Assignment ①, Name - Ishaan Bhadoo

① To show,

$p_c(\mathbb{Z}) = 1$

Proof: let X_n = # open paths of length n

Then $\mathbb{P}_p(0 \longleftrightarrow \infty) \leq \mathbb{P}_p(X_n > 0) \leq \mathbb{E}X_n$

$X_n = \sum_{\gamma \in S_n} \mathbb{1}(\gamma \text{ is open})$ (S_n: paths of length n) $\Rightarrow \mathbb{E}X_n = |S_n| p^n$

$= 2p^n$ (since $|S_n| = 2$)

Hence, $\mathbb{P}_p(0 \longleftrightarrow \infty) \leq 2 \cdot p^n \longrightarrow 0$

$\forall\, p \in [0,1)$

$\therefore$ $p_c(\mathbb{Z}) = 1$