SYSTEM ANALYSIS

Using MATLAB to determine the ROCs of rational Z-transforms.

The statement

[z,p,k]= tf2zp (num,den)

determines the zeros, poles and gain constants of a rational z-transform expressed as ratio of polynomials in descending powers of z.

Input arguments are the row vectors " num" and "den" containing the coefficients of the numerator and the denominator polynomials in descending powers of z.

The statement

[num,den]= zp2tf (z,p,k)

is used to reverse the process. Given the zeros, poles and gains it computes the coefficients of the numerator and denominator polynomials.

From the zero-pole description the factored form of the transfer function can be obtained using the function below:

sos = zp2sos(z,p,k)

This function computes the coefficients of each second-order factor given as an L×6 matrix

$$sos = \left[\begin{array}{ccc|ccc} b_{01} & b_{11} & b_{21} & a_{01} & a_{11} & a_{21} \\ b_{02} & b_{12} & b_{22} & a_{02} & a_{12} & a_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{0L} & b_{1L} & b_{2L} & a_{0L} & a_{1L} & a_{2L} \end{array}\right]$$

where k^{th} row contains the coefficients of the numerator and denominator of the k^{th} second order factor of the z-transform.

The pole-zero plot of a rational z-transform can also be plotted using the m-file function zplane.

zplane(zeros, poles) note: need to enter values as column vectors

zplane(num,den) note: input arguments need to be entered as row vectors.

Example :

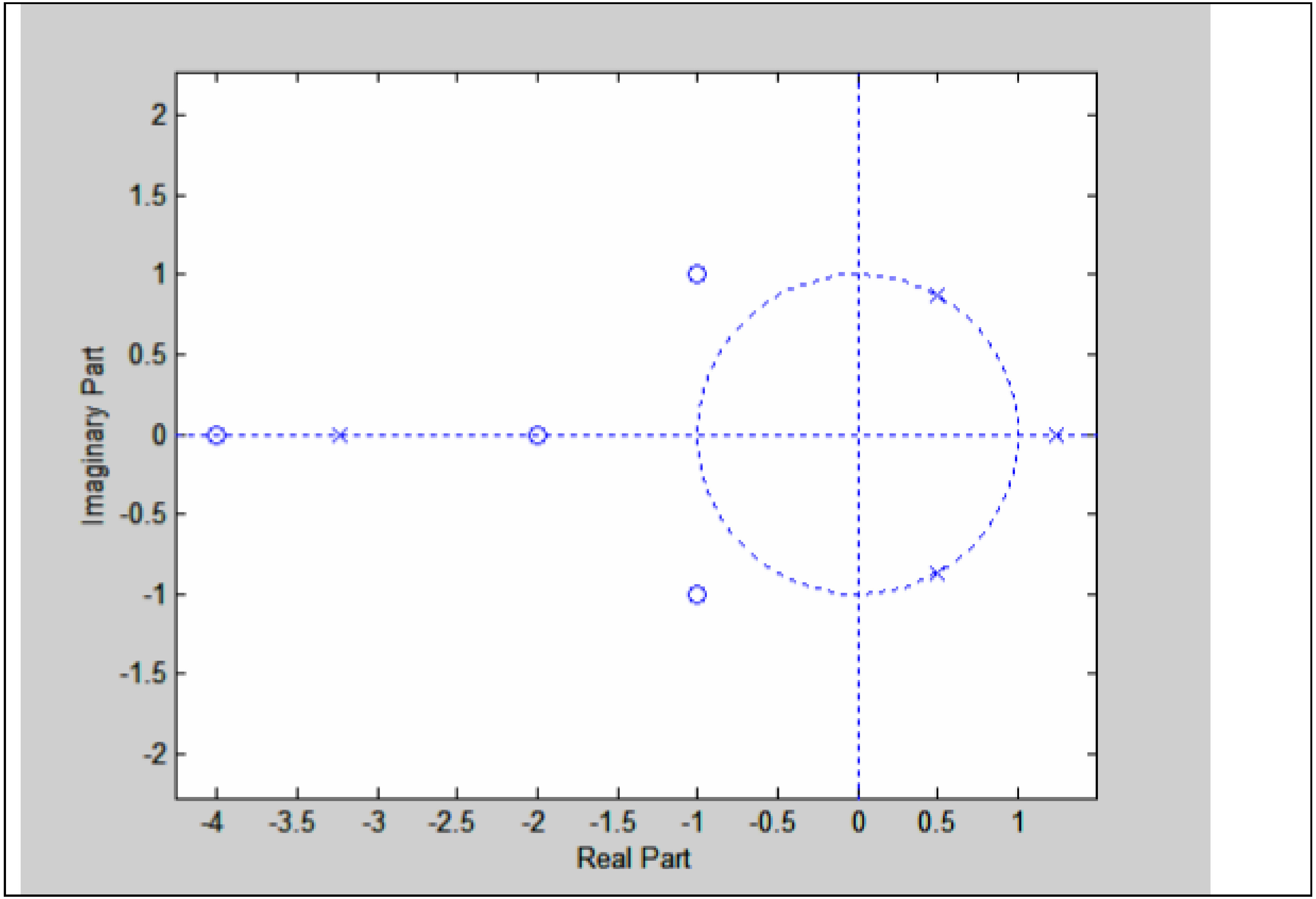
Express the following z-transform in factored form, plot its poles and zeros and then determine the ROC.

$$G[z] = \frac{2z^4 + 16z^3 + 44z^2 + 56z + 32}{3z^4 + 3z^3 - 15z^2 + 18z - 12}$$

we can use a matlab program like the one below:

```
num = input(`Type in the numerator coefficients =`)
den = input(`Type in the denominator coefficients =`)
[z,p,k]=tf2pz(num,den)
m = abs(p)   % to find distance from origin of poles
disp(`Zeros are at`); disp(z);
disp(`poles are at`); disp(p);
disp(`gain constant`); disp(k);
disp(`Radius of poles`); disp(m);

sos= zp2sos(z,p,k);
disp('Second order sections');disp(real(sos));
zplane(num,den)
```



Imaginary Part
Real Part
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1

```
>> sos= zp2sos(z,p,k);
>> sos

sos =

    0.6667    4.0000    5.3333    1.0000    2.0000   -4.0000
    1.0000    2.0000    2.0000    1.0000   -1.0000    1.0000
```

Now we can use the above information to write the factored form of the z-transform as :

$$G[z] = \frac{\left(0.0667 + 0.4z^{-1} + 0.5333z^{-2}\right)\left(10.0 + 20.0z^{-1} + 20z^{-2}\right)}{\left(1.0 + 2.0z^{-1} - 4.0z^{-2}\right)\left(1.0 - 1.0z^{-1} + 1.0z^{-2}\right)}$$

$$= (0.667)\frac{\left(1 + 6z^{-1} + 8z^{-2}\right)\left(1 + 2z^{-1} + 2z^{-2}\right)}{\left(1 + 2z^{-1} - 4z^{-2}\right)\left(1 - z^{-1} + z^{-2}\right)}$$

by observing each quantity we can find the ROCs

ROC regions has to be bounded by the poles

$$R1: \quad \infty \geq |z| > 3.2361$$

$$R2: \quad 3.2361 > |z| > 1.2361$$

$$R3: \quad 1.2361 > |z| > 1$$

$$R4: \quad 1 > |z| \geq 0$$

- Freqz: The function freqz is used to compute the frequency response of systems expressed by difference equations or rational transfer functions.
- [H,w]=freqz(b,a,N); where N is a positive integer, returns the frequency response H and the vector w with the N angular frequencies at which H has been calculated (i.e. N equispaced points on the unit circle, between 0 and π). If N is omitted, a default value of 512 is assumed. If no output argument is specified, the amplitude plot and the phase plot of the frequency response are directly displayed.
- The function **"freqz"** is used to compute the frequency response. Refer ***[H,w]=freqz(b,a,N);*** where N is a positive integer, returns the frequency response H and the vector w with the N angular frequencies at which H has been calculated (i.e. N equispaced points on the unit circle, between 0 and π). If N is omitted, a default value of 512 is assumed. If no output argument is specified, the amplitude plot and the phase plot of the frequency response are directly displayed.
- [H,w]=freqz(b,a,w); where w is a vector of frequencies (in radians, e.g. w=-pi:1/100:pi;) computes the frequency response at the frequencies specified by w. This function can be used to evaluate the DTFT of a sequence x on any desired set of frequencies w, e.g. with the command [X,w]=freqz(x,1,w); See help freqz for a complete reference.
- See ***[h,t]=impz(b,a)*** produces the impulse response in vector h and the time axis in vector t. If the output arguments h and t are omitted, a plot of the impulse response is directly displayed. If the impulse response is of infinite length, only its initial part is computed.

PRACTICE:1

Example : $H(z) = \frac{1}{1-0.9z^{-1}}$, i.e. a system with exponentially decaying impulse response

$h[n] = (0.9)^n u[n]$

```
b=1;
a=[1 -0.9];
[H,w]=freqz(b,a);
subplot(211)
plot(w/pi, 20*log10(abs(H))); % amplitude plot in decibel
```

```
xlabel('frequency in \pi units'); ylabel('Magnitude in dB');
title('Magnitude Response')
subplot(212)
plot(w/pi, angle(H)/pi);      % phase plot
xlabel('frequency in \pi units'); ylabel('Phase in radians/\pi');
title('Phase Response')
```

EXPERIMENT_3

- For the given Stable System, Plot The Pole-zero Plot, Impulse Response, Magnitude and Phase Response. Answer is given for your reference.

$$H(z)=\frac{2+2z^{-1}}{1-0.8z^{-1}}$$

```
b=[2,2];
a=[1,-0.8];
w=[-2.5:1/10:2.5];
[H,w]=freqz(b,a,w);
```