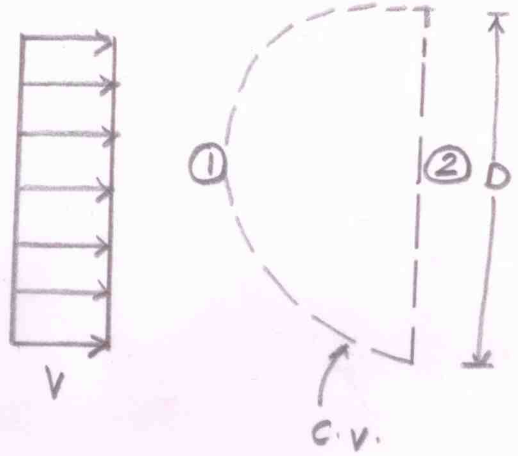
Soln 1



We are given a control surface as shown in fig, having unit depth into the page. Now, we consider the C.V. to be made of two faces namely ① and ②. As, the flow is ~~ce~~ incompressible and the C.V. is fixed and non-deforming.

$$\frac{d\rho\forall_{c.v.}}{dt} = \cancel{\frac{\partial}{\partial t}\iiint_{\forall}\rho\, d\forall}^{\,0} + \oiint_S \rho(\vec{V}\cdot\hat{n})\, ds = 0$$

$$\Rightarrow \quad \oiint_S (\vec{V}\cdot\hat{n})\rho\, ds = 0.$$

$$\cancel{\dot{\forall}_1 + \dot{\forall}_2 = 0}$$

$$\oiint_S (\vec{V}\cdot\hat{n})\rho\, ds\Big|_1 + \oiint_S (\vec{V}\cdot\hat{n})\rho\, ds\Big|_2 = 0$$

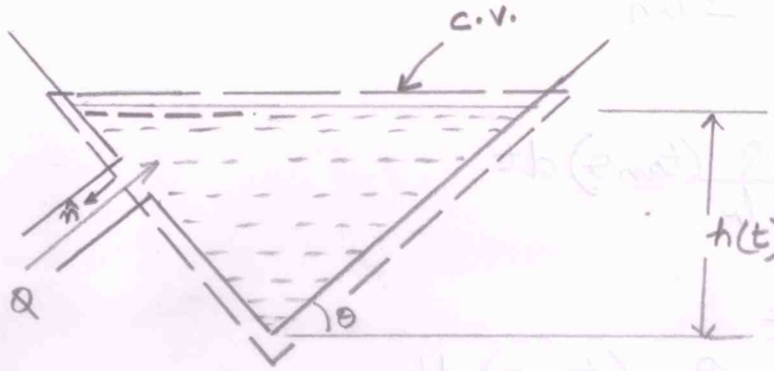
As, the flow is steady, so ~~V or volume~~ mass flowrate from the surface 1 would be equal to the mass flux from the surface 2.

$$\dot{m}_1 + \dot{m}_2 = 0$$

$$\dot{m}_2 = \rho V D$$

$$\Rightarrow \dot{m}_1 = -\rho V D$$

Soln 2



Consider the control volume as shown, which is fixed as a whole and is deforming / or chaning its dimensions in time.

By Reynolds transport theorm:—

$$\frac{\partial}{\partial t}\iiint_{\forall} \rho \, d\forall + \oiint_{S} \rho(\vec{V}\cdot\hat{n})\,ds = 0$$

$$\frac{\partial}{\partial t}\iiint_{\forall} d\forall + \oiint_{S} (\vec{V}\cdot\hat{n})\,ds = 0 \qquad \text{(as the flow is incompressible)}$$

$$\frac{d}{dt}\left[\forall_{c.v.}\right] + (-Q) = 0$$

$$\frac{d}{dt}\left[\frac{1}{2}\times b\times h\times\frac{2h}{\tan\theta}\right] - Q = 0 \qquad \left(\text{the base of the triangle is } \frac{2h}{\tan\theta}\right)$$

$$\frac{d}{dt}\left(\frac{b\,h^2}{\tan\theta}\right) - Q = 0$$

$$\frac{b}{\tan\theta}\,2h\,\frac{dh}{dt} - Q = 0$$

$$\Rightarrow \frac{dh}{dt} = \frac{Q(\tan\theta)}{2bh}$$

b)

$$\frac{dh}{dt} = \frac{Q(\tan\theta)}{2bh}$$

$$\frac{h\,dh}{h} = \frac{Q}{2b}(\tan\theta)\,dt$$

$$\int_1^2 \frac{h\,dh}{h} = \int_1^2 \frac{Q}{2b}(\tan\theta)\,dt$$

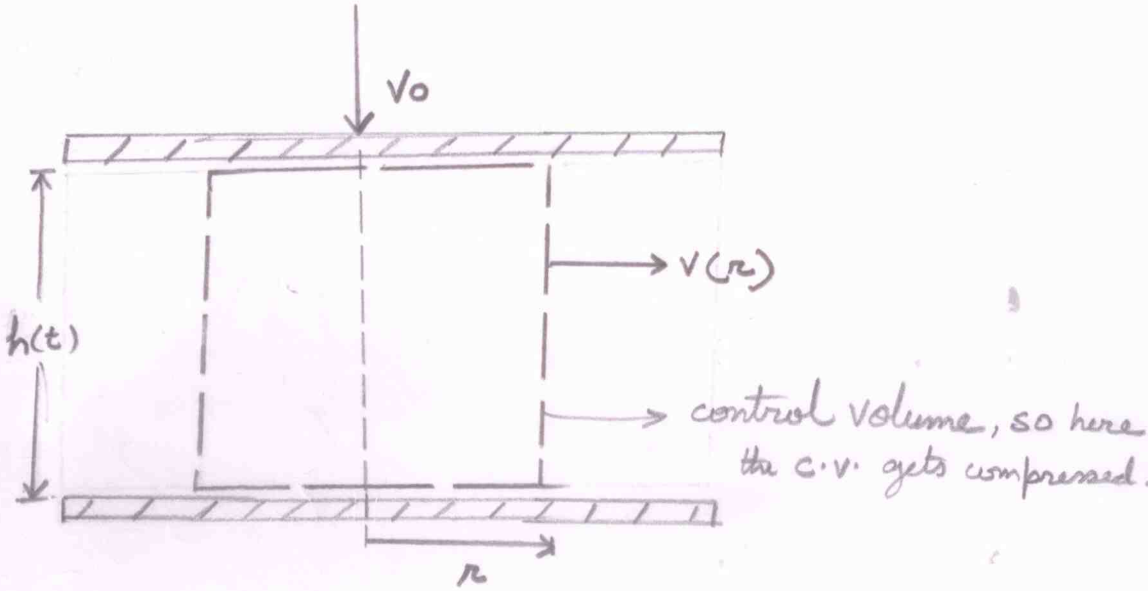
$$\left|\frac{h^2}{2}\right|_1^2 = \frac{Q}{2b}(\tan\theta)\left|t\right|_1^2$$

$$\frac{h_2^2 - h_1^2}{2} = \frac{Q}{2b}(\tan\theta)(t_2 - t_1)$$

$$\frac{(h_2^2 - h_1^2)\,b}{Q(\tan\theta)} = t_2 - t_1$$

$$\Rightarrow t_2 - t_1 = \frac{b(h_2^2 - h_1^2)}{Q(\tan\theta)}$$

Soln 3



$$\frac{\partial}{\partial t}\iiint_{\forall} \rho \, d\forall + \oiint_{S} \rho\,(\vec{V}\cdot\hat{n})\,ds = 0$$

As, the flow is incompressible,

$$\frac{\partial}{\partial t}\iiint_{\forall} d\forall + \oiint_{S} (\vec{V}\cdot\hat{n})\,ds = 0$$

$$\frac{d}{dt}\left[\pi r^2 h\right] + (2\pi r h)\,V(r) = 0$$

$$\pi r^2 \frac{dh}{dt} + 2\pi r h\, V(r) = 0$$

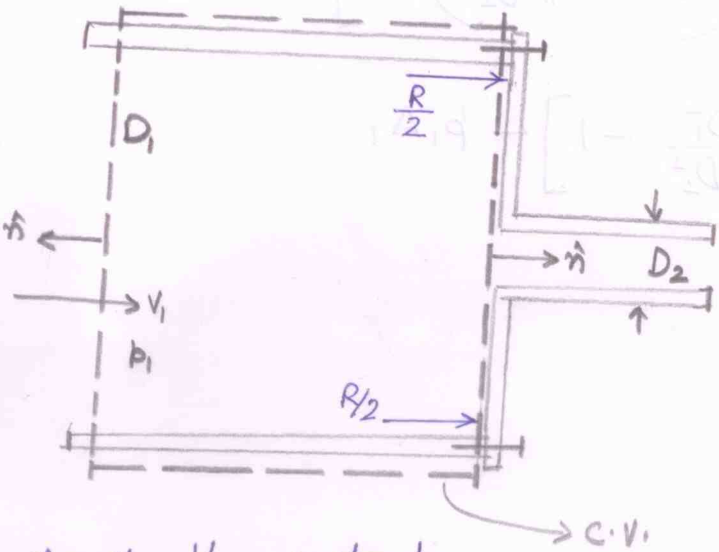
$$\frac{dh}{dt} + \frac{2h}{r} V(r) = 0$$

$$\text{As, } -\frac{dh}{dt} = V_0$$

$$\Rightarrow \quad -V_0 + \frac{2h}{r} V(r) = 0$$

$$V(r) = \frac{V_0 r}{2h}.$$

Soln 4



Continuity :– As, the flow is steady,

$$\frac{\partial}{\partial t}\int_{\forall} \rho\, d\forall^{\nearrow 0} + \oiint_{C.S.} \rho(\vec{V}\cdot\hat{n})\,ds = 0$$

$$\oiint_{C.S.} \rho(\vec{V}\cdot\hat{n})\,ds = 0$$

$$\oiint_{C.S.} (\vec{V}\cdot\hat{n})\,ds = 0 \qquad \text{(as flow is incompressible)}$$

$V_1 A_1 = V_2 A_2$ where $A_1 = \frac{\pi}{4} D_1^2$ and $A_2 = \frac{\pi}{4} D_2^2$

Momentum conservation :– $\frac{R}{2} + \frac{R}{2} = F_{ext}$ (Sum of the bolt reactions will be equal to the total external force).

$$p_1 A_1 + F_{ext} = -A_1 \rho V_1^2 + \rho V_2^2 A_2^2$$

$$\Rightarrow p_1 A_1 + F_{ext} = -A_1 \rho V_1^2 + \rho V_2^2 A_2^2$$

$$p_1 A_1 + F_{ext} = -A_1 \rho V_1^2 + \rho A_2 \left[\frac{V_1 A_1}{A_2}\right]^2$$

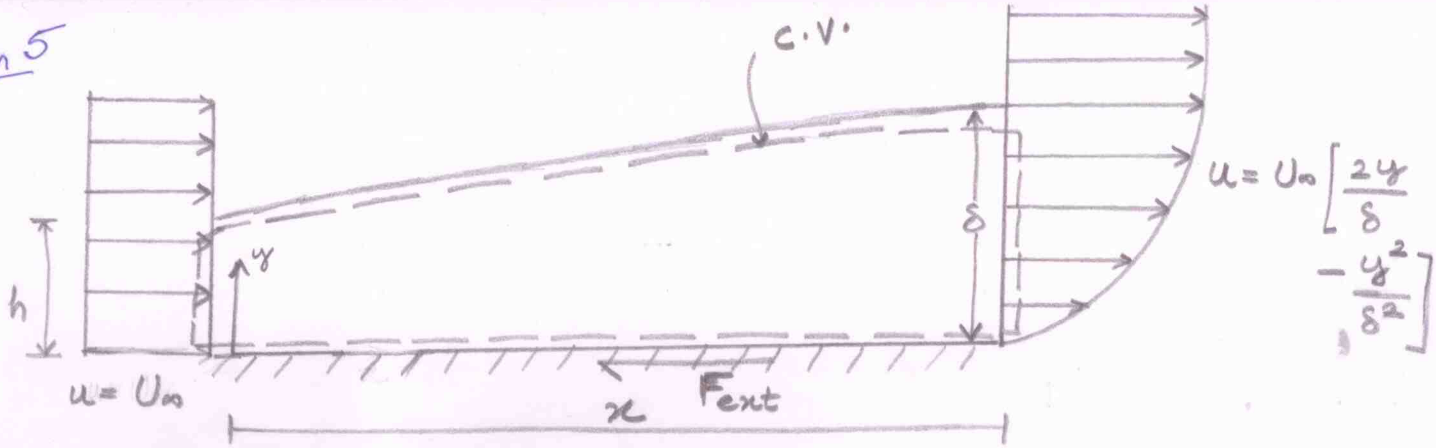
$$p_1 A_1 + F_{ext} = -A_1 \rho V_1^2 + \frac{\rho V_1^2 A_1^2}{A_2} = \rho V_1^2 A_1\left[-1 + \frac{A_1}{A_2}\right]$$

$$p_1 A_1 + F_{ext} = \rho V_1^2 A_1\left[-1 + \frac{A_1}{A_2}\right]$$

$$F_{ext} = \rho V_1^2 A_1 \left[\left(\frac{\pi}{4} D_1^2 \times \frac{4}{\pi D_2^2} \right) - 1 \right] - p_1 A_1$$

$$\Rightarrow F_{ext} = \rho V_1^2 A_1 \left[\frac{D_1^2}{D_2^2} - 1 \right] - p_1 A_1$$

Soln 5



Continuity :—

$$\rho(hb)U_\infty = \int_0^\delta \rho U_\infty\left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right] b\,dy$$

$$\rho U_\infty b h = \rho U_\infty\left[\frac{y^2}{\delta} - \frac{y^3}{3\delta^2}\right]\Bigg|_0^\delta b$$

$$h = \delta - \frac{\delta}{3} = \frac{2\delta}{3}$$

$$\Rightarrow h = \frac{2}{3}\delta$$

Momentum conservation :—

$$-F_{ext} = \oiint_S \rho(\vec{V}\cdot\hat{n})\vec{V}\,ds$$

$$-F_{ext} = -\rho U_\infty^2 h b + \int_0^\delta \rho U_\infty^2\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)^2 b\,dy$$

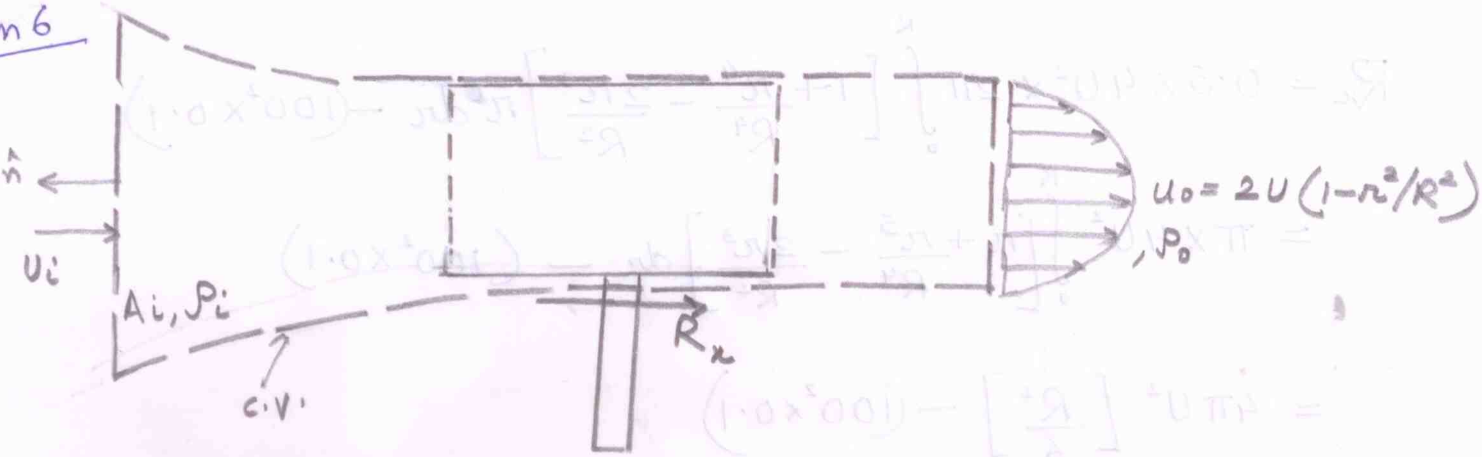
$$-F_{ext} = -\rho U_\infty^2\left(\frac{2}{3}\delta\right)b + \rho U_\infty^2\int_0^\delta\left[\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^4} - \frac{4y^3}{\delta^3}\right]dy$$

$$-F_{ext} = -\rho U_\infty^2\left(\frac{2\delta}{3}\right)b + \rho U_\infty^2\left[\frac{4y^3}{3\delta^2} + \frac{y^5}{5\delta^4} - \frac{4y^4}{4\delta^3}\right]\Bigg|_0^\delta$$

$$-F_{ext} = \rho U_\infty^2 b\left[\frac{-2\delta}{15}\right]$$

$$F_{ext} = \frac{2}{15}\rho U_\infty^2 b\delta \quad\Rightarrow\quad \vec{F}_{drag} = -\frac{2}{15}\rho U_\infty^2 b\delta\,\hat{i}$$

Soln 6



$U_i = 100 m/s$, $\rho_1 = 1 kg/m^3$, $A_1 = 0.1 m^2$, $U = 600 m/s$, $\rho_0 = 0.5 kg/m^3$

(a) average velocity at jet exit :–

$$V_{exit\ average} = \frac{\int_0^R 1200\left(1 - \frac{r^2}{R^2}\right) 2\pi r\,dr}{\pi R^2}$$

$$= \frac{1200 \times 2\pi \times \left[\frac{R^2}{2} - \frac{R^4}{4}\right]}{\pi R^2}$$

$$V_{exit\ average} = 600 m/s$$

(b) Thrust of the turbojet engine :–

$$\rho_i A_i U_i = \rho_0 A_e V_e \quad \text{(By continuity eqn.)}$$

$$100 \times 1 \times 0.1 = 0.5 \times A_e \times 0.5$$

$$A_e = \frac{10}{600 \times 0.5}$$

By momentum equation :–

$$R_x = \int_0^R 0.5\left[2U\left(1 - \frac{r^2}{R^2}\right)\right]^2 2\pi r\,dr - (1 \times 100^2 \times 0.1)$$

$$R_x = 0.5 \times 4U^2 \times 2\pi \int_0^R \left[1 + \frac{r^4}{R^4} - \frac{2r^2}{R^2}\right] r\, dr - (100^2 \times 0.1)$$

$$= \pi \times 4U^2 \int_0^R \left[r + \frac{r^5}{R^4} - \frac{2r^3}{R^2}\right] dr - (100^2 \times 0.1)$$

$$= 4\pi U^2 \left[\frac{R^2}{6}\right] - (100^2 \times 0.1)$$

$$= \left[\frac{4U^2}{6} \times (\pi R^2)\right] - 1000$$

$$= \left(\frac{4}{6} \times 600^2 \times \frac{10}{600 \times 0.5}\right) - 1000$$

$$= 7000\ kg\, m/s^2$$

$R_x = 7000N$

So, thrust force will be $\vec{F} = -R_x \hat{i} = -7000N\hat{i}$

c) Thrust with constant or uniform exit velocity :-

$$R_x = \rho_e U_e^2 A_e - \rho_i U_i^2 A_i$$

$$= 0.5 \times 600^2 \times \frac{10}{600 \times 0.5} - (100^2 \times 0.1)$$

$$R_x = 5000N$$

$$\vec{F} = -R_x \hat{i} = -5000N\hat{i}.$$