Shiv Nadar University

Department of Electrical Engineering-(SOE)

EED364-Graph Signal Processing- Monsoon 2019

Instructor: Prof. Vijay Kumar Chakka **Lab No.**: 4

Topic: Properties of Laplacian Eigenvalue Decomposition and Synthesizing A Graph From Different Eigenvector Spaces

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1. Verify the following properties of Laplacian eigenvalue decomposition

   (a) At least one eigenvalue of the Laplacian matrix is zero i.e., $\lambda_0 = 0$ and its corresponding constant unit energy eigenvector is $\mathbf{U}_0 = \frac{1}{\sqrt{(N)}}[1, 1, \ldots, 1]^T$.

   (b) The Laplacian spectrum of a complete unweighted graph, with N vertices, is $\lambda_k \in \{0, N, \ldots, N\}$.

   (c) For a $\mathcal{J}$-regular graph, the eigenvectors of the Laplacian and Adjacency matrices are identical, with the following relation for the eigenvalues: $\lambda_k^{(L)} = \mathcal{J} - \lambda_k^{(A)}$.

   (d) Eigenvalues of normalized Laplacian matrix ($\mathbf{L}_N$) are non-negative and bounded as follows: $0 \leq \lambda \leq 2$. The equality for the upper bound holds if and only if the graph is a bipartite graph.

   (e) Graph spectrum folding of the matrix $\mathbf{L}_N$ of a bipartite graph is $\lambda_k = 2 - \lambda_{N-k}$.

2. Consider the given orthogonal eigenvector matrix $[\mathbf{U}]_{N \times N}$ (*A1.mat*), where $N = 8$. Now construct different Laplacian matrices by considering the following set of eigenvalues

   - $\lambda_k = [1, 0, 0, 0, 0, 0, 0, 0]$, $\lambda_k = [0, 1, 0, 0, 0, 0, 0, 0]$, $\lambda_k = [0, 0, 1, 1, 0, 0, 0, 0]$.
   - $\lambda_k = [0, 0, 0, 0, 1, 1, 0, 0]$, $\lambda_k = [0, 0, 0, 0, 1, 1, 1, 1]$, $\lambda_k = [1, 1, 1, 1, 1, 1, 1, 1]$.

   Is that the constructed Laplacian matrix is valid? If so, plot the graph (Use Matlab built-in function as well as user-defined function).

3. Repeat question 2, for the given different orthogonal eigenvector matrices *A2.mat*, *A3.mat*, *A4.mat* and non-orthogonal eigenvector matrices *A5.mat*, *A6.mat*.

4. Write your overall observations and conclusion.

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