

Ishan Jain
Computer Problem Set 2

ChE 5126

Spring 2014

Due 4/08/13

- a. Input Program 10.3 in the Matlab handout and reproduce Table 10.5. Print out your solution. Note: to carry out this problem you will have to add a file for calculating the initial temperature at the interior nodes. You can call this $f = \text{fun}(x)$ and name the file `fun.m`. Also when you do the function evaluation you must replace f by “fun”.

Solution a)

columns 1 through 4

0	1.1180	1.5388	1.1180
0	0.6169	0.9288	0.8621
0	0.3942	0.6480	0.7186
0	0.2887	0.5067	0.6253
0	0.2331	0.4258	0.5560
0	0.1995	0.3720	0.4996
0	0.1759	0.3315	0.4511
0	0.1574	0.2981	0.4082
0	0.1419	0.2693	0.3698
0	0.1283	0.2437	0.3351
0	0.1161	0.2208	0.3038

Columns 5 through 8

0.3633	0	0.3633	1.1180
0.6177	0.4905	0.6177	0.8621
0.6800	0.6488	0.6800	0.7186
0.6665	0.6733	0.6665	0.6253
0.6251	0.6458	0.6251	0.5560
0.5754	0.6002	0.5754	0.4996
0.5253	0.5504	0.5253	0.4511
0.4778	0.5015	0.4778	0.4082
0.4338	0.4558	0.4338	0.3698
0.3936	0.4137	0.3936	0.3351
0.3570	0.3753	0.3570	0.3038

Columns 9 through 11

1.5388	1.1180	0
0.9288	0.6169	0
0.6480	0.3942	0
0.5067	0.2887	0
0.4258	0.2331	0
0.3720	0.1995	0
0.3315	0.1759	0
0.2981	0.1574	0
0.2693	0.1419	0
0.2437	0.1283	0
0.2208	0.1161	0

Step-1 Define fun.m

```
function ux=fun(x)
ux=sin(pi*x)+sin(3*pi*x);
```

Step-2 Forward Difference Program

Used program listed in reference material.

Step-3 Calling and listing the table 10.5

```
crnich('fun', 0, 0, 1, 1, 0.1, 11, 11)
```

b. Now adapt Program 10.3 to the case when:

$$\text{BC. } U(1,t) = U(0,t) = 0 \quad \text{IC. } U(x,0) = 1.0$$

Use the following data: $L = 3.175 \times 10^{-3}$ and $\alpha = 5.8 \times 10^{-8} \text{ m}^2/\text{s}$

Take U as the dimensionless temperature, t as real time and $\xi = x/L$. The differential equation now becomes:

$$\frac{\partial U}{\partial t} = \frac{\alpha}{L^2} \frac{\partial^2 U}{\partial \xi^2} \quad \text{with } c^2 = \alpha/L^2$$

Find the time required to reduce $U(0.5,t) = 0.5$. Adjust the time and spatial steps to obtain a solution accurate to the nearest 1.0 s. **How many time and spatial steps are needed to get a convergent solution?**

Solution b)

```
function U=crnich(fun,c1,c2,a,b,c,n,m)
%input = u(x,0) as a string 'f'
%c1=u(0,t) and c2=u(a,t)
% a and b are right end point of [0,a] and [0,b]
% c the constant in the heat equation
% n and m are numbe of grid pointd over [0,a] and [0,b]
%output - U solution matrix table 10.5
% initialize parameters and U
h=a/(n-1);
k=b/(m-1);
r=c^2*k/h^2;
U=zeros(n,m);
s1=2+2/r;
s2=2/r-2;
    %boundary conditions
U(1,1:m)=c1;
U(n,1:m)=c2;
%generate first row
U(2:n-1,1)=feval(fun,h:h:(n-2)*h)';
% form the diagonal and off-diagonal elements of A and the constant vector B
% and solve tridiagonal system AX=B
Vd(1,1:n)=s1*ones(1,n);
Vd(1)=1;
Vd(n)=1;

Va=-ones(1,n-1);
Va(n-1)=0;

Vc=-ones(1,n-1);
Vc(1)=0;

Vb(1)=c1;
Vb(n)=c2;
for j=2:m
    for i=2:n-1
        Vb(i)=U(i-1,j-1)+U(i+1,j-1)+s2*U(i,j-1);
    end
    X=trisys(Va,Vd,Vc,Vb);
    U(1:n,j)=X';
end
U=U'
```

```
function ux=fun(x)
%ux=sin(pi*x)+sin(3*pi*x);
%ux=4*x-4*x.^2;
ux=1;
```

crnich('fun',0,0,1,20,0.0759,11,81)

U =

Columns 1 through 9

0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0	0.8736	0.9920	0.9995	1.0000	1.0000	1.0000	0.9995	0.9920
0	0.7773	0.9718	0.9973	0.9998	1.0000	0.9998	0.9973	0.9718
0	0.7023	0.9455	0.9927	0.9992	0.9998	0.9992	0.9927	0.9455
0	0.6427	0.9167	0.9857	0.9980	0.9995	0.9980	0.9857	0.9167
0	0.5944	0.8873	0.9766	0.9960	0.9989	0.9960	0.9766	0.8873
0	0.5546	0.8585	0.9658	0.9932	0.9978	0.9932	0.9658	0.8585
0	0.5213	0.8309	0.9538	0.9895	0.9962	0.9895	0.9538	0.8309
0	0.4930	0.8048	0.9409	0.9849	0.9939	0.9849	0.9409	0.8048
0	0.4687	0.7803	0.9274	0.9794	0.9910	0.9794	0.9274	0.7803
0	0.4474	0.7574	0.9136	0.9732	0.9873	0.9732	0.9136	0.7574
0	0.4288	0.7360	0.8996	0.9663	0.9829	0.9663	0.8996	0.7360
0	0.4122	0.7160	0.8857	0.9588	0.9778	0.9588	0.8857	0.7160
0	0.3974	0.6973	0.8719	0.9508	0.9720	0.9508	0.8719	0.6973
0	0.3840	0.6798	0.8582	0.9422	0.9656	0.9422	0.8582	0.6798
0	0.3719	0.6634	0.8447	0.9333	0.9586	0.9333	0.8447	0.6634
0	0.3608	0.6480	0.8315	0.9240	0.9510	0.9240	0.8315	0.6480
0	0.3506	0.6335	0.8185	0.9145	0.9430	0.9145	0.8185	0.6335
0	0.3412	0.6198	0.8058	0.9046	0.9346	0.9046	0.8058	0.6198
0	0.3325	0.6068	0.7934	0.8946	0.9258	0.8946	0.7934	0.6068
0	0.3244	0.5945	0.7812	0.8845	0.9167	0.8845	0.7812	0.5945
0	0.3168	0.5828	0.7693	0.8742	0.9073	0.8742	0.7693	0.5828
0	0.3097	0.5716	0.7577	0.8638	0.8976	0.8638	0.7577	0.5716
0	0.3030	0.5609	0.7463	0.8534	0.8878	0.8534	0.7463	0.5609
0	0.2967	0.5507	0.7351	0.8429	0.8778	0.8429	0.7351	0.5507
0	0.2907	0.5409	0.7242	0.8324	0.8677	0.8324	0.7242	0.5409
0	0.2850	0.5314	0.7135	0.8219	0.8575	0.8219	0.7135	0.5314
0	0.2796	0.5223	0.7030	0.8114	0.8472	0.8114	0.7030	0.5223
0	0.2744	0.5135	0.6927	0.8010	0.8369	0.8010	0.6927	0.5135
0	0.2694	0.5050	0.6826	0.7906	0.8265	0.7906	0.6826	0.5050
0	0.2646	0.4968	0.6727	0.7802	0.8162	0.7802	0.6727	0.4968
0	0.2601	0.4888	0.6629	0.7700	0.8058	0.7700	0.6629	0.4888
0	0.2556	0.4810	0.6533	0.7597	0.7955	0.7597	0.6533	0.4810
0	0.2513	0.4735	0.6439	0.7496	0.7852	0.7496	0.6439	0.4735
0	0.2472	0.4661	0.6347	0.7396	0.7750	0.7396	0.6347	0.4661
0	0.2432	0.4590	0.6256	0.7296	0.7648	0.7296	0.6256	0.4590
0	0.2393	0.4520	0.6167	0.7198	0.7547	0.7198	0.6167	0.4520
0	0.2355	0.4451	0.6079	0.7100	0.7447	0.7100	0.6079	0.4451
0	0.2318	0.4385	0.5992	0.7003	0.7347	0.7003	0.5992	0.4385
0	0.2283	0.4319	0.5907	0.6908	0.7249	0.6908	0.5907	0.4319
0	0.2248	0.4255	0.5823	0.6813	0.7151	0.6813	0.5823	0.4255
0	0.2213	0.4193	0.5740	0.6720	0.7054	0.6720	0.5740	0.4193
0	0.2180	0.4131	0.5659	0.6627	0.6958	0.6627	0.5659	0.4131
0	0.2147	0.4071	0.5579	0.6536	0.6864	0.6536	0.5579	0.4071
0	0.2115	0.4012	0.5500	0.6446	0.6770	0.6446	0.5500	0.4012
0	0.2084	0.3953	0.5423	0.6357	0.6677	0.6357	0.5423	0.3953
0	0.2054	0.3896	0.5346	0.6269	0.6585	0.6269	0.5346	0.3896
0	0.2023	0.3840	0.5271	0.6182	0.6495	0.6182	0.5271	0.3840
0	0.1994	0.3785	0.5197	0.6097	0.6405	0.6097	0.5197	0.3785
0	0.1965	0.3731	0.5124	0.6012	0.6317	0.6012	0.5124	0.3731
0	0.1937	0.3678	0.5051	0.5929	0.6230	0.5929	0.5051	0.3678
0	0.1909	0.3625	0.4980	0.5846	0.6144	0.5846	0.4980	0.3625
0	0.1881	0.3574	0.4910	0.5765	0.6058	0.5765	0.4910	0.3574
0	0.1854	0.3523	0.4841	0.5685	0.5974	0.5685	0.4841	0.3523
0	0.1828	0.3473	0.4773	0.5605	0.5891	0.5605	0.4773	0.3473
0	0.1802	0.3424	0.4706	0.5527	0.5810	0.5527	0.4706	0.3424
0	0.1776	0.3375	0.4640	0.5450	0.5729	0.5450	0.4640	0.3375
0	0.1751	0.3327	0.4575	0.5374	0.5649	0.5374	0.4575	0.3327
0	0.1726	0.3280	0.4511	0.5299	0.5570	0.5299	0.4511	0.3280
0	0.1701	0.3234	0.4448	0.5225	0.5493	0.5225	0.4448	0.3234
0	0.1677	0.3188	0.4385	0.5152	0.5416	0.5152	0.4385	0.3188
0	0.1654	0.3144	0.4324	0.5080	0.5341	0.5080	0.4324	0.3144
0	0.1630	0.3099	0.4263	0.5009	0.5266	0.5009	0.4263	0.3099
0	0.1607	0.3056	0.4204	0.4940	0.5193	0.4940	0.4204	0.3056
0	0.1584	0.3013	0.4145	0.4870	0.5120	0.4870	0.4145	0.3013
0	0.1562	0.2970	0.4087	0.4802	0.5049	0.4802	0.4087	0.2970
0	0.1540	0.2929	0.4029	0.4735	0.4978	0.4735	0.4029	0.2929

0	0.1518	0.2887	0.3973	0.4669	0.4909	0.4669	0.3973	0.2887
0	0.1497	0.2847	0.3917	0.4604	0.4840	0.4604	0.3917	0.2847
0	0.1476	0.2807	0.3862	0.4539	0.4773	0.4539	0.3862	0.2807
0	0.1455	0.2768	0.3808	0.4476	0.4706	0.4476	0.3808	0.2768
0	0.1435	0.2729	0.3755	0.4413	0.4640	0.4413	0.3755	0.2729
0	0.1415	0.2690	0.3702	0.4352	0.4575	0.4352	0.3702	0.2690
0	0.1395	0.2653	0.3650	0.4291	0.4511	0.4291	0.3650	0.2653
0	0.1375	0.2615	0.3599	0.4231	0.4448	0.4231	0.3599	0.2615
0	0.1356	0.2579	0.3549	0.4172	0.4386	0.4172	0.3549	0.2579
0	0.1337	0.2543	0.3499	0.4113	0.4325	0.4113	0.3499	0.2543
0	0.1318	0.2507	0.3450	0.4056	0.4264	0.4056	0.3450	0.2507
0	0.1300	0.2472	0.3402	0.3999	0.4204	0.3999	0.3402	0.2472
0	0.1281	0.2437	0.3354	0.3943	0.4146	0.3943	0.3354	0.2437
0	0.1263	0.2403	0.3307	0.3888	0.4088	0.3888	0.3307	0.2403

Columns 10 through 11

1.0000	0
0.8736	0
0.7773	0
0.7023	0
0.6427	0
0.5944	0
0.5546	0
0.5213	0
0.4930	0
0.4687	0
0.4474	0
0.4288	0
0.4122	0
0.3974	0
0.3840	0
0.3719	0
0.3608	0
0.3506	0
0.3412	0
0.3325	0
0.3244	0
0.3168	0
0.3097	0
0.3030	0
0.2967	0
0.2907	0
0.2850	0
0.2796	0
0.2744	0
0.2694	0
0.2646	0
0.2601	0
0.2556	0
0.2513	0
0.2472	0
0.2432	0
0.2393	0
0.2355	0
0.2318	0
0.2283	0
0.2248	0
0.2213	0
0.2180	0
0.2147	0
0.2115	0
0.2084	0
0.2054	0
0.2023	0
0.1994	0
0.1965	0
0.1937	0
0.1909	0
0.1881	0
0.1854	0
0.1828	0
0.1802	0
0.1776	0
0.1751	0
0.1726	0
0.1701	0
0.1677	0

0.1654	0
0.1630	0
0.1607	0
0.1584	0
0.1562	0
0.1540	0
0.1518	0
0.1497	0
0.1476	0
0.1455	0
0.1435	0
0.1415	0
0.1395	0
0.1375	0
0.1356	0
0.1337	0
0.1318	0
0.1300	0
0.1281	0
0.1263	0

- c. Move the axes to $x = 0.5L$ and repeat the solution for the conditions in part b. **That is find how long it takes for the temperature at $\xi = 0$ to reach $U=0.5$.**

For this problem use $\xi = x/0.5L$. The B.C. 's become: $U(1,t) = 0$ and

$$\frac{\partial U(\xi, t)}{\partial \xi} = 0 \quad \text{I.C. } U(\xi, 0) = 1.$$

Solution:

```
function U=crnich(fun,c1,c2,a,b,c,n,m)
%input = u(x,0) as a string 'f'
%c1=u(0,t) and c2=u(a,t)
% a and b are right end point of [0,a] and [0,b]
% c the constant in the heat equation
% n and m are numbe of grid pointd over [0,a] and [0,b]
%output - U solution matrix table 10.5
% initialize parameters and U
h=a/(n-1);
k=b/(m-1);
r=c^2*k/h^2;
U=zeros(n,m);
s1=2+2/r;
s2=2/r-2;
%boundary conditions
U(1,1:m)=c1;
U(n,1:m)=c2;
%generate first row
U(2:n-1,1)=feval(fun,h:h:(n-2)*h)';
% form the diagonal and off-diagonal elements of A and the constant vector B
% and solve tridiagonal system AX=B
Vd(1,1:n)=s1*ones(1,n);
Vd(1)=1;
Vd(n)=1;

Va=-ones(1,n-1);
Va(n-1)=0;

Vc=-ones(1,n-1);
Vc(1)=0;

Vb(1)=c1;
Vb(n)=c2;
for j=2:m
    for i=2:n-1
        Vb(i)=U(i-1,j-1)+U(i+1,j-1)+s2*U(i,j-1);
    end
    X=trisys(Va,Vd,Vc,Vb);
    U(1:n,j)=X';
end
U=U'
```

```
function ux=fun(x)
%ux=sin(pi*x)+sin(3*pi*x);
%ux=4*x-4*x.^2;
ux=1;
```

crnich('fun',0.1,0.1,20,0.0759,11,41)

Columns 1 through 9

0.1000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1000	0.7962	0.9769	0.9974	0.9997	0.9999	0.9997	0.9971	0.9743
0.1000	0.6753	0.9264	0.9875	0.9981	0.9994	0.9979	0.9861	0.9183
0.1000	0.5968	0.8732	0.9698	0.9938	0.9978	0.9932	0.9664	0.8591
0.1000	0.5421	0.8243	0.9473	0.9864	0.9941	0.9850	0.9414	0.8048
0.1000	0.5015	0.7814	0.9226	0.9759	0.9879	0.9736	0.9140	0.7571
0.1000	0.4701	0.7441	0.8973	0.9629	0.9790	0.9593	0.8860	0.7156
0.1000	0.4450	0.7115	0.8723	0.9477	0.9675	0.9430	0.8584	0.6795
0.1000	0.4242	0.6828	0.8482	0.9310	0.9537	0.9250	0.8318	0.6477
0.1000	0.4067	0.6574	0.8249	0.9132	0.9381	0.9060	0.8062	0.6196
0.1000	0.3916	0.6346	0.8026	0.8946	0.9211	0.8862	0.7817	0.5944
0.1000	0.3784	0.6140	0.7811	0.8755	0.9030	0.8660	0.7584	0.5716
0.1000	0.3667	0.5951	0.7606	0.8562	0.8841	0.8456	0.7361	0.5508
0.1000	0.3562	0.5776	0.7408	0.8367	0.8648	0.8252	0.7147	0.5317
0.1000	0.3466	0.5614	0.7218	0.8172	0.8452	0.8048	0.6942	0.5140
0.1000	0.3378	0.5462	0.7034	0.7979	0.8255	0.7847	0.6746	0.4975
0.1000	0.3296	0.5318	0.6856	0.7787	0.8058	0.7649	0.6556	0.4819
0.1000	0.3220	0.5182	0.6684	0.7599	0.7863	0.7454	0.6374	0.4672
0.1000	0.3148	0.5053	0.6518	0.7413	0.7670	0.7263	0.6198	0.4533
0.1000	0.3080	0.4929	0.6356	0.7231	0.7480	0.7076	0.6028	0.4401
0.1000	0.3015	0.4809	0.6199	0.7053	0.7294	0.6893	0.5864	0.4274
0.1000	0.2953	0.4695	0.6047	0.6878	0.7111	0.6714	0.5705	0.4153
0.1000	0.2893	0.4584	0.5899	0.6708	0.6931	0.6540	0.5551	0.4036
0.1000	0.2836	0.4478	0.5755	0.6541	0.6756	0.6370	0.5402	0.3924
0.1000	0.2781	0.4375	0.5616	0.6379	0.6585	0.6205	0.5257	0.3816
0.1000	0.2728	0.4275	0.5480	0.6221	0.6418	0.6044	0.5117	0.3712
0.1000	0.2677	0.4178	0.5348	0.6066	0.6255	0.5887	0.4981	0.3611
0.1000	0.2627	0.4084	0.5220	0.5916	0.6097	0.5734	0.4850	0.3514
0.1000	0.2579	0.3993	0.5095	0.5769	0.5942	0.5586	0.4722	0.3420
0.1000	0.2532	0.3904	0.4973	0.5627	0.5792	0.5442	0.4597	0.3328
0.1000	0.2487	0.3818	0.4855	0.5488	0.5645	0.5301	0.4477	0.3240
0.1000	0.2443	0.3734	0.4740	0.5353	0.5503	0.5165	0.4360	0.3154
0.1000	0.2400	0.3653	0.4629	0.5221	0.5364	0.5032	0.4246	0.3071
0.1000	0.2358	0.3574	0.4520	0.5094	0.5229	0.4903	0.4135	0.2990
0.1000	0.2318	0.3497	0.4414	0.4969	0.5098	0.4778	0.4028	0.2911
0.1000	0.2279	0.3423	0.4311	0.4848	0.4970	0.4656	0.3924	0.2835
0.1000	0.2240	0.3350	0.4211	0.4730	0.4846	0.4537	0.3823	0.2761
0.1000	0.2203	0.3280	0.4114	0.4616	0.4726	0.4422	0.3724	0.2690
0.1000	0.2167	0.3211	0.4020	0.4505	0.4608	0.4310	0.3629	0.2620
0.1000	0.2132	0.3144	0.3928	0.4396	0.4494	0.4201	0.3536	0.2552
0.1000	0.2098	0.3079	0.3838	0.4291	0.4383	0.4095	0.3445	0.2486

Columns 10 through 11

1.0000	0
0.7735	0
0.6392	0
0.5520	0
0.4912	0
0.4461	0
0.4113	0
0.3833	0
0.3603	0
0.3408	0
0.3241	0
0.3095	0
0.2966	0
0.2850	0
0.2744	0

0.2648	0
0.2559	0
0.2476	0
0.2398	0
0.2324	0
0.2255	0
0.2189	0
0.2125	0
0.2065	0
0.2007	0
0.1951	0
0.1897	0
0.1845	0
0.1795	0
0.1747	0
0.1700	0
0.1654	0
0.1610	0
0.1568	0
0.1526	0
0.1486	0
0.1447	0
0.1409	0
0.1373	0
0.1337	0
0.1302	0

