Shiv Nadar University

Department of Electrical Engineering-(SOE)
EED364-Graph Signal Processing- Monsoon 2019

Instructor: Prof. Vijay Kumar Chakka **Lab No.**: 3

Topic: Influential Node Ranking of The Given Graph Using Degree Centrality and Entropy Measures

1. Consider an undirected, unweighted complete bipartite graph $\mathcal{G} = \{\mathcal{V}, \mathcal{B}\}$, characterized by its adjacency matrix $\mathbf{A_{N\times N}}$, where $\mathcal{V} = \{0, 1, 2, 3, 4\}$, which is partitioned into two disjoint sets $\mathcal{E} = \{0, 1\}$ and $\mathcal{H} = \{2, 3, 4\}$. Write a MATLAB program to rank the influential node of $\mathcal{G}$ using degree centrality (DC) approach.
   Note 1: In DC approach, the rank of a vertex is defined based on its degree. A vertex with highest degree is assigned as rank 1, the vertex with second highest degree assigned as rank 2 and so on. Same rank is assigned to the vertices with same degree.

2. Consider the graph $\mathcal{G}$ given in question 1 and determine the rank of the nodes using Structural Information Entropy (SIE) approach.
   Note 2: The structural information entropy of a vertex v_i, denoted as I_i^s with M neighbours is defined as follows:

$$I_i^s = -\sum_{i=1}^{M+1} \frac{DC_i}{\sum\limits_{i}^{M+1} DC_i} log \left(\frac{DC_i}{\sum\limits_{i}^{M+1} DC_i} \right), \tag{1}$$

   where DC_i indicates the Degree Centrality (DC) of vertex v_i. In SIE approach, the rank of a vertex is defined based on I_i^s value.

3. Consider a weighted, directed bipartite graph $\mathcal{G} = \{\mathcal{V}, \mathcal{B}\}$, with adjacency matrix

$$\mathbf{A} = [0\ 0\ 2\ 3\ 5; 0\ 0\ 3\ 2\ 2; 2\ 3\ 0\ 0\ 0; 2\ 4\ 0\ 0\ 0; 2\ 4\ 0\ 0\ 0]. \tag{2}$$

   Plot its 2D graph. Find the structure information entropy I_i^s of each vertex and rank each node based on this. **Hint:** Consider the following coordinates ($\mathbf{C}$) for plotting the above graph: $\mathbf{C} = [1\ 2; 1\ 4; 4\ 2;\ 4\ 3; 4\ 5]$.

4. Consider the graph $\mathcal{G}$ given in question 3 and perform the following:
   (a) Consider a vertex v_i and find the interaction frequency entropy, i.e., I_i^f using edge weights and structural information entropy, i.e., I_i^s.
   (b) Combine both the structure and interacted based entropies to get the total influence of a vertex v_i, i.e., I_i using the following equation:

$$I_i = w_1 I_i^s + w_2 I_i^f, \tag{3}$$

   where w_1 and w_2 are weight coefficients.
   (c) Rank each vertex using their corresponding total influence value.

   Note: Interaction frequency entropy of a vertex v_i having M neighbours with edge weights C_{ij} is given by

$$I_i^f = -\sum_{j=1}^{M} \frac{C_{ij}}{\sum\limits_{k=1}^{M} C_{ik}} log \left(\frac{C_{ij}}{\sum\limits_{k=1}^{M} C_{ik}} \right) \tag{4}$$

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