$$\frac{\partial E_{MSE}}{\partial b} = \frac{\frac{1}{N}\partial \sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\partial b}$$

$$= \frac{1}{N} \cdot \frac{\partial \sum_{i=1}^{N}(y_i - (ax_i + b))^2}{\partial b}$$

$$= \frac{1}{N}\sum_{i=1}^{N} 2(y_i - ax_i - b)(-1)$$

For critical point

$$\frac{1}{N} 2 \sum_{i=1}^{N} -(y_i - ax_i - b) = 0$$

$$\sum_{i=1}^{N} (-y_i + ax_i + b) = 0$$

$$b = \frac{y_i - ax_i}{N}$$

$$\frac{\partial E_{MSE}}{\partial a} = \frac{\frac{1}{N}\partial \sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\partial b}$$

$$= \frac{\frac{1}{N}\partial \sum_{i=1}^{N}(y_i - ax_i - b)^2}{\partial a}$$

$$= \frac{1}{N} \, 2\left(y_i - ax_i - b\right)\cdot\left(-x_i\right)$$

$$= \frac{2}{N}\sum_{i=1}^{N}\left(-y_i x_i + a x_i^2 + x_i b\right)$$

For Critical point

$$\frac{2}{N}\left(-y_i x_i + a x_i^2 + x_i b\right) = 0$$

$$\sum_{i=1}^{N}\left(-x_i y_i + a x_i^2 + b x_i\right) = 0$$

$$\frac{\overline{-x_i y_i + a x_i^2 + b x_i}}{N} = 0$$

$$\overline{-x_i y_i + a x_i^2 + b x_i} = 0$$

$$\Rightarrow \overline{-x_i y_i + a x_i^2 + \left(\frac{\overline{y_i - a x_i}}{N}\right) x_i} = 0$$

$$\Rightarrow \frac{\overline{-N x_i y_i + N \cdot a \cdot x_i^2 + (y_i - a x_i)(x_i)}}{N} = 0$$

$$\Rightarrow -N\overline{x_i y_i} + N a \overline{x_i^2} + \overline{y_i}\,\overline{x_i} - a\left(\overline{x_i}\right)^2 =$$

$$a = \frac{N.\overline{x_i y_i} - \cancel{\overline{y_i}\,\overline{x_i}}\ \overline{y_i}.\overline{x_i}}{N\overline{x_i^2} - \left(\overline{x_i}\right)^2}$$

Therfore

$$b = \frac{\overline{y_i} - \dfrac{N.\overline{x_i y_i} - \overline{y_i}.\overline{x_i}}{N.\overline{x_i^2} - \left(\overline{x_i}\right)^2}\left(\overline{x_i}\right)}{N}$$