

Assignment (3)

⑤ To show

$$\mathbb{P}_p(A) \leq \mathbb{P}_{p'}(A) \quad \text{for } A \text{ -inc and } 0 \leq p \leq p' \leq 1.$$

Proof: Recall the natural coupling.

For that coupling $G[p] \leq G[p']$

Since A is inc,

$$\mathbb{1}_A(G[p]) \leq \mathbb{1}_A(G[p'])$$

$$\text{i.e. } \mathbb{P}_p(A) \leq \mathbb{P}_{p'}(A)$$

□

⑥ let A be an inc event

To show, A^c is dec.

Proof : let $\omega \geq \omega'$ to show

$$\gamma_{A^c}(\omega') \geq \gamma_{A^c}(\omega)$$

Now $\gamma_{A^c} = 1 - \gamma_A$, Thus $\gamma_{A^c}(\omega) = 1 - \gamma_A(\omega)$
 $\leq 1 - \gamma_A(\omega')$
 $\leq \gamma_{A^c}(\omega')$

Hence A^c is dec

□