Mathematics Coursework

Real Analysis I

- Instructor: Prof. Rajarama Bhat
- Topics Studied: The language of sets and functions - countable and uncountable sets. Real numbers - least upper bounds and greatest lower bounds. Sequences - limit points of a sequence, convergent sequences; bounded and monotone sequences, the limit superior and limit inferior of a sequence. Cauchy sequences and the completeness of R. Series - convergence and divergence of series, absolute and conditional convergence. Riemann's rearrangement theorem. Various tests for convergence of series. Connection between infinite series and decimal expansions, ternary, binary expansions of real numbers. Cauchy product, Infinite products. calculus of a single variable - continuity; attainment of supremum and infimum of a continuous function on a closed bounded interval, uniform continuity. Differentiability of functions. Chain Rule, Rolle's theorem and mean value theorem. Higher derivatives, Leibniz formula, maxima and minima. Taylor's theorem - various forms of remainder, infinite Taylor expansions. L'Hospital Rule
- References: T. M. Apostol: Mathematical Analysis, Bartle and Sherbert: Introduction to Real Analysis, T.Tao: Analysis I & II.
- **Grade: 100/100**

Probability Theory I

- Instructor: Prof. D Yogeshwaran
- Topics Studied: Random experiments, outcomes, sample space, events. Discrete sample spaces and probability models . Equally likely setup (including examples such as Maxwell-Boltzmann, Bose-Einstein, Fermi-Dirac statistics). Gibbs distribution, Combination of events: inclusion/exclusion, Boole's inequality and Bonferroni's inequality. Conditional probability: independence, law of total probability and Bayes' theorem, Composite experiments: Polya's urn scheme. Discrete random variables. Standard discrete distributions. Convergence of Binomial to Poisson distribution and normal distribution.Continuous random variables. Introduction to cumulative distribution functions (CDF) and properties. Distributions with densities. Functions of random variables, Expectation, moments, variance, computations involving indicator random variables. Joint distributions of discrete random variables (multinomial distributions), linearity and monotonicity of expectations,independence, covariance, variance of a sum, computations involving indicator random variables, distribution of sum of two independent random variables. Conditional distributions, conditional expectation.
- References: W. Feller: Introduction to Probability: Theory and Applications - Vol. I and II, Santosh S. Venkatesh: Theory of Probability - Explorations and Applications, Sheldon Ross: Introduction to Probability.
- **Grade: 96/100**

Linear Algebra I

- Instructor: Prof. Aniruddha Naolekar
- Topics Studied: Homogeneous and non-homogeneous systems of linear equations, condition for consistency, solution set as a translate of a subspace. Vector spaces, subspaces, linear independence, span, basis and dimension, sum and intersection of subspaces, direct sum, complement and projection. Linear transformation and its matrix with respect to a pair of bases, properties of matrix operations, use of partitioned matrices. Column space and row space, rank of a matrix, nullity, rank of AA^* . g-inverse and its elementary properties, left inverse, right inverse and inverse, inverse of a partitioned matrix, lower and upper bounds for rank of a product, rank factorization of a matrix, rank of a sum. Elementary operations and

elementary matrices, Echelon form, Normal form, Hermite canonical form and their use in solving linear equations and in finding inverse or g-inverse. LDU-decomposition.
- References: M.Artin: Algebra, K. Hoffman and R. Kunze: Linear Algebra, P. R. Halmos: Finite Dimensional Vector Spaces.
- **Grade: 92/100**

Elementary Number Theory

- Instructor: Prof. Balasubramanian Sury
- Topics Studied: Basic set theory, Equivalence relations and partitions. Mathematical Induction, Binomial theorem, principle of inclusion-exclusion and pigeonhole. Divisibility, Division algorithm, Euclidean algorithm, Fundamental theorem of arithmetic and Sieve of Eratosthenes. Basic properties, Fermat's little theorem, Euler and Wilson congruences, RSA, Chinese remainder theorem, Group structure of U(Z/nZ), Primitive roots. Quadratic residues, Quadratic reciprocity law, Jacobi symbol, Binary Quadratic forms, Sum of two squares. Arithmetic Functions, Mobius inversion formula, Linear recurrences, Linear Diophantine equations.
- References: I. Niven, H. S. Zuckermann and H. L. Montgomery: An Introduction to the Theory of Numbers.
- **Grade: 100/100**

Linear Algebra II

- Instructor: Prof. Rajarama Bhat
- Topics Studied: Determinant of n-th order and its elementary properties, expansion by a row or column, statement of Laplace expansion, determinant of a product, statement of Cauchy-Binet theorem, inverse through classical adjoint, Cramers rule, determinant of a partitioned matrix, Idempotent matrices. Norm and inner product on Rn and Cn, norm induced by an inner product, Orthonormal basis, Gram-Schmidt orthogonalization starting from any finite set of vectors, orthogonal complement, orthogonal projection into a subspace, orthogonal projector into the column space of A, orthogonal and unitary matrices. Characteristic roots, relation between characteristic polynomials of AB and BA when AB is square, Cayley-Hamilton theorem, idea of minimal polynomial, eigenvectors, algebraic and geometric multiplicities, characterization of diagonalizable matrices, spectral representation of Hermitian and real symmetric matrices, singular value decomposition. Quadratic form, category of a quadratic form, use in classification of conics, Lagranges reduction to diagonal form, rank and signature, Sylvesters law, determinant criteria for n.n.d. and p.d. quadratic forms, Hadamards inequality, extrema of a p. d. quadratic form, simultaneous diagonalization of two quadratic forms one of which is p.d., simultaneous orthogonal diagonalization of commuting real symmetric matrices, square-root method.
- References: A. Ramachandra Rao and P. Bhimasankaram: Linear Algebra, S. Axler: Linear Algebra Done Right!
- **Grade: 100/100**

Real Analysis II

- Instructor: Prof. Jaydeb Sarkar
- Topics Studied: The existence of Riemann integral for sufficiently well behaved functions. Fundamental theorem of Calculus, computation of definite integrals, improper integrals, sequences and series of functions, double sequences, pointwise versus uniform convergence for a function defined on an interval of R, term by term differentiation and integration, the Weierstrass's theorem about uniform approximation of a continuous function by a sequence of polynomials on a closed bounded interval. Radius of convergence of power series and real analyticity of functions.
- References: W. Rudin: Principles of Mathematics Analysis, T. M. Apostol: Mathematical Analysis.
- **Grade: 100/100**

Probability Theory II

- Instructor: Prof. Parthanil Roy
- Topics Studied: Functions of more than one discrete random variables. Markov's inequality, Tchebyshev's inequality and Weak law of large numbers. Generating functions, Fluctuations in coin tossing and random walks, Review of conditional distributions and random sums of random variables. Review of probability densities on the real line, Bivariate continuous distributions, bivariate CDFs, independence, distribution of sums, products and quotients for bivariate continuous distributions, Examples: Bivariate Dirichlet and bivariate normal distributions. Independence and marginal distributions. Distributions of functions of bivariate continuous random vectors.Conditional distribution, conditional density, examples. Conditional distributions of bivariate normal distribution. Expectation of functions of random variables with densities, variance and moments of random variables. Conditional expectation and variance, illustrations. Discussion of a.s. convergence, convergence in probability and distribution. Statements of CLT and Strong law of large numbers for i.i.d. random variables.
- References: W. Feller: Introduction to Probability: Theory and Applications - Vol. I and II, P. G. Hoel, S. C. Port and C. J. Stones: Introduction to Probability Theory.
- **Grade: 100/100**

Analysis of Several Variables

- Instructor: Prof. Jaydeb Sarkar
- Topics Studied: Calculus of several variables: Differentiability of maps from Rm to Rn and the derivative as a linear map. Higher derivatives, Chain Rule, Taylor expansions in several variables, Local maxima and minima, Lagrange multiplier Multiple integrals, Existence of the Riemann integral for sufficiently well-behaved functions on rectangles, i.e. product of intervals. Multiple integrals expressed as iterated simple integrals. Brief treatment of multiple integrals on more general domains. Change of variables and the Jacobian formula, illustrated with plenty of examples. Inverse and implicit functions theorems (with proofs). More advanced topics in the calculus of one and several variables curves in R2 and R3 . Line integrals, Surfaces in R3 , Surface integrals, Divergence, Gradient and Curl operations, Green's, Strokes' and Gauss' (Divergence) theorems.
- References: M.Spivak: Calculus on Manifolds, J.Munkres: Analysis on Manifolds, T.M. Apostol: Mathematical Analysis, S. Dineen: Multivariate Calculus and Geometry.
- **Grade: 98/100**

Discrete Mathematics I

- Instructor: Prof. Shreedhar Inamdar
- Topics Studied: Basic counting techniques: Double-counting, Averaging principle, Inclusion-Exclusion principle. Euler indicator, Mobius function and inversion formula. Recursions and generating functions. The pigeonhole principle, Erdos-Szekeres theorem. Mantel's theorem. Turan's theorem, Dirichlet's theorem, Schur's theorem. Graphs: Euler's theorem and Hamilton Cycles. Spanning Trees. Cayley's theorem and Spanning trees. System of distinct representatives: Hall's marriage theorem, Applications to latin rectangles and doubly stochastic matrices, Konig-Egervary theorem, Dilworth's theorem, Sperner's theorem. Flows in networks: Max-flow min-cut theorem, Ford-Fulkerson theorem, Integrality theorem for max-flow. Latin squares and combinatorial design: Orthogonal Latin squares, Existence theorems and finite projective planes. Block designs. Hadamard designs, Incidence matrices. Steiner triple systems.
- References: J. H. van Lint & R. M. Wilson: A Course in Combinatorics, D. B. West: Introduction to Graph Theory.
- **Grade: 90/100**

Group Theory

- Instructor: Prof. Maneesh Thakur
- Topics Studied: Equivalence relations and partitions, Zorn's lemma, Axiom of choice, Principle of mathematical induction. Groups, subgroups, homomorphisms, Modular arithmetic, quotient groups, isomorphism theorems. Groups acting on sets, Sylow's theorems, Permutation groups, Semidirect products. Classification of groups of small order. Notion of solvable groups and proof of simplicity of A_n (for $n > 4$). Matrix groups O(2), U(2), SL(2, R), Matrix exponential.
- References: S. D. Dummit and M. R. Foote: Abstract Algebra, I. N. Herstein: Topics in Algebra, M.Artin: Algebra, M.Isaacs : Finite Group Theory.
- **Grade: 100/100**

Topology

- Instructor: Prof. Maneesh Thakur
- Topics Studied: Metric spaces: Elements of metric space theory. Sequences and Cauchy sequences and the notion of completeness, elementary topological notions for metric spaces i.e.open sets, closed sets, compact sets, connectedness, continuous and uniformly continuous functions on a metric space. The Bolzano - Weirstrass theorem, Supremum and infimum on compact sets, R n as a metric space. Topological spaces: Definitions and Examples; Bases and sub-bases; Subspace and metric topology; closed sets, limit points and continuous functions; product and quotient topology. Separation: Countability and Separation axioms, Normal spaces, Urysohn lemma, Tietze extension theorem. Connectedness and Compactness: Connected subspaces of the real line, Compact subspaces of the real line, limit point compactness, local compactness. Tychnoff's theorem. One point compactification. Topological groups
- References: J. Munkres: Topology a first course, M. A. Armstrong: Basic Topology, G. F. Simmons: Introduction to Topology and Modern Analysis, K. Janich: Topolgy.
- **Grade: 100/100**

Rings and Modules

- Instructor: Prof. B. Sury
- Topics Studied: Rings, Left and Right ideals, Examples of Polynomial rings, Matrix rings and Group rings, Quotient rings by two-sided ideals. Commutative rings: Units, Nilpotents, Adjunction of elements, Chinese remainder theorem, Maximal and prime ideals, Localization. Factorisation theory in domains: Irreducible and prime elements, Euclidean domains, Principal Ideal Domains, Unique Factorisation Domains, Gauss's lemma, Eisenstein's Criterion. Noetherian rings, Hilbert basis theorem. Modules: Structure of finitely generated modules over a PID and their representation matrices, Applications to Rational canonical form and Jordan form of a matrix.
- References: S. D. Dummit and M. R. Foote: Abstract Algebra. I. N. Herstein: Topics in Algebra. C. Musili: Rings and Modules.
- **Grade: 100/100**

Ordinary Differential Equations

- Instructor: Prof. CRE Raja
- Topics Studied: First order differential equations, Picard's theorem, existence and uniqueness of solution to first order ordinary differential equations (Peano's existence theorem, Osgood's uniqueness theorem), Systems of first order differential equations, higher order linear differential equations, solving higher order linear DE with constant coefficients. Introduction to power series solutions, Equations with regular singular points, Special ordinary differential equations arising in physics and some special functions (eg. Bessels functions, Legendre polynomials, Gamma functions). Sturm -Liouville problems, Sturm comparison principle, Critical points and stability in linear systems. Nonlinear equations - Lyapunov's method for detecting stability in systems, simple critical points of nonlinear systems, Periodic solutions,

- statement of the Poincare-Bendixson theorem (no proof). Numerical methods and error analysis - Euler method, Second order Taylor method, Trapezoid method, Improved Euler method, Runge-Kutta method.
- References: G.F. Simmons: Differential equations with applications and historical notes, Peter J. Olver: Lecture notes on Nonlinear Ordinary Differential Equations, Garrett Birkhoff and Gian-Carlo Rota: Ordinary Differential Equations.
- **Grade: 98/100**

Introduction to Differential Topology (Audit)

- Instructor: Prof. Suresh Nayak
- Curriculum: Manifolds. Inverse function theorem and immersions, submersions, transversality, homotopy and stability, Sards theorem and Morse functions, Embedding manifolds in Euclidean space, manifolds with boundary , intersection theory mod 2, winding numbers and Jordan- Brouwer separation theorem, Borsuk- Ulam fixed point theorem.
- References: V. Guillemin and Pollack: Differential Topology (Chapters I, II and Appendix 1, 2).

Field and Galois Theory

- Instructor: Prof. Anita Naolekar
- Curriculum: Algebraic extensions: degree, Splitting fields and normal extensions, Algebraic closure. Separable extensions, Fundamental theorem of Galois theory. Finite fields, Cyclic extensions, Kummer theory. Ruler and compass constructions, Solvability by radicals. Transcendental extensions: Transcendence bases and transcendence degree.
- References: P. Morandi: Field and Galois theory, Dummit and Foote: Abstract Algebra, D. Cox: Field and Galois theory.
- **Grade: 97/100**

Differential geometry and Lie groups

- Instructor: Prof. Maneesh Thakur
- Curriculum: Manifolds and Lie groups, Frobenius theorem, Tensors and Differential forms, Stokes theorem, Riemannian metrics, Levi-Civita connection, Curvature tensor and fundamental forms.
- References: John Lee: Introduction to Smooth Manifolds, S. Kumaresan:A Course in Differential Geometry and Lie Groups.
- **Grade: 99/100**

Probability III

- Instructor: Prof. Siva Athreya
- Curriculum: Sigma-algebras, axioms of probability, uniqueness of extension for probability measures. Examples of countable probability spaces, Borel sigma-algebra on the real line and standard probability distributions on the real line. Construction of Lebesgue measure. Random variables and examples. Push-forward of a probability measure. Borel probability measures on Euclidean spaces as push-forward of Lebesgue measure; General definition of expectation and properties. Change of variables. Review of conditional distribution and conditional expectation, General definition, Examples. Limit theorems: Monotone Convergence Theorem, Fatou's Lemma, Dominated Convergence Theorem. Different modes of convergence and their relations, Weak Law of large numbers, First and Second Borel-Cantelli Lemmas, Strong Law of large numbers (proof under finite variance). Characteristic functions, properties, Inversion formula and Levy continuity theorem, CLT in i.i.d. finite variance case. Slutsky's Theorem. Introduction to Finite Markov chains - Definition. Random mapping representation. Examples. Irreducibility and aperiodicity. Stationary distribution and reversibility. Random walks on graphs.
- References: S. R. Athreya and V. S. Sunder: Measure and Probability, Norris J.R. : Markov Chains.
- **Grade: 89/100**

Function Spaces

- Instructor: Prof. Soumyashant Nayak
- Curriculum: Convex Geometry, Hahn-Banach Theorems, Riesz Representation Theorem, Lebesgue Integrals, Levi's monotone convergence theorems, Lebesgue's dominated convergence theorem, measurable functions, Continuity of functions defined by integrals. Differentiation under the integral sign. Interchange of order of integrals. Riesz-Fischer Theorem. Fourier Series and Fourier Integrals, theorem on best approximation. Plancherel's theorem. Fourier series with respect to an orthogonal system. Convolution theorem for Fourier transforms. Fourier Integral theorem. Stone-Weierstrass theorem. Convergence properties of Fourier series. Riemann-Lebesgue lemma, Dirichlet and Fejer kernels, Dini-Lipschitz test, Riemann's localization theorem. Cesaro summability of Fourier series.
- References: T.M. Apostol: Mathematical Analysis, Introduction to Fourier Analysis on Euclidean Spaces,
- **Grade: 100/100**

Measure Theory (Audit)

- Instructor: Prof. Jaydeb Sarkar
- Curriculum: The concept of sigma-algebra, Borel subsets of R, Construction of Lebesgue and Lebesgue-Stieltjes measures on the real line following outer measure. Abstract measure theory: definition and examples of measure space, measur- able functions, Lebesgue integration, convergence theorems (Fatous Lemma, Monotone convergence and dominated convergence theorem). Caratheodory extension theorem, completion of measure spaces. Product measures and Fubinis theorem. L^p-spaces, Riesz-Fischer Theorem, approximation by step functions and con- tinuous functions. Absolute continuity, Hahn-Jordan decomposition, Radon-Nikodym theorem, Lebesgue decomposition theorem. Functions of bounded variation. Complex measures. Vitali covering lemma, differentiation and fundamental theorem of calculus.
- References: Gerald B. Folland, Real Analysis: Modern Techniques and Their Applications, Pure and Applied Mathematics, A Wiley Series.

Harmonic Analysis

- Instructor: Prof. Soumyashant Nayak
- Curriculum: Haar Measure, Banach Algebras, the Gelfand-Naimark theorem, Plancherel's theorem, Pontryagin Duality, Structure of locally compact abelian groups, Operators on Hilbert spaces, Representation theory of locally compact abelian groups, Peter-Weyl theorem, Representations of SU(2), Direct Integrals, the Selberg trace formula, the Heisenberg group, Wavelets.
- References: A. Deitmar and S. Echterhoff: Principles of Harmonic Analysis
- **Grade: 97/100**

Complex Analysis

- Instructor: Prof. Jaydeb Sarkar
- Curriculum: Holomorphic functions and the Cauchy-Riemann equations, Power series, Functions defined by power series as holomorphic functions, Complex line integrals and Cauchys theorem, Cauchys integral formula. Representations of holomorphic functions in terms of power series. Zeroes of analytic functions, Liouvilles theorem, The fundamental theorem of algebra, The maximum modulus principle, Schwarzs lemma, The argument principle, The open mapping property of holomorphic functions. The calculus of residues and evaluation of integrals using contour integration.
- References: T. W. Gamelin: Complex Analysis, J. B. Conway: Functions of one Complex Variable.

- **Grade: 100/100**

Geometry

- Instructor: Prof. Jishnu Biswas
- Curriculum: Affine Geometry: Affine spaces, mappings. Thales theorem, Pappus the orem, Desargues theorem. Convexity. Euclidean Geometry: Euclidean vector spaces, Euclidean affine spaces. Linear isometries and rigid motions. Spheres, Spherical triangles, Polyhedra and Eulers formula. Projective Geometry: Projective spaces, Pappus and Desargues theorem. Projective duality, Projective transformations. Cross Ratio. Complex projective line and circular group
- References: M. Audin: Geometry.
- **Grade: 72/100**

Stochastic Processes

- Instructor: Prof. Yogeshwaran D.
- Curriculum: Discrete time martingales: Optional Stopping theorem, Martingale convergence theorem, Doobs inequality and convergence.
- Branching Processes: Model definition. Connection with martingales. Probability of survival. Mean and variance of number of individuals.
- Discrete time markov chains: Classification of states, Stationary distribution, reversibility and convergence. Random walks and electrical networks. Collision and recurrence.
- Probabilistic Inequalities and Applications: First and Second Moment methods. Applications to Longest increasing subsequences, Random k-Sat problem and connectivity threshold for Erdos-Renyi graphs. Chernoff bounds and Johnson-Lindenstrauss lemma.
- References Sebastien Roch: Modern Discrete Probability, Stephane Boucheron, Gabor Lugosi, Pascal Massart: Concentration inequalities A nonasymptotic theory of independence.
- **Grade: 97/100**:

Statistics Coursework

Introduction to Statistics and Computation with Data

- Instructor: Prof. Siva Athreya
- Curriculum: R- Basics, Introduction to exploratory Data analysis using R: Descriptive statistics; Graphical representation of data: Histogram, Stem-leaf diagram, Box-plot; Visualizing categorical data. Review of Basic Probability: Basic distributions, properties; simulating samples from standard distributions using R commands. Sampling distributions based on normal populations: t, chi^2 and F distributions. Model fitting and model checking: Basics of estimation, method of moments, Basics of testing including goodness of fit tests, interval estimation; Distribution theory for transformations of random vectors; Nonparametric tests: Sign test, Signed rank test,Wilcoxon-Mann-Whitney test. Bivariate data: covariance, correlation and least squares. Resampling methods: Jackknife and Bootstrap.
- References: H Wickham and G Grolemund: R for Data Science, S. Athreya, D. Sarkar, S. Tanner: Probability and Statistics with Examples Using R.
- **Grade: 87/100**

Introduction to Statistical Inference

- Instructor: Prof. Rituparna Sen
- Curriculum: Sufficiency, Exponential family, Bayesian methods, Moment methods, Maximum likelihood estimation. Criteria for estimators; UMVUE, Fisher Information. Multivariate normal distribution: Marginals, Conditionals; Distribution of linear forms. Order statistics and their distributions. Large sample theory: Consistency, asymptotic normality, asymptotic relative efficiency. Elements of hypothesis testing; Neyman-Pearson Theory, UMP tests, Likelihood ratio and related tests, Large sample tests. Confidence intervals
- References: G. Casella and R. L. Berger: Statistical Inference.
- **Grade: 97/100**

Introduction to Linear Models and Regression

- Instructor: Prof. Mohan Delampady
- Curriculum: Multivariate distributions and properties; Multivariate densities; Independence, marginal and conditional distributions; Distributions of functions of continuous random vectors; Examples of multivariate densities: Dirichlet and multivariate normal distributions; Transformations and quadratic forms. Review of matrix algebra involving projection matrices and matrix decompositions; Fisher-Cochran Theorem. Simple linear regression and Analysis of variance.General linear model, Matrix formulation, Estimation in linear model, Gauss- Markov theorem, Estimation of error variance. Testing in the linear model, Analysis of variance. Partial and multiple correlations, Multiple comparisons. Stepwise regression, Regression diagnostics. Odds ratios, Logit model. Splines and Lasso.
- References: CR Rao: Linear Statistical Inference and Its Applications
- **Grade: 97/100**