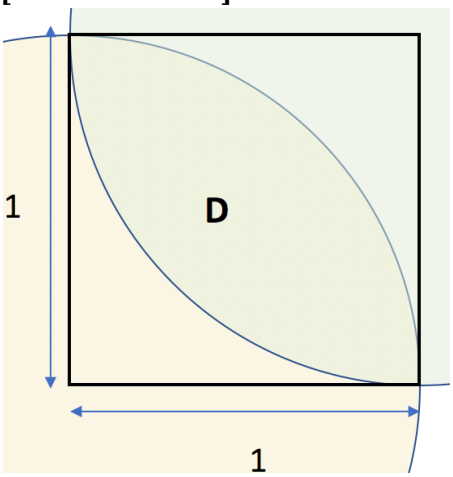
EE 511 Simulation Methods for Stochastic Systems
Project #3: Investigations on Monte Carlo Methods

[Area-Estimation]



i.) Generate n=500 samples (X, Y) of i.i.d 2-dimensional uniform random variables in the unit-square.
ii.) Write a function that counts how many of these samples fall within the region, D, formed by the intersection of two quarter unit-circles centered at the origin and at (1,1) as shown above.
iii.) Use these random samples to estimate the area of the inscribed region D. Use this area estimate to estimate the area of D. Do k=50 runs of these estimations. Plot the histogram of the k=50 area-estimates.
iv.) Repeat the experiment with different numbers of uniform samples, n. Plot the sample variance of the Monte Carlo estimates as a function of your sample size n. Keep k=50 for all these runs. Comment on the sample variance of your estimates.

[Monte Carlo Integration and Variance Reduction Strategies]

Use n=1000 random samples to obtain Monte Carlo estimates for the definite integrals:
(a) $[1+\sinh(2x)\ln(x)]^{-1}$, x in [0.8,3]
(b) $\text{Exp}[-x^4 - y^4]$, (x, y) in [-pi, pi]

Calculate the sample variance of the Monte Carlo estimates using a similar method as in problem 1.
Use the same number of random samples, n=1000, to obtain those Monte Carlo estimates. But this time incorporate stratification and importance sampling in the Monte Carlo estimation procedures. Compare the Monte Carlo estimates and their sample variances.

Discuss the quality of the Monte Carlo estimates from each method. Also discuss the strengths and weaknesses of stratification and importance sampling in Monte Carlo estimation.

Test your integral estimator on the following function with your own choice of n samples:

$$f(x,y) = 20 + x^2 + y^2 - 10(cos[2\pi \times x] + cos[2\pi \times y])$$

(x, y) in [-5, 5] for f(x,y)

Turn in:
• A summary of your experiments including plots and statistics,
• a brief discussion of the results for each question (max 1 page per problem), &
• a print out of your code. If you are working in Python/R, consider turning in a PDF of your jupyter/Rmarkdown notebook containing all of the above.