\theta=0\ \forall r$ [redacted]
$Z=h$, $V_r=0$, $V_\theta=\omega r$ [redacted]
$r=R_1$, $p=p_0$, $r=R_2$, $p=p_{atm}$ [redacted]

$V_z=0$; $\partial()/\partial\theta=0$, $\frac{\partial()}{\partial t}=0$, $g=0$

LHS of NS = 0 (inertia terms)

$V_r, V_\theta = f(r,z)$, $p(r,z)$

b) C.E. $\frac{1}{r}\frac{\partial}{\partial r}(rV_r)=0$

$\frac{\partial}{\partial r}(rV_r)=0 \Rightarrow rV_r=f(z)$

$V_r=\frac{f(z)}{r}$ [redacted]

c) r: $0=-\frac{\partial p}{\partial r}+\mu\frac{\partial^2 V_r}{\partial z^2}$ [redacted]

θ: $0=\mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right)+\frac{\partial^2 V_\theta}{\partial z^2}\right\}$ [redacted]

z: $0=-\frac{\partial p}{\partial z} \Rightarrow p=p(r)$ [redacted]

d) $\frac{\partial p}{\partial r}=\frac{\mu f''(z)}{r}$

$\Rightarrow \underbrace{r\frac{dp}{dr}}_{f(r)}=\underbrace{\mu f''(z)}_{f(z)}=$ const. say K_1

$p=K\ln r+C_1$ [redacted]

$\frac{dp}{dr}=\frac{K}{r}$

BC $r=R_1$, $p=p_0$
$r=R_2$, $p=p_{atm}$ [redacted]

$p_0=K\ln R_1+C_1$; $p_{atm}=K\ln R_2+C_1$

$p_0-p_{atm}=K\ln\frac{R_1}{R_2} \Rightarrow K=\frac{p_0-p_{atm}}{\ln R_1/R_2}$

$C_1=p_0-K\ln R_1$

e) $f''(z)=\frac{K}{\mu} \Rightarrow f'(z)=\frac{Kz}{\mu}+C_2$

$\cancel{f(z)=\frac{Kz^2}{2\mu}+C_2 z+C_3}$

$z=0$ $V_r=0 \Rightarrow f(0)=0$; $C_3=0$

$z=h$ $V_r=0 \Rightarrow f(h)=\frac{Kh^2}{2\mu}+C_2h=0$

$C_2=-\frac{Kh}{2\mu}$

$\cancel{V_z=\left(\frac{Kz^2}{2\mu}-\frac{Khz}{2\mu}\right)\frac{1}{r}=\frac{Kz}{2\mu r}(z-h)}$

f) Load $=\int_{R_1}^{R_2}(p-p_{atm})2\pi r\,dr$ [redacted]

$=\int_{R_1}^{R_2}K\left(\ln\frac{r}{R_2}\right)(2\pi r)\,dr$ $+p_0\pi R_1^2$

g) $\cancel{V_\theta=\frac{\omega r}{2}(1+z/h)}$

$\cancel{rV_\theta=\frac{\omega r^2}{2}(1+z/h);\ \frac{\partial}{\partial r}(rV_\theta)=\omega r\left(1+\frac{z}{h}\right)}$

$\cancel{\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)=\omega(1+z/h)}$

$\cancel{\partial/\partial r\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right)=0}$

$\cancel{\frac{\partial V_\theta}{\partial z}=\frac{\omega r}{2h},\ \frac{\partial^2 V_\theta}{\partial z^2}=0}$

$\Rightarrow$ N.S. Eqn for θ satisfied.

[redacted]