Eco 213: Basic Data Analysis and Econometrics
Lecture 8: Autocorrelation
March 30, 2019

Dr. Garima Malik
Department of Economics
Shiv Nadar University

Outline

- What is autocorrelation?
- Causes of Autocorrelation
- OLS and Autocorrelation
- Tests for Autocorrelation

Introduction

- Autocorrelation occurs in time-series studies when the errors associated with a given time period carry over into future time periods.
- For example, if we are predicting the growth of stock dividends, an overestimate in one year is likely to lead to overestimates in succeeding years.

Introduction

- Times series data follow a natural ordering over time.
- It is likely that such data exhibit intercorrelation, especially if the time interval between successive observations is short, such as weeks or days.

Introduction

- We expect stock market prices to move or move down for several days in succession.
- In situation like this, the assumption of no auto or serial correlation in the error term that underlies the CLRM will be violated.
- We experience autocorrelation when

$$E(u_i u_j) \neq 0$$

Causes of Autocorrelation

1. **Inertia -** Macroeconomics data experience cycles/business cycles.
2. **Specification Bias- Excluded variable**

- Appropriate equation:

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + \beta_4 X_{4t} + u_t$$

- Estimated equation

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + v_t$$

- Estimating the second equation implies

$$v_t = \beta_4 X_{4t} + u_t$$

Causes of Autocorrelation

3. Specification Bias- Incorrect Functional Form

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{2t}^2 + v_t$$

$$Y_t = \beta_1 + \beta_2 X_{2t} + u_t$$

$$u_t = \beta_3 X_{2t}^2 + v_t$$

4. **Cobweb Phenomenon**

- In agricultural market, the supply reacts to price with a lag of one time period because supply decisions take time to implement. This is known as the cobweb phenomenon.
- Thus, at the beginning of this year's planting of crops, farmers are influenced by the price prevailing last year.

Causes of Autocorrelation

5. **Lags**

$$Consumption_t = \beta_1 + \beta_2 Consumption_{t-1} + u_t$$

- The above equation is known as autoregression because one of the explanatory variables is the lagged value of the dependent variable.
- If you neglect the lagged the resulting error term will reflect a systematic pattern due to the influence of lagged consumption on current consumption.

Causes of Autocorrelation

6. Data Manipulation

$$Y_t = \beta_1 + \beta_2 X_t + u_t \qquad Y_{t-1} = \beta_1 + \beta_2 X_{t-1} + u_{t-1}$$

$$\Delta Y_t = \beta_2 \Delta X_t + v_t$$

- This equation is known as the first difference form and dynamic regression model. The previous equation is known as the level form.
- Note that the error term in the first equation is not autocorrelated but it can be shown that the error term in the first difference form is autocorrelated.

Causes of Autocorrelation

6. **Nonstationarity**

- When dealing with time series data, we should check whether the given time series is stationary.
- A time series is stationary if its characteristics (e.g. mean, variance and covariance) are time invariant; that is, they do not change over time.
- If that is not the case, we have a nonstationary time series.

OLS Estimation

$$Y_t = \beta_1 + \beta_2 X_t + u_t \qquad E(u_i u_j) \neq 0$$

ρ

- Assume that the error term can be modeled as follows:

  $$u_t = \rho u_{t-1} + \in_t \qquad -1 < \rho < 1$$

- ρ is known as the coefficient of autocovariance and the error term satisfies the OLS assumption.
- This Scheme is known as an Autoregressive (AR(1))process

OLS Estimation

$$Var(u_t) = E(u_t^2) = \frac{\sigma_\in^2}{1-\rho^2}$$

$$Cov(u_t, u_{t+s}) = E(u_t u_{t-s}) = \rho^s \frac{\sigma_\in^2}{1-\rho^2}$$

$$Cor(u_t, u_{t+s}) = \rho^s$$

$$Var(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_t^t}\left[1 + 2\rho\frac{\sum x_t x_{t-1}}{\sum x_t^2} + 2\rho^2\frac{\sum x_t x_{t-1}}{\sum x_t^2} + ... + 2\rho^{n-1}\frac{\sum x_t x_{t-1}}{\sum x_t^2}\right]$$

BLUE Estimator

- Under the AR (1) process, the BLUE estimator of β_2 is given by the following expression.

$$\hat{\beta}_2^{GLS} = \frac{\sum_{t-2}^{n} (x_t - \rho x_{t-1})(y_t - \rho y_{t-1})}{\sum_{t-2}^{n} (x_t - \rho x_{t-1})^2} + C$$

$$Var(\hat{\beta}_2^{GLS}) = \frac{\sigma^2}{\sum_{t-2}^{n} (x_t - \rho x_{t-1})^2} + D$$

BLUE Estimator

- The Gauss Theorem provides only the sufficient condition for OLS to be BLUE.
- Therefore, in some cases, it can happen that OLS is BLUE despite autocorrelation. But such cases are very rare.

OLS and Serial Correlation

- The implications of serial correlation for OLS are similar to those of heteroskedasticity:
  - OLS is still unbiased
  - OLS is inefficient
  - The OLS formula for estimated standard errors is incorrect
- “Fixes” are more complicated

Consequences of Using OLS

- *OLS Estimation Allowing for Autocorrelation*
- As noted, the estimator is no more not BLUE, and even if we use the variance, the confidence intervals derived from there are likely to be wider than those based on the GLS procedure.
- Hypothesis testing: we are likely to declare a coefficient statistically insignificant even though in fact it may be.
- One should use GLS and not OLS.

Consequences of Using OLS

- ***OLS Estimation Disregarding Autocorrelation***
- The estimated variance of the error is likely to overestimate the true variance
- Over estimate R-square
- Therefore, the usual *t* and *F* tests of significance are no longer valid, and if applied, are likely to give seriously misleading conclusions about the statistical significance of the estimated regression coefficients.

Detecting Autocorrelation

- *Graphical Method*
- There are various ways of examining the residuals.
- The time sequence plot can be produced.
- Alternatively, we can plot the standardized residuals against time.
- The standardized residuals is simply the residuals divided by the standard error of the regression.
- If the actual and standard plot shows a pattern, then the errors may not be random.
- We can also plot the error term with its first lag.

Detecting Autocorrelation

- *The Durbin Watson Test*

$$d = \frac{\sum_{t=2}^{t=n} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{t=n} \hat{u}_t^2}$$

- It is simply the ratio of the sum of squared differences in successive residuals to the RSS.
- The number of observation is n-1 as one observation is lost in taking successive differences.

Detecting Autocorrelation

- A great advantage of the Durbin Watson test is that based on the estimated residuals. It is based on the following assumptions:

1. The regression model includes the intercept term.
2. The explanatory variables are nonstochastic, or fixed in repeated sampling.
3. The disturbances are generated by the first order autoregressive scheme.

Detecting Autocorrelation

4. The error term is assumed to be normally distributed.
5. The regression model does not include the lagged values of the dependent an explanatory variables.
6. There are no missing values in the data.

- Durbin-Watson have derived a lower bound d_L and an upper bound d_U such that if the computed d lies outside these critical values, a decision can be made regarding the presence of positive or negative serial correlation.

Detecting Autocorrelation

$$d = \frac{\sum \hat{u}_t^2 + \sum \hat{u}_{t-1}^2 - 2\sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2} \approx 2\left(1 - \frac{\sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2}\right)$$

$$d \approx 2(1 - \hat{\rho})$$

- Where $\hat{\rho} = \frac{\sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2}$

- But since $-1 \le \rho \le 1$, this implies that $0 \le d \le 4$.

Detecting Autocorrelation

- If the statistic lies near the value 2, there is no serial correlation.
- But if the statistic lies in the vicinity of 0, there is positive serial correlation.
- The closer the d is to zero, the greater the evidence of positive serial correlation.
- If it lies in the vicinity of 4, there is evidence of negative serial correlation

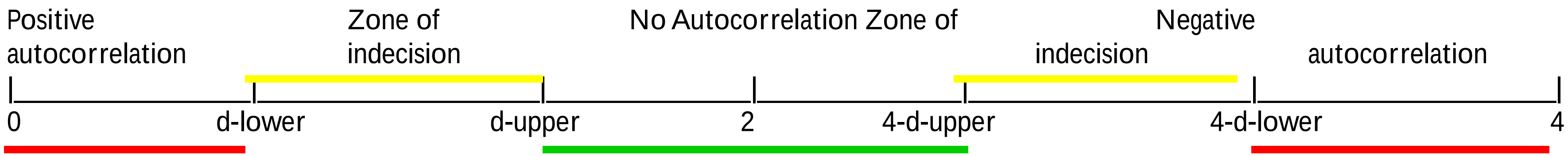
Detecting Autocorrelation

- If it lies between d_L and d_U / 4 –d_L and 4 – d_U, then we are in the zone of indecision.
- The mechanics of the Durbin-Watson test are as follows:
- Run the OLS regression and obtain the residuals
- Compute d
- For the given sample size and given number of explanatory variables, find out the critical d_L and d_U.
- Follow the decisions rule

Checking for Autocorrelation

- Test: Durbin-Watson statistic:

$$\boxed{d = \frac{\sum (e_i - e_{i-1})^2}{\sum e_i^2}, \text{ for n and K - 1 d.f.}}$$



- Autocorrelation is clearly evident
- Ambiguous – cannot rule out autocorrelation
- Autocorrelation in not evident

Detecting Autocorrelation

- Use Modified d test if d lies in the zone in the of indecision. Given the level of significance α,
- H_o: $\rho = 0$ versus H_1: $\rho > 0$, reject H_o at α level if $d < d_U$. That is there is statistically significant evidence of positive autocorrelation.
- H_o : $\rho = 0$ versus H_1 : $\rho < 0$, reject H_o at α level if 4- $d < d_U$. That is there is statistically significant evidence of negative autocorrelation.
- H_o : $\rho = 0$ versus H_1 : $\rho \neq 0$, reject H_o at 2α level if $d < d_U$ and 4- $d < d_U$. That is there is statistically significant evidence of either positive or negative autocorrelation.

Detecting Autocorrelation

- The Breusch – Godfrey
- The BG test, also known as the LM test, is a general test for autocorrelation in the sense that it allows for

1. nonstochastic regressors such as the lagged values of the regressand;
2. higher-order autoregressive schemes such as AR(1), AR (2)etc.; and
3. simple or higher-order moving averages of white noise error terms.

Detecting Autocorrelation

- The Breusch – Godfrey
- The BG test, also known as the LM test, is a general test for autocorrelation in the sense that it allows for

1. nonstochastic regressors such as the lagged values of the regressand;
2. higher-order autoregressive schemes such as AR(1), AR (2)etc.; and
3. simple or higher-order moving averages of white noise error terms.

Detecting Autocorrelation

- Consider the following model:

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \ldots\ldots.. \rho_p u_{t-p} + \in_t$$

$$H_o : \rho_1 = \rho_2 = \ldots.. = \rho_p = 0_t$$

- Estimate the regression using OLS
- Run the following regression and obtained the R-square

Detecting Autocorrelation

- If the sample size is large, Breusch and Godfrey have shown that $(n - p)\, R^2$ follow a chi-square
- If $(n - p)\, R^2$ exceeds the critical value at the chosen level of significance, we reject the null hypothesis, in which case at least one rho is statistically different from zero.

Detecting Autocorrelation

- Point to note:
- The regressors included in the regression model may contain lagged values of the regressand Y. In DW, this is not allowed.
- The BG test is applicable even if the disturbances follow a p^{th-}order moving averages (MA) process, that is u_t is integrated as follows:

$$u_t = \varepsilon_t + \lambda_1 \varepsilon_{t-1} + \lambda_2 \varepsilon_{t-2} + \dots\dots\dots \lambda_p \varepsilon_{t-p}$$

- A drawback of the BG test is that the value of p, the length of the lag cannot be specified as a priori.

Detecting Autocorrelation

- ***Model Misspecification vs. Pure Autocorrelation***
- It is important to find out whether autocorrelation is pure autocorrelation and not the result of mis-specification of the model.

Detecting Autocorrelation

- *The Method of GLS*

$$Y_t = \beta_1 + \beta_2 X_t + u_t \qquad u_t = \rho_1 u_{t-1} + \in_t \qquad -1 < \rho < 1$$

- There are two cases when (1) ρ is known and (2) ρ is not known

Detecting Autocorrelation

- When ρ is known
- If the regression holds at time t, it should hold at time t-1, i.e.

$$Y_{t-1} = \beta_1 + \beta_2 X_{t-1} + u_{t-1}$$

- Multiplying the second equation by ρ gives

$$\rho Y_{t-1} = \rho\beta_1 + \rho\beta_2 X_{t-1} + \rho u_{t-1}$$

- Subtracting (3) from (1) gives

$$Y_t - \rho Y_{t-1} = \beta_1(1-p) + \beta_2(X_t - \rho X_{t-1}) + \varepsilon$$

$$\varepsilon_t = (u_t - \rho u_{t-1})$$

Detecting Autocorrelation

- The equation can be

$$Y_t^* = \beta_t^* + \beta_t^* X_t^* + \varepsilon_t$$

- The error term satisfies all the OLS assumptions
- Thus we can apply OLS to the transformed variables *Y** and *X** and obtain estimation with all the optimum properties, namely BLUE
- In effect, running this equation is the same as using the GLS.

Detecting Autocorrelation

- When ρ is unknown, there are many ways to estimate it.
- Assume that $\rho = +1$ the generalized difference equation reduces to the first difference equation

$$Y_t - Y_t = \beta_2 (X_t - X_{t-1}) + (u_t - u_{t-1})$$

$$\Delta Y_t = \beta_2 \Delta X_t + \varepsilon_t$$

Detecting Autocorrelation

- The first difference transformation may be appropriate if the coefficient of autocorrelation is very high, say in excess of 0.8, or the Durbin-Watson *d* is quite low.

Detecting Autocorrelation

- ρ Based on Durbin-Watson *d* statistic
- From the Durbin – Watson Statistics, we know that

$$\rho \approx 1 - \frac{d}{2}$$

- In reasonably large samples one samples one can obtain rho from this equation and use it to transform the data as shown in the GLS.

Detecting Autocorrelation

- ρ Based on Durbin-Watson *d* statistic
- From the Durbin – Watson Statistics, we know that

$$\rho \approx 1 - \frac{d}{2}$$

- In reasonably large samples one samples one can obtain rho from this equation and use it to transform the data as shown in the GLS.

Detecting Autocorrelation

- ρ Based on the error terms

$$u_t = \rho_1 u_{t-1} + \in_t$$

- Estimate the following equation

$$\hat{u}_t = \hat{\rho}.\hat{u}_{t-1} + v_t$$