Important Questions for Class 9

Maths

Chapter 1 – Number Systems

Very Short Answer Questions **1 Mark**

1. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is irrational number.

Ans: We know that square root of every positive integer will not yield an integer.

We know that $\sqrt{4}$ is 2, which is an integer. But, $\sqrt{7}$ or $\sqrt{10}$ will give an irrational number.

Therefore, we conclude that square root of every positive integer is not an irrational number.

2. Write three numbers whose decimal expansions are non-terminating non-recurring.

Ans: The three numbers that have their expansions as non-terminating on recurring decimal are given below.

$0.04004000400004....$

$0.07007000700007....$

$0.13001300013000013....$

3. Find three different irrational numbers between the rational numbers $\frac{5}{11}$ and $\frac{9}{11}$.

Ans: Let us convert $\frac{5}{11}$ and $\frac{9}{11}$ into decimal form, to get

$\frac{5}{7} = 0.714285....\text{and}\frac{9}{11} = 0.818181....$

Three irrational numbers that lie between $0.714285....$ and $0.818181....$ are:

0.73073007300073....

0.74074007400074....

0.76076007600076....

4. Which of the following rational numbers have terminating decimal representation?

(i) $\frac{3}{5}$

(ii) $\frac{2}{13}$

(iii) $\frac{40}{27}$

(iv) $\frac{23}{7}$

Ans: (i) $\frac{3}{5}$

5. How many rational numbers can be found between two distinct rational numbers?

(i) Two

(ii) Ten

(iii) Zero

(iv) Infinite

Ans: (iv) Infinite

6. The value of $\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)$ in

(i) 1

(ii) -1

(iii) 2

(iv) none of these

Ans: (i) 1

7. $(27)^{-2/3}$ **is equal to**

(i) 9

(ii) 1/9

(iii) 3

(iv) none of these

Ans: (ii) $1/9$

8. Every natural number is

(i) not an integer

(ii) always a whole number

(iii) an irrational number

(iv) not a fraction

Ans: (ii) always a whole number

9. Select the correct statement from the following

(i) $\frac{7}{9} > \frac{4}{5}$

(ii) $\frac{2}{6} < \frac{3}{9}$

(iii) $\frac{-2}{3} > \frac{-4}{5}$

(iv) $\frac{-5}{7} < \frac{-3}{4}$

Ans: (iii) $\frac{-2}{3} > \frac{-4}{5}$

10. $7.\overline{2}$ **is equal to**

(i) $\mathbf{\frac{68}{9}}$

(ii) $\mathbf{\frac{64}{9}}$

(iii) $\mathbf{\frac{65}{9}}$

(iv) $\mathbf{\frac{63}{9}}$

Ans: (iii) $\frac{65}{9}$

11. 0.83458456...... is

(i) an irrational number

(ii) rational number

(iii) a natural number

(iv) a whole number

Ans: (i) an irrational number

12. A terminating decimal is

(i) a natural number

(ii) a rational number

(iii) a whole number

(iv) an integer.

Ans: (ii) a rationalnumber

13. The $\mathbf{\frac{p}{q}}$ form of the number 0.8 is

(i) $\mathbf{\frac{8}{10}}$

(ii) $\mathbf{\frac{8}{100}}$

(iii) $\mathbf{\frac{1}{8}}$

(iv) 1

Ans: (i) $\frac{8}{10}$

14. The value of $\sqrt[3]{1000}$ is

(i) 1

(ii) 10

(iii) 3

(iv) 0

Ans: (ii) 10

15. The sum of rational and an irrational number

(i) may be natural

(ii) may be irrational

(iii) is always irrational

(iv) is always rational

Ans: (iii) is always rational

16. The rational number not lying between $\frac{3}{5}$ and $\frac{2}{3}$ is

(i) $\mathbf{\frac{49}{75}}$

(ii) $\mathbf{\frac{50}{75}}$

(iii) $\frac{47}{75}$

(iv) $\frac{46}{75}$

Ans: (ii) $\frac{50}{75}$

17. $0.12\overline{3}$ **is equal to**

(i) $\frac{122}{90}$

(ii) $\frac{122}{100}$

(iii) $\frac{122}{99}$

(iv) None of these

Ans: (i) $\frac{122}{90}$

18. The number $\left(1+\sqrt{3}\right)^2$ **is**

(a) natural number

(b) irrational number

(c) rational number

(d) integer

Ans: (b) irrational number

19. The simplest form of $\sqrt{600}$ **is**

(i) $10\sqrt{60}$

(ii) $100\sqrt{6}$

(iii) $20\sqrt{3}$

(iv) $10\sqrt{6}$

Ans: (iv) $10\sqrt{6}$

20. The value of $0.\overline{23}+0.\overline{22}$ **is**

(i) $0.4\bar{5}$

(ii) $0.4\bar{4}$

(iii) $0.\overline{45}$

(iv) $0.\overline{44}$

Ans: (iii) $0.\overline{23}=0.232323....$

$0.\overline{22}=0.222222....$

$0.\overline{23}+0.\overline{22}=0.454545....$

$=0.\overline{45}$

21. The value of $2^{\frac{1}{3}}\times 2^{-\frac{4}{3}}$ **is**

(i) 2

(ii) $\frac{1}{2}$

(iii) 3

(iv) None of these

Ans: (i) $2^{\frac{1}{3}}\times 2^{-\frac{4}{3}}=2^{\frac{1}{3}-\frac{4}{3}}=2^{\frac{1-4}{3}}=2^{-\frac{3}{3}}$

22. $16\sqrt{13}\div 9\sqrt{52}$ **is equal to**

(i) $\frac{3}{9}$

(ii) $\mathbf{\frac{9}{8}}$

(iii) $\mathbf{\frac{8}{9}}$

(iv) None of these

Ans: $16\sqrt{13} \div 9\sqrt{52}$

$$\frac{16\sqrt{13}}{9\sqrt{52}} = \frac{16}{9}\sqrt{\frac{13}{52}} = \frac{8}{9}$$

23. $\mathbf{\sqrt{8}}$ **is an**

(i) natural number

(ii) rational number

(iii) integer

(iv) irrational number

Ans: (iv) $\sqrt{8}$ is an irrational number

$\therefore \sqrt{4 \times 2} = 2\sqrt{2}$

Short Answer Questions **2 Marks**

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Ans: Considerthedefinitionofarationalnumber.

A rational number is the one that can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Zero can be written as $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \frac{0}{5}$......

So, we arrive at the conclusion that 0 can be written in the form $\frac{p}{q}$, where q is any integer.

Therefore, zero is a rational number.

2. Find six rational numbers between 3 and 4.

Ans: We know that there are infinite rational numbers between any two numbers.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

We know that the numbers $3.1, 3.2, 3.3, 3.4, 3.5$ and 3.6 all lie between 3 and 4.

We need to rewrite the numbers $3.1, 3.2, 3.3, 3.4, 3.5$ and 3.6 in $\frac{p}{q}$ form to get the rational numbers between 3 and 4.

So, after converting we get $\frac{32}{10}, \frac{32}{10}, \frac{33}{10}, \frac{34}{10}, \frac{35}{10}$, and $\frac{36}{10}$, into lowest fractions.

On converting the fractions into lowest fractions, we get $\frac{16}{5}, \frac{17}{5}, \frac{7}{2}$ and $\frac{18}{5}$.

Therefore, six rational numbers between 3 and 4 are $\frac{31}{10}, \frac{16}{5}, \frac{33}{10}, \frac{17}{5}, \frac{7}{2}$ and $\frac{18}{5}$.

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Ans: We know that there are infinite rational numbers between any two numbers.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

We know that the numbers $\frac{3}{5}$ and $\frac{4}{5}$ can also be written as 0.6 and 0.8.

We can conclude that the numbers $0.61, 0.62, 0.63, 0.64$ and 0.65 in $\frac{p}{q}$ form to get the rational numbers between 3 and 4.

So, after converting, we get $\frac{61}{100}, \frac{62}{100}, \frac{63}{100}, \frac{64}{100}$ and $\frac{65}{100}$.

We can further convert the rational numbers $\frac{62}{100}, \frac{64}{100}$ and $\frac{65}{100}$ into lowest fractions.

On converting the fractions, we get $\frac{31}{50}, \frac{16}{25}$ and $\frac{13}{20}$.

Therefore, six rational numbers between 3 and 4 are $\frac{61}{100}, \frac{31}{50}, \frac{63}{100}, \frac{16}{50}$ and $\frac{13}{50}$.

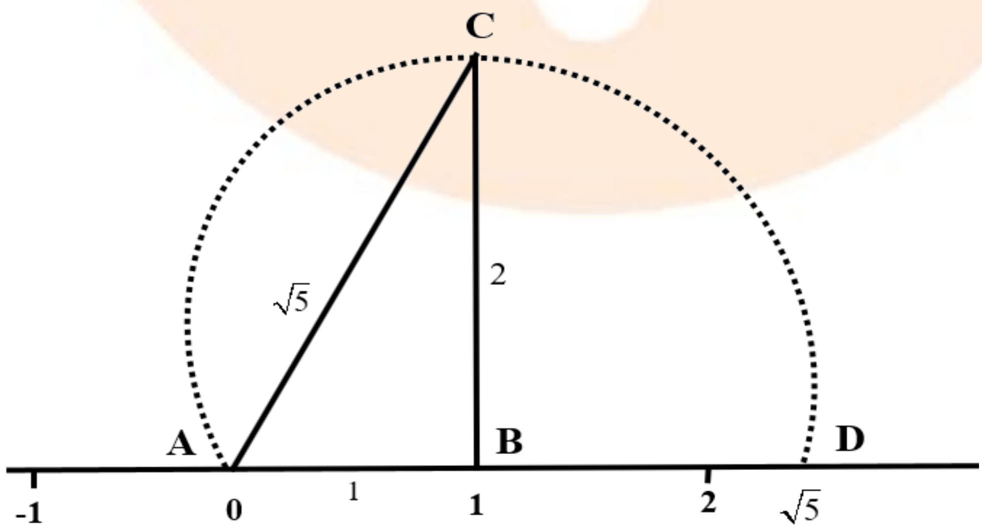
4. Show how $\sqrt{5}$ can be represented on the number line.

Ans: According to Pythagoras theorem, we can conclude that

$\left(\sqrt{5}\right)^2 = (2)^2 + (1)^2$.

We need to draw a line segment AB of 1unit on the number line. Then draw a straight line segment BC of 2 units. Then join the points C and A, to form a line segment BC.

Then draw the arc ACD, to get the number $\sqrt{5}$ on the number line.



5. You know that $\frac{1}{7}=0.142857.....$ Can you predict what the decimal expansion of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7},\frac{6}{7}$ are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Ans: We are given that $\frac{1}{7}=0.\overline{142857}$ or $\frac{1}{7}=0.142857....$

We need to find the value of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$, without performing long division.

We know that $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$ can be rewritten as

$2\times\frac{1}{7},3\times\frac{1}{7},4\times\frac{1}{7},5\times\frac{1}{7}$ and $6\times\frac{1}{7}$.

On substituting value of $\frac{1}{7}$ as $0.142857....$, we get

$$2\times\frac{1}{7}=2\times0.142857....=0.285714....$$

$$3\times\frac{1}{7}=3\times0.142857....=0.428571....$$

$$4\times\frac{1}{7}=4\times0.142857....=0.571428....$$

$$5\times\frac{1}{7}=5\times0.142857....=0.714285....$$

$$6\times\frac{1}{7}=6\times0.142857....=0.857142....$$

Therefore, we conclude that, we can predict the values of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$, without performing long division, to get

$$\frac{2}{7}=0.\overline{285714},\frac{3}{7}=0.\overline{428571},\frac{4}{7}=0.\overline{571428},\frac{5}{7}=0.\overline{714285},\frac{6}{7}=0.\overline{857142}$$

6. Express $0.99999....$ in the form $\frac{p}{q}$. Are you surprised by your answer? Discuss why the answer makes sense with your teacher and classmates.

Ans: Let $x = 0.99999....$ ......(a)

We need to multiply both sides by 10 to get

$10x = 9.9999....$ ......(b)

We need to subtract (a) from (b), to get

$$\begin{aligned} 10x &= 9.99999.... \\ -x &= 0.99999.... \\ \hline 9x &= 9 \end{aligned}$$

We can also write $9x = 9$ as $x = \frac{9}{9}$ or $x = 1$.

Therefore, on converting $0.99999....$ in the $\frac{p}{q}$ form, we get the answer as 1.

Yes,ataglancewearesurprisedat our answer.

But the answer makes sense when we observe that $0.99999....$ goes on forever. So there is not gap between 1 and $0.9999....$ and hence they are equal.

7. Visualize 3.765 on the number line using successive magnification.

Ans: We know that the number 3.765 will lie between 3.764 and 3.766.

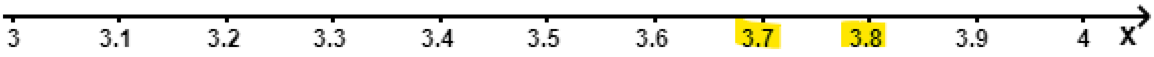
We know that the number 3.764 and 3.766 will lie between 3.76 and 3.77.

We know that the number 3.76 and 3.77. will lie between 3.7 and 3.8.

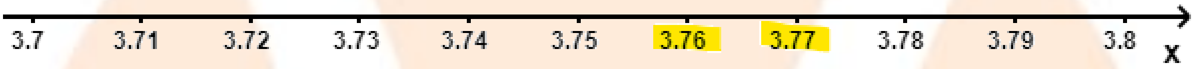
We know that the number 3.7 and 3.8 will lie between 3 and 4.

Therefore, we can conclude that we need to use the successive magnification, after locating numbers 3 and 4 on the number line

Apply magnification between 3 and 4



Apply magnification between 3.7 and 3.8



Apply magnification between 3.76 and 3.77

8. Visualize $4.\overline{26}$ on the number line, upto 4 decimal places.

Ans: We know that the number $4.\overline{26}$ can also be written as $4.262.....$

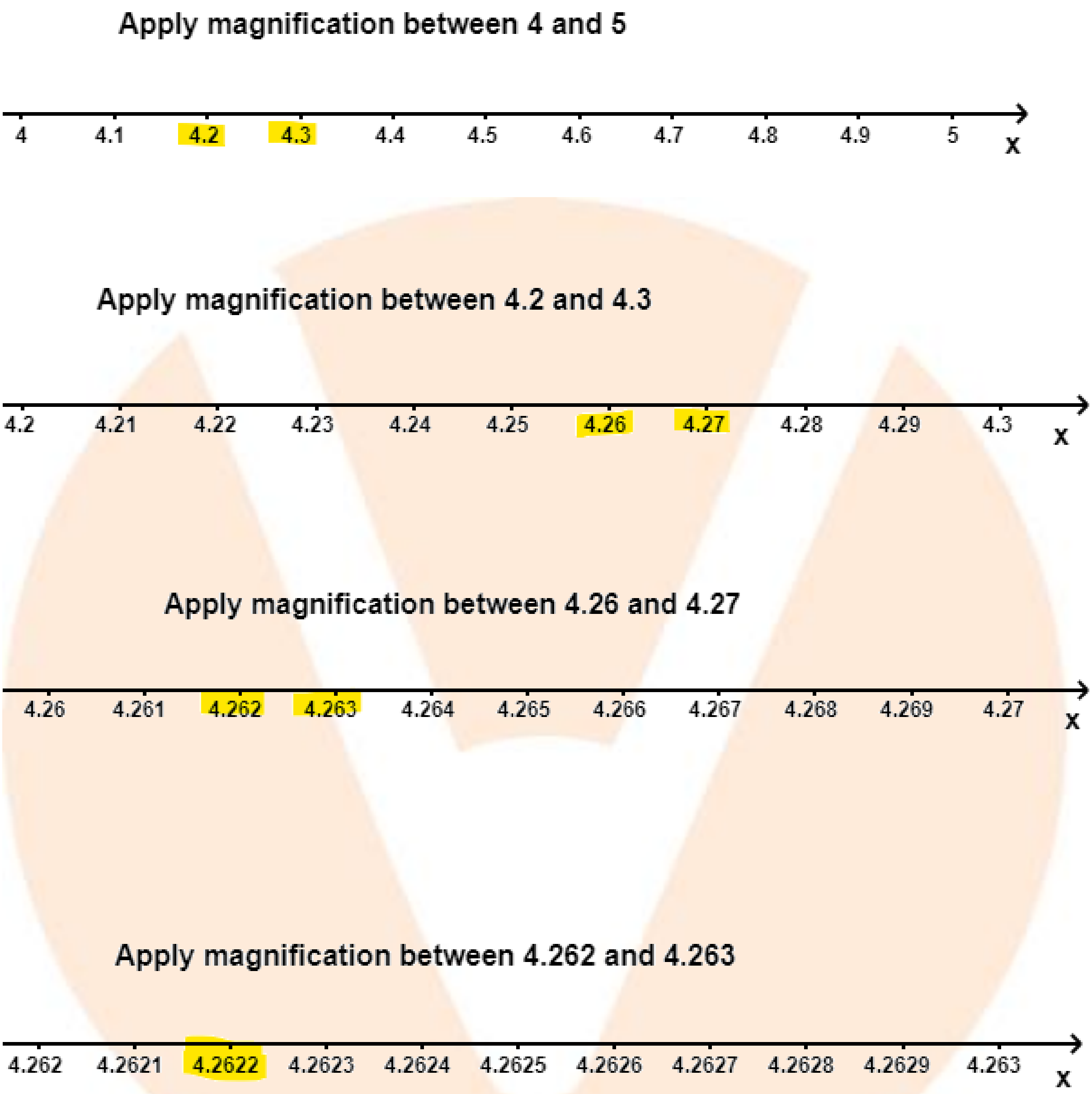
We know that the number $4.262....$ will lie between 4.261 and 4.263.

We know that the number 4.261 and 4.263 will lie between 4.26 and 4.27.

We know that the number 4.26 and 4.27 will lie between 4.2 and 4.3.

We know that the number 4.2 and 4.3 will lie between 4 and 5.

Therefore, we can conclude that we need to use the successive magnification, after locating numbers 4 and 5 on the number line.



9. Recall, π is defined as the ratio of the circumference (say c) of a circle of its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How you resolve this contradiction?

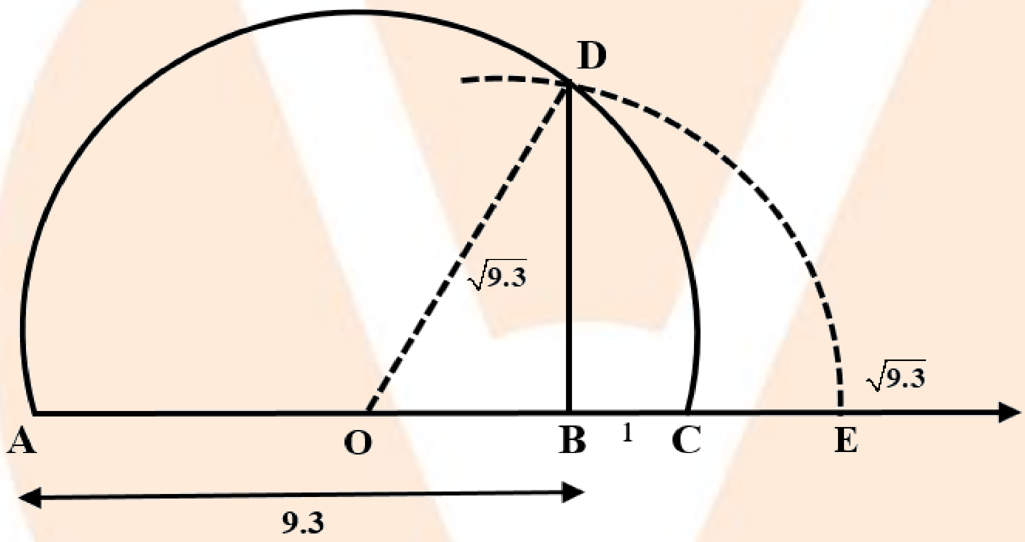
Ans: We know that when we measure the length of the line or a figure by using a scaleneory device, we do not get an exact measurement. In fact, we get an approximate rational value. So, we are not able to realize that either circumference (c) or diameter (d) of a circle is irrational.

Therefore, we can conclude that as such there is not any contradiction regarding the value of π and we realize that the value of π is irrational.

10. Represent $\sqrt{9.3}$ on the number line.

Ans: Mark the distance 9.3 units from a fixed point A on a given line to obtain a point B suchthat $AB = 9.3$ units. From B mark a distance of 1 unit and call the new point as C. Find themid-point of AC and call that point as O. Draw a semi-circle with centre O and radius $OC = 5.15$ units. Draw a line perpendicular to AC passing through B cutting the semi-circle at D.

Then $BD = \sqrt{9.3}$.



11. Find (i) $64^{\frac{1}{5}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Ans: (i) $64^{\frac{1}{2}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$

We conclude that $64^{\frac{1}{2}}$ can also be written as $\sqrt[2]{64} = \sqrt[2]{8 \times 8}$

$\sqrt[2]{64} = \sqrt[2]{8 \times 8} = 8$

Therefore, the value of $64^{\frac{1}{2}}$ will be 8.

Ans: (ii) $32^{\frac{1}{5}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$

We conclude that $32^{\frac{1}{5}}$ can also be written as $\sqrt[5]{32} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2}$

$\sqrt[5]{32} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2} = 2$

Therefore, the value of $32^{\frac{1}{5}}$ will be 2.

Ans: (iii) $125^{\frac{1}{3}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$

We conclude that $125^{\frac{1}{3}}$ can also be written as $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5}$

$\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$

Therefore, the value of $125^{\frac{1}{3}}$ will be 5.

12. Simplify $\sqrt[3]{2} \times \sqrt[4]{3}$

Ans: $\sqrt[3]{2} \times \sqrt[4]{3}$

$2^{\frac{1}{3}} \times 3^{\frac{1}{4}}$

The LCM of 3 and 4 is 12

$\therefore 2^{\frac{1}{3}} = 2^{\frac{4}{12}} = \left(2^4\right)^{\frac{1}{12}} = 16^{\frac{1}{12}}$

$3^{\frac{1}{4}} = 3^{\frac{3}{12}} = \left(3^3\right)^{\frac{1}{12}} = 27^{\frac{1}{12}}$

$2^{\frac{1}{3}} \times 3^{\frac{1}{4}} = 16^{\frac{1}{12}} \times 27^{\frac{1}{12}} = (16 \times 27)^{\frac{1}{12}}$

$= (432)^{\frac{1}{12}}$

13. Find the two rational numbers between $\frac{1}{2}$ and $\frac{1}{3}$.

Ans: First rational number between $\frac{1}{2}$ and $\frac{1}{3}$

$$=\frac{1}{2}\left[\frac{1}{2}+\frac{1}{3}\right]\Rightarrow\frac{1}{2}\left[\frac{3+2}{6}\right]\Rightarrow\frac{5}{12}$$

$$=\frac{1}{2},\frac{5}{12}\text{ and }\frac{1}{3}$$

Second rational number between $\frac{1}{2}$ and $\frac{1}{3}$

$$=\frac{1}{2}\left[\frac{1}{2}+\frac{5}{12}\right]\Rightarrow\frac{1}{2}\left[\frac{6+5}{12}\right]\Rightarrow\frac{11}{24}$$

$=\frac{5}{12}$ and $\frac{11}{24}$ are two rational numbers between $\frac{1}{2}$ and $\frac{1}{3}$.

14. Find two rational numbers between 2 and 3.

Ans: Irrational numbers between 2 and 3 is $\sqrt{2\times3}=\sqrt{6}$

Irrational number between 2 and 3 is $\sqrt{6}$.

$$\sqrt{2\times\sqrt{6}}=2^{\frac{1}{2}}\times6^{\frac{1}{4}}=2^{2\times\frac{1}{4}}\times6^{\frac{1}{4}}$$

$$=\left(2^2\right)^{\frac{1}{4}}\times6^{\frac{1}{4}}=4^{\frac{1}{4}}\times6^{\frac{1}{4}}=(24)^{\frac{1}{4}}=\sqrt[4]{24}$$

$\sqrt{6}$ and $\sqrt{24}$ are two rational numbers between 2 and 3.

15. Multiply $\left(3-\sqrt{5}\right)$ by $\left(6+\sqrt{2}\right)$.

Ans: $\left(3-\sqrt{5}\right)\left(6+\sqrt{2}\right)$

$$=3\left(6-\sqrt{2}\right)-\sqrt{5}\left(6+\sqrt{2}\right)$$

$=18+3\sqrt{2}-6\sqrt{5}-\sqrt{5}\times\sqrt{2}$

$=18+3\sqrt{2}-6\sqrt{5}-\sqrt{10}$

16. Evaluate (i) $\sqrt[3]{125}$ (ii) $\sqrt[4]{1250}$

Ans: (i) $\sqrt[3]{125}=(5\times5\times5)^{\frac{1}{3}}=\left(5^3\right)^{\frac{1}{3}}=5$

Ans: (ii) $\sqrt[4]{1250}$ $=(2\times5\times5\times5\times5)^{\frac{1}{4}}=\left(2\times5^4\right)^{\frac{1}{4}}$

$=2^{\frac{1}{4}}\times\left(5^4\right)^{\frac{1}{4}}=5\times\sqrt[4]{2}$

17. Find rationalizing factor of $\sqrt{300}$.

Ans: $\sqrt{300}=\sqrt{2\times2\times3\times5\times5}$

$=\sqrt{2^2\times3\times5^2}$

$=2\times5\sqrt{3}=10\sqrt{3}$

Rationalizing factor is $\sqrt{3}$

18. Rationalizing the denominator $\frac{1}{\sqrt{5}+\sqrt{2}}$ and subtract it from $\sqrt{5}-\sqrt{2}$.

Ans: $\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\sqrt{5}-\sqrt{2}}{5-2}=\frac{\sqrt{5}-\sqrt{2}}{3}$

Difference between $\left(\sqrt{5}-\sqrt{2}\right)$ and $\left(\sqrt{5}-\frac{\sqrt{2}}{3}\right)$

$=\sqrt{5}-\sqrt{2}-\left(\frac{\sqrt{5}-\sqrt{2}}{3}\right)$

$$=\sqrt{5}-\sqrt{2}-\frac{\sqrt{5}}{3}+\frac{\sqrt{2}}{3}$$

$$=\left(\sqrt{5}-\frac{\sqrt{5}}{3}\right)-\left(\sqrt{2}-\frac{\sqrt{2}}{3}\right)$$

$$=\frac{2\sqrt{5}}{3}-\frac{2\sqrt{2}}{3}=\frac{2}{3}\left(\sqrt{5}-\sqrt{2}\right)$$

19. Show that $\sqrt{7}-3$ is irrational.

Ans: Suppose $\sqrt{7}-3$ is rational

Let $\sqrt{7}-3=x$ (x is a rational number)

$\sqrt{7}=x+3$

x is a rational number 3 is also a rational number

$\therefore x+3$ is a rational number

But is $\sqrt{7}$ irrational number which is contradiction

$\therefore \sqrt{7}-3$ is irrational number.

20. Find two rational numbers between 7 and 5.

Ans: First rational number $=\frac{1}{2}[7+5]=\frac{12}{2}=6$

Second rational number $=\frac{1}{2}[7+6]=\frac{1}{2}\times 13=\frac{13}{2}$

Two rational numbers between 7 and 5 are 6 and $\frac{13}{2}$.

21. Show that $5+\sqrt{2}$ is not a rational number.

Ans: Let $5+\sqrt{2}$ is rational number.

Say $5+\sqrt{2=x}$ i.e., $\sqrt{2}=x-5$

x is a rational number 5 is also rational number

$\therefore x-5$ is also rational number.

But $\sqrt{2}$ is irrational number which is a contradiction

$\therefore 5+\sqrt{2}$ is irrational number.

22. Simplify $\left(\sqrt{5}+\sqrt{2}\right)^2$.

Ans: $\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\sqrt{5}\times\sqrt{2}=5+2+2\sqrt{10}=7+2\sqrt{2}$

23. Evaluate $\dfrac{11^{\frac{5}{2}}}{11^{\frac{3}{2}}}$.

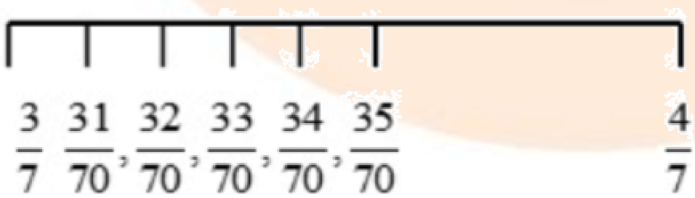
Ans: $\dfrac{11^{\frac{5}{2}}}{11^{\frac{3}{2}}}=11^{\frac{5}{2}-\frac{3}{2}}\left[\because \dfrac{a^m}{a^n}=a^{m-n}\right]$

$=11^{\frac{5-3}{2}}=11^{\frac{2}{2}}$

$=11$

24. Find four rational numbers between $\dfrac{3}{7}$ and $\dfrac{4}{7}$.

Ans:



$\dfrac{3}{7}\times\dfrac{10}{10}=\dfrac{30}{70}$ and $\dfrac{4}{7}\times\dfrac{10}{10}=\dfrac{40}{70}$

Take any four rational numbers between $\frac{30}{70}$ and $\frac{40}{70}$ i.e., rational numbers between $\frac{3}{7}$ and $\frac{4}{7}$ are $\frac{31}{70}, \frac{32}{70}, \frac{33}{70}, \frac{34}{70}, \frac{35}{70}$

25. Write the following in decimal form (i) $\frac{36}{100}$ (ii) $\frac{2}{11}$

Ans: (i) $\frac{36}{100} = 0.36$

Ans: (ii) $\frac{2}{11} = 0.\overline{18}$

26. Express $2.417\bar{8}$ in the form $\frac{a}{b}$

Ans: $x = 2.4\overline{178}$

$10x = 24.\overline{178}$ ......(1) [Multiplying both sides by 10]

$10x = 24.178178178....$

$1000 \times 10x = 1000 \times 24.178178178....$[Multiplying both sides by 1000]

$10,000x = 24178.178178....$

$10000x = 24178.\overline{178}$ ......(2)

Subtracting (1) from (2)

$10,000x - x = 24178.\overline{178} - 24.\overline{178}$

$9990x = 24154$

$x = \frac{24154}{9990}$

$2.4\overline{178} = \frac{24154}{9990} + \frac{12077}{4995}$

27. Multiply $\sqrt{3}$ by $\sqrt[3]{5}$.

Ans: $\sqrt{3}$ and $\sqrt[3]{5}$

Or $3^{\frac{1}{2}}$ and $5^{\frac{1}{3}}$

LCM of 2 and 3 is 6

$$3^{\frac{1}{2}} = 3^{\frac{1}{2}\times\frac{3}{3}} = \left(3^3\right)^{\frac{1}{6}} = \left(27\right)^{\frac{1}{6}}$$

$$5^{\frac{1}{3}} = 5^{\frac{1}{3}\times\frac{2}{2}} = \left(5^2\right)^{\frac{1}{6}} = \left(25\right)^{\frac{1}{6}}$$

$$\sqrt{3}\times\sqrt[3]{5} = \left(27\right)^{\frac{1}{6}}\times\left(25\right)^{\frac{1}{6}} = \left(27\times 25\right)^{\frac{1}{6}}$$

$$= 675^{\frac{1}{6}} = \sqrt[6]{675}$$

28. Find the value of $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{5}}$ if $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

Ans: $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}+5}{5} = \frac{8.162}{5} = 1.6324$

29. Convert $0.\overline{25}$ into rational number.

Ans: Let $x = 0.\overline{25}$ ......(i)

$x = 0.252525....$

Multiply both sides by 100

$100x = 25.252525....$

$100x = 25.\overline{25}$ ......(ii)

Subtract (i) from (ii)

$100x - x = 25.\overline{25} - 0.\overline{25}$

$99x = 25$

$$x = \frac{25}{99}$$

30. Simplify $\left(3\sqrt{3}+2\sqrt{2}\right)\left(2\sqrt{3}+3\sqrt{2}\right)$.

Ans: By multiplying each terms in the given product we have,

$$\left(3\sqrt{3}+2\sqrt{2}\right)\left(2\sqrt{3}+3\sqrt{2}\right)$$
$$=3\sqrt{3}\left(2\sqrt{3}+3\sqrt{2}\right)+2\sqrt{2}\left(2\sqrt{3}+3\sqrt{2}\right)$$
$$=18+9\sqrt{6}+4\sqrt{6}+12$$
$$=30+(9+4)\sqrt{6}$$
$$=30+13\sqrt{6}$$

31. Simplify $\dfrac{9^{\frac{3}{2}}\times 9^{-\frac{4}{2}}}{9^{\frac{1}{2}}}$.

Ans: By using the formulas of exponents with same base we get,

$$\frac{9^{\frac{3}{2}}\times 9^{-\frac{4}{2}}}{9^{\frac{1}{2}}}=\frac{9^{\frac{3}{2}-\frac{4}{2}}}{9^{\frac{1}{2}}}\left[a^m.a^n=a^{m-n}\right]$$

$$\frac{9^{-\frac{1}{2}}}{9^{\frac{1}{2}}}=\frac{1}{9^{\frac{1}{2}+\frac{1}{2}}}\left[a^{-m}=\frac{1}{a^m}\right]$$
$$=\frac{1}{9^{\frac{2}{2}}}=\frac{1}{9}$$

Long Answer Questions **3 Marks**

1. State whether the following statements are true or false. Give reasons for your answers.

i. Every natural number is a whole number.

Ans: Separately, consider whole numbers and natural numbers.

We know that whole number series is 0,1,2,3,4,5....

We know that natural number series is 0,1,2,3,4,5....

As a result, every number in the natural number series may be found in the whole number series.

Therefore, we can safely conclude that any natural number is a whole number.

ii. Every integer is a whole number.

Ans: Separately, consider whole numbers and integers.

We know that integers are those numbers that can be written in the form of $\frac{p}{q}$ where q=1.

In the case of an integer series, we now have.... 4,3,2,1,0,1,2,3,4....

We know that whole number series is 0,1,2,3,4,5....

We can conclude that all whole number series numbers belong to the integer series.

However, the whole number series does not contain every number of integer series.

As a result, we can conclude that no integer is a whole number.

iii. Every rational number is a whole number.

Ans: Separately, consider whole numbers and rational numbers.

We know that integers are those numbers that can be written in the form of $\frac{p}{q}$ where $q \neq 0$.

We know that whole number series is 0,1,2,3,4,5....

We know that every number of whole number series can be written in the form of $\frac{p}{q}$ as $\frac{0}{1},\frac{1}{1},\frac{2}{1},\frac{3}{1},\frac{4}{1},\frac{5}{1}...$

We conclude that every number of the whole number series is a rational number.

But, every rational number does not appear in the whole number series.

2. State whether the following statements are true or false. Justify your answers.

i. Every irrational number is a real number.

Ans: Separately, consider irrational numbers and real numbers.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

A real number is made up of both rational and irrational numbers, as we all know.

As a result, we might conclude that any irrational number is, in fact, a real number.

ii. Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

Ans: Consider a number line. We know that we can express both negative and positive numbers on a number line.

We know that when we take the square root of any number, we cannot receive a negative value.

Therefore, we conclude that not every number point on the number line is of the form $\sqrt{m}$, where m is a natural number.

iii. Every real number is an irrational number.

Ans: Separately, consider irrational numbers and real numbers.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

A real number is made up of both rational and irrational numbers, as we all know.

As a result, we can deduce that any irrational number is actually a real number. However, not every real number is irrational.

Therefore, we conclude that, every real number is not a rational number.

3. Express the following in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$

i. $0.\overline{6}$

Ans: Let $x = 0.\overline{6}$

$\Rightarrow x = 0.6666$ ......(a)

Multiplying both sides by 10 we get

$10x = 6.6666$ ......(b)

We need to subtract (a) from (b), to get

$9x = 6$

We can also write $9x = 6$ as $x = \frac{6}{9}$ or $x = \frac{2}{3}$.

Therefore, on converting $0.\overline{6}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{2}{3}$.

ii. $0.4\overline{7}$

Ans: Let $x = 0.4\overline{7} \Rightarrow x = 0.47777$ ......(a)

Multiplying both sides by 10 we get

$10x = 4.7777$ ......(b)

We need to subtract (a) from (b), to get

$9x = 4.3$

We can also write $9x = 4.3$ as $x = \frac{4.3}{9}$ or $x = \frac{43}{90}$

Therefore, on converting $0.4\overline{7}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{43}{90}$.

iii. $0.\overline{001}$

Ans: Let $x = 0.\overline{001} \Rightarrow x = 0.001001$ ......(a)

Multiplying both sides by 1000 we get

$1000x = 1.001001$ ......(b)

We need to subtract (a) from (b), to get

$999x = 1$

We can also write $999x = 1$ as $x = \frac{1}{999}$

Therefore, on converting $0.\overline{001}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{1}{999}$.

4. What can the maximum number of digits be in the recurring block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Ans: The number of digits in the recurring block of $\frac{1}{17}$ must be determined.

To acquire the repeating block of $\frac{1}{17}$ we'll use long division.

We need to divide 1 by 17, to get 0.0588235294117647.... and we got the remainder as 1, which will continue to be 1 after carrying out 16 continuous divisions.

Therefore, we conclude that

$\frac{1}{17} = 0.0588235294117647$ or $\frac{1}{17} = 0.\overline{0588235294117647}$ which is a non-terminating decimal and recurring decimal.

5. Look at several examples of rational numbers in the form $\frac{p}{q}(q \neq 0)$ where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Ans: Let us consider the examples of the form $\frac{p}{q}$ that are terminating decimals**.**

$\frac{5}{2} = 2.5$

$\frac{5}{4} = 1.25$

$\frac{2}{5} = 0.4$

$\frac{5}{16} = 0.3125$

It can be observed that the denominators of the above rational numbers have powers of 2,5 or both.

Therefore, we can conclude that property, which q must satisfy in $\frac{p}{q}$, so that the rational number $\frac{p}{q}$ is a terminating decimal is that q must have powers of 2,5 or both.

6. Classify the following numbers as rational or irrational:

i. $\mathbf{2-\sqrt{5}}$

Ans: $2-\sqrt{5}$

We know that $\sqrt{5} = 2.236....$, which is an irrational number.

$2-\sqrt{5} = 2-2.236....$

$= -0.236...$, which is also an irrational number.

As a result, we can deduce that $2-\sqrt{5}$ is an irrational number.

ii. $\mathbf{\left(3+\sqrt{23}\right)-\sqrt{23}}$

Ans: $\left(3+\sqrt{23}\right)-\sqrt{23}$

$\left(3+\sqrt{23}\right)-\sqrt{23} = 3+\sqrt{23}-\sqrt{23} = 3$

As a result, we can deduce that $\left(3+\sqrt{23}\right)-\sqrt{23}$ is a rational number.

iii. $\mathbf{\frac{2\sqrt{7}}{7\sqrt{7}}}$

Ans: $\frac{2\sqrt{7}}{7\sqrt{7}}$

We can cancel $\sqrt{7}$ in the numerator and denominator to get $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$, because $\sqrt{7}$ is a common number in both the numerator and denominator.

iv. $\mathbf{\frac{1}{\sqrt{2}}}$

Ans: $\frac{1}{\sqrt{2}}$

We know that $\sqrt{2} = 1.4142...$, which is an irrational number.

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$= \frac{1.4142...}{2} = 0.707...$ which is also an irrational number.

As a result, we can deduce that $\frac{1}{\sqrt{2}}$ is an irrational number.

v. $\mathbf{2\pi}$

Ans: 2π

We know that $\pi = 3.1415....,$ which is an irrational number.

We can conclude that 2π will also be an irrational number.

As a result, we can deduce that 2π is an irrational number.

7. Simplify each of the following expression:

i. $\mathbf{\left(3+3\sqrt{3}\right)\left(2+\sqrt{2}\right)}$

Ans: $(3+3\sqrt{3})(2+\sqrt{2})$

Applying distributive law,

$$(3+3\sqrt{3})(2+\sqrt{2})=3(2+\sqrt{2})\sqrt{3}(2+\sqrt{2})$$

$$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$$

ii. $\left(\mathbf{3+3\sqrt{3}}\right)\mathbf{3-\sqrt{3}}$

Ans: $(3+3\sqrt{3})(3-\sqrt{3})$

Applying distributive law,

$$(3+3\sqrt{3})(3-\sqrt{3})=(3-\sqrt{3})+\sqrt{3}(3-\sqrt{3})$$

$$=9-3\sqrt{3}+3\sqrt{3}-3$$

$$=6$$

iii. $\left(\mathbf{\sqrt{5}+\sqrt{2}}\right)^2$

Ans: $\left(\sqrt{5}+\sqrt{2}\right)^2$

Applying the formula $\left(a+b\right)^2=a^2+2ab+b^2$

$$\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+2\times\sqrt{5}\times\sqrt{2}+\left(\sqrt{2}\right)^2$$

$$=5+2\sqrt{10}+2$$

$$=7+2\sqrt{10}$$

iv. $\left(\mathbf{5+\sqrt{2}}\right)\left(\mathbf{5+\sqrt{2}}\right)$

Ans: $\left(5+\sqrt{2}\right)\left(5+\sqrt{2}\right)$

Applying the formula $\left(a-b\right)\left(a+b\right)=a^2-b^2$

$$\left(5+\sqrt{2}\right)\left(5+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$$

$= 5 - 2$

$= 3$

8. Find

i. $\mathbf{9^{\frac{3}{2}}}$

Ans: We know that $a^{\frac{1}{n}} = \sqrt[n]{a}, a > 0$

As a result, we can deduce that $9^{\frac{3}{2}}$ can also be written as

$\sqrt[2]{(9)^3} = \sqrt[2]{9 \times 9 \times 9} = \sqrt[2]{3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= 3 \times 3 \times 3$

$= 27$

Therefore, the value of $9^{\frac{3}{2}}$ will be 27.

ii. $\mathbf{32^{\frac{2}{5}}}$

Ans: We know that $a^{\frac{1}{n}} = \sqrt[n]{a}, a > 0$

As a result, we can deduce that $32^{\frac{2}{5}}$ can also be written as

$\sqrt[5]{(32)^2} = \sqrt[5]{(2 \times 2 \times 2 \times 2 \times 2)(2 \times 2 \times 2 \times 2 \times 2)}$

$= 2 \times 2$

$= 4$

Therefore, the value of $32^{\frac{2}{5}}$ will be 4.

iii. $\mathbf{16^{\frac{3}{4}}}$

Ans: We know that $a^{\frac{1}{n}} = \sqrt[n]{a}, a > 0$

As a result, we can deduce that $16^{\frac{3}{4}}$ can also be written as

$$\sqrt[4]{(16)^3} = \sqrt[4]{(2\times2\times2\times2)(2\times2\times2\times2)(2\times2\times2\times2)}$$
$$=2\times2\times2$$
$$=8$$

Therefore, the value of $16^{\frac{3}{4}}$ will be 8.

iv. $\mathbf{125^{-\frac{1}{3}}}$

Ans: We know that $a^{-n} = \dfrac{1}{a^n}$

As a result, we can deduce that $125^{-\frac{1}{3}}$ can also be written as $\dfrac{1}{125^{\frac{1}{3}}}$, or $\left(\dfrac{1}{125}\right)^{\frac{1}{3}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}, a > 0$

$$\sqrt[3]{\frac{1}{125}} = \sqrt[3]{\left(\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\right)}$$
$$=\frac{1}{5}$$

Therefore, the value of $125^{-\frac{1}{3}}$ will be $\dfrac{1}{5}$.

9. Simplify

i. $\mathbf{2^{\frac{2}{3}}.2^{\frac{1}{5}}}$

Ans: We know that $a^m.a^n = a^{(m+n)}$

As a result, we can deduce that $2^{\frac{2}{3}}.2^{\frac{1}{5}} = (2)^{\frac{2}{3}+\frac{1}{5}}$

$$2^{\frac{2}{3}}.2^{\frac{1}{5}} = (2)\frac{10+3}{15} = (2)^{\frac{13}{15}}$$

Therefore, the value of $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ will be $(2)^{\frac{13}{15}}$.

ii. $\left(3^{\frac{1}{3}}\right)^7$

Ans: We know that $a^m.a^n = a^{(m+n)}$

As a result, we can deduce that $\left(3^{\frac{1}{3}}\right)^7$ can also be written as $3^{\frac{7}{3}}$

iii. $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

Ans: We know that $\frac{a^m}{a^n} = a^{(m-n)}$

As a result, we can deduce that $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}} - 11^{\frac{1}{4}}$

$$= 11^{\frac{2-1}{4}}$$

$$= 11^{\frac{1}{4}}$$

Therefore, the value of $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ will be $11^{\frac{1}{4}}$.

iv. $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

Ans: We know that $a^m.b^m = (a \times b)^m$

As a result, we can deduce that $7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}}$.

$7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}} = (56)^{\frac{1}{2}}$.

Therefore, the value of $7^{\frac{1}{2}}.8^{\frac{1}{2}}$ will be $(56)^{\frac{1}{2}}$.

10. Express $0.8888....$ in the form $\frac{p}{q}$.

Ans: Let us assume that the given decimal as,

$x = 0.8888$

$x = 0.\bar{8}......(1)$

$10x = 10 \times 0.8888$ (Multiply both sides by 10)

$10x = 8.8888$

$10x = 8.\bar{8}.....(2)$

$10x - x = 8.\bar{8} - 0.\bar{8}$ (Subtracting (1) from (2))

$9x = 8$

$x = \frac{8}{9}$

11. Simplify by rationalizing denominator $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}$.

Ans: We are given the fraction to rationalize. By rationalizing the denominator we get,

$$\frac{7+3\sqrt{5}}{7-3\sqrt{5}} = \frac{7+3\sqrt{5}}{7-3\sqrt{5}} \times \frac{7+3\sqrt{5}}{7+3\sqrt{5}}$$

$$= \frac{\left(7+3\sqrt{5}\right)^2}{7^2 - \left(3\sqrt{5}\right)^2}$$

$$= \frac{7^2 + \left(3\sqrt{5}\right)^2 + 2 \times 7 \times 3\sqrt{5}}{49 - 3^2 \times 5}$$

$$= \frac{49 + 9 \times 5 + 42\sqrt{5}}{49 - 45}$$

$$=\frac{49+45+42\sqrt{5}}{4}$$

$$=\frac{94+42\sqrt{5}}{4}$$

$$=\frac{94}{4}+\frac{42}{4}\sqrt{5}$$

$$=\frac{47}{2}+\frac{21}{2}\sqrt{5}$$

12. Simplify $\left\{\left[625^{-\frac{1}{2}}\right]^{-\frac{1}{4}}\right\}^{2}$.

Ans: Let us take the given expression to simplify and using the exponents formulas we get,

$$\left\{\left[625^{-\frac{1}{2}}\right]^{-\frac{1}{4}}\right\}^{2}$$

$$=\left\{\left(\frac{1}{625^{\frac{1}{2}}}\right)^{-\frac{1}{4}}\right\}^{2}$$

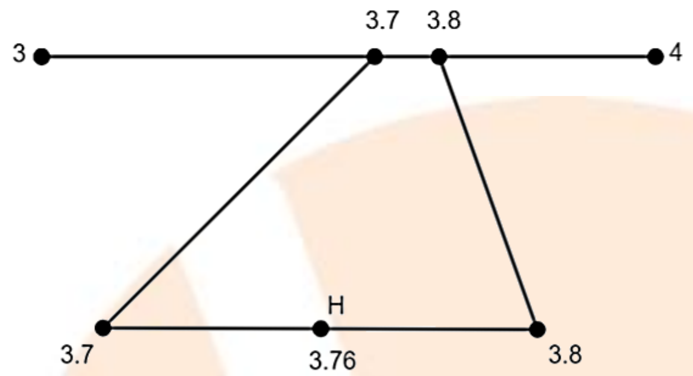
$$=\left\{\left(\frac{1}{\left(25^{2}\right)^{\frac{1}{2}}}\right)^{-\frac{1}{4}}\right\}^{2}\left(a^{-m}=\frac{1}{a^{m}}\right)$$

$$=\left\{\left(\frac{1}{25}\right)^{-\frac{1}{4}\times 2}\right\}$$

$$=\left(\frac{1}{25^{-\frac{1}{2}}}\right)=\frac{1}{\left(5^{2}\right)^{-\frac{1}{2}}}=\frac{1}{5^{-1}}=5$$

13. Visualize 3.76 on the number line using successive magnification.

Ans:



14. Prove that $\mathbf{\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}=1}$

Ans: We are asked to prove the expression,

$$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}=1$$

Let us take the LHS of the given expression that is,

$$\text{LHS}=\frac{1}{1+x^{b}.x^{-a}+x^{c}.x^{-a}}+\frac{1}{1+x^{a}.x^{-b}+x^{c}.x^{-b}}+\frac{1}{1+x^{a}.x^{-c}+x^{b}.x^{-c}}$$

$$=\frac{1}{x^{-a}.x^{a}+x^{b}.x^{-a}+x^{c}.x^{-a}}+\frac{1}{x^{b}.x^{-b}+x^{a}.x^{-b}+x^{c}.x^{-b}}+\frac{1}{x^{c}.x^{-c}+x^{a}.x^{-c}+x^{b}.x^{-c}}$$

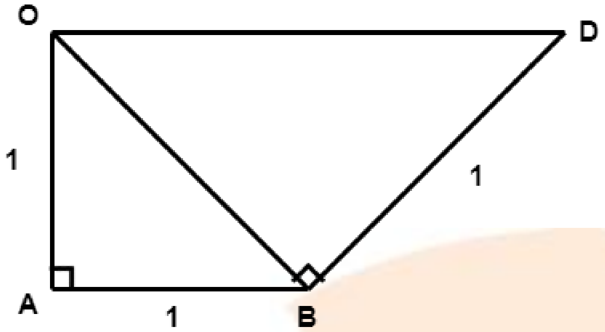
$$=\frac{1}{x^{-a}\left(x^{a}+x^{b}+x^{c}\right)}+\frac{1}{x^{-b}\left(x^{a}+x^{b}+x^{c}\right)}+\frac{1}{x^{-c}\left(x^{a}+x^{b}+x^{c}\right)}$$

$$=\frac{x^{a}}{\left(x^{a}+x^{b}+x^{c}\right)}+\frac{x^{b}}{\left(x^{a}+x^{b}+x^{c}\right)}+\frac{x^{c}}{\left(x^{a}+x^{b}+x^{c}\right)}$$

$$=\frac{\left(x^{a}+x^{b}+x^{c}\right)}{\left(x^{a}+x^{b}+x^{c}\right)}=1$$

15. Represent $\mathbf{\sqrt{3}}$ **on number line.**

Ans: Consider a number line OD such that the construction to form two triangles is done as shown below.



Take $OA = AB = 1$ unit.

And $\angle A = 90°$

In ΔOAB, by using the Pythagorean theorem we get,

$OB^2 = 1^2 + 1^2$

$OB^2 = 2$

$OB = \sqrt{2}$

Now from triangle ΔOBD, using the Pythagorean theorem we get,

$OD^2 = OB^2 + BD^2$

$OD^2 = \left(\sqrt{2}\right)^2 + (1)^1$

$OD^2 = 2 + 1 = 3$

$OD = \sqrt{3}$

Now, is point O is 0 units then the point D represents $\sqrt{3}$ units.

16. Simplify $\left(3\sqrt{2}+2\sqrt{3}\right)^2\left(3\sqrt{2}-2\sqrt{3}\right)^2$.

Ans: We are given the expression as,

$\left(3\sqrt{2}+2\sqrt{3}\right)^2\left(3\sqrt{2}-2\sqrt{3}\right)^2$

Now, by regrouping the terms in the above expression we have,

$= \left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)$

$= \left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)$

$$=\left[\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2\right]\left[\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2\right]$$
$$=[9\times2-4\times3][9\times2-4\times3]$$
$$=[18-12][18-12]$$
$$=6\times6=36$$

17. Express $2.\overline{4178}$ in the form $\frac{p}{q}$.

Ans: Let $\frac{p}{q}=2.\overline{4178}$

$$\frac{p}{q}=2.4178178178$$

Multiply by 10

$$10\frac{p}{q}=24.178178$$

Multiply by 1000

$$10000\frac{p}{q}=1000\times24.178178$$

$$1000\frac{p}{q}-\frac{p}{q}=24178.178178-14.178178$$

$$9999\frac{p}{q}=24154$$

$$\frac{p}{q}=\frac{24154}{9999}$$

18. Simplify $(27)^{-\frac{2}{3}}\div9^{\frac{1}{2}}.3^{-\frac{3}{2}}$.

Ans: $(27)^{-\frac{2}{3}}\div9^{\frac{1}{2}}.3^{-\frac{3}{2}}$

$$=\frac{(3\times3\times3)^{-\frac{2}{3}}\times3^{\frac{3}{2}}}{(3\times3)^{\frac{1}{2}}}\left[a^{-m}=\frac{1}{a^{m}}\right]$$

$$=\frac{(3^{3})^{-\frac{2}{3}}\times3^{\frac{3}{2}}}{(3^{2})^{\frac{1}{2}}}$$

$$=\frac{3^{\frac{3}{2}-2}}{3}=\frac{3^{-\frac{1}{3}}}{3}$$

$$=\frac{1}{3^{\frac{4}{3}}}=\frac{1}{\sqrt[3]{81}}$$

19. Find three rational numbers between $2.\overline{2}$ and $2.\overline{3}$.

Ans: The irrational numbers are the numbers that do not end after the decimal point nor repeat its numbers in a sequence.

Representing the given numbers in decimal form we have,

$2.\overline{2}=2.222222222......$

$2.\overline{3}=2.333333333.......$

So any numbers between these two numbers that do not end nor repeat in any sequence gives the required irrational numbers.

Three rational numbers between $2.\overline{2}$ and $2.\overline{3}$ are $2.222341365....$, $2.28945187364....$ and $2.2321453269....$

20. Give an example of two irrational numbers whose

i. Sum is a rational number

Ans: The required two irrational numbers are $2+\sqrt{2}$ and $2-\sqrt{2}$

Sum $2+\sqrt{2}+2-\sqrt{2}=4$ which is a rational number.

ii. Product is a rational number

Ans: The required two irrational numbers are $3\sqrt{2}$ and $6\sqrt{2}$

Product $3\sqrt{2}\times 6\sqrt{2}=18\times 2=36$ which is rational.

iii. Quotient is a rational number

Ans: The required two irrational numbers are $2\sqrt{125}$ and $3\sqrt{5}$

Quotient $\frac{2\sqrt{125}}{3\sqrt{5}}=\frac{2}{3}\sqrt{\frac{125}{5}}=\frac{2}{3}\times 5=\frac{10}{3}$

21. If $\sqrt{2}=1.414$ and $\sqrt{3}=1.732$, find the value of $\frac{5}{\sqrt{2}+\sqrt{3}}$.

Ans: First let us take the given expression and by rationalizing the denominator we get,

$$\frac{5}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$=\frac{5\left(\sqrt{2}-\sqrt{3}\right)}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{5\left(\sqrt{2}-\sqrt{3}\right)}{2-3}$$

Now, substituting the required values of irrational numbers we get,

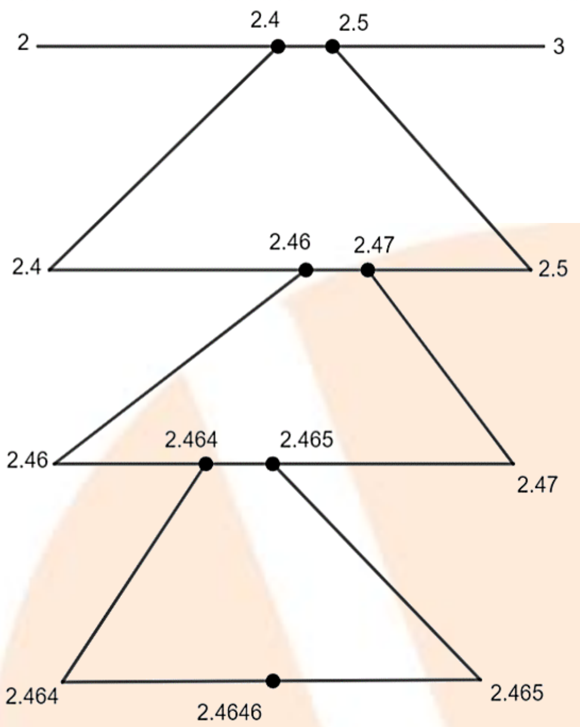
$$=-5[1.414-1.732]$$

$$=-5\times -0.318$$

$$=1.59$$

22. Visualize 2.4646 on the number line using successive magnification.

Ans:



23. Rationalizing the denominator of $\frac{1}{4+2\sqrt{3}}$.

Ans: First let us take the given expression and rationalizing the denominator by multiplying the numerator and denominator with its conjugate we get,

$$\frac{1}{4+2\sqrt{3}}=\frac{1}{4+2\sqrt{3}}\times\frac{4-2\sqrt{3}}{4+2\sqrt{3}}$$

$$=\frac{4-2\sqrt{3}}{(4)^2-\left(2\sqrt{3}\right)^2}$$

$$=\frac{4-2\sqrt{3}}{16-\left(2\sqrt{3}\right)^2}$$

$$=\frac{4-2\sqrt{3}}{16-12}$$

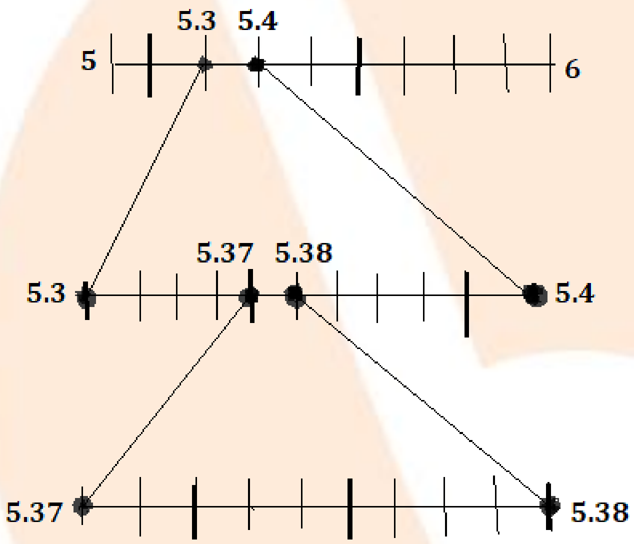
$$=\frac{4-2\sqrt{3}}{4}$$

$$=\frac{2\left(2-\sqrt{3}\right)}{4}$$

$$=\frac{2-\sqrt{3}}{2}$$

24. Visualize the representation of $5.3\overline{7}$ on the number line up to 3 decimal places.

Ans: The representation of $5.3\overline{7}$ on the number line is given below:



25. Show that $5\sqrt{2}$ is not a rational number.

Ans: Let us assume that $5\sqrt{2}$ is a rational number.

Take $x=5\sqrt{2}$, with x being rational as well.

Now,

$$x=5\sqrt{2}$$

$$\Rightarrow \frac{x}{5}=\sqrt{2}$$

Let us compare the terms in LHS and RHS.

In LHS, we have $\frac{x}{5}$, with x and 5 being rational numbers [Here x is rational, based on our assumption]. So $\frac{x}{5}$ is a rational number.

In RHS, we have $\sqrt{2}$, which is not a rational number, but an irrational number.

This is a contradiction, i.e. $\text{LHS} \neq \text{RHS}$.

So, we can conclude that $5\sqrt{2}$ is not a rational number.

26. Simplify $3\sqrt[3]{250}+7\sqrt[3]{16}-4\sqrt[3]{54}$.

Ans: Let us first find the cube roots of given numbers to their simplest forms by using the prime factorization then we get,

$$3\sqrt[3]{250}+7\sqrt[3]{16}-4\sqrt[3]{54}=3\sqrt[3]{5\times5\times5\times2}+7\sqrt[3]{2\times2\times2\times2}-4\sqrt[3]{3\times3\times3\times2}$$

$$=\left(3\times5\sqrt[3]{2}\right)+\left(7\times2\sqrt[3]{2}\right)-\left(4\times3\sqrt[3]{2}\right)$$

$$=\left(15\sqrt[3]{2}\right)+\left(14\sqrt[3]{2}\right)-\left(12\sqrt[3]{2}\right)$$

$$=(15+14-12)\sqrt[3]{2}$$

$$=17\sqrt[3]{2}$$

Thus, we get $3\sqrt[3]{250}+7\sqrt[3]{16}-4\sqrt[3]{54}=17\sqrt[3]{2}$

27. Simplify $3\sqrt{48}-\frac{5}{2}\sqrt{\frac{1}{3}}+4\sqrt{3}$.

Ans: Let us first find the square roots of given numbers to their simplest forms by using the prime factorization then we get,

$$3\sqrt{48}-\frac{5}{2}\sqrt{\frac{1}{3}}+4\sqrt{3}=\left(3\sqrt{2\times2\times2\times2\times3}\right)-\left[\frac{5}{2}\left(\sqrt{\frac{1}{3}}\times\frac{\sqrt{3}}{\sqrt{3}}\right)\right]+\left(4\sqrt{3}\right)$$

$$=\left(3\times2\times2\sqrt{3}\right)-\left[\frac{5}{2}\left(\frac{\sqrt{3}}{3}\right)\right]+\left(4\sqrt{3}\right)$$

$$=\left(12\sqrt{3}\right)-\left(\frac{5\sqrt{3}}{6}\right)+\left(4\sqrt{3}\right)$$

$$=\left(12-\frac{5}{6}+4\right)\sqrt{3}$$

$$=\left(\frac{72-5+24}{6}\right)\sqrt{3}$$

$$=\frac{91}{6}\sqrt{3}$$

Thus, we get $3\sqrt{48}-\frac{5}{2}\sqrt{\frac{1}{3}}+4\sqrt{3}=\frac{91}{6}\sqrt{3}$

28. If $\frac{1}{7}=0.\overline{142857}$. Find the value of $\frac{2}{7},\frac{3}{7},\frac{4}{7}$

Ans: It is given that – $\frac{1}{7}=0.\overline{142857}$

Now,

(i) $\frac{2}{7}=2\times\frac{1}{7}$

$=2\times0.\overline{142857}$

$=0.\overline{285714}$

$\Rightarrow\frac{2}{7}=0.\overline{285714}$

(ii) $\frac{3}{7}=3\times\frac{1}{7}$

$=3\times0.\overline{142857}$

$=0.\overline{428571}$

$\Rightarrow\frac{3}{7}=0.\overline{428571}$

(iii) $\frac{4}{7}=4\times\frac{1}{7}$

$=4\times 0.\overline{142857}$

$=0.\overline{571428}$

$\Rightarrow \frac{4}{7}=0.\overline{571428}$

29. Find 6 rational numbers between $\frac{6}{5}$ and $\frac{7}{5}$

Ans: It is possible to divide the interval between $\frac{6}{5}$ and $\frac{7}{5}$ into 10 equal parts.

Then we will have – $\frac{6}{5},\frac{6.1}{5},\frac{6.2}{5},\frac{6.3}{5},\frac{6.4}{5},\frac{6.5}{5},\frac{6.6}{5},\frac{6.7}{5},\frac{6.8}{5},\frac{6.9}{5},\frac{7}{5}$

i.e. $\frac{60}{50},\frac{61}{50},\frac{62}{50},\frac{63}{50},\frac{64}{50},\frac{65}{50},\frac{66}{50},\frac{67}{50},\frac{68}{50},\frac{69}{50},\frac{70}{50}$

From these fractions, it is possible to choose 6 rational numbers between $\frac{6}{5}$ and $\frac{7}{5}$

Thus, 6 rational numbers between $\frac{6}{5}$ and $\frac{7}{5}$ are $\frac{61}{50},\frac{62}{50},\frac{63}{50},\frac{64}{50},\frac{65}{50},\frac{66}{50}$

30. Show how $\sqrt{4}$ can be represented on the number line.

Ans: Take $AB=OA=1\text{ unit}$ on a number line.

Also, $\angle A=90^{\circ}$

In $\triangle OAB$, apply Pythagoras Theorem,

$\therefore OA^2+AB^2=OB^2$

$\Rightarrow OB^2=1^2+1^2$

$\Rightarrow OB^2 = 1+1$

$\Rightarrow OB^2 = 2$

$\Rightarrow OB = \sqrt{2}$

Now, draw $OB = OA_1 = \sqrt{2}$

And, $A_1B_1 = 1$ unit with $\angle A_1 = 90^\circ$

In $\triangle OA_1B_1$, apply Pythagoras Theorem,

$\therefore OA_1^{\ 2} + A_1B_1^{\ 2} = OB_1^{\ 2}$

$\Rightarrow OB_1^{\ 2} = \left(\sqrt{2}\right)^2 + 1^2$

$\Rightarrow OB_1^{\ 2} = 2+1$

$\Rightarrow OB_1^{\ 2} = 3$

$\Rightarrow OB_1 = \sqrt{3}$

Now, draw $OB_1 = OA_2 = \sqrt{3}$

And, $A_2B_2 = 1$ unit with $\angle A_2 = 90^\circ$

In $\triangle OA_2B_2$, apply Pythagoras Theorem,

$\therefore OA_2^{\ 2} + A_2B_2^{\ 2} = OB_2^{\ 2}$

$\Rightarrow OB_2^{\ 2} = \left(\sqrt{3}\right)^2 + 1^2$

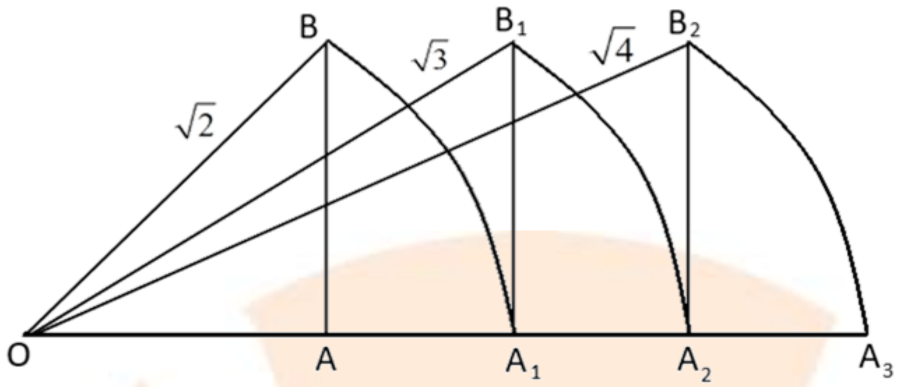
$\Rightarrow OB_2^{\ 2} = 3+1$

$\Rightarrow OB_2^{\ 2} = 4$

$\Rightarrow OB_2 = \sqrt{4}$

Now, draw $OB_2 = OA_3 = \sqrt{4}$

Thus line segment $OA_3 = \sqrt{4}$



Short Answer Questions **4 Marks**

1. Write the following in decimal form and say what kind of decimal expansion each has:

i. $\mathbf{\frac{36}{100}}$

Ans: Performing long division of 36 by 100

$$\begin{array}{r|l} & 0.36 \\ \hline 100 & 36 \\ & \underline{00} \\ & 360 \\ & \underline{300} \\ & 60 \\ & \underline{60} \\ & 0 \end{array}$$

Thus, $\frac{36}{100} = 0.36$ - this is a terminating decimal.

ii. $\mathbf{\frac{1}{11}}$

Ans: Performing long division of 1 by 11

$$
\begin{array}{r|l}
 & 0.0909.. \\
11 & 1 \\
 & \underline{0} \\
 & 10 \\
 & \underline{0} \\
 & 100 \\
 & \underline{99} \\
 & 10 \\
 & \underline{0} \\
 & 100 \\
 & \underline{99} \\
 & 1
\end{array}
$$

It can be seen that performing further division will produce a reminder of 1 continuously.

Thus, $\frac{1}{11} = 0.09090...$ i.e. $\frac{1}{11} = 0.\overline{09}$, this is a non-terminating, but recurring decimal.

iii. $\mathbf{4\frac{1}{8}}$

Ans: First convert the mixed fraction into an improper fraction –

$$4\frac{1}{8} = \frac{(4 \times 8) + 1}{8} = \frac{33}{8}$$

Performing long division of 33 by 8

$$
\begin{array}{r|l}
 & 4.125 \\
8 & 33 \\
 & \underline{32} \\
 & 10 \\
 & \underline{8} \\
 & 20
\end{array}
$$

$\underline{16}$

40

$\underline{40}$

0

Thus, $4\frac{1}{8} = 4.125$ - this is a terminating decimal.

iv. $\mathbf{\frac{3}{13}}$

Ans: Performing long division of 3 by 13

$$
\begin{array}{r}
0.230769.. \\
13\overline{)\quad 3} \\
\underline{0} \\
30 \\
\underline{26} \\
40 \\
\underline{39} \\
10 \\
\underline{0} \\
100 \\
\underline{91} \\
90 \\
\underline{78} \\
120 \\
\underline{117} \\
3
\end{array}
$$

It can be seen that performing further division will produce a reminder of 3 periodically, after every six divisions.

Thus, $\frac{3}{13} = 0.230769...$ i.e. $\frac{3}{13} = 0.\overline{230769}$, this is a non-terminating, but recurring decimal.

v. $\mathbf{\frac{2}{11}}$

Ans: Performing long division of 2 by 11

$$
\begin{array}{r}
0.1818.. \\
11\overline{)\quad 2} \\
\underline{0} \\
20 \\
\underline{11} \\
90 \\
\underline{88} \\
20 \\
\underline{11} \\
90 \\
\underline{88} \\
2
\end{array}
$$

It can be seen that performing further division will produce a reminder of 2 followed by 9 alternatively.

Thus, $\frac{2}{11} = 0.181818...$ i.e. $\frac{2}{11} = 0.\overline{18}$ this is a non-terminating, but recurring decimal.

vi. $\mathbf{\frac{329}{400}}$

Ans: Performing long division of 33 by 8

$$
\begin{array}{r}
0.8225 \\
400\overline{)\quad 329} \\
\underline{000} \\
3290 \\
\underline{3200} \\
900
\end{array}
$$

$$\begin{array}{r} \underline{800} \\ 1000 \\ \underline{800} \\ 2000 \\ \underline{2000} \\ 0 \end{array}$$

Thus, $\frac{329}{400} = 0.8225$ - this is a terminating decimal.

2. Classify the following as rational or irrational:

i. $\sqrt{23}$

Ans: It is known that the root of 23 will produce a non-terminating and non-recurring decimal number [it is not a perfect square value], also it cannot be represented as a fraction. Thus we can say that $\sqrt{23}$ is an irrational number.

ii. $\sqrt{225}$

Ans: It is known that $\sqrt{225} = 15$, which is an integer.

Thus $\sqrt{225}$ is a rational number.

iii. 0.3796

Ans: Here, 0.3796 is a terminating decimal number, and also it can be expressed as a fraction.

i.e. $0.3796 = \frac{3796}{10000} = \frac{949}{2500}$

Thus 0.3796 is a rational number.

iv. $7.478478...$

Ans: Here, $7.478478...$ is a non-terminating, but recurring decimal number, and also it can be expressed as a fraction.

i.e. $7.478478... = 7.\overline{487}$

Converting it into fraction

If $x = 7.478478...$ (1)

Then $1000x = 7478.478478...$ (2)

Subtract equations $(2)-(1)$

$$\begin{array}{r} 1000x = 7478.478478... \\ -\quad x = \quad 7.478478... \\ \hline 999x = 7471 \end{array}$$

Now, $999x = 7471$

$$\Rightarrow x = \frac{7471}{999}$$

i.e. $7.\overline{478} = \frac{7471}{999}$

Thus $7.478478...$ is a rational number.

v. 1.101001000100001...

Ans: Here, $1.101001000100001...$ is a non-terminating and non-recurring decimal number and also it cannot be represented as a fraction. Thus we can say that $1.101001000100001...$ is an irrational number.

3. Rationalize the denominator of the following:

i. $\frac{1}{\sqrt{7}}$

Ans: In order to rationalize the denominator, we multiply and divide $\frac{1}{\sqrt{7}}$ by $\sqrt{7}$

$$\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

Rationalizing the denominator of $\frac{1}{\sqrt{7}}$ produces $\frac{\sqrt{7}}{7}$.

ii. $\mathbf{\dfrac{1}{\sqrt{7}-\sqrt{6}}}$

Ans: In order to rationalize the denominator, we multiply and divide $\dfrac{1}{\sqrt{7}-\sqrt{6}}$ by $\sqrt{7}+\sqrt{6}$

$$\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}-\sqrt{6}\right)\left(\sqrt{7}+\sqrt{6}\right)}$$

Using the identity - $(a+b)(a-b)=a^2-b^2$

$$=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{1}$$

$$\Rightarrow\frac{1}{\sqrt{7}-\sqrt{6}}=\sqrt{7}+\sqrt{6}$$

Rationalizing the denominator of $\dfrac{1}{\sqrt{7}-\sqrt{6}}$ produces $\sqrt{7}+\sqrt{6}$.

iii. $\mathbf{\dfrac{1}{\sqrt{5}+\sqrt{2}}}$

Ans: In order to rationalize the denominator, we multiply and divide $\dfrac{1}{\sqrt{5}+\sqrt{2}}$ by $\sqrt{5}-\sqrt{2}$

$$\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$$

Using the identity - $(a+b)(a-b)=a^2-b^2$

$$=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}$$

$$\Rightarrow \frac{1}{\sqrt{5}+\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{3}$$

Rationalizing the denominator of $\frac{1}{\sqrt{5}+\sqrt{2}}$ produces $\frac{\sqrt{5}-\sqrt{2}}{3}$.

iv. $\mathbf{\frac{1}{\sqrt{7}-2}}$

Ans: In order to rationalize the denominator, we multiply and divide $\frac{1}{\sqrt{7}-2}$ by $\sqrt{7}+2$

$$\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}=\frac{\sqrt{7}+2}{\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)}$$

Using the identity - $(a+b)(a-b)=a^2-b^2$

$$=\frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-(2)^2}$$

$$=\frac{\sqrt{7}+2}{7-4}$$

$$=\frac{\sqrt{7}+2}{3}$$

$$\Rightarrow \frac{1}{\sqrt{7}+2}=\frac{\sqrt{7}+2}{3}$$

Rationalizing the denominator of $\frac{1}{\sqrt{7}-2}$ produces $\frac{\sqrt{7}+2}{3}$.

Long Answer Questions **5 Marks**

1. Write the following in decimal form and say what kind of decimal expansion each has:

i. $\frac{36}{100}$

Ans: Performing long division of 36 by 100

$$\begin{array}{r|l} & 0.36 \\ 100 & 36 \\ & \underline{00} \\ & 360 \\ & \underline{300} \\ & 60 \\ & \underline{60} \\ & 0 \end{array}$$

Thus, $\frac{36}{100}=0.36$ - this is a terminating decimal.

ii. $\frac{1}{11}$

Ans: Performing long division of 1 by 11

$$\begin{array}{r|l} & 0.0909.. \\ 11 & 1 \\ & \underline{0} \\ & 10 \\ & \underline{0} \\ & 100 \\ & \underline{99} \\ & 10 \end{array}$$

$$\begin{array}{r} \underline{0} \\ 100 \\ \underline{99} \\ 1 \end{array}$$

It can be seen that performing further division will produce a reminder of 1 continuously.

Thus, $\frac{1}{11}=0.09090...$ i.e. $\frac{1}{11}=0.\overline{09}$, this is a non-terminating, but recurring decimal.

iii. $\mathbf{4\frac{1}{8}}$

Ans: First convert the mixed fraction into an improper fraction –

$$4\frac{1}{8}=\frac{(4\times 8)+1}{8}=\frac{33}{8}$$

Performing long division of 33 by 8

$$\begin{array}{r} 4.125 \\ 8\overline{)\ \ 33} \\ \underline{32} \\ 10 \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Thus, $4\frac{1}{8}=4.125$ - this is a terminating decimal.

iv. $\mathbf{\frac{3}{13}}$

Ans: Performing long division of 3 by 13

$$
\begin{array}{r}
0.230769.. \\
13\overline{)\quad 3} \\
\underline{0} \\
30 \\
\underline{26} \\
40 \\
\underline{39} \\
10 \\
\underline{0} \\
100 \\
\underline{91} \\
90 \\
\underline{78} \\
120 \\
\underline{117} \\
3
\end{array}
$$

It can be seen that performing further division will produce a reminder of 3 periodically, after every six divisions.

Thus, $\frac{3}{13} = 0.230769...$ i.e. $\frac{3}{13} = 0.\overline{230769}$, this is a non-terminating, but recurring decimal.

v. $\mathbf{\frac{2}{11}}$

Ans: Performing long division of 2 by 11

$$
\begin{array}{r}
0.1818.. \\
11\overline{)\quad 2} \\
\underline{0} \\
20
\end{array}
$$

$$
\begin{array}{l}
\underline{11} \\
\quad 90 \\
\quad \underline{88} \\
\qquad 20 \\
\qquad \underline{11} \\
\qquad\quad 90 \\
\qquad\quad \underline{88} \\
\qquad\qquad 2
\end{array}
$$

It can be seen that performing further division will produce a reminder of 2 followed by 9 alternatively.

Thus, $\frac{2}{11} = 0.181818...$ i.e. $\frac{2}{11} = 0.\overline{18}$ this is a non-terminating, but recurring decimal.

vi. $\mathbf{\frac{329}{400}}$

Ans: Performing long division of 33 by 8

$$
\begin{array}{r|l}
 & 0.8225 \\
400 & 329 \\
 & \underline{000} \\
 & 3290 \\
 & \underline{3200} \\
 & \quad 900 \\
 & \quad \underline{800} \\
 & \quad 1000 \\
 & \qquad \underline{800} \\
 & \qquad 2000 \\
 & \qquad \underline{2000} \\
 & \qquad\quad 0
\end{array}
$$

Thus, $\frac{329}{400} = 0.8225$ - this is a terminating decimal.

2. Repeated question

3. Repeated question

4. If $\sqrt{5}=2.236$ and $\sqrt{3}=1.732$. Find the value of $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{7}{\sqrt{5}-\sqrt{3}}$.

Ans: It is given that –

$\sqrt{5}=2.236$

$\sqrt{3}=1.732$

Now, $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{7}{\sqrt{5}-\sqrt{3}}$

Taking LCM

$$\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{7}{\sqrt{5}-\sqrt{3}}=\left[\frac{2}{\left(\sqrt{5}+\sqrt{3}\right)}\times\frac{\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)}\right]+\left[\frac{7}{\left(\sqrt{5}-\sqrt{3}\right)}\times\frac{\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)}\right]$$

$$=\left[\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}\right]+\left[\frac{7\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}\right]$$

$$=\left[\frac{\left(2\sqrt{5}-2\sqrt{3}\right)+\left(7\sqrt{5}+7\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}\right]$$

Using the identity - $(a+b)(a-b)=a^2-b^2$

$$=\left[\frac{2\sqrt{5}-2\sqrt{3}+7\sqrt{5}+7\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}\right]$$

$$=\left[\frac{(2+7)\sqrt{5}+(7-2)\sqrt{3}}{5-3}\right]$$

$$=\left[\frac{9\sqrt{5}+5\sqrt{3}}{2}\right]$$

Since, $\sqrt{5}=2.236$ and $\sqrt{3}=1.732$

$$=\left[\frac{(9\times 2.236)+(5\times 1.732)}{2}\right]$$

$$=\left[\frac{20.124+8.66}{2}\right]$$

$$=\left[\frac{28.784}{2}\right]$$

$=14.392$

Thus, $\dfrac{2}{\sqrt{5}+\sqrt{3}}+\dfrac{7}{\sqrt{5}-\sqrt{3}}=14.392$

5. Find the value of $\dfrac{3}{\sqrt{5}+\sqrt{2}}+\dfrac{7}{\sqrt{5}-\sqrt{2}}$, if $\sqrt{5}=2.236$ and $\sqrt{2}=1.414$.

Ans: It is given that –

- $\sqrt{5}=2.236$
- $\sqrt{2}=1.414$

Now, $\dfrac{3}{\sqrt{5}+\sqrt{2}}+\dfrac{7}{\sqrt{5}-\sqrt{2}}$

Taking LCM

$$\frac{3}{\sqrt{5}+\sqrt{2}}+\frac{7}{\sqrt{5}-\sqrt{2}}=\left[\frac{3}{\left(\sqrt{5}+\sqrt{2}\right)}\times\frac{\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}-\sqrt{2}\right)}\right]+\left[\frac{7}{\left(\sqrt{5}-\sqrt{2}\right)}\times\frac{\left(\sqrt{5}+\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)}\right]$$

$$=\left[\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}\right]+\left[\frac{7\left(\sqrt{5}+\sqrt{2}\right)}{\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)}\right]$$

$$=\left[\frac{\left(3\sqrt{5}-3\sqrt{2}\right)+\left(7\sqrt{5}+7\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}\right]$$

Using the identity - $(a+b)(a-b)=a^2-b^2$

$$=\left[\frac{3\sqrt{5}-3\sqrt{2}+7\sqrt{5}+7\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}\right]$$

$$=\left[\frac{(3+7)\sqrt{5}+(7-3)\sqrt{2}}{5-2}\right]$$

$$=\left[\frac{10\sqrt{5}+4\sqrt{2}}{3}\right]$$

Since, $\sqrt{5}=2.236$ and $\sqrt{2}=1.414$

$$=\left[\frac{(10\times 2.236)+(4\times 1.414)}{3}\right]$$

$$=\left[\frac{22.36+5.656}{3}\right]$$

$$=\left[\frac{28.016}{3}\right]$$

Thus, $\frac{3}{\sqrt{5}+\sqrt{2}}+\frac{7}{\sqrt{5}-\sqrt{2}}=\frac{28.016}{3}$

6. Simplify $\frac{2+\sqrt{5}}{2-\sqrt{5}}+\frac{2-\sqrt{5}}{2+\sqrt{5}}$

Ans: $\frac{2+\sqrt{5}}{2-\sqrt{5}}+\frac{2-\sqrt{5}}{2+\sqrt{5}}$

Taking LCM

$$\frac{2+\sqrt{5}}{2-\sqrt{5}}+\frac{2-\sqrt{5}}{2+\sqrt{5}}=\left[\frac{2+\sqrt{5}}{2-\sqrt{5}}\times\frac{\left(2+\sqrt{5}\right)}{\left(2+\sqrt{5}\right)}\right]+\left[\frac{2-\sqrt{5}}{2+\sqrt{5}}\times\frac{\left(2-\sqrt{5}\right)}{\left(2-\sqrt{5}\right)}\right]$$

$$=\left[\frac{\left(2+\sqrt{5}\right)\left(2+\sqrt{5}\right)}{\left(2-\sqrt{5}\right)\left(2+\sqrt{5}\right)}\right]+\left[\frac{\left(2-\sqrt{5}\right)\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}\right]$$

$$=\left[\frac{\left(2+\sqrt{5}\right)^2+\left(2-\sqrt{5}\right)^2}{\left(2-\sqrt{5}\right)\left(2+\sqrt{5}\right)}\right]$$

Using the identities –

- $(a+b)(a-b)=a^2-b^2$
- $(a+b)^2=a^2+b^2+2ab$
- $(a-b)^2=a^2+b^2-2ab$

$$=\left[\frac{\left((2)^2+\left(\sqrt{5}\right)^2+\left(2\times2\times\sqrt{5}\right)\right)+\left((2)^2+\left(\sqrt{5}\right)^2-\left(2\times2\times\sqrt{5}\right)\right)}{(2)^2-\left(\sqrt{5}\right)^2}\right]$$

$$=\left[\frac{\left(4+5+\left(4\sqrt{5}\right)\right)+\left(4+5-\left(4\sqrt{5}\right)\right)}{4-5}\right]$$

$$=\left[\frac{9+9}{-1}\right]$$

$$=\left[\frac{18}{-1}\right]$$

$$=-18$$

Thus, $\dfrac{2+\sqrt{5}}{2-\sqrt{5}}+\dfrac{2-\sqrt{5}}{2+\sqrt{5}}=(-18)$

7. Find a and b, if $\dfrac{3-\sqrt{6}}{3+2\sqrt{6}}=a\sqrt{6}-b$

Ans: $\dfrac{3-\sqrt{6}}{3+2\sqrt{6}}=a\sqrt{6}-b$

Here,

$$\text{LHS} = \frac{3-\sqrt{6}}{3+2\sqrt{6}}$$

$$\text{RHS} = a\sqrt{6} - b$$

Start by rationalizing the denominator in LHS

In order to rationalize the denominator, we multiply and divide $\frac{3-\sqrt{6}}{3+2\sqrt{6}}$ by $3+2\sqrt{6}$

$$\frac{3-\sqrt{6}}{3+2\sqrt{6}} \times \frac{3-2\sqrt{6}}{3-2\sqrt{6}} = \frac{\left(3-\sqrt{6}\right)\left(3-2\sqrt{6}\right)}{\left(3+2\sqrt{6}\right)\left(3-2\sqrt{6}\right)}$$

Using the identity - $(a+b)(a-b) = a^2 - b^2$

$$= \frac{(3\times 3) - \left(3\times 2\sqrt{6}\right) - \left(\sqrt{6}\times 3\right) + \left(\sqrt{6}\times 2\sqrt{6}\right)}{(3)^2 - \left(2\sqrt{6}\right)^2}$$

$$= \frac{(9) - \left(6\sqrt{6}\right) - \left(3\sqrt{6}\right) + (12)}{9-24}$$

$$= \frac{(21) - \left(9\sqrt{6}\right)}{-15}$$

$$= \frac{(21)}{-15} - \frac{\left(9\sqrt{6}\right)}{-15}$$

They are all divisible by 3

$$= -\frac{7}{5} + \frac{\left(3\sqrt{6}\right)}{5}$$

Thus, $\text{LHS} = \frac{3}{5}\sqrt{6} - \frac{7}{5}$

Comparing with RHS, we get –

$\text{RHS} = a\sqrt{6} - b$

Thus,

$a = \frac{3}{5}$

$b = \frac{7}{5}$