EE 511 Simulation Methods for Stochastic Systems
Project #2: Rejection and Independence

[Double Rejection]

The random variable X has a bimodal distribution made up of an equally weighted, convex summation of a beta and a triangle distribution:

$$f(x) = \begin{cases} 0.5 \times Beta(8,5), 0 < x \leq 1 \\ 0.5 \times (x-4), 4 < x \leq 5 \\ -0.5 \times (x-6), 5 < x \leq 6 \\ 0, else \end{cases}$$

Implement rejection sampling routines for X. Generate 1000 samples of the random variable using each envelope. Track the rejection rate of your rejection sampling RNGs. The rejection rate is the average number of rejected candidates per sample. This is a measure of the efficiency of your RNG.

[Independence: Internally and Externally]

1. Take 1000 samples of the bimodal distribution above. Use the covariance statistic to test the independence between X_k and the lagged version X_{k+5}.
2. Generate 1000 samples of the random variables X~Beta(8,5) and Y~Beta(4,7). Use 2-way contingency tables to argue for the independence of X and Y.

[Network Fit]

Note: you need a network grapher e.g. networkx (python) and igraph (R)

Given n people in a social network. Suppose any given unordered pair of two people are connected at random and independently with probability p

- Generate and plot three network samples for each value of p=0.03 and p=0.12. Briefly discuss the structure of these sample graphs.
- Generate a network with (n, p) = (100, 0.06). The number of connections for network node *i* is called the *degree* d_i of the node. Count the degree of each node in the network and plot the histogram of degrees. Use goodness-of-fit tests to check how well the network degree distribution fits a binomial(n=100, p=0.06) or a Poisson(λ=np=6).

Turn in:

- A summary of your experiments including plots and statistics,
- a brief discussion of the results for each question (max 1 page per problem), &
- a print out of your code. If you are working in Python/R, consider turning in a PDF of your jupyter/Rmarkdown notebook containing all of the above.