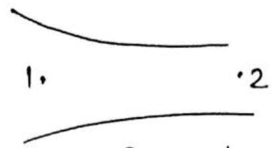
ASSIGNMENT 10
APL106

Sol 1 → $P_2 = 240$ kPa, $P_1 = P_0 = 406$ kPa

$T_1 = 95 + 273 = 368$K



1. Stagnant
2. exit

$\frac{P_2}{P_1} = 0.5911$

Since $P_1 = P_0 \Rightarrow Ma_2 = 0.9$ (from tables)

So flow not choked $\dot{m} = \rho_2 V_2 A_2$

$\rho_0 = \frac{P_0}{RT_0} = \frac{406 \times 10^3}{287 \times 368} = 3.8441$ kg/m^3

$\frac{\rho_2}{\rho_0} = 0.6870$ (table) $\Rightarrow \rho_2 = 2.6409$ kg/m^3

$V_2 = Ma_2\, a_2 = Ma \sqrt{KRT_2}$ $\frac{T_2}{T_0} = 0.8606$ (Table)

$T_2 = 316.7$ K

$V_2 = 0.9 \sqrt{1.4 \times 287 \times 316.7} = 321.04$ m/s

$\dot{m} = 2.6409 \times 321.04 \times 0.61 = 8.478$ kg/s

Sol 2 → $P_{exit} = 101 \times 10^3$ KPa

for sonic flow $\frac{P_{exit}}{P_0} = 0.5283$

$P_0 = 600$KPa
$T_0 = 600$K

$\frac{101 \times 10^3}{0.5283} = P_0$ (for sonic flow)

for $P_{tank} > 191.18$ KPa (flow in nozzle sonic)

Given $P_0 = 600$ KPa > 191.18 KPa, so flow choked and sonic condition at exit

$\dot{m} = \rho_* V_* A_*$

$\Rightarrow P_0 = 600$KPa $P_2 \Rightarrow$ obtained from $Ma = 1$

$\frac{P_e}{P_0} = 0.5283 \Rightarrow P_e = 0.5283 \times 600 = 316.98$ KPa

$\frac{T_e}{T_0} = 0.8333 \Rightarrow T_e = 600 \times 0.8333 = 500$K

$\rho_e = P_e/RT_e = \frac{316.98 \times 10^3}{287 \times 500} = 2.209$ kg/m^3

$V_e = (Ma)_e \sqrt{KRT_e} = 1 \sqrt{1.4 \times 287 \times 500} = 448.2$ m/s

$\dot{m} = 2.209 \times 448.2 \times 1.29 \times 10^{-3} = 1.277$ kg/m^3

Sol 3 → $P_1 = P_0 = (620 + 101)\ \text{KPa}$ (abs)

$P_2 = 101\ \text{kPa}$, $T_1 = 400\text{K} = T_0$

at this stage $P_2/P_0 = \frac{101}{721} = 0.41 < 0.5283$

So flow choked

$Ma_{\text{exit plane}} = 1 \Rightarrow \frac{P_{exit}}{P_0} = 0.5283$ or $P_e = 721 \times 0.5283$
$= 380.9\ \text{kPa}$

As flow take place P_{tank} reduces

$\frac{P_{exit}}{P_{tank}} = 0.5283 \Rightarrow P_{tank} = \frac{P_{exit}}{0.5283} = \frac{101\ \text{KPa}}{0.5283}$

Flow choked till P_{tank} reduces to 191.18 kPa

After this flow rate in nozzle will reduce and Ma_{exit} will reduce from 1 to zero (no flow).

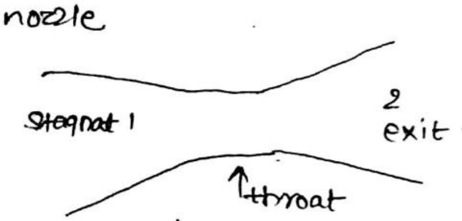
To find Temp in tank when no flow,

Gas in tank is following reversible adiabatic process

So $\frac{P_f}{P_0} = \left(\frac{T_f}{T_0}\right)^{k/k-1}$

$\Rightarrow T_f = T_0\left(\frac{P_f}{P_0}\right)^{k-1/k} = 400\left(\frac{101}{721}\right)^{0.286} = 227\text{K} = -45^\circ\text{C}$

Sol 4 → Isentropic flow through CD nozzle



$P_1 = 1.1 \times 10^6\ \text{Pa}$, $P_e = 141 \times 10^3\ \text{Pa}$

$T_1 = T_0 = 115 + 273 = 388\ \text{K}$

$\frac{P_e}{P_0} = \frac{141 \times 10^3}{1.1 \times 10^6} = 0.1282$

$\dot{m} = 2\ \text{kg/s}$

looking at tables (isentropic flow throughout) $A_2 = ??$

So $(Ma)_2 = 2.0$

$\Rightarrow$ At throat we have $Ma = 1$, $A_t = A^*$

at exit $P_2 = 141 \times 10^3\ \text{Pa}$ $Ma_2 = 2.0$

$T_2/T_0 = 0.5556 \Rightarrow T_2 = 388 \times 0.5556 = 215.6\ \text{K}$

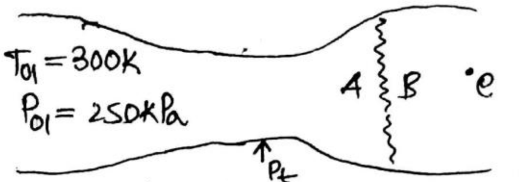
$\rho_2 = \frac{141 \times 10^3}{287 \times 215.6} = 2.279\ \text{kg/m}^3$, $a_2 = \sqrt{kRT_2} = \sqrt{1.4 \times 287 \times 215.6}$
$= 294.3\ \text{m/s}$.

$V_2 = Ma_2 a_2 = 2 \times 294.3 = 588.6\ \text{m/s}$

$\dot{m} = \rho_2 V_2 A_2 = 2 \Rightarrow A_2 = 2/(2.279 \times 588.6) = 1.491 \times 10^{-3}\ \text{m}^2$

$\frac{A^*}{A} = 0.5926$ (for $Ma = 2$) $\Rightarrow A^* = 0.5926 \times 1.491 \times 10^{-3}\ \text{m}^2$
$= 8.884 \times 10^{-4}\ \text{m}^2$

Sol→5 find Ma_B, P_B, Δs across shock isentropic flow from 1 to A



A before shock
B after shock.
$P_A = 68.1$ kPa
$P_e = 180$ kPa.
$P_t = 132$ kPa.

Since shock at A, $Ma_t = 1.0$
A–B shock (flow not isentropic)
from B to e flow again isentropic

$P_A = 681.$ kPa $\Rightarrow P_A/P_0 = \frac{681}{250} = 0.2724$

$\Rightarrow (Ma)_A = 1.5$ (from table of isentropic flow).

$\Rightarrow (Ma)_B = 0.7011$ (from shock wave table)

$P_B/P_A = 2.458 \Rightarrow P_B = P_A \times 2.458 = 167.4$ kPa

for Δs across shock

$$ds = C_p\frac{dT}{T} - R\frac{dp}{P} = S_B - S_A = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

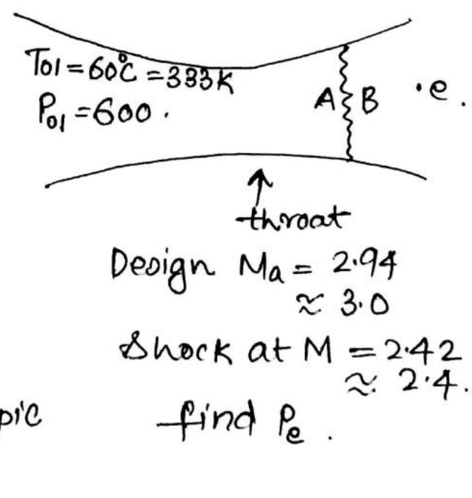
$$C_p = \frac{k}{k-1}R = 1004.5$$

$$\frac{T_2}{T_1} = 1.320,\quad \frac{P_2}{P_1} = 2.458 \text{ (shock tables)}.$$

$$S_B - S_A = 1004.5\ \ln 1.320 - 287 \ln 2.458$$
$$= 278.88 - 258.11 = 20.76\ J/kg\,K$$

Sol→6

Design $Ma = 3.0 \Rightarrow$ for isentropic flow from 1 to e (with supersonic condition at exit)



$A^*/A = 0.2362 = A^*_A/A_e$

At given condition shock at A
isentropic flow from 1 to A, with $Ma_t = 1.0$, shock from A to B
isentropic from B to e

Design $Ma = 2.94$
≈ 3.0
Shock at $M = 2.42$
$\approx 2.4.$
find P_e.

$Ma_A = 2.4 \Rightarrow P_A/P_0 = 0.6840$
$\frac{A^*_A}{A_A} = 0.4161$
} from isentropic flow

From shock table, $Ma_B = 0.5231$;
isentropic flow from B to e (refer as B flow).

For $Ma_B = 0.50$ (instead of 0.5231) $\frac{A^*_B}{A_B} = 0.7464$

(note $A^*_B \neq A^*_A$ as they are 2 different isentropic flows.

Sol→6 (contd.) flow from B to E isentropic

$$\frac{A_B^*}{A_e} = \frac{A_A^*}{A_e} \times \frac{A_B^*}{A_B} \times \frac{A_A}{A_A^*} \quad \text{(note } A_B = A_A\text{)}.$$

for $\frac{A^*}{A} = 0.4237$, $M_a = 0.25$ (in subsonic range).

$$\frac{P_e}{P_{B_0}} = 0.9575 \qquad (P_{B_0} = ??).$$

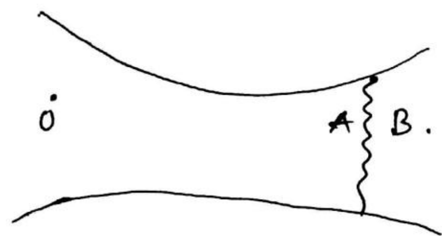
$$\frac{P_{B_0}}{P_{A_0}} = 0.5401 \text{ (shock table)} \Rightarrow P_{B_0} = 0.5401 \times 600\,\text{kPa}.$$

$$\therefore \quad P_e = P_{B_0} \times 0.9575 = 310.3\,\text{kPa}.$$

Sol→7

$M_{a_A} = 3.0$

$A_A = 500 \times 10^{-6}\,m^2$



$P_{0A} = 1000\,\text{kPa} + 101\,\text{kPa}$

$T_{0A} = 400\,K$

$P_A/P_{0A} = 0.02722$

$T_A/T_{0A} = 0.3571$

$M_{a_B} = 0.4752$ (from shock table).

$$P_B/P_A = 10.33 \Rightarrow P_B = 10.33 \times 0.02722 \times P_{0A}$$
$$= 10.33 \times (0.02722)(1101)\,\text{kPa}$$
$$= 309.6\,\text{kPa (abs)}$$

$$\frac{P_{0B}}{P_{0A}} = 0.3283 \Rightarrow P_{0B} = 0.3283 \times (1101) = 361.5\,\text{kPa (abs)}.$$

Nozzle throat Area: $M_{a_A} = 3.0$, $A^*/A = 0.2362$.

$$\Rightarrow A^* = 0.2362 \times 500\,mm^2 = 118.1\,mm^2$$

To find A_e, $\frac{A_B^*}{A_B} = 0.7464$ (corresponding $M_a = 0.5$ closest to 0.475)

$$\frac{A_e}{A_B^*} = \frac{A_e}{A_B} \times \frac{A_B}{A_B^*} = \frac{600}{500} \times \frac{1}{0.7464} = 1.608$$

$$\Rightarrow M_{a_e} = \left(\text{correspond}^n \text{ to subsonic flow for } \frac{A^*}{A} = \frac{1}{1.608} = 0.622\right)$$
$$= 0.4$$