Banking on Resolution: Portfolio Effects of Bail-in vs. Bailout*

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Abstract

This paper investigates the impact of supervisory resolution tools, specifically bail-ins versus bailouts, on the ex-ante banks' portfolio composition and resulting default probabilities in the presence of both idiosyncratic and systematic shocks. Banks make decisions regarding short-term versus long-term asset investments while considering the expected supervisory resolution policy. I find that market expectations can generate financial instability, which the two resolution tools address through distinct channels. Creditor bailouts, acting as debt insurance, reduce funding costs and by that eliminate the equilibrium with bank defaults. Whereas, bail-ins alter bank payoffs, induce banks to invest less in the risky short-term asset, and possibly prevent defaults. In the presence of aggregate risk, I show that creditor bailouts can deter systemic events. In contrast, bail-ins are less effective in preventing systemic defaults and could even contribute to systemic risk.

JEL Classification: G21, G28, G33

Keywords: bailouts, bail-ins, bank resolution, systemic risk, bank portfolio allocation, fire sales.

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1 Introduction

Despite advocating bail-ins as the primary resolution tool for over a decade, supervisors still often favor bailouts in various instances, among others driven by concerns of contagion (e.g., Silicon Valley and Signature Bank in the US in 2023), households and retail investors holding bail-inable debt (e.g., Italy in 2015), or economic contractions (e.g., Banco Espirito Santio in Portugal in 2016).[1] This cautious approach towards employing bail-ins indicates that the trade-offs between bail-ins and bailouts necessitate further examination. This paper aims to contribute to the discourse by investigating, from a theoretical perspective, the ex-ante impact of supervisory resolution tools on banks' portfolio choices and, consequently, on the equilibrium default outcomes in the presence of idiosyncratic and systematic shocks. Its key contribution is to highlight the distinct channels through which the resolution anticipation affects bank defaults.

I model a large number of banks that operate over two periods. These banks are financed through a combination of insured short-term debt and fairly priced long-term debt subject to default costs. The banks choose to invest their funds in a short-term asset with idiosyncratic risk and a long-term common asset. The presence of short-term assets with idiosyncratic risk introduces heterogeneity in banks' short-term liquidity as well as in their long-term solvency. The common risk associated with the long-term asset allows for a systematic shock to the banking system. Additionally, the common asset can be traded in the first period, serving as an endogenous source of liquidity that influences banks' portfolio choices. Bank defaults incur deadweight losses suggesting supervisory intervention. The supervisor can prevent second-period defaults by either bailing out the creditors or bailing them in. In this context, the choice of the supervisory resolution strategy affects banks' ex-ante portfolio decisions, influencing the market price of the long-term asset and highlighting the pecuniary costs of the resolution policy, particularly during fire sales.

[1] See the Word Bank's case study by, Manuilov Oleksiy (2017) for a selection of bank resolution cases in the EU post the Global Financial Crisis.

In the baseline model with no aggregate risk, the short-term asset has an uncertain idiosyncratic return, while the long-term asset is safe. Banks collect one unit of endowment and decide on their short-term risky investment. Since banks are opaque their portfolio composition is unobservable to the market and the price of the long-term asset as well as the gross return of the long-term debt is defined by market expectations on the banks' short-term investment. A larger short-term investment translates to a larger supply of liquidity in the first period, increasing the cash-in-the-market price of the long-term asset. However, a larger investment in the risky asset raises the likelihood of bank defaults and thus increases the gross return of the long-term debt for which the consumers are willing to lend.

In a symmetric equilibrium, market expectations can influence banks' investment decisions and generate multiple equilibria. In one equilibrium, the market anticipates banks to remain solvent which reduces the cost of long-term funding. In this case, each individual bank chooses a short-term investment that leads to no defaults. In another equilibrium, the market expects the banks that face a negative short-term shock to default, raising the gross return of long-term debt. In this case, each bank invests more in the risky asset and defaults after a low short-term return. The fact that, following pessimistic market expectations, banks choose portfolios that lead to defaults can be interpreted as a source of financial instability.

In the presence of multiple equilibria in laissez-faire, I demonstrate that the prospect of creditor bailouts, in which the supervisor insures the long-term debt, does not alter the trade-off faced by banks in their investment decisions. However, it does lead to a reduction in banks' funding costs. Cheaper funding increases bank payoffs when it is expected that creditors will be bailed out to prevent defaults. Subsequently, each individual bank prefers to remain solvent and earn higher profits. This choice contradicts market expectations, thereby ruling out the equilibrium with bank defaults. In other words, when in laissez-faire depending on market expectations banks may remain solvent or default in equilibrium, in anticipation of bailouts banks remain solvent. On the other hand, following a bail-in the long-term debt is converted into equity while the banks' shareholders are not entirely wiped out. As a result, each bank takes the downside of its risk-taking into account and invests less in the risky short-term asset. This portfolio reallocation towards a less risky portfolio can remove the equilibrium with defaults. In other words, if in anticipation of bail-ins the risk reduction is large enough, banks will remain solvent in equilibrium. In summary, when the economy is facing no aggregate risk, the anticipation of both supervisory interventions reduces the likelihood of defaults and hence the need for resolution, but through distinct channels.

Next, I introduce aggregate risk to the baseline model by assuming that the long-term asset, which is common to all banks, has also uncertain returns. The realization of the second-period return is observable in the first-period, depressing the market price of the long-term asset when a low return is anticipated. The depressed asset price, in anticipation of an aggregate shock, leaves banks with a low holding of the short-term asset, and hence lower liquidity in the first period, vulnerable to defaults. I define scenarios in which all banks default simultaneously as systemic events. Macroprudential supervisors are concerned with how the expectation of a resolution tool influences systemic risk, whether it alleviates it or, conversely, exacerbates it.

In the case of creditor bailouts, the anticipation of bailouts reduces banks' funding costs and, hence increases bank payoffs while anticipating supervisory interventions. This change in funding costs may incentivize banks to choose a portfolio that ensures solvency, particularly when the long-term asset has a low return. Thus, the anticipation of bailouts may prevent banks from defaulting simultaneously during fire sales, while without supervisory intervention such an event would occur. In essence, bailouts have the potential to eliminate systemic risk. Conversely, bail-ins impact the ex-ante portfolio composition, potentially towards the lower holding of the short-term asset. This portfolio reallocation effect may not be enough to prevent systemic events or can even trigger systemic defaults, while without supervisory intervention such an event would not occur. In essence, the portfolio reallocation effect related to bail-ins is not as effective as the funding cost effect related to bailouts in reducing systemic risk.

The baseline model without aggregate risk is connected to the literature on supervisory interventions under idiosyncratic risk. The consideration of future profits while anticipating bailouts often motivates banks to engage in value-creating projects (Lambrecht and Tse, 2023). However, this pursuit of profit may also lead to increased portfolio risk and leverage among banks (Lambrecht and Tse, 2023; Leanza, Sbuelz, and Tarelli, 2021). More precisely, if the supervisor cannot commit to refrain from bailouts, this may generate a "too-big-too-fail" problem, since banks internalize their size effect on the supervisory intervention and increase their leverage (Davila and Walther, 2020). This commitment issue cannot be resolved when alongside bailouts, bail-ins are promised (Chari and Kehoe, 2016). However, distributing bailout transfers across banks (Philippon and Wang, 2023) and uncertainty about the timing of the bailout (Nosal and Ordoñez, 2016) can mitigate the moral hazard problem by incentivizing banks to avoid becoming the worst performer. My paper adds to the existing literature by demonstrating that bailouts and their associated ex-ante lower funding cost effectively prevents defaults when, depending on market expectations, both an equilibrium with default alongside an equilibrium with no defaults exist without supervisory intervention.

In the presence of systemic risk, supervisors might be inclined to resort to bailouts out of fear of contagion, i.e. "too-many-to-fail" problem (Acharya and Yorulmazer, 2007). This can encourage banks to correlate their portfolios in a way that prompts the supervisor to bail them out during adverse times, contributing to a collective moral hazard problem (Farhi and Tirole, 2012). Wagner and Zeng (2023) argue that a targeted bailout policy, in which banks are assigned to bailout groups, will solve the "too-many-to-fail" problem. Finally, Keister (2016) demonstrates that a strict no-bailout policy may not be welfare-enhancing because higher investor losses could lead to runs. I show that a bailout resolution policy is effective in preventing systemic events.

When considering bail-ins under idiosyncratic risk, Berger, Himmelberg, Roman, and Tsyplakov (2022) shows that shareholders are more likely to consider recapitalization and may engage less in risk-shifting when anticipating bail-ins. Nonetheless, the higher funding costs associated with bail-ins can introduce moral hazard problems (Pandolfi, 2022). When examining private bail-ins, where shareholders initiate the bail-in process, the lack of supervisory commitment to refrain from bailouts can distort private incentives to engage in bail-ins (Keister and Mitkov, 2023). This lack of commitment may also prolong the restructuring process (Colliard and Gomb, 2020) and create a moral hazard for lending banks to accept privately negotiated bail-in offers (Benoit and Riabi, 2020). Moreover, when designing bail-ins, supervisors should take into account the potential disclosure of negative information to the market, which could trigger runs (Walther and White, 2020). I argue that when bank shareholders are not entirely wiped out after a bail-in, banks tend to invest less in short-term risky assets, and hence bank defaults may be prevented.

Avgouleas and Goodhart (2015) underscore that in the presence of aggregate risk relying solely on bail-ins as a resolution tool may exacerbate systemic crises. Dewatripont (2014) suggests that both tools, bail-ins, and bailouts, should complement each other during crisis periods. Farmer, Goodhart, and Kleinnijenhuis (2021) further argues that poorly designed bail-ins, especially in bank networks, can result in losses for other interconnected banks, leading to multiple layers of contagion. Bernard, Capponi, and Stiglitz (2022) posit that when interconnected banks participate in a private bail-in, the prospect of a supervisory bailout may undermine the negotiation process, particularly if banks are less exposed to contagion risk. Finally, Clayton and Schaab (2022) suggests that the higher the fire-sale risk, the more bail-inable debt banks should hold, and the greater the magnitude of write-downs that should be applied. This paper shows that bail-ins due to the ex-ante portfolio reallocation may cause systemic events.

The paper is organized as follows. Section 2 describes the model. In Section 3, I begin by establishing the market price of the long-term asset in the first period for the case of no aggregate risk. Following this, I describe the equilibria under no supervisory intervention, in anticipation of bailouts, and in anticipation of bail-ins. Section 4 modifies the baseline model by incorporating aggregate risk stemming from uncertain second-period asset returns. Within this context, I establish the market price of the long-term asset in the presence of aggregate risk and describe the equilibria under no supervisory intervention, in anticipation of bailouts, and in anticipation of bail-ins. Section 5 concludes. Proofs of the analytical results are in Appendix A.

2 Model setup

Consider an economy with three dates $t = 0, 1, 2$, and a large number of islands. In each island i there is a single risk-neutral *bank* that issues *short-term insured debt* that matures at $t = 1$ and *long-term uninsured debt* that matures at $t = 2$ to a set of risk-neutral consumers located in the island. There is also a bank *supervisor* that insures the short-term debt and either bails-in or bails out failing banks.

In each island i, there is a unit measure of consumers who possess in total a unit endowment at time $t = 0$. Among these consumers, a fraction θ, referred to as the *early consumers*, only values consumption at $t = 1$, whereas the remaining fraction $1 - \theta$, referred to as *late consumers*, only values consumption at $t = 2$. The early consumers invest in bank's short-term debt, while the late consumers invest in the long-term debt. Both types of consumers have access to a safe asset with a zero net return, guaranteeing a net-zero return from their investments in the bank.

Banks are identical ex-ante. They each raise capital by offering short-term and long-term debt. The gross return on the short-term debt is set at one, considering that consumers can invest in the safe asset and there is deposit insurance. The long-term debt is fairly priced and is subject to default costs. Thus, the gross return on the long-term deb D_2 is determined by the late consumers' binding participation constraint. Once the bank collects one unit of funds, it can invest in two assets, a *short island-specific asset*, and a *long common (to all islands) asset*. Specifically, if bank i chooses to invest a fraction λ_i of its portfolio in the short-term asset, it yields a return of $h(\lambda_i)X_i$, where $h(\lambda_i)$ takes the simple quadratic form

$$h(\lambda_i) = \lambda_i - \lambda_i^2/2\,,$$

which is increasing and concave in λ_i. The short-term asset return is either high X_h with probability $1-\alpha$ or low X_ℓ with probability α, that is

$$X_i = \begin{cases} X_\ell, & \text{with probability } \alpha \\ X_h, & \text{with probability } 1-\alpha \end{cases}$$

where $X_\ell < X_h$, the expected asset return is $\overline{X} = \alpha X_\ell + (1-\alpha)X_h$, and X_i is independent and identically distributed across islands. I call banks with $X_i = X_\ell$ *weak banks* and banks with $X_i = X_h$ *strong banks*. The long-term asset has return Z with cdf $G(Z)$, which I will detail in Sections 3 and 4. I assume bank portfolios are opaque. This means λ_i is unobservable to both the supervisor and the consumers. Hence, the gross return on long-term debt D_2 is a function of the expected market investment in the short-term asset λ. I assume consumers' expectation regarding the equilibrium short-term asset investment is rational.

At $t = 1$ the anticipated return on the long-term asset Z for $t = 2$ becomes observable and there is an economy-wide market in which the banks can trade the long-term asset at a price p. There is also a demand for this asset by outside investors

$$d(p, Z) = \frac{Z - p}{p}\,. \tag{1}$$

which is decreasing in p and satisfies $d(p, Z) = 0$ for $p \geq Z$. Additionally, banks have the option to invest in the safe asset at time $t = 1$. Therefore, the price of the common asset cannot exceed Z. If banks manage to secure enough liquidity to repay early consumers, they will continue their operations until $t = 2$. However, if they fail to do so, they are liquidated. In such a case, the supervisor will sell off the long-term asset holdings of the defaulting bank and repay the early consumers as the deposit insurer. In addition to the possibility of default at $t = 1$, banks may also continue their operations until $t = 2$ and default on their long-term debt. I assume bank defaults, either at $t = 1$ and $t = 2$, generate deadweight losses, with a fraction $1 - c$ of the asset returns being lost.[2]

The supervisor can prevent the default losses at $t = 2$ by either bailing out or bailing-in banks. In a creditor bailout, the supervisor promises to repay the late consumers. On the other hand in bail-ins, the long-term debt is converted to equity with a conversion rate γ. I analyze each resolution policy separately to assess its ex-ante effect on banks' investment decision and their likelihood of default. To this aim, I have established a simplified bank model with essential characteristics. The defaults at $t = 2$ trigger supervisory interventions, while the price of the tradable long-term asset represents the pecuniary costs of the resolution policy. The common return of the long-term asset introduces a systematic shock to the system, while the idiosyncratic return of the short-term asset generates heterogeneous liquidity needs at $t = 1$, prompting banks to fire sale assets.

To analyze the different supervisory resolution tools, I first look at a model with no aggregate risk in which the short-term asset return is random but the long-term asset return is deterministic. I first characterize the market price of the long-term asset at $t = 1$, then compare banks' investment decisions under no supervisory intervention, with bailouts, and with bail-ins. After that, in Section 4 I introduce aggregate risk in the model by assuming that the long-term asset return is also random and compare the two resolution tools. Section 5 concludes.

3 The model without aggregate risk

In this Section, I assume the return on the long-term asset is $Z = \overline{Z}$ with probability 1. At $t = 1$, given price p, the bank in island i has to pay θ to the early consumers. If the combined liquidity from the short-term asset and the sale of the long-term asset is not

[2] For example Bernard et al. (2022); Chari and Kehoe (2016); Leanza et al. (2021) assume similar bank default costs proportional to asset values.

enough to repay the early consumers the bank fails at $t = 1$ and the supervisor proceeds to liquidate the long-term asset holdings. Alternatively, the bank may accumulate additional liquidity by selling the long-term asset and successfully repaying the early consumers, or may even hold excess liquidity after covering the short-term debt and can buy the long-term asset in the market. In all these scenarios, besides banks with excess liquidity, the outside investors are willing to buy the asset, provided its price does not exceed its return of $\overline{Z}$.

In a symmetric equilibrium, banks in all islands choose the same λ, which is the market short-term investment. Then, the liquidity in the market generated by the short-term asset defines the cash-in-the-market price of the long-term asset $p(\lambda)$. Proposition 1 characterized the market price for the case of no aggregate risk.

Proposition 1. *The market price of the long-term asset, for any value λ of the banks' expected investment in the short-term asset, is*

$$p(\lambda) = \min\{\max\{p^c(\lambda), p^\ell(\lambda)\}, \overline{Z}\} \, ,$$

where $p^c(\lambda)$ is the continuation price, when no bank defaults at $t = 1$,

$$p^c(\lambda) = h(\lambda)\overline{X} + \overline{Z} - \theta \, ,$$

and $p^\ell(\lambda)$ is the liquidation price, when weak banks default at $t = 1$,

$$p^\ell(\lambda) = \frac{(1-\alpha)[h(\lambda)X_h - \theta] + \overline{Z}}{1 + \alpha(1-\lambda)} \, .$$

For parameters such that

$$\frac{X_\ell}{2} < \theta < \max\left\{\frac{\overline{Z}}{2}, \frac{X_h}{2}\right\} \, ,$$

strong banks survive the first period, but weak banks may default at $t = 1$ if the ex-ante investment in the short-term asset is too large.

Figure 1 illustrates the result in Proposition 1, showing the market price $p(\lambda)$, alongside the continuation price $p^c(\lambda)$, and the liquidation price $p^\ell(\lambda)$. For low values of λ weak banks cannot repay the early consumers at $t = 1$ with the return of the short-term asset. Therefore, they need to sell a portion of their long-term asset to survive. Then, the continuation price $p^c(\lambda)$ defines the market price. This price is, as depicted in Figure 1, increasing in λ. Specifically, larger investments in the short-term asset lead to higher average first-period returns. This increase in returns translates to larger liquidity in the market, thereby raising the cash-in-the-market price of the long-term asset.

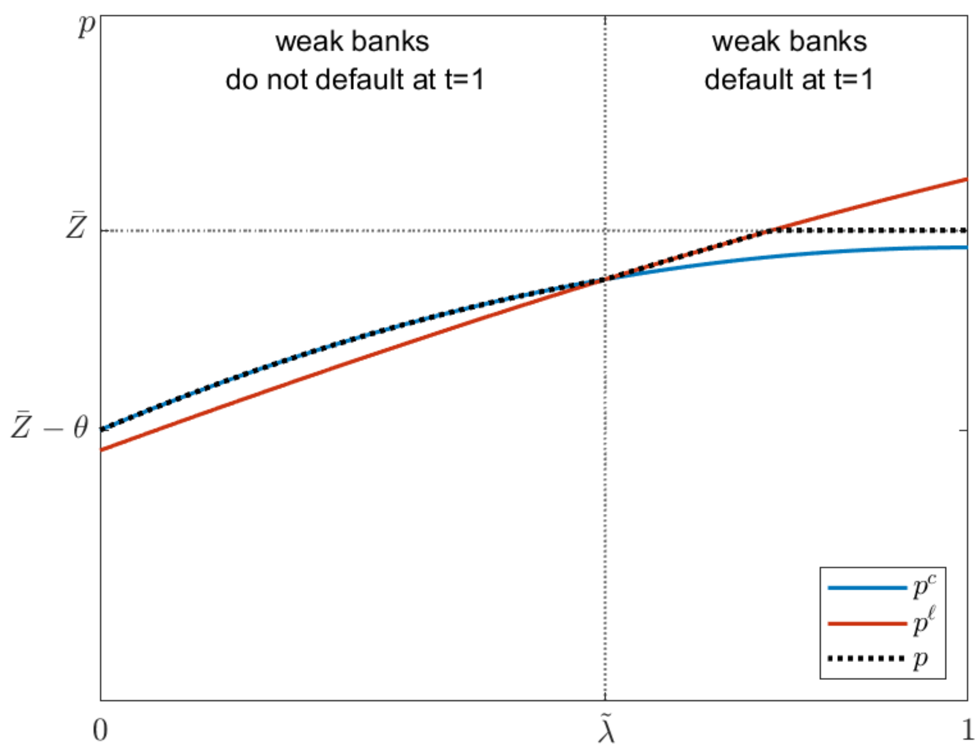


Figure 1 – Market price of the long-term asset

The solid blue line is the continuation price $p^c(\lambda)$ when weak banks can sell assets to repay early consumers. The solid red line is the liquidation price $p^\ell(\lambda)$ when weak banks cannot repay early consumers and are liquidated. The dotted black line is the market price of the long-term asset $p(\lambda)$ at $t = 1$. The threshold $\widetilde{\lambda}$ is the intersection of the continuation and liquidation price, above which weak banks default at $t = 1$. The parameter values are $\theta = 0.70$, $\alpha = 0.40$, $X_\ell = 0.20$, $X_h = 2.0$, and $\overline{Z} = 1.65$.

However, when banks allocate large investments to the risky short-term asset, even selling the entire holding of the long-term asset is not enough for the weak banks to repay early consumers, leading to their failure at $t = 1$. In Figure 1, when the banks invest more than $\widetilde{\lambda}$, which is the intersection of the liquidation price and the continuation price, weak banks default at $t = 1$. Then, the liquidation price $p^{\ell}(\lambda)$ defines the market price. This price is, as depicted in Figure 1, increasing in λ. Specifically, as banks hold a large proportion of short-term assets, the strong banks will have more net liquidity to buy the asset and the proportion of liquidated assets for sale will be smaller. In other words, there are fewer assets for sale and larger cash in the market to buy the asset. This raises the liquidation price.

Finally, the market price, as defined in Preposition 1, cannot exceed the long-term asset return due to the possibility of investing in the safe asset with zero net return. If $p(\lambda) = \overline{Z}$ the weak banks sell the long-term asset without a discount, strong banks are indifferent between buying the asset and investing in the safe asset, and the outside investors do not enter the market.

If, given bank i's short-term investment λ_i and the market price $p(\lambda)$, the bank defaults at $t = 1$ the supervisor sells the bank's long-term assets $1 - \lambda_i$ and there are no second-period returns for the bank. If on the other hand, the bank survives at $t = 1$, it will yield a return

$$\left(1 - \lambda_i + \frac{h(\lambda_i)X_i - \theta}{p(\lambda)}\right)\overline{Z}$$

at $t = 2$, with the value inside the parentheses representing the quantity of long-term assets the bank holds at $t = 2$. This comprises the bank's initial investment of $1 - \lambda_i$ at $t = 0$ and the volume of long-term assets the bank trades at $t = 1$.

In sum, bank i's second-period return can be generalized as

$$R(\lambda_i, p, X_i) = (1 - \lambda_i + a_i)\overline{Z}$$

where the volume traded is

$$a_i(\lambda_i, p, X_i) = \max\left\{\frac{h(\lambda_i)X_i - \theta}{p(\lambda)}, -(1 - \lambda_i)\right\},$$

which depends on the bank's investment choice, the bank's individual short-term asset return, and market price. The maximum operator ensures that the bank cannot sell more long-term assets than it actually owns. In other words, if the bank must sell more long-term assets at

$t = 1$ to continue operating than it actually possesses, the bank faces liquidation.

To simplify the notation, let's denote the second-period return of bank i as $R_h(\lambda_i)$ when its short-term asset return is X_h and as $R_\ell(\lambda_i)$ when the short-term asset return is X_ℓ. I next characterize banks' equilibrium portfolio choice in laissez-faire without supervisory intervention.

3.1 Equilibrium with no supervisory intervention

The late consumers' expectation of the banks' short-term investment λ determines the gross return of long-term debt $D_2(\lambda)$ to which they are willing to lend their endowments. Moreover, in a symmetric equilibrium with rational expectations, the market price of the long-term asset is characterized by λ. Given the market price $p(\lambda)$ and the gross return of the long-term asset $D_2(\lambda)$, bank i chooses its short-term investment λ_i to maximize expected payoffs

$$\max_{\lambda_i \in [0,1)} \mathbb{E}\Big[\max\{R(\lambda_i) - (1-\theta)D_2(\lambda), 0\}\Big],$$

when by limited liability the payoff either is the net second-period return after repaying the late consumers or zero if the bank defaults either at $t = 1$ or $t = 2$. The bank chooses a short investment that leads to at least strong banks being solvent at $t = 2$. Otherwise, the bank's payoffs in all states would be zero, making it indifferent to its portfolio composition. Consequently, bank i's expected payoff can be defined as

$$\max_{\lambda_i \in [0,1)} (1-\alpha)[R_h(\lambda_i) - (1-\theta)D_2(\lambda)] + \alpha\Big[\max\{R_\ell(\lambda_i) - (1-\theta)D_2(\lambda), 0\}\Big].$$

If in equilibrium the bank never defaults, that is when

$$R_h(\lambda_i^*) > R_\ell(\lambda_i^*) > (1-\theta)D_2(\lambda) \tag{2}$$

then, bank i's expected payoff is

$$(1-\alpha)R_h(\lambda_i) + \alpha R_\ell(\lambda_i) - (1-\theta)D_2(\lambda)$$

and the first-order condition is

$$h'(\lambda_i^*)\overline{X} = p(\lambda)\,,$$

where $h'(\lambda_i) = (1 - \lambda_i)$. The portfolio solving the first-order condition is

$$\lambda_i^*(p) = \frac{\overline{X} - p(\lambda)}{\overline{X}}\,,$$

which defines the solution to the bank's problem if it satisfies the equilibrium condition (2). The first-order condition can be interpreted as the bank selecting a portfolio that makes the expected marginal return of the short-term asset equal to the marginal value of the long-term asset at $t = 1$, which corresponds to its market price.

Conversely, if in equilibrium the bank stays solvent at $t = 2$ when the high return state X_h occurs, but defaults either at $t = 1$ or $t = 2$ in the low return state X_ℓ, that is when

$$R_h(\lambda_i^{**}) > (1 - \theta)D_2(\lambda) > R_\ell(\lambda_i^{**})\,, \tag{3}$$

then, bank i's expected payoff is

$$(1 - \alpha)[R_h(\lambda_i) - (1 - \theta)D_2(\lambda)]\,,$$

and the first-order condition

$$h'(\lambda_i^{**})X_h = p(\lambda)$$

defines the solution to the bank's problem if

$$\lambda_i^{**} = \frac{X_h - p(\lambda)}{X_h}$$

satisfies the equilibrium condition (3). In this case, the bank receives a positive payoff when the short-term asset return is X_h. Therefore, the bank solely considers the marginal value of the short-term asset in the high return state, while choosing its short-term investment. When comparing the portfolio $\lambda_i^*(p)$ that results in no defaults with the portfolio $\lambda_i^{**}(p)$ that leads to weak banks defaulting, as $X_h > \overline{X}$, the equilibrium investment in the risky short-term asset is higher when the bank chooses $\lambda_i^{**}(p)$. This underscores the moral hazard problem arising from opaque portfolios and limited liability.

Finally, if both portfolios $\lambda_i^*(p)$ and $\lambda_i^{**}(p)$ are solutions to bank i's problem, the bank chooses the portfolio that generates the highest payoff. More precisely, given market expectations regarding λ, the portfolio with $\lambda_i^{**}(p)$ short investment, that leads to the bank defaulting when X_ℓ occurs, is the local solution to bank i's problem when the resulting payoff is larger than that after choosing $\lambda_i^*(p)$ and staying solvent, that is when

$$\alpha[R_\ell(\lambda_i^*) - (1-\theta)D_2(\lambda)] < (1-\alpha)[R_h(\lambda_i^{**}) - R_h(\lambda_i^*)]\,. \tag{4}$$

Condition (4) illustrates the trade-off confronting the bank when choosing a risky portfolio that results in default. On one hand, there's the forgone payoff the bank could have received in the low return state if it had stayed solvent by choosing $\lambda_i^*(p)$. On the other hand, there are the higher payoffs it obtains in the high return state when increasing its short-term investment to $\lambda_i^{**}(p)$. Condition (4) can be rewritten as

$$D_2(\lambda) > \frac{\mathbb{E}[R(\lambda_i^*)] - \mathbb{E}[R(\lambda_i^{**})]}{\alpha(1-\theta)}\,. \tag{5}$$

This means when the gross return of long-term debt is large enough the bank prefers to choose $\lambda_i^{**}(p)$ short-term investment and will default in the low return state X_ℓ.

Whether the equilibrium conditions (2) or (3) are satisfied and the portfolios $\lambda_i^*(p)$ or $\lambda_i^{**}(p)$ are an equilibrium depends on the market short investment λ and late consumers' expectation. In a symmetric equilibrium with no defaults, all banks would opt for a portfolio that avoids default. In this scenario, late consumers anticipate full repayment in each state. Therefore, given an expectation of zero default probability, the gross return on the long-term debt is $D_2(\lambda) = 1$. In this case, if $\lambda_i^*(p)$ is the solution to the bank's problem, which ensures the bank's solvency, based on the assumption of rational expectations, the short-term investment $\lambda_i^*(p)$ represents an equilibrium with no defaults and by symmetry $\lambda = \lambda_i^*$. However, if the bank decides, contrary to market expectations of no defaults, to invest $\lambda_i^{**}(p)$

in the short-term asset, which leads to the bank's default in the low state X_ℓ, by symmetry it would result in all banks defaulting in the low state. This contradicts the assumption of rational expectations and cannot be considered an equilibrium.

On the other hand, when the market expects weak banks to default at $t = 2$, the gross return on long-term debt increases as late consumers anticipate weak banks to default. This gross return is determined by the binding late consumers' participation constraint

$$\alpha c R_\ell(\lambda) + (1-\alpha)(1-\theta)D_2(\lambda) = 1-\theta\,, \tag{6}$$

where they are repaid the face value of debt by strong banks and receive the second-period returns after weak banks have defaulted, resulting in a loss of a fraction of $1-c$ of the asset returns. In this case, the gross return $D_2(\lambda)$ decreases with the second-period return $R_\ell(\lambda)$. Essentially, if late consumers expect to receive less in the event of default, the required gross return of the debt increases to break even in expectation. Finally, when the market anticipates the weak banks to default at $t = 1$, the gross return of long-term debt, based on the binding late consumers' participation constraint, reaches its maximum

$$D_2(\lambda) = \frac{1}{1-\alpha}\,.$$

That is the late consumers expect to be repaid only by the strong banks.

If in a symmetric equilibrium the market expects weak banks to default either at $t = 1$ or $t = 2$ and $\lambda_i^{**}(p)$ is the solution to the bank's problem, which results in the bank defaulting in the low state X_ℓ, based on the assumption of rational expectations, the short-term investment $\lambda_i^{**}(p)$ stands as an equilibrium with bank defaults and by symmetry $\lambda = \lambda_i^{**}(p)$. However, should the bank, contrary to market expectations of no default, opt for investing $\lambda_i^{*}(p)$ in the short-term asset, which leads to the bank staying solvent, by asymmetry all banks would stay solvent. This is inconsistent with market expectations and cannot be considered an equilibrium.

The above analysis reveals that banks' equilibrium portfolio composition depends on market expectations regarding their portfolio choices and, consequently, default probabilities. When the market anticipates that weak banks are likely to default, the market price of the long-term asset and long-term funding costs increase. Subsequently, even though individual banks' portfolio choices are not observable, they opt to invest more in the short-term asset compared to what they would choose if the market expected no defaults. This portfolio choice leads to bank defaults when short-term return is low, which confirms the market expectations.

Figure 2 illustrates an example for bank i's response function $\lambda_i(p)$, which is its short-term investment given any market expectation λ. For a low λ, it is expected that no bank defaults. However, each individual bank prefers to deviate from this and invest λ_i^{**} in the short-term asset, leading to the bank defaulting at $t = 2$, which contradicts market expectations. Hence, the dotted red line represents the region where the banks' preferred portfolio choice is not an equilibrium due to rational market expectations. For intermediate levels of λ, the bank chooses a short-term investment λ_i^*, resulting in all banks staying solvent. By symmetry, the intersection of the green line and the 45-degree line defines an equilibrium with no defaults in laissez-faire. For larger values of λ, the market expects weak banks to default at $t = 2$, and individual banks choose to invest λ_i^{**} in the short-term asset, which leads to the bank defaulting when X_ℓ. Hence, the intersection of the red line and the 45-degree line defines another symmetric equilibrium with bank defaults in laissez-faire.

As illustrated in Figure 2, the fact that banks' equilibrium portfolio depends on market expectations can lead to the existence of multiple equilibria, which can be seen as a potential source of financial instability. One equilibrium corresponds to a scenario in which the market expects no defaults. In this case, the gross return of the long-term debt is at its lowest. Furthermore, with low market investment λ, the price of the long-term asset remains low, which can result in fire sales. Consequently, banks prefer to invest less in the risky short-term asset to remain solvent. In the second equilibrium, the market expects weak banks to default. Then, the gross return of the long-term debt increases, and for larger values of λ,

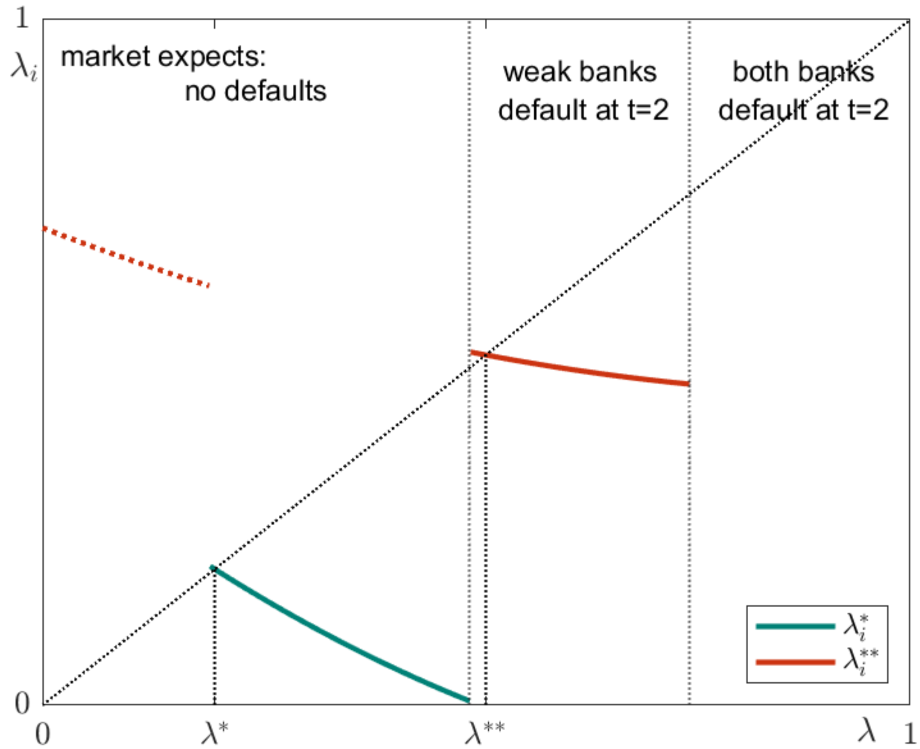


Figure 2 – Bank's response function given market expectations

The figure illustrates the banks' choice of short-term investment based on their market expectations of the bank's portfolio λ in laissez-faire without supervisory intervention. The green line $\lambda_i^*(p)$ represents a local equilibrium portfolio that ensures the bank remains solvent, while the red line $\lambda_i^{**}(p)$ represents a local equilibrium portfolio that leads to the bank's default in the low state X_ℓ. The dotted line indicates situations where the bank deviates from market expectations, thus violating the principle of rational expectations. The intersection of the banks' response function with the 45-degree line defines the symmetric equilibrium of the depicted example. Parameter values are $\theta = 0.65$, $\alpha = 0.70$, $X_h = 2.80$, $X_\ell = 0.75$, $Z_g = 1.75$, $Z_b = 1.5$, and $c = 0.65$.

the market price of the long-term asset is also higher. In this scenario, banks invest more in the risky short-term asset, which leads to defaults when their short-term return is low.

3.2 Equilibrium with bailout

At $t = 1$ the supervisor observes the short-term asset return and can bail out the creditors of the weak banks when the bank is expected to default at $t = 2$. After a creditor bailout bank shareholders are wiped out, similar to what occurs in the event of defaults. As a result, the bank's optimization problem is identical to the laissez-faire case. Therefore, given market portfolio λ which defines the market price $p(\lambda)$ and $D_2(\lambda)$, the portfolios $\lambda_i^*(p)$ and $\lambda_i^{**}(p)$, as defined in laissez-faire, are local solutions to the banks problem if they satisfy the equilibrium conditions (2) and (3). Finally, if both portfolios are solutions to the bank's problem, the bank chooses the risky portfolio $\lambda_i^{**}(p)$ if the gross return of the long-term debt is high enough as defined by condition (5).

Regarding the long-term funding costs, in a creditor bailout the supervisor commits to repaying the debt to the late consumers. More precisely, the supervisor transfers the amount

$$(1-\theta)D_2(\lambda) - R_\ell(\lambda)$$

when weak banks default at $t = 2$. As a result, late consumers always receive the face value of the debt, either from the bank or the supervisor. This resembles deposit insurance and renders the long-term debt risk-free, with a gross return $D_2^{out}(\lambda) = 1$. Thus, the gross return of the long-term debt is lower in anticipation of bailouts than in laissez-faire as defined in (6). The fact that banks now benefit from cheaper funding costs has an impact on the individual bank's local choice of short-term investment, as defined by condition (5). More precisely, if both portfolios $\lambda_i^*(p)$ and $\lambda_i^{**}(p)$ are a solution to the bank's maximization problem since the left-hand side of the inequality is weakly lower than in laissez-faire, the bank might find the portfolio with no defaults, $\lambda^*(p)$, preferable when anticipating bailouts. Whereas in laissez-faire without supervisory intervention, it would have chosen $\lambda^{**}(p)$, which includes default risk.

In the case of multiple equilibria in laissez-faire, this shift in an individual bank's local portfolio preference, which stems from cheaper funding facilitated by creditor bailouts, eliminates the equilibrium with defaults. Without supervisory intervention banks would choose a safe portfolio $\lambda^*(p)$ when the market expects no defaults and a risky portfolio $\lambda^{**}(p)$ when the market expects the weak banks to default at $t = 2$. This translates in terms of the condition 5 into the inequality

$$D_2(\lambda) > \frac{\mathbb{E}[R(\lambda_i^*)] - \mathbb{E}[R(\lambda_i^{**})]}{\alpha(1-\theta)} > 1\,,$$

where $D_2(\lambda)$ is the gross return of long-term debt in laissez-faire, as in (6), when defaults are expected at $t = 2$ and 1 is the gross-return of long-term debt when no defaults are expected. In other words, given parameter values, when the left inequality holds there exists an equilibrium with bank default at $t = 2$ and when concurrently the right inequality holds, there exists a second equilibrium with no defaults. If bailout is expected, the gross return is $D_2(\lambda) = 1$ and the above condition is violated. More precisely, even if the market expects weak banks to default at $t = 2$ banks still prefer to stay solvent and hence the equilibrium with defaults is ruled out.

Expanding on the example depicted in Figure 2, when the market expects weak banks to default at $t = 2$ for an intermediate level of market investment λ, the gross return of debt remains unchanged due to the insurance expected from the supervisor by late consumers. Then, the bank prefers to investment in $\lambda_i^*(p)$, as the expected payoff from staying solvent increases in comparison to the payoffs the bank would receive by choosing $\lambda_i^{**}(p)$ and defaulting when X_ℓ. However, the choice of remaining solvent contradicts market expectations and cannot be an equilibrium. As a result, as depicted in Figure 3 in a scenario with multiple equilibria, the anticipation of bailouts excludes the second equilibrium with defaults. Therefore, creditor bailouts mitigate the financial instability that arises from market expectations through their effect on funding costs.

3.3 Equilibrium with bail-in

As an alternative to a creditor bailout, the supervisor can bail-in the weak banks that would otherwise default at $t = 2$. At $t = 1$, the supervisor observes the short-term returns and, given market expectations of λ, converts the long-term debt of the weak banks into equity using a conversion rate $\gamma(\lambda)$.

The choice of the conversion rate is constrained by the No Creditor Worse Off (NCWO) rule. Accordingly, a bail-in should not result in weak bank creditors experiencing losses greater than what they would face in a liquidation scenario at $t = 1$. In other words, the

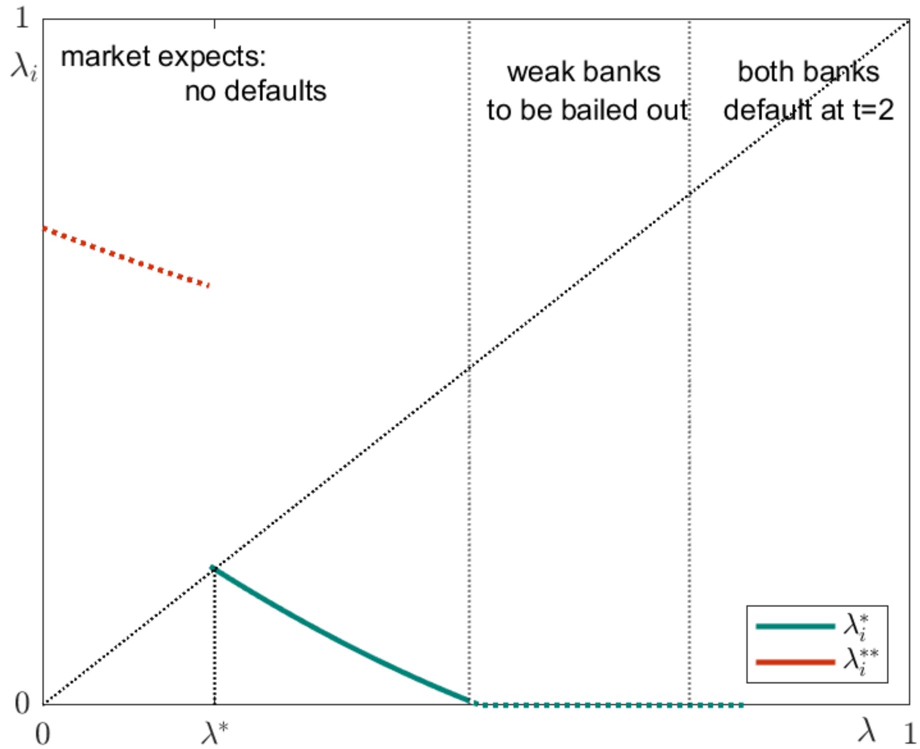


Figure 3 – Bank's response function in anticipation of bailouts

The figure illustrates the banks' choice of short-term investment based on their market expectations of the bank's portfolio λ and in anticipation of bailouts. The green line $\lambda_i^*(p)$ represents a local equilibrium portfolio that ensures the bank remains solvent, while the red line $\lambda_i^{**}(p)$ represents a local equilibrium portfolio that leads to the bank's being bailed out in the low state X_ℓ. The dotted line indicates situations where the bank deviates from market expectations, thus violating the principle of rational expectations. The intersection of the banks' response function with the 45-degree line defines the symmetric equilibrium of the depicted example. Parameter values are $\theta = 0.65$, $\alpha = 0.70$, $X_h = 2.80$, $X_\ell = 0.75$, $Z_g = 1.75$, $Z_b = 1.5$, and $c = 0.65$.

late consumers should receive a payoff at least equal to the proceeds from closing the bank, selling the long-term asset in the market, and repaying the early consumers first, that is

$$\gamma \left(1 - \lambda + \frac{h(\lambda)X_\ell - \theta}{p^c(\lambda)} \right) \overline{Z} \geq (1-\lambda)p^\ell(\lambda) + h(\lambda)X_\ell - \theta \,.$$

The left-hand side of the inequality is a fraction γ of the second-period return that late consumers receive after the bail-in, where due to no defaults at $t = 1$ the market price of the long-term asset is $p^c(\lambda)$. The right-hand side is the payoff late consumers receive when weak banks are liquidated, and the market price is $p^\ell(\lambda)$.

To demonstrate the largest ex-ante effect of bail-ins on bank portfolios, I assume that the supervisor selects the conversion rate that exactly satisfies the NCWO rule, which gives

$$\gamma(\lambda) = \left[\frac{(1-\lambda)p^{\ell}(\lambda) + h(\lambda)X_{\ell} - \theta}{(1-\lambda)p^{c}(\lambda) + h(\lambda)X_{\ell} - \theta}\right] \frac{p^{c}(\lambda)}{\overline{Z}} \, .$$

The conversion rate $\gamma(\lambda)$ is smaller than one. To see this, note that the term in brackets is the pecuniary cost of selling the long-term asset in the market after liquidating the weak banks at $t = 1$, which reduces the market price from the continuation price $p^{c}(\lambda)$ to the liquidation price $p^{\ell}(\lambda)$, and is born by the late consumers. This is because according to Proposition 1, the continuation price $p^{c}(\lambda)$ is higher than the liquidation price $p^{\ell}(\lambda)$ whenever the weak banks survive at $t = 1$. Moreover, the fraction $p^{c}(\lambda)/\overline{Z}$ is smaller than or equal to one and strictly smaller than one in case of fire sales, i.e. whenever the cash-in-the-market price of the long-term asset is smaller than the asset return.

Given the market price $p(\lambda)$ and the gross return on the long-term asset $D_2(\lambda)$, bank i's maximization problem at $t = 0$ while anticipating bail-in is

$$\begin{aligned}
\max_{\lambda_i \in [0,1)} \; & (1-\alpha)[R_h(\lambda_i) - (1-\theta)D_2(\lambda)] \\
& + \alpha\Big[\mathbf{1}\{R_{\ell}(\lambda_i) \geq (1-\theta)D_2(\lambda)\}\Big[R_{\ell}(\lambda_i) - (1-\theta)D_2(\lambda)\Big] \\
& + \mathbf{1}\{0 < R_{\ell}(\lambda_i) < (1-\theta)D_2(\lambda)\}[1-\gamma(\lambda)]R_{\ell}(\lambda_i)\Big] \, ,
\end{aligned}$$

where $\mathbf{1}\{\cdot\}$ is an indicator function with the condition for which it turns one being in the curly brackets. The bank's payoff consists of the net second-period returns in the case of X_h, the net second-period returns when X_{ℓ} while the bank remains solvent, or a share $1 - \gamma(\lambda)$ of the second-period return if it is bailed-in, or zero if it defaults on the short-term debt.

If in equilibrium the bank never defaults, that is if

$$R_h(\lambda_i^*) > R_{\ell}(\lambda_i^*) > (1-\theta)D_2(\lambda) \tag{7}$$

the portfolio

$$\lambda_i^*(p) = \frac{\overline{X} - p(\lambda)}{\overline{X}} \, ,$$

which is identical to the laissez-faire case, defines the solution to the bank's problem if it satisfies the equilibrium condition (7).

If, on the other hand, in equilibrium, the bank remains solvent at $t = 2$ in the high return state X_h, but is going to default at $t = 2$ in the low return state X_ℓ and hence is bailed-in, that is when

$$R_h(\lambda_i^{in}) > (1-\theta)D_2(\lambda) > R_\ell(\lambda_i^{in}) > 0 \tag{8}$$

then, bank i's expected payoff is

$$(1-\alpha)[R_h(\lambda_i) - (1-\theta)D_2(\lambda)] + \alpha[1-\gamma(\lambda)]R_\ell(\lambda_i)$$

and the first-order condition is

$$(1-\alpha)\frac{\partial R_h(\lambda_i)}{\partial \lambda} + \alpha[1-\gamma(\lambda)]\frac{\partial R_\ell(\lambda_i)}{\partial \lambda} = 0\,,$$

which simplifies to

$$h'(\lambda_i^{in})\left[\frac{\overline{X} - \alpha\gamma(\lambda)X_\ell}{1-\alpha\gamma(\lambda)}\right] = p(\lambda)\,.$$

Then, the solution to the bank's problem is

$$\lambda_i^{in}(p) = 1 - p(\lambda)\left[\frac{\overline{X} - \alpha\gamma(\lambda)X_\ell}{1-\alpha\gamma(\lambda)}\right]^{-1}\,,$$

if it satisfies the equilibrium condition (8). In this case, one can show that the term in the brackets is smaller than X_h.[3] This implies that the short-term investment in anticipation of bail-ins is lower than it would be without supervisory intervention. Given that the bank receives a positive payoff after a bail-in, it accounts for the downside of its risk-taking and selects a less risky short-term investment relative to laissez-faire. In essence, bail-ins mitigate the moral hazard problem arising from limited liability and opaque portfolios.

If the bank defaults at $t = 1$ when it receives a low short-term investment X_ℓ, that is

[3]Note that $\frac{\overline{X}-\alpha\gamma(\lambda)X_\ell}{1-\alpha\gamma(\lambda)} < X_h \Leftrightarrow \alpha\gamma(X_h - X_\ell) < X_h - \overline{X} \Leftrightarrow \alpha\gamma(X_h - X_\ell) < \alpha(X_h - X_\ell) \Leftrightarrow \gamma < 1$.

when $R_\ell(\lambda_i^{**}) = 0$, then, bank i's expected payoff is

$$(1-\alpha)[R_h(\lambda_i) - (1-\theta)D_2(\lambda)]$$

and the first-order condition

$$h'(\lambda_i^{**})X_h = p(\lambda)$$

defines the solution to the bank's problem if

$$\lambda_i^{**}(p) = \frac{X_h - p(\lambda)}{X_h}$$

satisfies the equilibrium condition $R_\ell(\lambda_i^{**}) = 0$.

Regarding the gross return of the long-term debt, the binding participation constraint of the late consumers when bail-in is anticipated is

$$\alpha\gamma(\lambda)R_\ell(\lambda) + (1-\alpha)(1-\theta)D_2(\lambda) = 1-\theta\,. \tag{9}$$

That is the later consumers receive the face value of their debt from strong banks and a fraction $\gamma(\lambda)$ of the second-period returns from weak banks. In the absence of supervisory intervention, late consumers would have received a fraction c of the second-period return if weak banks had defaulted at $t = 2$. However, when late consumers are bailed-in, they receive a fraction $\gamma(\lambda)$ of the returns, which can be either larger or smaller than c. As a result, the impact of bail-ins on ex-ante funding costs is ambiguous.

In cases where multiple equilibria exist in laissez-faire, the anticipation of bail-ins can eliminate the equilibrium with defaults, if the reduction in short-term investment is large enough in magnitude. To illustrate this with the example presented in Figure 2, when the market expects weak banks to default at $t = 2$ for an intermediate level of market investment λ, the bank might prefer to invest $\lambda_i^{in}(p)$ in the short-term asset while anticipating bail-ins. Consequently, the short-term investment is lower than what it would have been in laissez-faire $\lambda_i^{**}(p)$. For the example illustrated, $\lambda_i^{in}(p)$ is low enough to rule out a symmetric equilibrium where all banks invest in a risky portfolio. In other words, the anticipation of bail-ins can

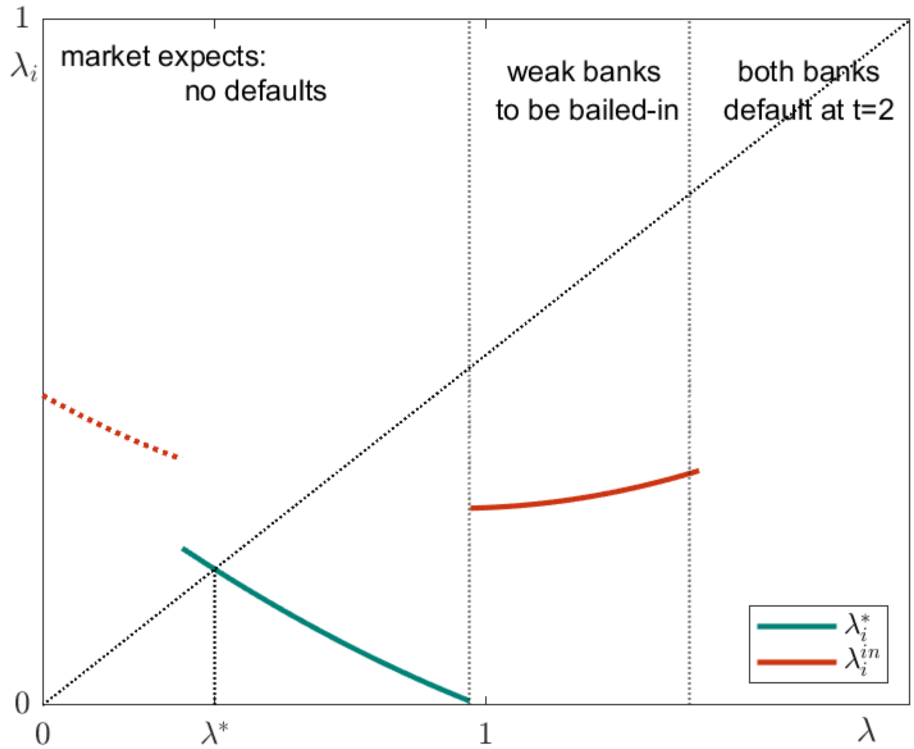


Figure 4 – Bank's response function in anticipation of bail-ins

The figure illustrates the banks' choice of short-term investment based on their market expectations of the bank's portfolio λ and in anticipation of bail-ins. The green line $\lambda_i^*(p)$ represents a local equilibrium portfolio that ensures the bank remains solvent, while the red line $\lambda_i^{in}(p)$ represents a local equilibrium portfolio that leads to the bank's being bailed-in in the low state X_ℓ. The dotted line indicates situations where the bank deviates from market expectations, thus violating the principle of rational expectations. The intersection of the banks' response function with the 45-degree line defines the symmetric equilibrium of the depicted example. Parameter values are $\theta = 0.65$, $\alpha = 0.70$, $X_h = 2.80$, $X_\ell = 0.75$, $Z_g = 1.75$, $Z_b = 1.5$, and $c = 0.65$.

reduce the riskiness of banks' portfolios to the extent that in a symmetric equilibrium all banks remain solvent.

Figure 5 summarizes the finding in the model without aggregate risk. The figure illustrates the symmetric equilibrium across the range of possible values for the probability of a low short-term asset return α in case of no supervisory intervention, in anticipation of bailouts, and in anticipation of bail-ins. The green line represents the equilibrium in which banks' choose a portfolio that ensures no bank defaults, while the red line illustrates the equilibrium in which banks' choose a portfolio that results in weak banks defaulting at

$t = 2$.

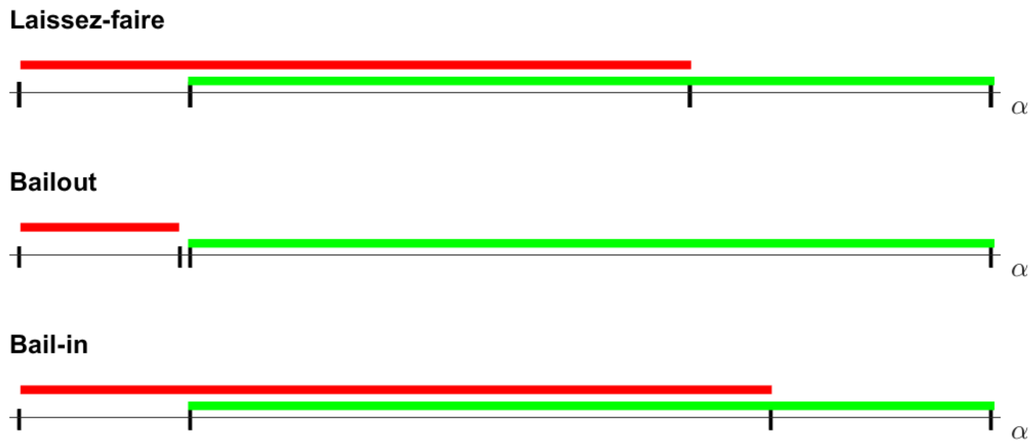


Figure 5 – Multiple equilibrium in anticipation of resolution

The figure depicts the symmetric equilibrium across the range of possible values for the probability of a low short-term asset return α in case of no supervisory intervention, in anticipation of bailouts, and in anticipation of bail-ins. The green line signifies the equilibrium with no defaults and the red line signifies the equilibrium in which weak banks default at $t = 2$. The overlapping area between the red and green lines denotes situations with multiple equilibria. Parameter values are $\theta = 0.30$, $\alpha = 0.50$, $X_h = 3.00$, $X_\ell = 0.60$, $\overline{Z} = 1.40$, and $c = 0.65$.

In case of laissez-faire, an increasing probability of a low short-term asset return, i.e. rising α, leads to banks investing less in the risky short-term asset and, hence transitioning from an equilibrium in which they default to another one in which they stay solvent. For intermediate levels of α, the presence of market expectations allows for the coexistence of both equilibria, reflecting instability. When banks and the market expect bailouts, the banks prefer to stay solvent over a larger range of α, thus eliminating the default equilibrium in cases of multiple equilibria. However, if the portfolio reallocation effect of bail-ins is not significant enough, as it is the case in this example, the expectation of bail-ins cannot rule out the equilibrium with defaults, and even it may broaden the region where multiple equilibria exist, amplifying instability.

4 The model with aggregate risk

In this Section, I assume the long-term asset return is either high Z_g with probability $1-\beta$ or low Z_b with probability β, that is

$$Z_j = \begin{cases} Z_b, & \text{with probability } \beta \\ Z_g, & \text{with probability } 1-\beta \end{cases}$$

where $Z_b < Z_g$ and $\overline{Z} = (1-\beta)Z_g + \beta Z_b$. I use the subscript $j = \{b, g\}$ to refer to the systematic return realization. A high long-term asset return Z_g will be called *good times* and a low realization Z_b will be called *bad times*.

At $t = 1$, before banks trade the long-term asset, I assume that the long-term asset's return, which will be realized at $t = 2$, is observable, thus eliminating all uncertainty about its fundamental value.[4] Consequently, the market price of the long-term asset does not exceed its fundamental value, due to the available safe asset. Thus, banks will not incur losses from trading the asset under uncertainty. In the following, I begin by defining the market price of the long-term asset at $t = 1$, given any short-term market investment λ. Then, I describe banks' investment decisions at $t = 0$.

Proposition 2. *The market price of the long-term asset, for any value λ of the banks' investment in the short-term asset and given the long-term asset return Z_j, is*

$$p(\lambda, Z_j) = \min\left\{ \max\left\{p^c(\lambda, Z_j), p^\ell(\lambda, Z_j), p^b(\lambda, , Z_j)\right\}, Z_j \right\},$$

where $p^c(\lambda, Z_j)$ is the continuation price, when no bank defaults at $t = 1$,

$$p^c(\lambda, Z_j) = h(\lambda)\overline{X} + Z_j - \theta\,,$$

the $p^\ell(\lambda, Z_j)$ is liquidation price, when weak banks default at $t = 1$,

$$p^\ell(\lambda, Z_j) = \frac{(1-\alpha)[h(\lambda)X_h - \theta] + Z_j}{1 + \alpha(1-\lambda)}\,,$$

[4]This assumption aligns with models of financial intermediaries with a market to trade banks' assets, as seen in Allen and Gale (2004).

and $p^b(\lambda, Z_j)$ is the crisis price, when both banks default at $t = 1$,

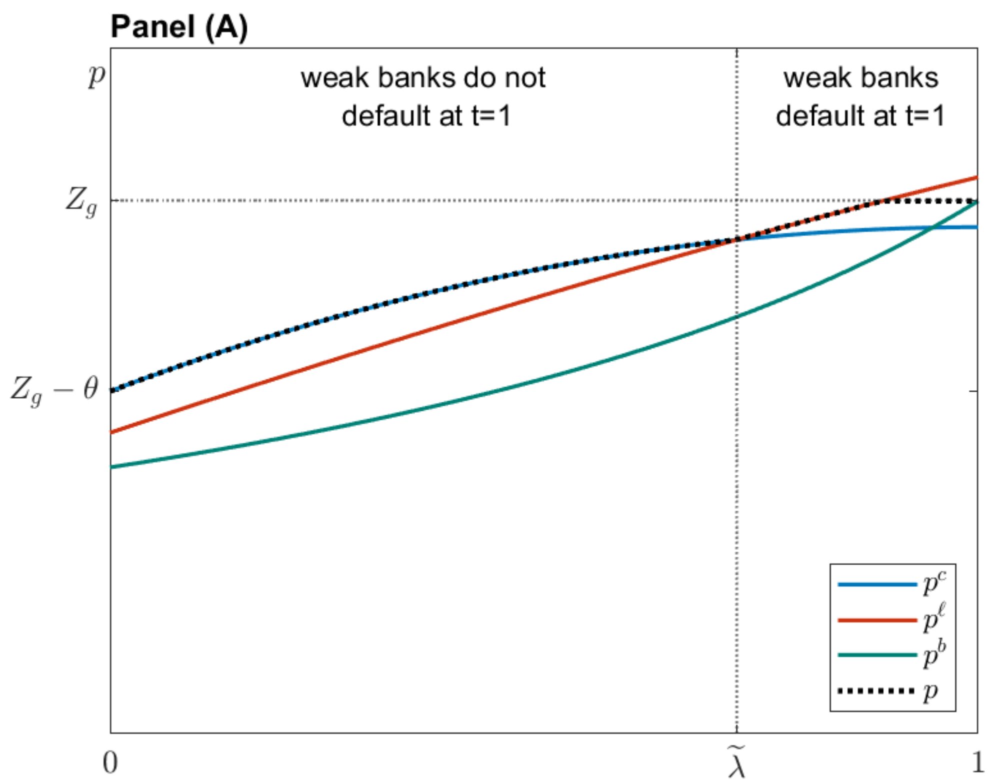
$$p^b(\lambda, Z_j) = \frac{Z_j}{2 - \lambda}\,.$$

For parameters such that

$$\frac{X_\ell}{2} < \theta < \max\left\{\frac{\overline{Z}}{2}, \frac{X_h}{2}\right\},$$

strong banks survive at $t = 1$ in good times, but weak banks may default at $t = 1$ if the ex-ante investment in the short-term asset is too large. However, in bad times, both may default at $t = 1$ if their ex-ante investment in the short-term asset is too low.

Figure 6 illustrates the result in Proposition 2, showing the market price $p(\lambda, Z_j)$, alongside the continuation price $p^c(\lambda, Z_j)$, the liquidation price $p^\ell(\lambda, Z_j)$, and the crisis price $p^b(\lambda, Z_j)$ for high long-term asset return Z_g in Panel (A) and low long-term asset return Z_b in Panel (B).



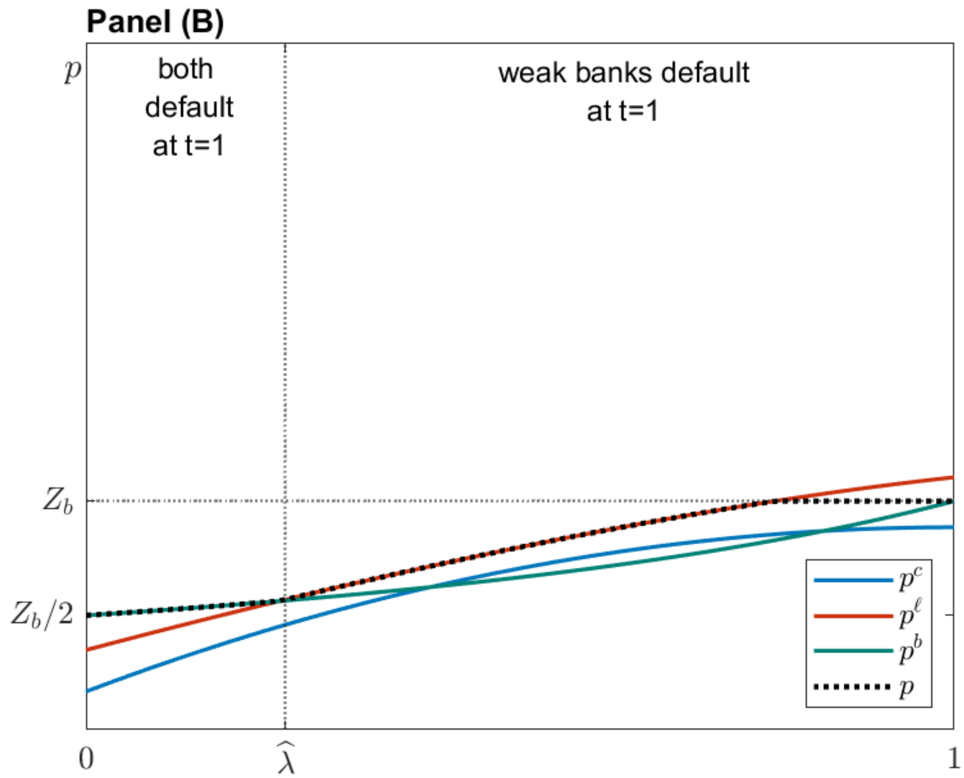


Figure 6 – Market price of the long-term asset

Panel (A) illustrates the long-term asset price in good times Z_g and Panel (B) illustrates the price in bad times Z_b. The solid blue lines are the continuation prices $p^c(\lambda, Z_j)$ when weak banks can sell assets to repay early consumers. The solid red lines are the liquidation prices $p^\ell(\lambda, Z_j)$ when weak banks cannot repay early consumers and are liquidated. The green solid lines are the crisis prices $p^b(\lambda, Z_j)$ when both banks default at $t = 1$ and are liquidated. The dotted black line is the market price of the long-term asset $p(\lambda, Z_j)$ at $t = 1$. The threshold $\widetilde{\lambda}$ is the intersection of $p^c(\lambda, Z_g)$ and $p^\ell(\lambda, Z_g)$ above which weak banks default at $t = 1$. The threshold $\widehat{\lambda}$ is the intersection of $p^c(\lambda, Z_b)$ and $p^b(\lambda, Z_b)$ below which both banks default at $t = 1$. Parameter values are $\theta = 0.625$, $\alpha = 0.375$, $\beta = 0.250$, $X_h = 1.50$, $X_\ell = 0.375$, $Z_g = 1.75$, $Z_b = 0.75$.

In good times, for low values of λ weak banks cannot repay the early consumers at $t = 1$ and need to sell a portion of their long-term assets to survive. Then, the continuation price $p^c(\lambda, Z_g)$ defines the market price. However, when banks allocate large investments into the risky short-term asset, even selling the entire holding of the long-term asset is not enough for the weak banks to repay early consumers, leading to their failure at $t = 1$ and the liquidation price $p^\ell(\lambda, Z_g)$ defining the market price. In Figure 6 Panel (A), when the banks invest more than $\widetilde{\lambda}$, which is the intersection of the liquidation price $p^\ell(\lambda, Z_g)$ and the continuation price $p^c(\lambda, Z_g)$, weak banks are going to default at $t = 1$. Finally, in good times strong banks are solvent at $t = 1$, regardless of the short-term investment. This can be seen in Figure 3 Panel (A) as the crisis price $p^b(\lambda, Z_g)$ not intersecting with the liquidation price $p^\ell(\lambda, Z_g)$.

While transitioning from good times to bad times, one can observe a decrease in the market price. As illustrated in Figure 6, when the long-term asset return is low the market price shown in Panel (B) shifts downward in comparison to the price in Panel (A). In essence, this reflects the fact that the market price as in Proposition 2 is increasing in the long-term asset return Z_j. This is because, for any market price lower than the long-term asset return Z_j, the outside investors' demand for the long-term asset (1) increases with the long-term asset return. In other words, the higher the return generated by the long-term asset, the more the outside investors are willing to buy, which increases the market price. The decrease in prices in bad times increases the likelihood of weak banks defaulting at $t = 1$. In Figure 6 Panel (B), the liquidation price $p^\ell(\lambda, Z_b)$ stays above the continuation price $p^c(\lambda, Z_b)$, which means weak banks default in bad times for any value of short-term investment.

Moreover, just like the market price outlined in Proposition 1 and for the same reason, the continuation prices $p^c(\lambda, Z_j)$ and the liquidation prices $p^\ell(\lambda, Z_j)$ are increasing in the short-term asset investment. Furthermore, the crisis price $p^b(\lambda, Z_b)$ increases with λ. When both banks are liquidated, an increase in short-term asset holdings results in fewer long-term assets being sold in the market, consequently leading to an increase in price. The combination of the two factors of lower prices in bad times and prices being increasing in λ, may result in strong banks defaulting at $t = 1$ in bad times. This is because, for low values of λ, the fire sales are so deep that even strong banks may have insufficient amounts of long-term asset to sell and repay early consumers. Then, both banks are liquidated, the crisis price $p^b(\lambda, Z_b)$ defines the market price, and the outside investors are the only buyers of the asset in the market. In Figure 3 Panel (B), when the banks invest less than $\widehat{\lambda}$, which is the intersection of the liquidation price $p^\ell(\lambda, Z_b)$ and the crisis price $p^b(\lambda, Z_b)$, both banks are going to default at $t = 1$.

If, given bank i's short-term investment λ_i, long-term asset return Z_j, and the market price $p(\lambda)$, the bank defaults at $t = 1$ the supervisor sells the bank's long-term assets $1 - \lambda_i$ and there are no second-period returns for the bank. If on the other hand, the bank survives at $t = 1$, it will yield a return

$$\left(1 - \lambda_i + \frac{h(\lambda_i)X_\ell - \theta}{p^c(\lambda, Z_j)}\right) Z_j$$

at $t = 2$ if the bank has a short-term return X_ℓ, and

$$\left(1 - \lambda_i + \frac{h(\lambda_i)X_h - \theta}{\max\{p^c(\lambda, Z_j), p^\ell(\lambda, Z_j)\}}\right) Z_j$$

if the bank has a short-term return X_h. The values inside the parentheses represent the quantity of long-term assets the bank holds at $t = 2$. This comprises the bank's initial investment of $1 - \lambda_i$ at $t = 0$ and the volume of long-term assets the bank trades at $t = 1$.

In sum, bank i's second-period return can be generalized as

$$R(\lambda_i, p, X_i, Z_j) = (1 - \lambda_i + a_i)Z_j$$

where the volume traded is

$$a_i(\lambda_i, p, X_\ell, Z_j) = \max\left\{\frac{h(\lambda_i)X_\ell - \theta}{p^c(\lambda, Z_j)}, -(1 - \lambda_i)\right\},$$

for the weak bank and

$$a_i(\lambda_i, p, X_h, Z_j) = \max\left\{\frac{h(\lambda_i)X_h - \theta}{\max\{p^c(\lambda, Z_j), p^\ell(\lambda, Z_j)\}}, -(1 - \lambda_i)\right\},$$

for the strong bank. Thus, the volume traded depends on the bank's investment choice, the bank's individual short-term asset return, the long-term asset return, and market price. The maximum operator ensures that the bank cannot sell more long-term assets than it owns. In other words, if the bank must sell more long-term assets at $t = 1$ to continue operating than it possesses, the bank faces liquidation.

To simplify the notation, given long-term asset return Z_j, let's denote the second-period return of bank i as $R_{h\,j}(\lambda_i)$ when its short-term asset return is X_h and as $R_{\ell\,j}(\lambda_i)$ when the short-term asset return is X_ℓ.

Systemic events: In the model with aggregate risk, banks may choose an investment in the short-term asset λ_i which leads to a symmetric equilibrium where both strong and weak banks face defaults either at $t = 1$ or $t = 2$, particularly in bad times. This means all banks may default in bad times regardless of their short-term return. I refer to these cases as *systemic* events. If the supervisor has a macroprudential mandate, her primary concern lies with systemic events rather than individual bank defaults. Therefore, the key questions revolve around whether a supervisory resolution can prevent systemic events or whether they might inadvertently contribute to such events.

When in laissez-faire, with no supervisory intervention, the banks choose a portfolio that results in all banks simultaneously defaulting in bad times, the anticipation of either bail-in or bailout can influence banks' short-term asset investment and their resulting payoffs. This adjustment can potentially prevent a systemic event from occurring. On the other hand, if banks, under laissez-faire, already choose a portfolio that leads to strong banks remaining solvent, the anticipation of bail-in or bailout may push the equilibrium towards a systemic event. In this case, the supervisory resolution tool itself becomes a factor contributing to systemic risk.

4.1 Equilibrium with no supervisory intervention

In the laissez-faire case with no supervisory intervention, given the market expectation λ which defines the market price $p(\lambda, Z_i)$ and the gross return of the long-term debt $D_2(\lambda)$, the bank either receives the net second-period return after repaying the late consumers or zero in case of default either at $t = 1$ or $t = 2$. Then, the maximization problem of the bank at $t = 0$ is

$$\max_{\lambda_i \in [0,1)} \Pr\Big(R_{ij}(\lambda_i) \geq (1-\theta)D_2(\lambda)\Big)\mathbb{E}\Big[R_{ij}(\lambda_i) - (1-\theta)D_2(\lambda) | R_{ij}(\lambda_i) \geq (1-\theta)D_2(\lambda)\Big].$$

Since the bank chooses a λ_i that at least leads to staying solvent in good times, the first-order condition to the bank's problem is

$$\begin{aligned} &(1-\alpha)(1-\beta)\frac{\partial R_{h\,g}(\lambda_i)}{\partial \lambda_i}+ \\ &\qquad \Pr\Big(R_{i\,j}(\lambda_i) \geq (1-\theta)D_2(\lambda)\Big)\mathbb{E}\Big[\frac{\partial R_{i\,j}(\lambda_i)}{\partial \lambda_i}\Big|R_{i\,j}(\lambda_i) \geq (1-\theta)D_2(\lambda)\Big] = 0\,, \end{aligned} \tag{10}$$

where

$$\frac{\partial R_{i\,j}(\lambda_i)}{\partial \lambda_i} = \left[-1 + \frac{(1-\lambda_i)X_i}{p(\lambda, Z_j)}\right] Z_j\,.$$

The portfolio satisfying the first-order condition is a solution to the bank's problem if the corresponding equilibrium condition in terms of survival or default in each state is satisfied. Lastly, if more than one portfolio satisfies the first-order condition the bank chooses the portfolio that generates the highest payoff.

Regarding the gross return of the long-term debt, late consumers receive either the full face value of debt when the bank stays solvent, a fraction c of the second-period return when the bank defaults at $t = 2$, or zero when the bank defaults at $t = 1$. Hence, their binding participation constraint is

$$\begin{aligned} &\Pr\Big(R_{i\,j}(\lambda) \geq (1-\theta)D_2(\lambda)\Big)(1-\theta)D_2(\lambda)+ \\ &\qquad c\Pr\Big(0 < R_{i\,j} < (1-\theta)D_2(\lambda)\Big)\mathbb{E}\Big[R_{i\,j}(\lambda)|0 < R_{i\,j}(\lambda) < (1-\theta)D_2(\lambda)\Big] = 1-\theta\,, \end{aligned}$$

given market expectations λ about banks short-term investment and the corresponding default probability.

4.2 Equilibrium with bailout

If the supervisor commits to bailout creditors, the shareholders are wiped out and their payoff is identical to the case of the bank defaulting. Hence, the bank's maximization problem at $t = 0$ is identical to laissez-faire and the solutions to the problem are the same. However, since the supervisor transfers the late consumers,

$$(1-\theta)D_2(\lambda) - R_{i\,j}(\lambda)\,,$$

they receive the face value of their debt whenever the bank survives at $t = 1$. Thus, the late consumers' binding participation constraint is

$$(1-\theta)D_2(\lambda)\Pr\Big(R_{i\,j}(\lambda) > 0\Big) = 1-\theta\,.$$

Since late consumers receive weakly more compared to the laissez-faire case, the gross return of long-term debt is lower in anticipation of bailouts than in laissez-faire, $D_2^{out}(\lambda) \leq D_2(\lambda)$. Thus, similar to the model without aggregate risk the cheaper funding in anticipation of bailouts, may change the bank's portfolio choice.

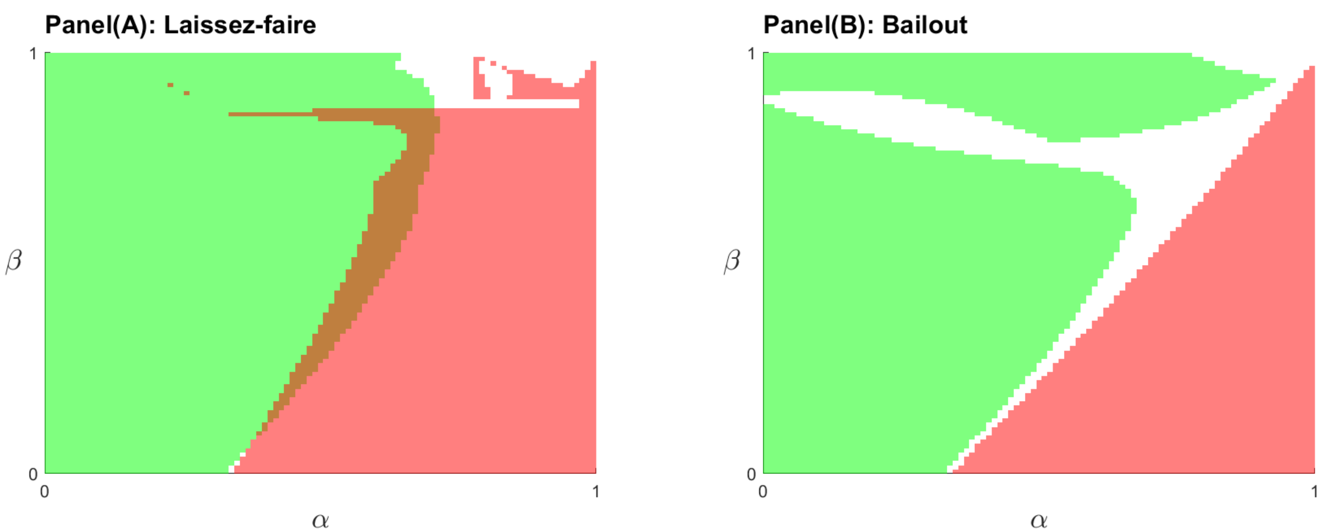


Figure 7 – Systemic risk in anticipation of bailouts

Pane (A) illustrates the systemic risk in laissez-faire, and Panel (B) illustrates the systemic risk in anticipation of bailouts for the range of possible values of low short-term asset return α and possible values of low long-term asset return β. The green area signifies an equilibrium with no systemic risk. The red region signifies an equilibrium with systemic risk. When these two areas overlap, there exists multiple equilibria. Parameter values are $\theta = 0.40$, $X_h = 3.00$, $X_\ell = 0.60$, $Z_g = 2.60$, $Z_b = 1.20$.

Figure 7 depicts the systemic risk for the range of possible values of low short-term asset return α and possible values of low long-term asset return β. The green area signifies an equilibrium in which the strong bank remains solvent in dependent of the long-term asset return. Whereas, the red area signifies an equilibrium in which both banks default when the long-term asset is expected to have a low return. When these two areas overlap, it indicates the presence of multiple equilibria. In laissez-faire, for low values of α banks' portfolio choice leads to no simultaneous defaults, that is there is no systemic risk. However, as the probability of low short-term asset return increases, banks reduce their investment in the short-term asset, getting into the region in which both banks default in bad times irrespective of their short-term asset return. The anticipation of bailouts, insures the long-term debt, and hence reduces the funding costs. This, as illustrated in Figure 7, may remove the equilibrium with systemic bank defaults, thus reducing the red area.

4.3 Equilibrium with bail-in

If the supervisor bails-in banks that are going to default at $t = 2$, she first observes both the short-term asset return X_i realized at $t = 1$ and the long-term asset return Z_j which is going to realize at $t = 2$. Then, she converts the long-term debt of the failing banks into equity with a conversion rate equal to $\gamma_{i\,j}$. The NCWO rule restricts the conversion rate such that creditors do not experience losses greater than what they would face in a liquidation scenario at $t = 1$. In the presence of aggregate risk, this means in good times if the weak banks are going to default at $t = 2$, a bail-in should generate payoffs for the late consumers equal to what they would receive when weak banks are liquidated at $t = 1$. In bad times, after a bail-in late consumers should receive payoffs at least equal to proceeds from either liquidating weak banks or, in case of a systemic default, the proceeds from liquidating both banks. Hence, the NCWO rule can be generalized as

$$\gamma_{i\,j}\left(1-\lambda+\frac{h(\lambda)X_i-\theta}{p(\lambda,Z_j)}\right)Z_j \geq (1-\lambda)\max\left\{p^{\ell}(\lambda,Z_j),p^{b}(\lambda,Z_j)\right\}+h(\lambda)X_i-\theta\,.$$

The left-hand side of the inequality is a fraction $\gamma_{i\,j}$ of the second-period return that late consumers receive after the bail-in as shareholders. In this case, the market price is either $p^c(\lambda, Z_i)$ when both banks survive at $t = 1$ and either (or both) of them are bailed-in, or $p^{\ell}(\lambda, Z_i)$ when only strong banks survive at $t = 1$ and are bailed-in. The right-hand side is the payoff late consumers receive when the defaulting bank is liquidated. The market price is either $p^\ell(\lambda, Z_j)$ if only weak banks are liquidated or $p^b(\lambda, Z_j)$ if both banks are liquidated.

I assume the supervisor chooses a conversion rate equal to

$$\gamma_{i\,j}(\lambda) = \left[\frac{(1-\lambda)\max\{p^\ell(\lambda, Z_j), p^b(\lambda, Z_j)\} + h(\lambda)X_i - \theta}{(1-\lambda)p(\lambda, Z_j) + h(\lambda)X_i - \theta}\right]\frac{p(\lambda, Z_j)}{Z_j}\,,$$

which just satisfies the NCWO rule. Then, the banks' maximization problem at $t = 0$ is

$$\begin{aligned}\max_{\lambda_i\in[0,1)} \Pr\Big(0 < R_{i\,j}(\lambda_i) \le (1-\theta)D_2(\lambda)\Big)\mathbb{E}\Big[\Big(1-\gamma_{i\,j}(\lambda)\Big)R_{i\,j}(\lambda_i)|0 < R_{i\,j}(\lambda_i) \le (1-\theta)D_2(\lambda)\Big]\\ + \Pr\Big(R_{i\,j}(\lambda_i) \ge (1-\theta)D_2(\lambda)\Big)\mathbb{E}\Big[R_{i\,j}(\lambda_i) - (1-\theta)D_2(\lambda)|R_{i\,j}(\lambda_i) \ge (1-\theta)D_2(\lambda)\Big]\,,\end{aligned}$$

where the first term means banks receive a fraction $1-\gamma_{i\,j}(\lambda)$ of the second-period asset return if the bank is bailed-in. The second term means when the bank is solvent at $t = 2$, banks receive the excess second-period return after repaying the late consumers. The corresponding first-order condition is

$$\begin{aligned}\Pr\Big(0 < R_{i\,j}(\lambda_i) \le (1-\theta)D_2(\lambda)\Big)\mathbb{E}\Big[\Big(1-\gamma_{i\,j}(\lambda)\Big)\frac{\partial R_{i\,j}(\lambda_i)}{\partial\lambda_i}\Big|0 < R_{i\,j}(\lambda_i) \le (1-\theta)D_2(\lambda)\Big]\\ + \Pr\Big(R_{i\,j}(\lambda_i) \ge (1-\theta)D_2(\lambda)\Big)\mathbb{E}\Big[\frac{\partial R_{i\,j}(\lambda_i)}{\partial\lambda_i}\Big|R_{i\,j}(\lambda_i) \ge (1-\theta)D_2(\lambda)\Big] = 0\,,\end{aligned} \tag{11}$$

which captures the effect of the short-term investment λ_i on the second-period return when banks are bailed-in and when banks are solvent at $t = 2$, respectively. When comparing the first-order condition in anticipation of bail-ins with the one in laissez-faire (10), the additional first term in (11) underscores that banks' now endogenous the effect of their portfolio choice on the second-period asset return following a bail-in, which could be positive or negative. As a result, resembling the situation without aggregate risk, bail-ins introduce ex-ante portfolio reallocation. If the expectation of bail-ins causes banks to invest less in the short-term asset, they might enter a region where even strong banks lack a sufficient first-period liquidity to repay early consumers during bad times, leading to all banks default simultaneously when the long-term asset return is low Z_b. In other words, the impact of bail-ins on portfolios may not prevent systemic events, and in some cases, it could potentially contribute to systemic events.

Regarding the gross return of long-term debt given the market expectation λ, the late consumers' binding participation constraint is

$$\Pr\Big(0 < R_{ij}(\lambda_i) \leq (1-\theta)D_2(\lambda)\Big)\mathbb{E}\Big[\gamma_{ij}(\lambda)R_{ij}(\lambda_i)|0 < R_{ij}(\lambda_i) \leq (1-\theta)D_2(\lambda)\Big] + (1-\theta)D_2(\lambda)\Pr\Big[R_{ij}(\lambda_i) \geq (1-\theta)D_2(\lambda)\Big] = 1-\theta\,, \tag{12}$$

where the first term means late consumers receive equity payoffs when the bank is bailed-in. The second term means when the bank is solvent at $t = 2$, the late consumers receive the face value of long-term debt. When compared to laissez-faire, the impact of the expectation of bail-ins on the gross return of long-term debt is uncertain, and it depends on the conversion rate $\gamma_{i,j}(\lambda)$ relative to the recovery rate after default c. Given market short-term investment λ, if late consumers receive lower payoffs after a bail-in than after a default at $t = 2$, i.e., $\gamma_{i,j}(\lambda) < c$, then the gross return of long-term debt while anticipating bail-ins is higher than when no supervisory intervention is expected.

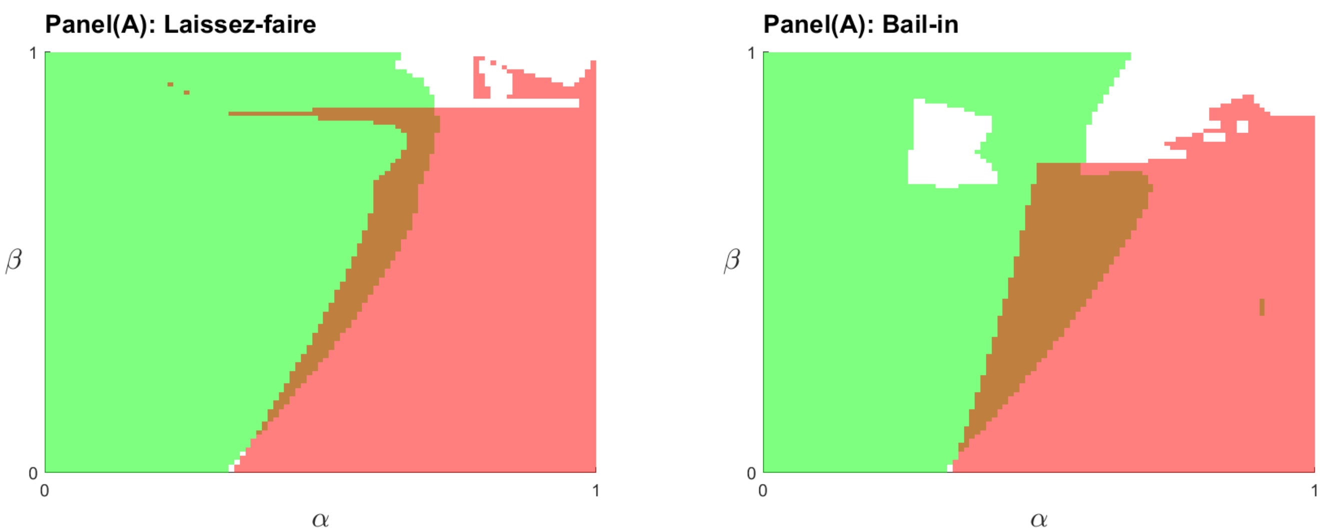


Figure 8 – Systemic risk in anticipation of bail-ins

Pane (A) illustrates the systemic risk in laissez-faire, and Panel (B) illustrates the systemic risk in anticipation of bail-ins for the range of possible values of low short-term asset return α and possible values of low long-term asset return β. The green area signifies an equilibrium with no systemic risk. The red region signifies an equilibrium with systemic risk. When these two areas overlap, there are multiple equilibria. Parameter values are $\theta = 0.40$, $X_h = 3.00$, $X_\ell = 0.60$, $Z_g = 2.60$, $Z_b = 1.20$.

Finally, Figure 8 depicts the systemic risk for the range of possible values of low short-term asset return α and possible values of low long-term asset return β. Bail-ins alter banks' payoffs and hence trigger ex-ante portfolio reallocation. Figure 8 illustrates that for intermediate levels of α, this portfolio reallocation effect creates the possibility of banks investing less in the short-term asset compared to the laissez-faire scenario. This adjustment can push them into the region where they default during bad times. In essence, the anticipation of bail-ins can introduce multiple equilibria and generate systemic risk. As a result, bail-ins are less effective in averting systemic risk and might even contribute to it.

5 Conclusion

This paper contributes to the ongoing debate about supervisory resolution tools, particularly the choice between bail-ins and bailouts. In a model without aggregate risk, the anticipation of bailouts leads to reduced funding costs for banks. The resulting perspective of higher bank payoffs may incentivize each individual bank to choose a portfolio that ensures its solvency, thereby eliminating bank defaults. Similarly, the anticipation of bail-ins leads to reduced investments in the short-term risky asset, potentially preventing banks from defaulting. Although both interventions have a similar impact on bank defaults, they influence the equilibrium through different channels. The introduction of aggregate risk increases the vulnerability of both weak and strong banks to defaults after a negative systematic shock. In such scenarios, bailouts may prevent simultaneous defaults during bad times, effectively eliminating systemic risk. In contrast, bail-ins have the potential to shift the equilibrium towards systemic defaults.

So far in this model, the supervisory resolution policy is treated as an exogenous variable to assess the positive effects of each policy on bank portfolios and defaults. A natural next step would involve explicitly defining the supervisory mandate, i.e. objective function, to analyze the (social) costs associated with bailouts versus bail-ins, and hence to formalize the supervisor's preferred resolution tool. Existing literature emphasizes the commitment challenge faced by supervisors in refraining from bailouts. Investigating supervisory preferences will help determine whether the supervisor can genuinely commit to a specific policy.

Appendix A

Proof of Proposition 1. When weak banks cannot repay early consumers out of the first-period return, that is when

$$\theta > h(\lambda_i)X_\ell \,,$$

they need to sell a fraction of their long-term asset holding to prevent a default at $t = 1$. In this case, if the short-term asset return plus the proceeds from selling the long-term asset is enough to repay the early consumers,

$$h(\lambda)X_\ell + p(1-\lambda) < \theta \,,$$

that is when

$$p \geq \widetilde{p}(\lambda) = \frac{\theta - h(\lambda)X_\ell}{1-\lambda} \,,$$

the market clearing condition,

$$\alpha[h(\lambda)X_\ell - \theta] + (1-\alpha)[h(\lambda)X_h - \theta] + (\overline{Z} - p) = 0 \,,$$

defines the continuation price,

$$p^c(\lambda) = h(\lambda)\overline{X} + \overline{Z} - \theta \,,$$

where superscript c indicates that all banks continue to operate until $t = 2$.

When weak banks fail to repay early consumers, that is when $p < \widetilde{p}(\lambda)$, the supervisor liquidates weak banks' assets, $1 - \lambda_i$. In this case, the market-clearing condition

$$-\alpha(1-\lambda) + (1-\alpha)\frac{h(\lambda)X_h - \theta}{p} + \frac{\overline{Z} - p}{p} = 0 \,,$$

defines the liquidation price,

$$p^{\ell}(\lambda) = \frac{(1-\alpha)[h(\lambda)X_h - \theta] + \overline{Z}}{1+\alpha(1-\lambda)}\,,$$

where superscript ℓ indicates that the weak banks are liquidated at $t = 1$. Note that the liquidation price can be rewritten as

$$p^{\ell}(\lambda) = p^{c}(\lambda) + \frac{\alpha(1-\lambda)}{1+\alpha(1-\lambda)}[\widetilde{p}(\lambda) - p^{c}(\lambda)] \tag{A1}$$

or as

$$p^{\ell}(\lambda) = \widetilde{p}(\lambda) + \frac{[p^{c}(\lambda) - \widetilde{p}(\lambda)]}{1+\alpha(1-\lambda)}\,. \tag{A2}$$

If for some λ we have $\widetilde{p}(\lambda) \leq p^{c}(\lambda)$, then (A1) implies $p^{\ell}(\lambda) \leq p^{c}(\lambda)$ and (A2) implies $\widetilde{p}(\lambda) \leq p^{\ell}(\lambda)$, that is

$$\widetilde{p}(\lambda) \leq p^{\ell}(\lambda) \leq p^{c}(\lambda). \tag{A3}$$

And if for some λ we have $\widetilde{p}(\lambda) > p^{c}(\lambda)$, then (A1) implies $p^{\ell}(\lambda) > p^{c}(\lambda)$ and (A2) implies $\widetilde{p}(\lambda) > p^{\ell}(\lambda)$, that is

$$\widetilde{p}(\lambda) > p^{\ell}(\lambda) > p^{c}(\lambda). \tag{A4}$$

In the first case, one cannot have weak banks defaulting, because the liquidation price $p^{\ell}(\lambda)$ is above the threshold $\widetilde{p}(\lambda)$, so the continuation price $p^{c}(\lambda)$ (which is above the threshold $\widetilde{p}(\lambda)$) becomes the market price $p(\lambda)$. In the second case, one cannot have weak banks surviving, because the continuation price $p^{c}(\lambda)$ is below the threshold $\widetilde{p}(\lambda)$, so the liquidation price $p^{\ell}(\lambda)$ (which is below the threshold $\widetilde{p}(\lambda)$) becomes the market price $p(\lambda)$. Hence, it follows that $p(\lambda) = \max\{p^{c}(\lambda), p^{\ell}(\lambda)\}$. Finally, the market price $p(\lambda)$ cannot exceed the return of the long-term asset $\overline{Z}$, which defines the market price as

$$p(\lambda) = \min\{\max\{p^{c}(\lambda), p^{\ell}(\lambda)\}, \overline{Z}\}\,.$$

The model focuses on cases in which strong banks are always solvent, whereas weak banks need to sell long-term asset to repay early consumers and may be solvent at $t = 2$ or default either at $t = 1$ or $t = 2$. To restrict attention to these cases, I make the following parameter assumptions:

First, note that the continuation price

$$p^c(\lambda) = h(\lambda)\overline{X} + \overline{Z} - \theta$$

is increasing and concave in short-term asset investment λ. The default threshold $\widetilde{p}(\lambda)$ is convex in λ,

$$\frac{\partial^2 \widetilde{p}}{\partial \lambda^2} = \frac{2\theta - X_\ell}{(1-\lambda)^3} \, ,$$

for $\theta > X_\ell/2$. Moreover, for a portfolio with no short-term assets $\lambda = 0$ the continuation price is higher than the default threshold,

$$p^c(0) = \overline{Z} - \theta > \theta = \widetilde{p}(0) \, .$$

when $\theta < \overline{Z}/2$. Consequently, in the parameter range

$$\frac{X_\ell}{2} < \theta < \frac{\overline{Z}}{2} \, ,$$

the continuation price $p^c(\lambda)$ and the default threshold $\widetilde{p}(\lambda)$ intersect once at $\tilde{\lambda}$. For $\lambda < \tilde{\lambda}$ the continuation price is above the threshold and defines the market price, case (A3). For $\lambda \geq \tilde{\lambda}$, the continuation price falls below the threshold and the liquidation price defines the market price, case (A4).

Finally note that for $\theta > X_\ell/2$, the weak banks' first-period net liquidity

$$h(\lambda)X_\ell - \theta = -\frac{\lambda_i^2}{2}X_\ell + \lambda_i X_\ell - \theta$$

is negative for any short-term asset investment λ. This means weak banks need to sell their long-term asset at $t = 1$ independent of their portfolio.

Strong banks default at $t = 1$ if selling the entire asset holding is not enough to repay early consumers,

$$h(\lambda)X_h + p(1-\lambda) < \theta\,,$$

that is when

$$p \geq \widehat{p}(\lambda) = \frac{\theta - h(\lambda)X_h}{1-\lambda}\,.$$

Note that for $\theta < X_h/2$ the default threshold for the strong banks is concave and decreasing in λ. Moreover, for a portfolio with no short-term assets $\lambda = 0$ and $\theta < \overline{Z}/2$ weak banks will not default at $t = 1$, therefore strong banks, which have a higher first-period return, will also not default. Hence, for

$$\theta < \max\left\{\frac{\overline{Z}}{2}, \frac{X_h}{2}\right\}\,,$$

the strong banks do not default at $t = 1$ regardless of λ, because the threshold $\widehat{p}(\lambda)$ does not intersect with the continuation price $p^c(\lambda)$. To summarize, for the parameters

$$\frac{X_\ell}{2} < \theta < \max\left\{\frac{\overline{Z}}{2}, \frac{X_h}{2}\right\}\,,$$

strong banks do not default at $t = 1$, but weak banks may default if banks' ex-ante investment in the short-term asset is too large, $\lambda > \tilde{\lambda}$. □

Proof of Proposition 2. Strong banks fail at $t = 1$ when the short-term asset return plus the proceeds from selling the entire holding of the long-term asset are not enough to repay the early consumers,

$$h(\lambda)X_h + p(1-\lambda) < \theta\,.$$

That is when

$$p \leq \widehat{p}(\lambda) = \frac{\theta - h(\lambda)X_h}{1-\lambda}\,.$$

When strong banks fail, weak banks are also going to fail because

$$\widehat{p}(\lambda) = \frac{\theta - h(\lambda)X_h}{1-\lambda} < \frac{\theta - h(\lambda)X_\ell}{1-\lambda} = \widetilde{p}(\lambda)\,.$$

Then, both banks are liquidated, and the market clearing condition

$$-\alpha(1-\lambda) - (1-\alpha)(1-\lambda) + \frac{Z_j - p}{p} = 0\,.$$

defines the crisis liquidation price

$$p^b(\lambda, Z_j) = \frac{Z_j}{2-\lambda}\,,$$

with the superscript h indicating that both the weak and strong banks are liquidated at $t = 1$.

If strong banks can successfully repay the early consumers and continue to operate until $t = 2$, but weak banks are going to fail at $t = 1$,

$$\widehat{p}(\lambda) < p < \widetilde{p}(\lambda)\,,$$

then weak banks' liquidation price

$$p^\ell(\lambda, Z_j) = \frac{(1-\alpha)[h(\lambda)X_h - \theta] + Z_j}{1 + \alpha(1-\lambda)}$$

is the market price, where superscript ℓ indicates that only the weak banks are liquidated at $t = 1$. Note that the liquidation price can be rewritten as

$$p^\ell(\lambda, Z_i) = p^b(\lambda, Z_i) + \frac{(1-\alpha)(1-\lambda)}{1+\alpha(1-\lambda)}[p^b(\lambda, Z_i) - \widehat{p}(\lambda)] \tag{A5}$$

or as

$$p^\ell(\lambda, Z_i) = \widehat{p}(\lambda) + \frac{2-\lambda}{1+\alpha(1-\lambda)}[p^b(\lambda, Z_i) - \widehat{p}(\lambda)]\,. \tag{A6}$$

Finally, if both banks are solvent at $t = 2$,

$$\widehat{p}(\lambda) < \widetilde{p}(\lambda) < p\,,$$

the continuation price

$$p^c(\lambda, Z_j) = h(\lambda)\overline{X} + Z_j - \theta\,,$$

is the market price, where superscript c indicates that all banks continue to operate until $t = 2$. Note that the liquidation price can be rewritten as

$$p^\ell(\lambda, Z_j) = p^c(\lambda, Z_j) + \frac{\alpha(1-\lambda)}{1+\alpha(1-\lambda)}[\widetilde{p}(\lambda) - p^c(\lambda, Z_j)] \tag{A7}$$

or as

$$p^\ell(\lambda, Z_j) = \widetilde{p}(\lambda) + \frac{[p^c(\lambda, Z_j) - \widetilde{p}(\lambda)]}{1+\alpha(1-\lambda)}\,. \tag{A8}$$

If for some λ we have $\widehat{p}(\lambda) \geq p^b(\lambda, Z_j)$, then (A5) implies $p^b(\lambda, Z_j) \geq p^\ell(\lambda, Z_j)$ and (A6) implies $\widehat{p}(\lambda) \geq p^\ell(\lambda, Z_j)$, that is

$$\widehat{p}(\lambda) \geq p^b(\lambda, Z_j) \geq p^\ell(\lambda, Z_j).$$

Since $\widetilde{p}(\lambda) > \widehat{p}(\lambda)$, the above inequalities imply $\widetilde{p}(\lambda) \geq p^\ell(\lambda, Z_j)$. Then (A8) implies $\widetilde{p}(\lambda) > p^c(\lambda, Z_j)$ and then (A7) implies $p^\ell(\lambda, Z_j) > p^c(\lambda, Z_j)$. That is,

$$\widetilde{p}(\lambda) > \widehat{p}(\lambda) \geq p^b(\lambda, Z_j) \geq p^\ell(\lambda, Z_j) > p^c(\lambda, Z_j).$$

In this case, weak banks cannot survive at $t = 2$, because the continuation price $p^c(\lambda, Z_j)$ is below the default threshold $\widetilde{p}(\lambda)$ and the strong banks cannot survive at $t = 2$ because the weak banks' liquidation price $p^\ell(\lambda, Z_j)$ is below the default threshold $\widehat{p}(\lambda)$. Then crisis price $p^b(\lambda, Z_j)$ (which is below the threshold $\widehat{p}(\lambda)$) becomes the market price $p(\lambda, Z_j)$.

If for some λ we have $\widehat{p}(\lambda) < p^b(\lambda, Z_j)$, then (A5) implies $p^b(\lambda, Z_j) < p^\ell(\lambda, Z_j)$ and (A6) implies $\widehat{p}(\lambda) < p^\ell(\lambda, Z_j)$, that is

$$\widehat{p}(\lambda) < p^b(\lambda, Z_j) < p^\ell(\lambda, Z_j). \tag{A9}$$

Additionally, if we have $p^\ell(\lambda, Z_j) < \widetilde{p}(\lambda)$, then (A8) implies $p^c(\lambda, Z_j) < \widetilde{p}(\lambda, Z_j)$, and then

(A7) implies $p^c(\lambda, Z_j) < p^\ell(\lambda, Z_j)$, that is

$$p^c(\lambda, Z_j) < p^\ell(\lambda, Z_j) < \widetilde{p}(\lambda). \tag{A10}$$

From the combination of the two inequalities (A9) and (A10) it follows that weak banks cannot survive at $t = 2$, because the continuation price $p^c(\lambda, Z_j)$ is below the default threshold $\widetilde{p}(\lambda)$ and strong banks cannot default because the crisis price $p^b(\lambda, Z_j)$ is above the threshold $\widehat{p}(\lambda)$. Consequently, the weak banks' liquidation price $p^\ell(\lambda, Z_j)$ (with is above the threshold $\widehat{p}(\lambda)$ and below the threshold $\widetilde{p}(\lambda)$) becomes the market price $p(\lambda)$.

However, if $p^\ell(\lambda, Z_j) \geq \widetilde{p}(\lambda)$, then (A7) implies $p^c(\lambda, Z_j) \geq \widetilde{p}(\lambda, Z_j)$ and then (A8) implies $p^c(\lambda, Z_j) \geq p^\ell(\lambda, Z_j)$, that is

$$p^c(\lambda, Z_j) \geq p^\ell(\lambda, Z_j) \geq \widetilde{p}(\lambda, Z_j). \tag{A11}$$

From the combination of the two inequalities (A9) and (A11) it follows that weak banks cannot default, because the weak banks' liquidation price $p^\ell(\lambda, Z_j)$ is above the threshold $\widetilde{p}(\lambda)$ and strong banks cannot default because the crisis price $p^b(\lambda, Z_j)$ is above the threshold $\widehat{p}(\lambda)$, so the continuation price $p^c(\lambda, Z_j)$ (which is above the threshold $\widetilde{p}(\lambda)$ becomes the market price $p(\lambda, Z_j)$.

Then, in sum, the market price for the long-term asset is

$$p(\lambda, Z_j) = \min\left\{\max\left\{p^c(\lambda, Z_j), p^b(\lambda, Z_j), p^\ell(\lambda, Z_j)\right\}, Z_j\right\}.$$

where the market price $p(\lambda, Z_j)$ cannot exceed the return of the long-term asset Z_j. Finally, note that for the parameter range defined in Proposition 1, both banks survive at $t = 1$ when $Z_j = Z_g$ because $Z_g > \overline{Z}$. However, for $Z_j = Z_b$ either bank may default at $t = 1$. □

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