

$$\begin{aligned}\frac{\partial E_{MSE}}{\partial b} &= \frac{\frac{1}{N} \frac{\partial \sum_{i=1}^N (y_i - \hat{y}_i)^2}{\partial b}}{\partial b} \\ &= \frac{\frac{1}{N} \frac{\partial \sum_{i=1}^N (y_i - (ax_i + b))^2}{\partial b}}{\partial b} \\ &= \frac{1}{N} \sum_{i=1}^N 2(y_i - ax_i - b)(-1)\end{aligned}$$

For critical point

$$\frac{1}{N} 2 \sum_{i=1}^N -(y_i - ax_i - b) = 0$$

$$\sum_{i=1}^N (y_i + ax_i + b) = 0$$

$$b = \frac{\bar{y}_i - a\bar{x}_i}{N}$$

$$\begin{aligned}\frac{\partial E_{MSE}}{\partial a} &= \frac{\frac{1}{N} \frac{\partial \sum_{i=1}^N (y_i - \hat{y}_i)^2}{\partial a}}{\partial a} \\ &= \frac{\frac{1}{N} \frac{\partial \sum_{i=1}^N (y_i - ax_i - b)^2}{\partial a}}{\partial a}\end{aligned}$$



$$= \frac{1}{N} \sum_{i=1}^N 2 (y_i - ax_i - b) \cdot (-x_i)$$

$$= \frac{2}{N} \sum_{i=1}^N (-y_i x_i + ax_i^2 + x_i b)$$

For Critical point

$$\frac{2}{N} (-y_i x_i + ax_i^2 + x_i b) = 0$$

$$\sum_{i=1}^N (-x_i y_i + ax_i^2 + x_i b) = 0$$

$$\frac{-\sum x_i y_i + a \sum x_i^2 + b \sum x_i}{N} = 0$$

$$-\sum x_i y_i + a \sum x_i^2 + b \sum x_i = 0$$

\Rightarrow

$$-\sum x_i y_i + a \sum x_i^2 + \left(\frac{\sum y_i - a \sum x_i}{N} \right) \sum x_i = 0$$

$$\Rightarrow \frac{-N \sum x_i y_i + N a \sum x_i^2 + (\sum y_i - a \sum x_i) \sum x_i}{N} = 0$$

$$\Rightarrow -N \bar{x} \bar{y} + N a \bar{x}^2 + \bar{y} \bar{x} - a (\bar{x})^2 = 0$$

$$a = \frac{N \cdot \bar{x} \bar{y} - \sum x_i y_i}{N \bar{x}^2 - (\bar{x})^2}$$

Therefore, $b = \frac{N \cdot \bar{x} \bar{y} - \sum x_i y_i}{N \bar{x}^2 - (\bar{x})^2} (\bar{x} - \bar{x})$

$$0 = (100 + 100 + 100) \cdot \frac{2}{4}$$

$$0 = (100 + 100 + 100) \cdot \frac{2}{4}$$

$$0 = (100 + 100 + 100) \cdot \frac{2}{4}$$

$$0 = (100 + 100 + 100) \cdot \frac{2}{4}$$

$$0 = (100 + 100 + 100) \cdot \frac{2}{4}$$

$$= 100 \left(\frac{100 - 100}{4} \right) + 100 + 100$$

$$(100)(100 - 100) + 100 + 100$$

$$(100)0 - 100 \cdot 100 + 100 + 100$$