# PROJECT – SUDOKU

## INTRODUCTON

Sudoku is a logic-based, combinatorial number-placement puzzle. In classic sudoku, the objective is to fill a 9×9 grid with digits so that each column, each row, and each of the nine 3×3 sub-grids that compose the grid contain all of the digits from 1 to 9. The puzzle setter provides a partially completed grid, which for a well-posed puzzle has a single solution. A number should only appear once in a row, column and 3 x 3 sub-grid.

The game in its current form was invented by American Howard Garns in 1979 and published by Dell Magazines as "Numbers in Place." In 1984, Maki Kaji of Japan published it in the magazine of his puzzle company Nikoli. He gave the game its modern name of Sudoku, which means "Single Numbers." The puzzle became popular in Japan and was discovered there by New Zealander Wayne Gould, who then wrote a computer program that would generate Sudokus. He was able to get some puzzles printed in the London newspaper The Times beginning in 2004. Soon after, Sudoku-fever swept England. The puzzle finally became popular in the U.S. in 2005. It has become a regular feature in many newspapers and magazines and is enjoyed by people all over the globe.

The standard version of Sudoku consists of a 9×9 square grid containing 81 cells. The grid is subdivided into nine 3×3 blocks. Some of the 81 cells are filled in with numbers from the set {1,2,3,4,5,6,7,8,9}. These filled-in cells are called **givens**. The goal is to fill in the whole grid using the nine digits so that each row, each column, and each block contains each number exactly once. We call this constraint on the rows, columns, and blocks the **One Rule**.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | | | 7 | | | | |
| 6 | | | 1 | 9 | 5 | | | |
| | 9 | 8 | | | | | 6 | |
| 8 | | | | 6 | | | | 3 |
| 4 | | | 8 | | 3 | | | 1 |
| 7 | | | | 2 | | | | 6 |
| | 6 | | | | | 2 | 8 | |
| | | | 4 | 1 | 9 | | | 5 |
| | | | | 8 | | | 7 | 9 |

*Figure 17. Sudoku*

Sudoku is one of the most popular puzzles in the world. Every person has tried their luck at solving sudoku at least once in their life. I used to try solving it when I was in school but was never able to completely solve a sudoku.

During my training when I was learning algorithmic concepts like recursion and backtracking it clicked in my mind that a sudoku is solved using the same concepts. So, I decided to use the knowledge I acquired to make a sudoku generator and solver as well; the user can also play Sudoku.

This project is based on recursion and backtracking. It uses brute force method and can solve every level of sudoku. We solve the sudoku by filling a square with a possible number and then go on repeating the process. If at any point we violate the ‘One Rule’ we go back and check for our latest mistake and correct it.

# THE PROBLEM

Generating and solving a Sudoku puzzle.

## CONSTRAINTS

- Each row has unique numbers from 1 – 9.
- Each column has unique numbers from 1 – 9.
- Each sub-grid has unique numbers from 1 – 9.

## ALGORITHM USED

**Backtracking** is a general algorithm for finding all (or some) solutions to a problem that incrementally builds candidates to the solution. As soon as it determines that a candidate cannot possibly be the solution to the problem, it abandons it ("backtracks").

When the algorithm abandons a candidate, it will typically return to a previous stage in the problem-solving process. This is the key to the algorithm and also where it gets its name.

In a sudoku puzzle, for each square we have possible candidates from 1 - 9 that can be filled in the square. In the algorithm used in this project, to fill an empty box we try all the numbers starting from 1 and going till 9. Therefore, the square is filled with the lowest possible number from 1 - 9 that can be filled in the square without violating the 'One Rule'. We continue along the row and repeat the same process for every empty square until the 'One Rule' is violated i.e. a same number appears again in a row, column or sub-grid.

Now at the current square, where the repetition occurred, we try the number just greater than the current number which violated the 'One Rule'. For example, if we filled 4 in the square but it violated the 'One Rule' then we try filling it with 5. We keep trying numbers from 1 - 9 chronologically until the number which does not violate the 'One Rule' is found. If no number is found that does not violate the 'One Rule' then it means that we made a mistake somewhere on our way to the current square. Now we go back to the previous square and also try all the other possible options it can be filled with until we find a valid number or all the possible numbers have been tested.

We continue this process of backtracking and trying all possible values until the solution is found.

Now, for generating sudoku of N x N size, first a random permutation of 1 to N is created. It is then filled against the diagonal of the sudoku. The solve function is called which solves the sudoku. Now, numbers from squares (chosen randomly) are removed to create an unfilled sudoku.

**TIME COMPLEXITY:**

The time complexity of the algorithm is O(*n* ^ *m*) where *n* is the number of possibilities for each square (i.e., 9 in classic Sudoku) and *m* is the number of spaces that are blank.

**EXAMPLE**

An example of the algorithm –

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | | | 7 | | | | |
| 6 | | | 1 | 9 | 5 | | | |
| | 9 | 8 | | | | | 6 | |
| 8 | | | | 6 | | | | 3 |
| 4 | | | 8 | | 3 | | | 1 |
| 7 | | | | 2 | | | | 6 |
| | 6 | | | | | 2 | 8 | |
| | | | 4 | 1 | 9 | | | 5 |
| | | | | 8 | | | 7 | 9 |

We start from the first row and continue row wise. The first empty square is highlighted. We start by testing all numbers from 1 - 9 chronologically. As '1' does not violate the 'One Rule' we fill the square with '1'.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 1 | 2 | 7 | | | | |
| 6 | | | 1 | 9 | 5 | | | |
| | 9 | 8 | | | | | 6 | |
| 8 | | | | 6 | | | | 3 |
| 4 | | | 8 | | 3 | | | 1 |
| 7 | | | | 2 | | | | 6 |
| | 6 | | | | | 2 | 8 | |
| | | | 4 | 1 | 9 | | | 5 |
| | | | | 8 | | | 7 | 9 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 1 | | 7 | | | | |
| 6 | | | 1 | 9 | 5 | | | |
| | 9 | 8 | | | | | 6 | |
| 8 | | | | 6 | | | | 3 |
| 4 | | | 8 | | 3 | | | 1 |
| 7 | | | | 2 | | | | 6 |
| | 6 | | | | | 2 | 8 | |
| | | | 4 | 1 | 9 | | | 5 |
| | | | | 8 | | | 7 | 9 |

Now in the next blank square we repeat the process and test the numbers 1 - 9 chronologically for the 'One Rule'. We see that 2 is the first number that can be filled. So, we fill the square with 2

Now, we repeat the same process for every blank square, going row wise.

Now in the last square of first row there is no number from 1 to 9 that satisfies the ‘One Rule’. It means that we have not filled the previous squares correctly.

| 5 | 3 | 1 | 2 | 7 | 4 | 8 | 9 | |
|---|---|---|---|---|---|---|---|---|
| 6 | | | 1 | 9 | 5 | | | |
| | 9 | 8 | | | | | 6 | |
| 8 | | | | 6 | | | | 3 |
| 4 | | | 8 | | 3 | | | 1 |
| 7 | | | | 2 | | | | 6 |
| | 6 | | | | | 2 | 8 | |
| | | | 4 | 1 | 9 | | | 5 |
| | | | | 8 | | | 7 | 9 |

| 5 | 3 | 1 | 2 | 7 | 4 | 8 | | |
|---|---|---|---|---|---|---|---|---|
| 6 | | | 1 | 9 | 5 | | | |
| | 9 | 8 | | | | | 6 | |
| 8 | | | | 6 | | | | 3 |
| 4 | | | 8 | | 3 | | | 1 |
| 7 | | | | 2 | | | | 6 |
| | 6 | | | | | 2 | 8 | |
| | | | 4 | 1 | 9 | | | 5 |
| | | | | 8 | | | 7 | 9 |

Now we go back to the previous square and try filling it with some another valid number. As the previous square contains 9 it means that all the numbers from 1 - 9 have been tested and none satisfies the ‘One Rule’. Therefore, it means that the previous squares have not been rightly filled. So, we clear out the current square and go back.

Now we are at the 7th column of the 1st row. The only number left to try is 9. As 9 satisfies the ‘One Rule’ we fill the square with 9 instead of 8.

| 5 | 3 | 1 | 2 | 7 | 4 | 9 | | |
|---|---|---|---|---|---|---|---|---|
| 6 | | | 1 | 9 | 5 | | | |
| | 9 | 8 | | | | | 6 | |
| 8 | | | | 6 | | | | 3 |
| 4 | | | 8 | | 3 | | | 1 |
| 7 | | | | 2 | | | | 6 |
| | 6 | | | | | 2 | 8 | |
| | | | 4 | 1 | 9 | | | 5 |
| | | | | 8 | | | 7 | 9 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 1 | 2 | 7 | 4 | 9 | 8 | |
| 6 | | | 1 | 9 | 5 | | | |
| | 9 | 8 | | | | | 6 | |
| 8 | | | | 6 | | | | 3 |
| 4 | | | 8 | | 3 | | | 1 |
| 7 | | | | 2 | | | | 6 |
| | 6 | | | | | 2 | 8 | |
| | | | 4 | 1 | 9 | | | 5 |
| | | | | 8 | | | 7 | 9 |

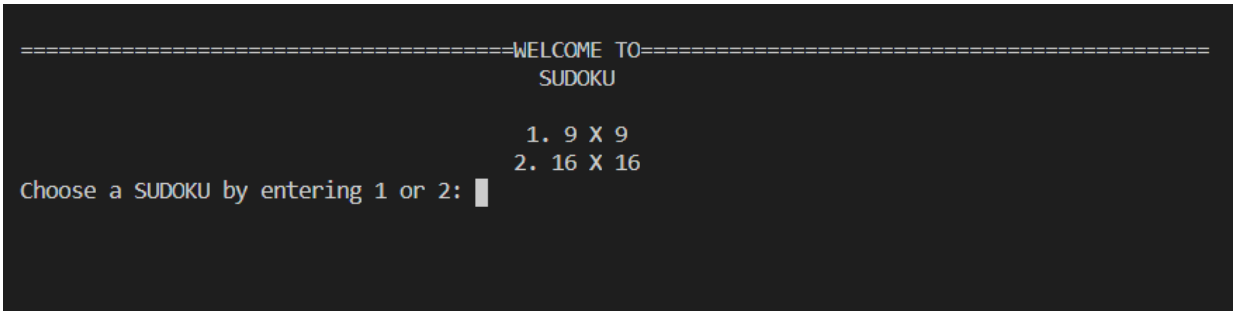
Now we go to the next square and check for a valid number between 1 - 9. We find that 8 is the first valid number and fill it with 8.

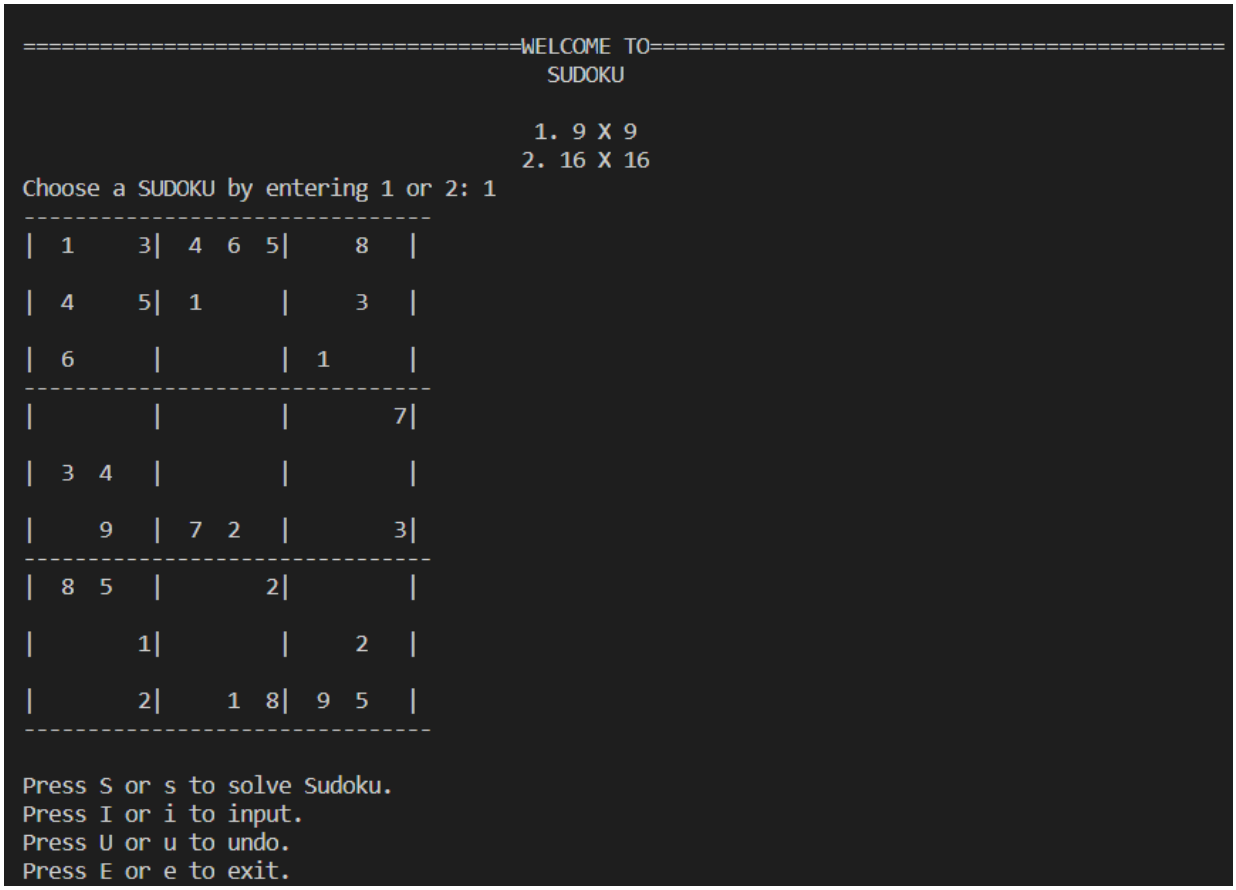
Now we can see that the next square does not have any number from 1 - 9 that satisfies the ‘One Rule’. Hence, we go back and try other values in the previous squares.

We continue this process until we solve the sudoku.

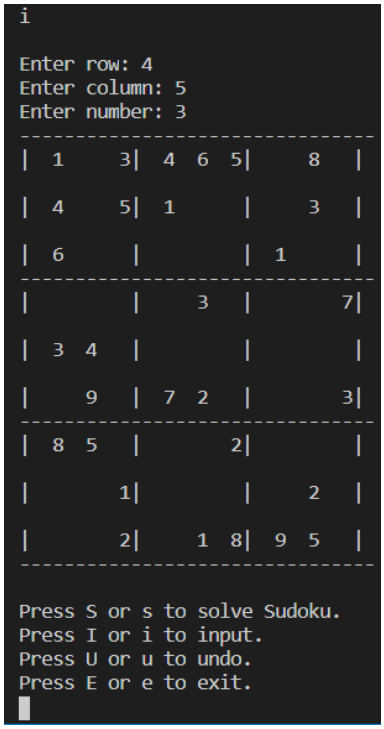
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 4 | 6 | 7 | 8 | 9 | 1 | 2 |
| 6 | 7 | 2 | 1 | 9 | 5 | 3 | 4 | 8 |
| 1 | 9 | 8 | 3 | 4 | 2 | 5 | 6 | 7 |
| 8 | 5 | 9 | 7 | 6 | 1 | 4 | 2 | 3 |
| 4 | 2 | 6 | 8 | 5 | 3 | 7 | 9 | 1 |
| 7 | 1 | 3 | 9 | 2 | 4 | 8 | 5 | 6 |
| 9 | 6 | 1 | 5 | 3 | 7 | 2 | 8 | 4 |
| 2 | 8 | 7 | 4 | 1 | 9 | 6 | 3 | 5 |
| 3 | 4 | 5 | 2 | 8 | 6 | 1 | 7 | 9 |

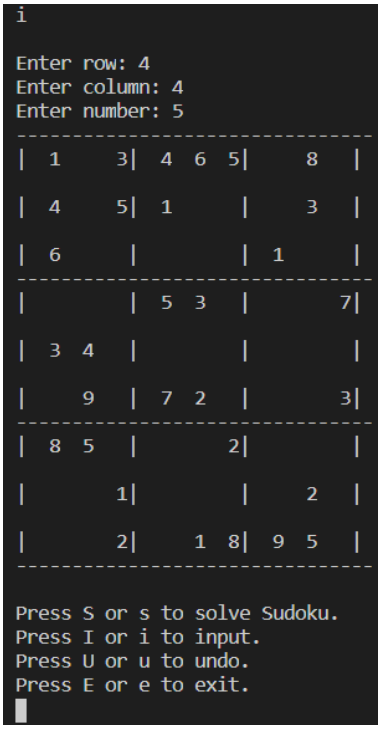
# SNAPSHOTS

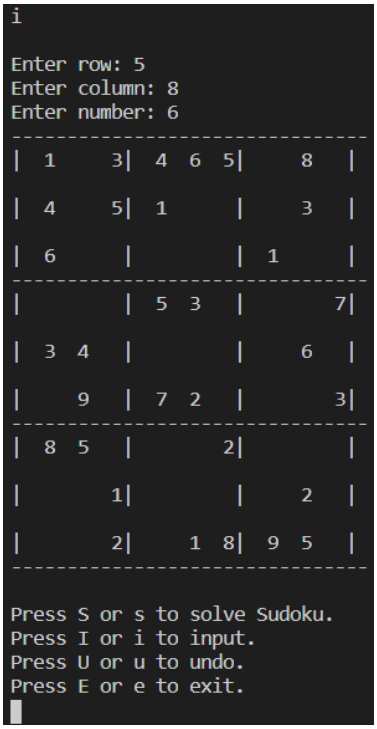

==========WELCOME TO==========
SUDOKU
1. 9 X 9
2. 16 X 16
Choose a SUDOKU by entering 1 or 2:

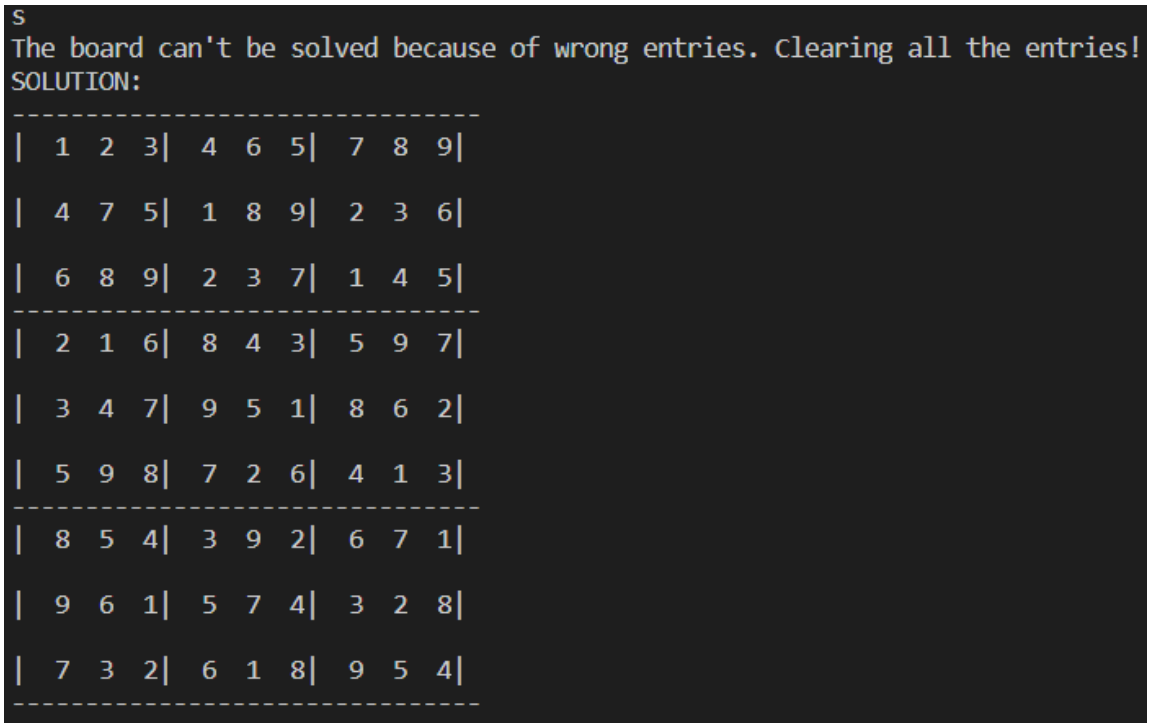

==========WELCOME TO==========
SUDOKU
1. 9 X 9
2. 16 X 16
Choose a SUDOKU by entering 1 or 2: 1
| 1 3| 4 6 5| 8 |
| 4 5| 1 | 3 |
| 6 | | 1 |
| | | 7|
| 3 4 | | |
| 9 | 7 2 | 3|
| 8 5 | 2| |
| 1| | 2 |
| 2| 1 8| 9 5 |
Press S or s to solve Sudoku.
Press I or i to input.
Press U or u to undo.
Press E or e to exit.

```
=====================================WELCOME TO=============================================
                                       SUDOKU

                                    1. 9 X 9
                                   2. 16 X 16
Choose a SUDOKU by entering 1 or 2: 2
-----------------------------------------------------
| 14        |           |       10  |    13 15  |
|  5        |       8   |    12   15|       11 14|
|           | 12        |           |        7  |
| 12        |      11   |       7   |           |
-----------------------------------------------------
|          3|    6  9  8|     5     | 14       13|
|          9|  3        | 14        |  4        |
|           |       4 14|  3  1   10|  6  5     |
|           |           |           |       1   |
-----------------------------------------------------
|  2        |      10  9|          7| 16        6|
|           |           |          8|           |
|           |  6       1|       12  |    7      |
|      13   |       3   |  6        |          1|
-----------------------------------------------------
|  3        |  8    7   | 11        | 13    12 10|
| 11  7     |    4      |   16  6   |           2|
|          2| 16        |    7  4   |   14  6   |
| 16        |       2   |       8   |         11|
-----------------------------------------------------

Press S or s to solve Sudoku.
Press I or i to input.
Press U or u to undo.
Press E or e to exit.
```


i
Enter row: 4
Enter column: 5
Enter number: 3
| 1 3| 4 6 5| 8 |
| 4 5| 1 | 3 |
| 6 | | 1 |
| | 3 | 7|
| 3 4 | | |
| 9 | 7 2 | 3|
| 8 5 | 2| |
| 1| | 2 |
| 2| 1 8| 9 5 |
Press S or s to solve Sudoku.
Press I or i to input.
Press U or u to undo.
Press E or e to exit.


i
Enter row: 4
Enter column: 4
Enter number: 5
| 1 3| 4 6 5| 8 |
| 4 5| 1 | 3 |
| 6 | | 1 |
| | 5 3 | 7|
| 3 4 | | |
| 9 | 7 2 | 3|
| 8 5 | 2| |
| 1| | 2 |
| 2| 1 8| 9 5 |
Press S or s to solve Sudoku.
Press I or i to input.
Press U or u to undo.
Press E or e to exit.


i
Enter row: 5
Enter column: 8
Enter number: 6
| 1 3| 4 6 5| 8 |
| 4 5| 1 | 3 |
| 6 | | 1 |
| | 5 3 | 7|
| 3 4 | | 6 |
| 9 | 7 2 | 3|
| 8 5 | 2| |
| 1| | 2 |
| 2| 1 8| 9 5 |
Press S or s to solve Sudoku.
Press I or i to input.
Press U or u to undo.
Press E or e to exit.


S
The board can't be solved because of wrong entries. Clearing all the entries!
SOLUTION:
| 1 2 3| 4 6 5| 7 8 9|
| 4 7 5| 1 8 9| 2 3 6|
| 6 8 9| 2 3 7| 1 4 5|
| 2 1 6| 8 4 3| 5 9 7|
| 3 4 7| 9 5 1| 8 6 2|
| 5 9 8| 7 2 6| 4 1 3|
| 8 5 4| 3 9 2| 6 7 1|
| 9 6 1| 5 7 4| 3 2 8|
| 7 3 2| 6 1 8| 9 5 4|

# REFERENCES


- http://pi.math.cornell.edu/~mec/Summer2009/Mahmood/Home.html