Linear Optimization Assignment

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Instructions

- The complete submission should include: (a) the report (in PDF format), (b) the source code(s), compressed into one zip file (YourFullName_StudentID.zip).
- You can use either Python or MATLAB for the coding part.
- You are strictly prohibited from copying solutions from another person. Violation of this would be considered an act of plagiarism.
- Note that there will be a demo where you will be called individually to discuss your codes and solutions. The schedule will be released shortly.

Question 1.

Write a code for the Revised Simplex method from scratch. **(10 marks)**

Question 2.

Given a graph $G = (V;E)$, a matching $M \subset E$ of G is a collection of vertex disjoint edges. The size of the matching M is the number of edges present in M and is denoted by $\|M\|$. Maximum matching of G is a matching of maximum size. Given a graph G with $\|V\|$ vertices and $\|E\|$ edges, formulate an ILP(integer linear program) to solve the maximum matching problem. For the given adjacency matrix of network 1, solve the problem using the Simplex code you have written. **(4 marks)**

Question 3.

Given a graph $G = (V;E)$, a vertex-cover $V^1 \subset V$ of G is a collection of vertices such that each edge in E is incident to at least one of the vertex in $V^{'}$. The size of a vertex cover is the number of vertices present in the cover. Write an ILP to find a minimum size vertex cover for G. For the given adjacency matrix of network 1, solve the problem using the Simplex code you have written. **(4 marks)**

Question 4.

We are given an adjacency matrix representation of the flow network 2. Each edge (i, j) is a directed edge from i to j. The capacity of each edge is given in the adjacency matrices. Node S represents the source, and node T represents the sink in both networks. Note that each capacity is an integer. Both the flow networks satisfy the following properties

- Conservation of flow at each node except Source (i.e., Total incoming flow at a node must equal total outgoing flow at the node).
- For each edge, any flow respects the capacity constraint of the edge.

Do the following

- Write a code for the Interior point method (IPOPT, based on newton iteration) from scratch. **(15 marks)**
- Formulate LPs for flow network 2 to find out the maximum flow from S to T and solve it using the IPOPT code you have written. **(3 marks)**
- Take the Duals of each LP which corresponds to $S-T$ Min-Cut in the respective network and solve it using the IPOPT code you have written. **(4 marks)**

Network 1	1	2	3	4
1	0	1	0	0
2	1	0	1	1
3	0	1	0	1
4	0	1	1	0

Network 2	S	A	B	C	D	E	F	G	H	I	J	T
S	0	11	15	10	0	0	0	0	0	0	0	0
A	0	0	0	0	0	18	4	0	0	0	0	0
B	0	3	8	5	0	0	0	0	0	0	0	0
C	0	0	0	0	6	0	0	3	11	0	0	0
D	0	0	0	4	0	0	0	17	6	0	0	0
E	0	0	0	0	3	16	0	0	0	13	0	0
F	0	12	0	0	4	0	0	0	0	0	0	21
G	0	0	0	0	0	0	0	0	4	9	4	3
H	0	0	0	0	0	0	0	4	0	0	5	4
I	0	0	0	0	0	0	0	0	0	0	7	9
J	0	0	0	0	0	0	0	0	2	0	0	15
T	0	0	0	0	0	0	0	0	0	0	0	0

Question 5.

Write the codes in AMPL to solve all LPs (you formed). Show that min-cut is equal to max-flow for question 4(i.e., an LP and its dual have the same optimal). **(10 marks)**