CBSE Class 9 Mathemaics

Important Questions

Chapter 2

Polynomials

1 Marks Questions

1. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Ans. The binomial of degree 35 can be $x^{35}+9$.

The binomial of degree 100 can be t^{100}.

2. Which of the following expression is a polynomial

(a) $x^3 - 1$

(b) $\sqrt{x}+2$

(c) $x^2 - \frac{1}{x^2}$

(d) $\sqrt{t}+5t-1$

Ans. (a) $x^3 - 1$

3. A polynomial of degree 3 in x has at most

(a) 5 terms

(b) 3 terms

(c) 4 terms

(d) 1 term

Ans. (b) 3 terms

4. The coefficient of x^2 in the polynomial $2x^3 + 4x^2 + 3x + 1$ is

(a) 2

(b) 3

(c) 1

(d) 4

Ans. (d) 4

5. The monomial of degree 50 is

(a) $x^{50} + 1$

(b) $2x^{50}$

(c) x+50

(d) 50

Ans. (b) $2x^{50}$

6. Divide f(x) by g(x) and verify the remainder $f(x) = x^3 + 4x^2 - 3x - 10$ **, g(x) = x + 4**

Ans. Dividend $= x^3 + 4x^2 - 3x - 10$, divisor = x + 4

Quotient $= x^3 - 3$, Remainder = 2

Dividend = Divisor $\times$ quotient + Remainder

$= (x + 4)(x^2 - 3) + 2$

$= x^3 - 3x + 4x^2 - 12 + 2$

$= x^3 + 4x^2 - 3x - 10$

7. **Which of the following expression is a monomial**

(a) 3 + x

(b) $4x^3$

(c) $x^6 + 2x^2 + 2$

(d) None of these

Ans. (a) 3 + x

8. A linear polynomial

(a) May have one zero

(b) has one and only one zero

(c) May have two zero

(d) May have more than one zero

Ans. (b) has one and only one zero

9. If $P(x) = x^3 - 1$ **, then the value of P(1) + P(-1) is**

(a) 0

(b) 1

(c) 2

(d) – 2

Ans. (d) -2

10. when polynomial $x^3 + 3x^2 + 3x + 1$ **is divided by x + 1, the remainder is**

(a) 1

(b) 0

(c) 8

(d) – 6

Ans. (b) -6

11. Factories $x^2 + y - xy - x$

Ans. $x^2 + y - xy - x$

$x^2 - x + y - xy = x^2 - x - xy + y$

= x (x – 1) – y (x – 1)

= (x – 1) (x – y)

12. The value of K for which x – 1 is a factor of the polynomial $4x^3 + 3x^2 - 4x + K$ **is**

(a) 0

(b) 3

(c) – 3

(d) 1

Ans. (c) – 3

13. The factors of $12x^2 - x - 6$ **are**

(a) (3x – 2) (4x + 3)

(b) (12x + 1) (x – 6)

(c) (12x – 1) (x + 6)

(d) (3x + 2) (4x – 3)

Ans. (d) (3x + 2) (4x – 3)

14. $x^3 + y^3 + z^3 - 3xyz$ is

(a) $(x+y-z)^3$

(b) $(x-y+z)^3$

(c) $(x+y+z)^3 - 3xyz$

(d) $(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

Ans. (d) $(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

15. The expanded form of $(x+y-z)^2$ is

(a) $x^2+y^2+z^2+2xy+2yz+2zx$

(b) $x^2+y^2-z^2+2xy-2yz-2xz$

(c) $x^2+y^2+z^2+2xy-2yz-2zx$

(d) $x^2+y^2+z^2+2xy+2yx+2xz$

Ans. (c) $x^2+y^2+z^2+2xy-2yz-2zx$

16. Find the integral zeroes of the polynomial x^3+3x^2-x-3

Ans. Given polynomial $P(x) = x^3+3x^2-x-3$

$p(x) = x^2(x+3) - 1(x+3)$

$= (x+3)(x^2-1)$

For zeros $p(x) = 0$

$(x+3)(x^2-1)=0$

$(x+3)(x+1)(x-1)=0$

$x=-3, x=-1, x=1$

Zeroes of polynomial -1, 1, and -3.

17. The value of $(102)^3$ is

(a) 1061208

(b) 1001208

(c) 1820058

(d) none of these

Ans. (a) 1061208

18. $(a-b)^3+(b-c)^3+(c-a)^3$ **is equal to**

(a) 3abc

(b) 3(a-b) (b-c) (c-a)

(c) $3a^3b^3bc^3$

(d) $\left[a-(b+c)\right]^3$

Ans. (b) 3(a-b) (b-c) (c-a)

19. The zeroes of the polynomial p(x) = x (x-2) (x+3) are

(a) 0

(b) 0, 2, 3

(c) 0, 2, -3

(d) none of these

Ans. (c) 0, 2, -3

20. If (x+1) and (x-1) are factors of Px^3+x^2-2x+9 then value of p and q are

(a) p = -1, q = 2

(b) p = 2, q = -1

(c) p = 2, q = 1

(d) p = -2, q = -2

Ans. (b) p = 2, q = -1

21. If x+y+z = 0, then $x^3+y^3+z^3$ is

(a) xyz

(b) 2xyz

(c) 3xyz

(d) 0

Ans. (b) 2xyz

22. The value of $(x-a)^3 + (x-b)^3 + (x-c)^3 - 3(x-a)(x-b)(x-c)$ when a + b + c= 3x, is

(a) 3

(b) 2

(c) 1

(d) 0

Ans. (c) 1

23. Factors of x^2 + $3\sqrt{2}x+4$ are

(a) $(x+2\sqrt{2})\ (x-\sqrt{2})$

(b) $(x+2\sqrt{2})\ (x+\sqrt{2})$

(c) $(x-2\sqrt{2})\ (x+\sqrt{2})$

(d) $(x-2\sqrt{2})\ (x-\sqrt{2})$

Ans. (b) $(x+2\sqrt{2})\ (x+\sqrt{2})$

24. The degree of constant function is

(a) 1

(b) 2

(c) 3

(d) 0

Ans. (d) 0

CBSE Class 9 Mathemaics

Important Questions

Chapter 2

Polynomials

2 Marks Questions

1. Write the coefficients of x^2 in each of the following:

(i) $2+x^2+x$

(ii) $2-x^2+x^3$

(iii) $\frac{\pi}{2}x^2+x$

(iv) $\sqrt{2}x-1$

Ans. (i) $2+x^2+x$

The coefficient of x^2 in the polynomial $2+x^2+x$ is 1.

(ii) $2-x^2+x^3$

The coefficient of x^2 in the polynomial $2-x^2+x^3$ is -1.

(iii) $\frac{\pi}{2}x^2+x$

The coefficient of x^2 in the polynomial $\frac{\pi}{2}x^2+x$ is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1$

The coefficient of x^2 in the polynomial $\sqrt{2}x-1$ is 0.

2. Find the value of the polynomial $5x-4x^2+3$ at

(i) $x = 0$

(ii) $x = -1$

(iii) $x = 2$

Ans. (i) Let $f(x) = 5x - 4x^2 + 3$.

We need to substitute 0 in the polynomial $f(x) = 5x - 4x^2 + 3$ to get

$f(0) = 5(0) - 4(0)^2 + 3$

$= 0 - 0 + 3$

= 3

Therefore, we conclude that at $x = 0$, the value of the polynomial $5x - 4x^2 + 3$ is 3.

(ii) Let $f(x) = 5x - 4x^2 + 3$.

We need to substitute -1 in the polynomial $f(x) = 5x - 4x^2 + 3$ to get

$f(-1) = 5(-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3$

= -6

Therefore, we conclude that at $x = -1$, the value of the polynomial $5x - 4x^2 + 3$ is -6

(iii) Let $f(x) = 5x - 4x^2 + 3$.

We need to substitute 0 in the polynomial $f(x) = 5x - 4x^2 + 3$ to get

$f(2) = 5(2) - 4(2)^2 + 3$

$= 10 - 16 + 3$

= -3

Therefore, we conclude that at $x = 2$, the value of the polynomial $5x-4x^2+3$ is -3.

3. Find the remainder when x^3-ax^2+6x-a is divided by $x-a$.

Ans. We need to find the zero of the polynomial $x-a$.

$x-a=0$
$\Rightarrow \ x=a$

While applying the remainder theorem, we need to put the zero of the polynomial $x-a$ in the polynomial x^3-ax^2+6x-a, to get

$p(x)=x^3-ax^2+6x-a$

$p(a)=(a)^3-a(a)^2+6(a)-a$

$=a^3-a^3+6a-a$

= 5a

Therefore, we conclude that on dividing the polynomial x^3-ax^2+6x-a by $x-a$, we will get the remainder as $5a$.

4. Evaluate the following products without multiplying directly:

(i) 103×107

(ii) 98×96

(iii) 104×96

Ans. (i) 103×107

103×107 can also be written as $(100+3)(100+7)$.

We can observe that we can apply the identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$(100+3)(100+7)=(100)^2+(3+7)(100)+3\times7$

$=10000+1000+21$

= 11021

Therefore, we conclude that the value of the product 103×107 is 11021.

(ii) 95×96

95×96 can also be written as $(100-5)(100-4)$

We can observe that we can apply the identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$(100-5)(100-4)=(100)^2+[(-5)+(-4)](100)+(-5)\times(-4)$

$=10000-900+20$

= 9120

Therefore, we conclude that the value of the product 95×96 is 9120.

(iii) 104×96

104×96 can also be written as $(100+4)(100-4)$.

We can observe that, we can apply the identity $(x+y)(x-y)=x^2-y^2$ with respect to the expression $(100+4)(100-4)$, to get

$(100+4)(100-4)=(100)^2-(4)^2$

$=10000-16$

= 9984

Therefore, we conclude that the value of the product 104×96 is 9984.

5. Factorize the following using appropriate identities:

(i) $9x^2+6xy+y^2$

(ii) $4y^2-4y+1$

(iii) $x^2-\frac{y^2}{100}$

Ans. (i) $9x^2+6xy+y^2$

$9x^2+6xy+y^2=(3x)^2+2\times3x\times y+(y)^2$

We can observe that we can apply the identity $(x+y)^2=x^2+2xy+y^2$

$\Rightarrow(3x)^2+2\times3x\times y+(y)^2=(3x+y)^2.$

(ii) $4y^2-4y+1$

$4y^2-4y+1=(2y)^2-2\times2y\times1+(1)^2$

We can observe that we can apply the identity $(x-y)^2=x^2-2xy+y^2$

$\Rightarrow(2y)^2-2\times2y\times1+(1)^2=(2y-1)^2.$

(iii) $x^2-\frac{y^2}{100}$

We can observe that we can apply the identity $(x)^2-(y)^2=(x+y)(x-y)$

$$\Rightarrow(x)^2-\left(\frac{y}{10}\right)^2=\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right).$$

6. Verify:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Ans. (i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

We know that $(x+y)^3=x^3+y^3+3xy(x+y)$.

$\Rightarrow x^3+y^3=(x+y)^3-3xy(x+y)$

$=(x+y)\left[(x+y)^2-3xy\right]$

$\because$ We know that $(x+y)^2=x^2+2xy+y^2$

$\therefore x^3+y^3=(x+y)(x^2+2xy+y^2-3xy)$

$=(x+y)(x^2-xy+y^2)$

Therefore, the desired result has been verified.

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

We know that $(x-y)^3=x^3-y^3-3xy(x-y)$.

$\Rightarrow x^3-y^3=(x-y)^3+3xy(x-y)$

$=(x-y)\left[(x-y)^2+3xy\right]$

$\because$ We know that $(x-y)^2=x^2-2xy+y^2$

$\therefore x^3-y^3=(x-y)(x^2-2xy+y^2+3xy)$

$=(x-y)(x^2+xy+y^2)$

Therefore, the desired result has been verified.

7. Factorize:

(i) $27y^3+125z^3$

(ii) $64m^3 - 343n^3$

Ans.

(i) $27y^3 + 125z^3$

The expression $27y^3 + 125z^3$ can also be written as $(3y)^3 + (5z)^3$.

We know that $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$.

$\therefore (3y)^3 + (5z)^3 = (3y+5z)\left[(3y)^2 - 3y \times 5z + (5z)^2\right]$

$= (3y+5z)(9y^2 - 15yz + 25z^2)$.

(ii) $64m^3 - 343n^3$

The expression $64m^3 - 343n^3$ can also be written as $(4m)^3 - (7n)^3$.

We know that $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$.

$\therefore (4m)^3 - (7n)^3 = (4m - 7n)\left[(4m)^2 + 4m \times 7n + (7n)^2\right]$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

Therefore, we conclude that after factorizing the expression $64m^3 - 343n^3$, we get $(4m - 7n)(16m^2 + 28mn + 49n^2)$.

8. Factorize: $27x^3 + y^3 + z^3 - 9xyz$

Ans. The expression $27x^3 + y^3 + z^3 - 9xyz$ can also be written as

$(3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z$.

We know that $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

$$\therefore (3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z$$

$$= (3x + y + z)\left[(3x)^2 + (y)^2 + (z)^2 - 3x \times y - y \times z - z \times 3x\right]$$

$$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz).$$

Therefore, we conclude that after factorizing the expression $27x^3 + y^3 + z^3 - 9xyz$, we get $(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$.

9. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\left[(x - y)^2 + (y - z)^2 + (z - x)^2\right]$

Ans.

LHS is $x^3 + y^3 + z^3 - 3xyz$ and RHS is $\frac{1}{2}(x + y + z)\left[(x - y)^2 + (y - z)^2 + (z - x)^2\right]$.

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

And also, we know that $(x - y)^2 = x^2 - 2xy + y^2$.

$$\frac{1}{2}(x + y + z)\left[(x - y)^2 + (y - z)^2 + (z - x)^2\right]$$

$$\frac{1}{2}(x + y + z)\left[(x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2xz + x^2)\right]$$

$$\frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$$

$$(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx).$$

Therefore, we can conclude that the desired result is verified

10. If $x + y + z = 0$**, show that** $x^3 + y^3 + z^3 = 0$.

Ans. We know that $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.

We need to substitute $x^3+y^3+z^3=0$ in

$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$, to get

$x^3+y^3+z^3-3xyz=(0)(x^2+y^2+z^2-xy-yz-zx)$, or

$x^3+y^3+z^3-3xyz=0$

$\Rightarrow x^3+y^3+z^3=3xyz.$

Therefore, the desired result is verified

11. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

Ans. (i) $(-12)^3+(7)^3+(5)^3$

Let $a=-12, b=7$ and $c=5$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=-12+7+5=0$

$\therefore (-12)^3+(7)^3+(5)^3=3(-12)(7)(5)$

$= -1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Let $a=28, b=-15$ and $c=-13$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=28-15-13=0$

$\therefore (28)^3+(-15)^3+(-13)^3=3(28)(-15)(-13)$

$= 16380$

12. Find the value of K if x – 2 is factor of $4x^3 + 3x^2 - 4x + K$

Ans. x – 2 is factor of $4x^3 + 3x^2 - 4x + K$

x – 2 = 0

$\Rightarrow$ x = 2

$\therefore 4(2)^3+3(2)^2-4\times 2+k=0$

$32+12-8+k=0$

$44-8+k=0$

$36+k=0$

K = -36

13. Factories the polynomial $x^3 + 8y^3 + 64z^3 - 24xyz$

Ans. $x^3+8y^3+64z^3-24xyz$

$x^3+(2y)^3+(4z)^3-3\times x\times(2y)\times(4z)$

$=(x+2y+4z)[x^2+(2y)^2+(4z)^2-x\times 2y-2y\times 4z-x\times 4z]$

$=(x+2y+4z)(x^2+4y^2+16z^2-2xy-8yz-4xz)$

14. Without actually Calculating the cubes, find the value of $(-12)^3 + (7)^3 + (5)^3$

Ans. $a^3+b^3+c^3=3abc$

$if\ \ a+b+c=0$

$(-12)^3+(7)^3+(5)^3=3\times-12\times7\times5$

= -1260

$\because -12+7+5=-12+12=0$

15. If x – 3 and $x-\frac{1}{3}$ are both factors of px^2 + 5x + r, then show that p = r

Ans. $\because x-3$ and $x-\frac{1}{3}$ are factors of $px^2+5x+r \ \ \therefore x=3, x=\frac{1}{3}$

zero of px^2+5x+r

$\therefore p(3)^2+5\times3+r=0$

$9p+15+4=0$

$9p+r=-14-----(1)$

$p\left(\frac{1}{3}\right)^2+5\times\frac{1}{3}+r=0$

$\frac{p}{9}+\frac{5}{3}+r=0$

$\frac{p+15+9r}{9}=0$

$p+9r=-15------(2)$

$9p+r=p+9r$

From (1) and (2),

9p+r=p+9r

9p-p=9r-r

8p=8r

P=r

Hence prove.

16. Show that 5 is a zero of polynomial $2x^3 - 7x^2 - 16x + 5$

Ans. Put x = 5 in $2x^3 - 7x^2 - 16x + 5$

$2\times 5^3 - 7\times 5^2 - 16\times 5 + 5$

= 250-175-80+5

= 255-255 = 0

$\therefore x = 5$ is zero of polynomial $2x^3 - 7x^2 - 16x + 5$

17. Using remainder theorem find the remainder when f(x) is divided by g(x)

$f(x) = x^{24} - x^{19} - 2$ g(x) = x + 1

Ans. When f(x) is divided by g(x)

Then remainder f(-1)

$F(-1) = (-1)^{24} - (-1)^{19} - 2 = 1 - (-1) - 2$

= 1+ 1 – 2 = 0

18. Find K if x + 1 is a factor of $P(x) = Kx^2 - x + 2$

Ans. Here $P(x)Kx^2 - \sqrt{2}x + 2$

$\because x + 1$ **is factor of P(x)**

$\therefore P(-1)=0$

$K(-1)^2-\sqrt{2}(-1)+2=0$

$K+\sqrt{2}+2=0$

$K=-(2+\sqrt{2})$

19. Find the values of m and n if the polynomial $\mathbf{2x^3+mx^2+nx-14}$ has $\mathbf{x-1}$ and x + 2 as its factors.

Ans. x – 1 and x + 2 are factor of $\mathbf{2x^3+mx^2+nx-14}$

x = 1, x = -2

$\therefore 2(1)^3+m(1)^2+n(1)-14=0$

$2+m+n-14=0$

m+n-12=0

$m+n=12------(1)$

$2(2)^3+m(2)^2+n(2)-14=0$

16+4m+2n-14= 0

4m+2n+2 =0

4m+2n= -2

2m+n= -1------(2)

Subtracting (2) from (1)

-m = 13 $\Rightarrow m=-13$

Put m = -13 in (1)

-13+n=12

N=12+13=25

20. Check whether 7+ 3x is a factor of $3x^2 + 7x$

Ans. Let $p(x) = 3x^2 + 7x$

7 + 3x is factor of p(x)

Remainder = 0

Remainder = $P\left(-\frac{7}{3}\right)$

$= 3\left(-\frac{7}{3}\right)^2 + 7\left(-\frac{7}{3}\right)$

$= \cancel{3} \times \frac{49}{\cancel{9}} - \frac{49}{3}$

= 0

Hence 7 + 3x is factor of p(x)

21. Factories $\frac{3}{2}x^2 - x - \frac{4}{3}$

Ans. $\frac{3}{2}x^2 - x - \frac{4}{3}$

$\frac{3}{2} \times \frac{-4}{3} = -2$

We factories by splitting middle term

$-2 + 1 = -1$

$\frac{3}{2}x^2 - 2x + x - \frac{4}{3}$

$$=\frac{3}{2}x\left(x-\frac{4}{3}\right)+1\left(x-\frac{4}{3}\right)$$

$$=\left(\frac{3}{2}x+1\right)\left(x-\frac{4}{3}\right)$$

22. Evaluate $(101)^2$ by using suitable identity

Ans. $(101)^2=(100+1)^2$

$(a+b)^2=a^2+2ab+b^2$

her a = 100, b =1

$(101)^2=(100+1)^2=100^2+2\times100\times1+1^2$

= 10000+200+1

= 10201

23. Find m and n if x – 1 and x – 2 exactly divide the polynomial $\mathbf{x^3+mx^2-nx+10}$

Ans. Let $p(x)=x^3+mx^2-nx+1$

x – 1 and x – 2 exactly divide p(x)

$\therefore p(1)=0\ and\ p(2)=0$

$p(1)=1^3+m\times1^2-n\times1+10=0$

1+m-n+10=0

m-n+11=0

m-n= -11 -----(1)

$m-n=-11-----(1)$

$p(2)=2^3+m\times 2^2-n\times 2+10=0$

8+4m-2n+10=0

4m-2n=-18

2m-n=-9 ----{dividing by 2}

Subtracting eq. (2) form (1). We get

-m=-2

M=2

Put m = 2 in eq. (1). We get

2-n=-11

-n=-11-2

+n=+13

N=13

M =2

24. Factories $\mathbf{8a^3-b^3-12a^2b+6ab^2}$

Ans. $8a^3-b^3-12a^2b+6ab^2$

$=(2a)^3-b^3-6ab(2a-b)$

$=(2a)^3-b^3-3(2a)(b)(2a-b)$

$=(2a-b)^3$

= (2a-b) (2a-b) (2b-b)

25. Evaluate $(99)^3$

Ans. $99^3 = (100-1)^3$

We know that $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Take a = 100, b =1

$(100\text{-}1)^3 = 100^3 - 3\times100^2\times1 + 3\times100\times1^2 - 1^3$

$= 1000000\text{-}30000\text{+}300\text{-}1$

= 1000300-30001

=970299

26. Find the value of k, if x-1 is factor of P(x) and P(x) = 3x^2+kx+ $\sqrt{2}$

Ans. x-1 is factor of p(x)

$\therefore\ p(1) = 0$

$3\times1 + k\times1 + \sqrt{2} = 0$

$3 + k + \sqrt{2} = 0$

$k = -(3+\sqrt{2})$

27. Expand $\left[\frac{2}{3}x+1\right]^3$

Ans. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$a = \frac{2}{3}x,\ b = 1$

$\left(\frac{2}{3}x+1\right)^3 = \left(\frac{2}{3}x\right)^3 + 1^3 + 3\times\frac{2}{3}x\times1\left(\frac{2}{3}x+1\right) = \frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2$

28. Factories $27x^3+y^3+z^3-9xyz$

Ans. $27x^3+y^3\ +z^3-9xyz$

$(3x)^3+(y)^3+(z)^3-3\times(3x)\times y\times z$

$(3x+y+z)[(3x)^2+(y)^2-3xy-yz-3xz]$

$(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$

29. Evaluate 105×95

Ans. 105×95

= (100+5) (100-5)

$=100^2-5^2\quad[(a+b)(a-b)=a^2-b^2]$

=10000 -25 = 9975

30. Using factor theorem check whether g(x) is factor of p(x)

p(x) = $x^3-4x^2+x+6,$ $\qquad g(x) = x\text{-}3$

Ans. Given g(x) =X-3, X-3=0

Put x=3 in p (x)

$P\ (3)\ =3^3-43^2+3+6$

= 27+9 -4$\times$ 9 =36-36 =0

Remainder =0

$\therefore$ By factor theorem g(x) is factor of P (X)

31. Expand $\left(x-\frac{2}{3}y\right)^3$

Ans. $x-\left(\frac{2}{3}y\right)^3$

$\therefore (a-b)^3 = a^3 - b^3 - 3ab(a-b)$

Hence $a=x$, $b=\frac{2}{3}y$

$\therefore (x-\frac{2}{3}y)^3 = x^3 - (\frac{2}{3}y)^3 - 3x\times\frac{2}{3}y(x-\frac{2}{3}y)$

$=x^3 - \frac{8}{27}y^3 - 2xy(x-\frac{2}{3}y)$

$=x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}Xy^2$

CBSE Class 9 Mathemaics

Important Questions

Chapter 2

Polynomials

3 Marks Questions

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2-3x+7$

(ii) $y^2+\sqrt{2}$

(iii) $3\sqrt{t}+t\sqrt{2}$

(iv) $y+\frac{2}{y}$

(v) $x^{10}+y^3+t^{50}$

Ans. (i) $4x^2-3x+7$

We can observe that in the polynomial $4x^2-3x+7$, we have *x* as the only variable and the powers of *x* in each term are a whole number.

Therefore, we conclude that $4x^2-3x+7$ is a polynomial in one variable.

(ii) $y^2+\sqrt{2}$

We can observe that in the polynomial $y^2+\sqrt{2}$, we have *y* as the only variable and the powers of *y* in each term are a whole number.

Therefore, we conclude that $y^2+\sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t}+t\sqrt{2}$

We can observe that in the polynomial $3\sqrt{t}+t\sqrt{2}$, we have t as the only variable and the powers of t in each term are not a whole number.

Therefore, we conclude that $3\sqrt{t}+t\sqrt{2}$ is not a polynomial in one variable.

(iv) $y+\frac{2}{y}$

We can observe that in the polynomial $y+\frac{2}{y}$, we have y as the only variable and the powers of y in each term are not a whole number.

Therefore, we conclude that $y+\frac{2}{y}$ is not a polynomial in one variable.

(v) $x^{10}+y^{3}+t^{50}$

We can observe that in the polynomial $x^{10}+y^{3}+t^{50}$, we have x, y and t as the variables and the powers of x, y and t in each term is a whole number.

Therefore, we conclude that $x^{10}+y^{3}+t^{50}$ is a polynomial but not a polynomial in one variable.

2. Write the degree of each of the following polynomials:

(i) $p(x)=5x^3+4x^2+7x$

(ii) $p(y)=4-y^2$

(iii) $f(t)=5t-\sqrt{7}$

(iv) $f(x)=3$

Ans.

(i) $5x^3+4x^2+7x$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial $5x^3 + 4x^2 + 7x$, the highest power of the variable *x* is 3.

Therefore, we conclude that the degree of the polynomial $5x^3 + 4x^2 + 7x$ is 3.

(ii) $4 - y^2$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial $4 - y^2$, the highest power of the variable *y* is 2.

Therefore, we conclude that the degree of the polynomial $4 - y^2$ is 2.

(iii) $5t - \sqrt{7}$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We observe that in the polynomial $5t - \sqrt{7}$, the highest power of the variable *t* is 1.

Therefore, we conclude that the degree of the polynomial $5t - \sqrt{7}$ is 1.

(iv) 3

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial 3, the highest power of the assumed variable *x* is 0.

Therefore, we conclude that the degree of the polynomial 3 is 0.

3. Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$

(iv) $p(x) = (x-1)(x+1)$

Ans. (i) $p(y) = y^2 - y + 1$

At $p(0)$:

$p(0) = (0)^2 - 0 + 1 = 1$

At $p(1)$:

$p(1) = (1)^2 - 1 + 1 = 1 - 0 = 1$

At $p(2)$:

$p(2) = (2)^2 - 2 + 1 = 4 - 1 = 3$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

At $p(0)$:

$p(0) = 2 + (0) + 2(0)^2 - (0)^3 = 2$

At $p(1)$:

$p(1) = 2 + (1) + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$

At $p(2)$:

$p(2) = 2 + (2) + 2(2)^2 - (2)^3 = 4 + 8 - 8 = 4$

(iii) $p(x) = (x)^3$

At $p(0)$:

$$p(0)=(0)^3=0$$

At $p(1)$:

$$p(1)=(1)^3=1$$

At $p(2)$:

$$p(2)=(2)^3=8$$

(iv) $p(x)=(x-1)(x+1)$

At $p(0)$:

$$p(0)=(0-1)(0+1)=(-1)(1)=-1$$

At $p(1)$:

$$p(1)=(1-1)(2+1)=(0)(3)=0$$

At $p(2)$:

$$p(2)=(2-1)(2+1)=(1)(3)=3$$

4. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x)=2x^3+x^2-2x-1, g(x)=x+1$

(ii) $p(x)=x^3+3x^2+3x+1, g(x)=x+2$

(iii) $p(x)=x^3-4x^2+x+6, g(x)=x-3$

Ans.

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a) = 0$.

We can conclude that *g(x)* is a factor of *p(x)*, if *p(-1)*=0.

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1$

= 0

Therefore, we conclude that the *g(x)* is a factor of *p(x)*.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a) = 0$.

We can conclude that *g(x)* is a factor of *p(x)*, if *p(-2)*=0.

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

= -8+12-6+1

=-1

Therefore, we conclude that the *g(x)* is not a factor of *p(x)*.

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a) = 0$.

We can conclude that *g(x)* is a factor of *p(x)*, if *p(3)*=0.

$p(3) = (3)^3 - 4(3)^2 + (3) + 6$

= 27-36+3+6

= 0

Therefore, we conclude that the *g(x)* is a factor of *p(x)*.

5. Find the value of *k*, if *x* – 1 is a factor of *p*(*x*) in each of the following cases:

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Ans. (i) $p(x) = x^2 + x + k$

We know that according to the factor theorem

$p(a) = 0,$ if $x - a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x) = x^2 + x + k$, then $p(1) = 0$.

$p(1) = (1)^2 + (1) + k = 0,$ or

K+2=0

K=-2

Therefore, we can conclude that the value of *k* is -2.

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

We know that according to the factor theorem

$p(a) = 0,$ if $x - a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$, then $p(1) = 0$.

$p(1)=2(1)^2+k(1)+\sqrt{2}=0,$ or

$2+k+\sqrt{2}=0$

$k=-\left(2+\sqrt{2}\right).$

Therefore, we can conclude that the value of k is $-\left(2+\sqrt{2}\right)$.

(iii) $p(x)=kx^2-\sqrt{2}x+1$

We know that according to the factor theorem

$p(a)=0,$ if $x-a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x)=kx^2-\sqrt{2}x+1$, then $p(1)=0$.

$p(1)=k(1)^2-\sqrt{2}(1)+1=0,$ or

$k-\sqrt{2}+1=0$

$k=\sqrt{2}-1.$

Therefore, we can conclude that the value of k is $\sqrt{2}-1$.

(iv) $p(x)=kx^2-3x+k$

We know that according to the factor theorem

$p(a)=0,$ if $x-a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x)=kx^2-3x+k$, then $p(1)=0$.

$p(1)=k(1)^2-3(1)+k,$ or $2k-3=0 \quad \Rightarrow \quad k=\frac{3}{2}$

Therefore, we can conclude that the value of k is $\frac{3}{2}$.

6. Factorize:

(i) $12x^2 - 7x + 1$

(ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$

(iv) $3x^2 - x - 4$

Ans. (i) $12x^2 - 7x + 1$

$12x^2 - 7x + 1 = 12x^2 - 3x - 4x + 1$

$= 3x(4x - 1) - 1(4x - 1)$

$= (3x - 1)(4x - 1).$

Therefore, we conclude that on factorizing the polynomial $12x^2 - 7x + 1$, we get $(3x - 1)(4x - 1)$.

(ii) $2x^2 + 7x + 3$

$2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (2x + 1)(x + 3).$

Therefore, we conclude that on factorizing the polynomial $2x^2 + 7x + 3$, we get $(2x + 1)(x + 3)$.

(iii) $6x^2 + 5x - 6$

$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$=(3x-2)(2x+3).$

Therefore, we conclude that on factorizing the polynomial $6x^2+5x-6$, we get $(3x-2)(2x+3)$.

(iv) $3x^2-x-4$

$3x^2-x-4=3x^2+3x-4x-4$

$=3x(x+1)-4(x+1)$

$=(3x-4)(x+1).$

Therefore, we conclude that on factorizing the polynomial $3x^2-x-4$, we get $(3x-4)(x+1)$.

7. Use suitable identities to find the following products:

(i) $(x+4)(x+10)$

(ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

Ans. (i) $(x+4)(x+10)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab$.

We need to apply the above identity to find the product $(x+4)(x+10)$

$(x+4)(x+10)=x^2+(4+10)x+(4\times10)$

$= x^2 + 14x + 40.$

Therefore, we conclude that the product $(x+4)(x+10)$ is $x^2 + 14x + 40$.

(ii) $(x+8)(x-10)$

We know that $(x+a)(x+b) = x^2 + (a+b)x + ab$.

We need to apply the above identity to find the product $(x+8)(x-10)$

$(x+8)(x-10) = x^2 + [8+(-10)]x + [8\times(-10)]$

$= x^2 - 2x - 80.$

Therefore, we conclude that the product $(x+8)(x-10)$ is $x^2 - 2x - 80$.

(iii) $(3x+4)(3x-5)$

We know that $(x+a)(x+b) = x^2 + (a+b)x + ab$.

We need to apply the above identity to find the product $(3x+4)(3x-5)$

$(3x+4)(3x-5) = (3x)^2 + [4+(-5)]3x + [4\times(-5)]$

$= 9x^2 - 3x - 20.$

Therefore, we conclude that the product $(3x+4)(3x-5)$ is $9x^2 - 3x - 20$.

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

We know that $(x+y)(x-y) = x^2 - y^2$.

We need to apply the above identity to find the product $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

$$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)=\left(y^2\right)^2-\left(\frac{3}{2}\right)^2$$

$$=y^4-\frac{9}{4}.$$

Therefore, we conclude that the product $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$ is $\left(y^4-\frac{9}{4}\right)$.

(v) $(3+2x)(3-2x)$

We know that $(x+y)(x-y)=x^2-y^2$.

We need to apply the above identity to find the product $(3+2x)(3-2x)$

$$(3+2x)(3-2x)=(3)^2-(2x)^2$$

$$=9-4x^2.$$

Therefore, we conclude that the product $(3+2x)(3-2x)$ **is** $\left(9-4x^2\right)$.

8. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $\left(\frac{3}{2}x+1\right)^3$

(iv) $\left(x-\frac{2}{3}y\right)^3$

Ans.

(i) $(2x+1)^3$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$\therefore (2x+1)^3 = (2x)^3 + (1)^3 + 3\times 2x\times 1(2x+1)$

$= 8x^3 + 1 + 6x(2x+1)$

$= 8x^3 + 12x^2 + 6x + 1$.

Therefore, the expansion of the expression $(2x+1)^3$ is $8x^3 + 12x^2 + 6x + 1$.

(ii) $(2a-3b)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$\therefore (2a-3b)^3 = (2a)^3 - (3b)^3 - 3\times 2a\times 3b(2a-3b)$

$= 8a^3 - 27b^3 - 18ab(2a-3b)$

$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$.

Therefore, the expansion of the expression $(2a-3b)^3$ is $8a^3 - 36a^2b + 54ab^2 - 27b^3$.

(iii) $\left(\frac{3}{2}x+1\right)^3$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\times\frac{3}{2}x\times 1\left(\frac{3}{2}x+1\right) \therefore$

$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$

$$=\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1.$$

Therefore, the expansion of the expression $\left(\frac{3}{2}x+1\right)^3$ is $\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$.

(iv) $\left(x-\frac{2}{3}y\right)^3$

We know that $(x-y)^3=x^3-y^3-3xy(x-y)$.

$$\therefore\left(x-\frac{2}{3}y\right)^3=(x)^3-\left(\frac{2}{3}y\right)^3-3\times x\times\frac{2}{3}y\left(x-\frac{2}{3}y\right)=x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right)$$

$$=x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3.$$

Therefore, the expansion of the expression $\left(x-\frac{2}{3}y\right)^3$ is $x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3$.

9. Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Ans. (i) $(99)^3$

$(99)^3$ can also be written as $(100-1)^3$.

Using identity, $(x-y)^3=x^3-y^3-3xy(x-y)$

$$(100-1)^3 = (100)^3 - (1)^3 - 3\times100\times1(100-1)$$

= 1000000-1-300(99)

= 999999-29700

= 970299.

(ii) $(102)^3$

$(102)^3$ can also be written as $(100+2)^3$.

Using identity $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$$(100+2)^3 = (100)^3 + (2)^3 + 3\times100\times2(100+2)$$

= 1000000+8+600(102)

= 1000008+61200

=1061208

(iii) $(998)^3$

$(998)^3$ can also be written as $(1000-2)^3$.

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$$(1000-2)^3 = (1000)^3 - (2)^3 - 3\times1000\times2(1000-2)$$

= 1000000000-8-6000(998)

= 999999992-5988000

= 994011992

10. Factorize each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Ans.

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

The expression $8a^3 + b^3 + 12a^2b + 6ab^2$ can also be written as

$= (2a)^3 + (b)^3 + 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$

$= (2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b).$

Using identity $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$ with respect to the expression $(2a)^3 + (b)^3 + 3 \times 2a \times b(2a+b)$, we get $(2a+b)^3$.

Therefore, after factorizing the expression $8a^3 + b^3 + 12a^2b + 6ab^2$, we get $(2a+b)^3$.

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

The expression $8a^3 - b^3 - 12a^2b + 6ab^2$ can also be written as

$= (2a)^3 - (b)^3 - 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$

$= (2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b).$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression $(2a)^3 - (b)^3 - 3 \times 2a \times b(2a-b)$, we get $(2a-b)^3$.

Therefore, after factorizing the expression $8a^3-b^3-12a^2b+6ab^2$, we get $(2a-b)^3$.

(iii) $27-125a^3-135a+225a^2$

The expression $27-125a^3-135a+225a^2$ can also be written as

$$=(3)^3-(5a)^3-3\times3\times3\times5a+3\times3\times5a\times5a$$

$$=(3)^3-(5a)^3+3\times3\times5a(3-5a).$$

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(3)^3-(5a)^3+3\times3\times5a(3-5a)$, we get $(3-5a)^3$.

Therefore, after factorizing the expression $27-125a^3-135a+225a^2$, we get $(3-5a)^3$.

(iv) $64a^3-27b^3-144a^2b+108ab^2$

The expression $64a^3-27b^3-144a^2b+108ab^2$ can also be written as

$$=(4a)^3-(3b)^3-3\times4a\times4a\times3b+3\times4a\times3b\times3b$$

$$=(4a)^3-(3b)^3-3\times4a\times3b(4a-3b).$$

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(4a)^3-(3b)^3-3\times4a\times3b(4a-3b)$, we get $(4a-3b)^3$.

Therefore, after factorizing the expression $64a^3-27b^3-144a^2b+108ab^2$, we get $(4a-3b)^3$.

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

The expression $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$ can also be written as

$$=(3p)^3-\left(\frac{1}{6}\right)^3-3\times 3p\times 3p\times\frac{1}{6}+3\times 3p\times\frac{1}{6}\times\frac{1}{6}$$

$$=(3p)^3-\left(\frac{1}{6}\right)^3-3\times 3p\times\frac{1}{6}\left(3p-\frac{1}{6}\right).$$

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(3p)^3-\left(\frac{1}{6}\right)^3-3\times 3p\times\frac{1}{6}\left(3p-\frac{1}{6}\right)$, to get $\left(3p-\frac{1}{6}\right)^3$.

Therefore, after factorizing the expression $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$, we get $\left(3p-\frac{1}{6}\right)^3$.

11. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2-35a+12$

(ii) Area : $35y^2+13y-12$

Ans.

(i) Area : $25a^2-35a+12$

The expression $25a^2-35a+12$ can also be written as $25a^2-15a-20a+12$.

$$25a^2-15a-20a+12=5a(5a-3)-4(5a-3)$$

$$=(5a-4)(5a-3).$$

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $25a^2-35a+12$ is $\text{Length}=(5a-4)$ and $\text{Breadth}=(5a-3)$.

(ii) Area : $35y^2+13y-12$

The expression $35y^2+13y-12$ can also be written as $35y^2+28y-15y-12$.

$35y^2+28y-15y-12=7y(5y+4)-3(5y+4)$

$=(7y-3)(5y+4)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2+13y-12$ is $\text{Length}=(7y-3)$ and $\text{Breadth}=(5y+4)$.

12. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) $\text{Volume}: 3x^2-12x$

(ii) $\text{Volume}: 12ky^2+8ky-20k$

Ans.

(i) $\text{Volume}: 3x^2-12x$

The expression $3x^2-12x$ can also be written as $3\times x\times(x-4)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $3x^2-12x$ is $3, x$ and $(x-4)$.

(ii) $\text{Volume}: 12ky^2+8ky-20k$

The expression $12ky^2+8ky-20k$ can also be written as $k(12y^2+8y-20)$.

$k(12y^2+8y-20)=k(12y^2-12y+20y-20)$

$=k[12y(y-1)+20(y-1)]$

$=k(12y+20)(y-1)$

$=4k\times(3y+5)\times(y-1)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $12ky^2+8ky-20k$ is $4k, (3y+5)$ and $(y-1)$.

13. Using suitable identity expand $\left(\frac{5}{4}x+\frac{3}{4}\right)^3$

Ans. $\left(\frac{5}{2}x+\frac{3}{4}\right)^3$

$(a+b)^3 = a^3+b^3+3ab(a+b)$

$$\left(\frac{5}{2}x+\frac{3}{4}\right)^3 = \left(\frac{5}{2}x\right)^3\left(\frac{3}{4}\right)^3 + 3\times\frac{5}{2}x\times\frac{3}{4}\left(\frac{5}{2}x+\frac{3}{4}\right)$$

$$=\frac{125x^3}{8}+\frac{27}{64}+\frac{45}{8}x\left(\frac{5}{2}x+\frac{3}{4}\right)$$

$$=\frac{125x^3}{8}+\frac{27}{64}+\frac{225}{16}x^2+\frac{135}{32}x$$

$$=\frac{125x^3}{8}+\frac{225}{16}x^2+\frac{135}{32}x+\frac{27}{64}$$

14. Using factor theorem factories $\mathbf{f(x) = x^2 - 5x + 6}$

Ans. $f(x) = x^2 - 5x + 6$

Put x = 1

$f(1) = 1^2 - 5\times 1 + 6 = 2 \neq 0$

Put x=3 f(2) $= 2^2 - 5\times 2 + 6 = 4 - 10 + 6 = 0$

$\therefore x-2$ is factor of f(x)

$$
\begin{array}{r}
x-3 \\
x-2\overline{)\ x^2-5x+6} \\
x^2-2x \\
-\quad + \\
\hline
-3x+6 \\
-3x+6 \\
\hline
0
\end{array}
$$

15. I thought actual division, prove that the polynomial $2x^3 + 4x^2 + x - 34$ is exactly divisible by (x – 2)

Ans. Let $f(x) = 2x^3 + 4x^2 + x - 34$

$x-2$ Is factor of $f(x)$

$x = 2$ Zero of $f(x)$

$f(2) = 2\times 2^3 + 4\times 2^2 + 2 - 34$

= 16+16+2-34

=34-34=0

$2x^3 + 4x^2 + x - 34$ is divisible by x-2

16. Factories $1 - a^2 - b^2 - 2ab$

Ans. $1 - a^2 - b^2 - 2ab$

$1-(a^2+b^2+2ab) = 1^2 - (a+b)^2$

$= (1+a+b)(1-a-b)$

17. Expand $\left(\frac{1}{2}a-\frac{1}{3}b+1\right)^2$

Ans. $\left(\frac{1}{2}a-\frac{1}{3}b+1\right)^2$

$$=\left(\frac{1}{2}a\right)^2+\left(-\frac{1}{3}b\right)^2+1^2+2\times\frac{1}{2}a\times\left(\frac{-1}{3}b\right)+2\times\left(\frac{-1}{3}b\right)\times1+2\times\left(\frac{1}{2}a\right)\times1$$

$$=\frac{a^2}{4}+\frac{b^2}{9}+1-\frac{ab}{3}-\frac{2b}{3}+a$$

18. Verify each of the following identities

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Ans. (i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

Taking R.H.S

$(x+y)(x^2-xy+y^2)$

$=x(x^2-xy+y^2)+y(x^2-xy+y^2)$

$=x^3-\cancel{x^2y}+\cancel{xy^2}+\cancel{yx^2}-\cancel{xy^2}+y^3$

$=x^3+y^3=L.H.S.$

$L.H.S=R.H.S.$

Verified

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

$R.H.S=x(x^2+xy+y^2)-y(x^2+xy+y^2)$

$= x^3 + \cancel{x^2y} + \cancel{xy^2} - \cancel{yx^2} - \cancel{xy^2} - y^3$

$= x^3 - y^3$

= L.H.S.

$L.H.S. = R.H.S.$

Verified

19. Using identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ derive the formula $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Ans. given $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$\therefore a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$= (a+b)[(a+b)^2 - 3ab]$

$= (a+b)[a^2 + b^2 + 2ab - 3ab]$

$= (a+b)(a^2 + b^2 - ab)$

$= (a+b)(a^2 - ab + b^2)$

20. Factories

(i) $64y^3 + 125z^3$

(ii) $27m^3 - 343n^3$

Ans. Solution

(i) $64y^3 + 125z^3$

$(4y)^3 + (5z)^3$

$(4y+5z)[(4y)^2-4y\times 5z+(5z)^2]$

$\left[\because a^3+b^3=(a+b)(a^2-ab+b^2)\right]$

$=(4y+5z)(16y^2-20yz+25z^2)$

(ii) $27m^3 - 343n^3$

$=(3m)^3-(7n)^3$

$=(3m-7n)[(3m)^2+3m\times 7n+(7n)^2]$

$[\because a^3-b^3=(a-b)(a^2+ab+b^2)]$

$(3m-7n)(9m^2+21mn+49n^2)$

21. Without actually calculating the cubes. Find the value of $(26)^3$ + $(-15)^3$ + $(11)^3$

Ans. Let a = 26, b = -15, c = -11

a + b + c = 26 – 15 – 11 = 0

Then $a^3 + b^3 + c^3 = 3abc$

$(26)^3+(-15)^3+(-11)^3$

$=3\times 26\times -15\times -11$

=12870

22. Find the values of m and n so that the polynomial x^3-mx^2-13x+n has x-1 and x+3 as factors.

Ans. Let polynomial be

$p(x)=x^3-mx^2-13x+n$

If $x-1$ *is* factor of p(x)

$\therefore \; p(1)=0$

$(1)^3-m(1)^2-13\times 1+n=0$

1-m-13+n=0

-m+n-12 = 0

-12 =m-n ....(1)

And if x-3 is factor of p(x)

$\therefore p(-3)=0$

$(-3)^3-m(-3)^2-13\times(-3)+n=0$

-27-9m+39+n=0

-9m+n+12=0

12=9m+n=0

12=9m-n

Subtracting (1) from (2),

8m=24

$m=\frac{24}{8}$

m=3

Put m = 3 in (1),

3-n=-12

-n=-12-3

-n=-15

N=15

$\therefore m=3$ and $n=15$

23. Prove that x^2+6x+15 has no zero.

Ans. $x^2+6x+15$

$=x^2+2\times 3x+3^2+6$

$=(x+3)^2+6$

$(x+3)^2$ is positive and 6 is positive

$\therefore\ (x+3)^2+6$ has no zero.

$x^2+6x+15$ has no zero.

24. Factories 3 $(x+y)^2$ - 5(x+y) + 2

Ans. $3(x+y)^2-5(x+y)+2$

Let x +y =z

$=3z\,(z-1)\ -2\,(z-1)$

$=(3z-2)\,(z-1)$

Put z = x+y

$\therefore\ 3(x+y)^2-5(x+y)+2$

$=[3(x+y)-2]\ [x+y-1]$

$=[3x+3y-2]\ [x+y-1]$

$=3z^2-5z+2$

25. The volume of a cuboid is given by the expression $3x^3$-12x. Find the possible expressions for its dimensions

Ans. The volume of cuboid is given by

$$3x^3-12x=3x(x^2-4)=3x\ (x+2)\ (x-2)$$

Dimensions of the cuboid are given by 3x, (x=2) and (x-2)

$$P(1)=1^3-m\times1^2-13\times1+n=0$$

$$=1\text{-}m\text{-}13+n=0$$

$$=-m+n=12 \qquad (1)$$

x+3 is factor of P(x)

$$\therefore \quad P(-3)=0$$

$$P(-3)=(3)^3-m(-3)^2-13\times(-3)+n=0$$

=-27-93+39+n=0

=-9m+n 12=0 (2)

=-9m+n=-12

Subtracting eq. (2) from (1)

8m = 24, m = 3

Put m = 3 in eq(1)

m =3 and n =15

26. Using remainder theorem factories

$x^3 - 3x^2 - x + 3$

Ans. $x^3 - 3x^2 - x + 3$

Coefficient of X^3 is 1

Constant =3

$3 \times 1 = 3$

$\therefore$ We can Put x=$\pm$ 3 and ($\overline{X}$) and check

Put= x=1

$1^3 - 3 \times 1^2 - 1 + 3$

1 – 3 – 1 + 3 = 0

Remainder =0

$\because x - 1$ *is factor of* $x^3 - 3x^2 - x + 3$

$$
\begin{array}{r|l}
 & x^2 - 2x - 3 \\
\hline
x - 1 & x^3 - 3x^2 + 3 \\
 & x^3 - x^2 \\
\hline
 & -2x^2 - x + 3 \\
 & -2x^2 + 2x \\
\hline
 & -3x + 3 \\
 & -3x + 3 \\
\hline
 & 0
\end{array}
$$

$\therefore$ $x^3 - 3x^2 x + 3 = (x - 1)\ (x^2 - 2x - 3)$

$= \ (x - 1)\ (x^2 - 3x + x - 3)$

$= \ (x - 1)\ [x(x - 3)\ + 1\ (x - 3)]$

$= \ (x - 1)\ (x - 3)\ (x + 1)$

27. If $y^3 + ay^2 + by + 6$ is divisible by y – 2 and leaves remainder 3 when divided by y – 3, find the values of a and b.

Ans. Let

$$p(y) = y^3 + ay^2 + by + 6$$

p(y) is divisible by y – 2

Then P (2) = 0

$$2^3 + a \times 2^2 + b \times 2 + 6 = 0$$

$$8 + 4a + 2b + 6 = 0$$

$$4a+2b = -14$$

$$2a+b = -7 \quad (i)$$

If p (y) is divided by y-3 remainder is 3

$$\therefore \ p(3)=3$$

$$3^3 + a \times 3^2 + b \times 3 + 6 = 3$$

9a+3b=-30

3a+b=-10 ---(ii)

Subtracting (i) from (ii)

-a = 3 and a = -3

Put a = -3 in eq (i)

$$2 \times -3 + b = -7$$

-6+b=-7

B=-7+6

B=-1

28. Factories $x^6 - 64$

Ans. $x^6 - 64$

$= (x^2)^3 - (2^2)^3$

$= (x^2-2^2)\,[x^4+4x^2+16]$

$= (x+2)\,(x-2)\,(x^4+4x^2+16)$

29. The volume of a cuboid is given by the algebraic expression ky^2-6ky+8k. Find the possible expressions for the dimensions of the cuboid.

Ans. Given volume of cuboid

$ky^2 - 6ky + 8k$

$= k\,[y^2 - 6y + 8]$

$k\,[y^2 - 4y - 2y + 8]$

$= k\,[y\,(y-4) -2\,(y-4)] = k\,(y-2)\,(y-4)$

Thus dimension of cuboid

k, (y-2) and (y-4)

CBSE Class 9 Mathemaics

Important Questions

Chapter 2

Polynomials

4 Marks Questions

1. Classify the following as linear, quadratic and cubic polynomials:

(i) $x^2 + x$

(ii) $x - x^3$

(iii) $y + y^2 + 4$

(iv) $1 + x$

(v) 3*t*

(vi) r^2

(vii) $7x^3$

Ans.

(i) $x^2 + x$

We can observe that the degree of the polynomial $x^2 + x$ is 2.

Therefore, we can conclude that the polynomial $x^2 + x$ is a quadratic polynomial.

(ii) $x - x^3$

We can observe that the degree of the polynomial $x - x^3$ is 3.

Therefore, we can conclude that the polynomial $x - x^3$ is a cubic polynomial.

(iii) $y + y^2 + 4$

We can observe that the degree of the polynomial $y+y^2+4$ is 2.

Therefore, the polynomial $y+y^2+4$ is a quadratic polynomial.

(iv) $1+x$

We can observe that the degree of the polynomial $(1+x)$ is 1.

Therefore, we can conclude that the polynomial $1+x$ is a linear polynomial.

(v) $3t$

We can observe that the degree of the polynomial $(3t)$ is 1.

Therefore, we can conclude that the polynomial $3t$ is a linear polynomial.

(vi) r^2

We can observe that the degree of the polynomial r^2 is 2.

Therefore, we can conclude that the polynomial r^2 is a quadratic polynomial.

(vii) $7x^3$

We can observe that the degree of the polynomial $7x^3$ is 3.

Therefore, we can conclude that the polynomial $7x^3$ is a cubic polynomial.

2. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x)=3x+1,\ x=-\frac{1}{3}$

(ii) $p(x)=5x-\pi,\ x=\frac{4}{5}$

(iii) $p(x)=x^2-1,\ x=-1,1$

(iv) $p(x)=(x+1)(x-2),\ x=-1,2$

(v) $p(x)=x^2, \quad x=0$

(vi) $p(x)=lx+m, \; x=-\frac{m}{l}$

(vii) $p(x)=3x^2-1, \quad x=-\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x)=2x+1, \; x=-\frac{1}{2}$

Ans. (i) $p(x)=3x+1, \quad x=-\frac{1}{3}$

We need to check whether $p(x)=3x+1$ at $x=-\frac{1}{3}$ is equal to zero or not.

$$p\left(-\frac{1}{3}\right)=3x+1=3\left(-\frac{1}{3}\right)+1=-1+1=0$$

Therefore, we can conclude that $x=-\frac{1}{3}$ is a zero of the polynomial $p(x)=3x+1$.

(ii) $p(x)=5x-\pi, \; x=\frac{4}{5}$

We need to check whether $p(x)=5x-\pi$ at $x=\frac{4}{5}$ is equal to zero or not.

$$p\left(\frac{4}{5}\right)=5\left(\frac{4}{5}\right)-\pi=4-\pi$$

Therefore, $x=\frac{4}{5}$ is not a zero of the polynomial $p(x)=5x-\pi$.

(iii) $p(x)=x^2-1, \quad x=-1,1$

We need to check whether $p(x)=x^2-1$ at $x=-1,1$ is equal to zero or not.

At $x=-1$

$$p(-1)=(-1)^2-1=1-1=0$$

At $x=1$

$$p(1)=(1)^2-1=1-1=0$$

Therefore, $x=-1,1$ are the zeros of the polynomial $p(x)=x^2-1$.

(iv) $p(x)=(x+1)(x-2)$, $x=-1,2$

We need to check whether $p(x)=(x+1)(x-2)$ at $x=-1,2$ is equal to zero or not.

At $x=-1$

$$p(-1)=(-1+1)(-1-2)=(0)(-3)=0$$

At $x=2$

$$p(2)=(2+1)(2-2)=(3)(0)=0$$

Therefore, $x=-1,2$ are the zeros of the polynomial $p(x)=(x+1)(x-2)$.

(v) $p(x)=x^2$, $x=0$

We need to check whether $p(x)=x^2$ at $x=0$ is equal to zero or not.

$$p(0)=(0)^2=0$$

Therefore, we can conclude that $x=0$ is a zero of the polynomial $p(x)=x^2$.

(vi) $p(x)=lx+m$, $x=-\frac{m}{l}$

We need to check whether $p(x)=lx+m$ at $x=-\frac{m}{l}$ is equal to zero or not.

$$p\left(-\frac{m}{l}\right)=l\left(-\frac{m}{l}\right)+m=m+m=0$$

Therefore, $x=-\frac{m}{l}$ is a zero of the polynomial $p(x)=lx+m$.

(vii) $p(x)=3x^2-1,\quad x=-\frac{1}{\sqrt{3}},\frac{2}{\sqrt{3}}$

We need to check whether $p(x)=3x^2-1$ at $x=-\frac{1}{\sqrt{3}},\frac{2}{\sqrt{3}}$ is equal to zero or not.

At $x=\frac{-1}{\sqrt{3}}$

$$p\left(-\frac{1}{\sqrt{3}}\right)=3\left(-\frac{1}{\sqrt{3}}\right)^2-1=3\left(\frac{1}{3}\right)-1=1-1=0$$

At $x=\frac{2}{\sqrt{3}}$

$$p\left(\frac{2}{\sqrt{3}}\right)=3\left(\frac{2}{\sqrt{3}}\right)^2-1=3\left(\frac{4}{3}\right)-1=4-1=3$$

Therefore, we can conclude that $x=\frac{-1}{\sqrt{3}}$ is a zero of the polynomial $p(x)=3x^2-1$ but $x=\frac{-1}{\sqrt{3}}$ is not a zero of the polynomial $p(x)=3x^2-1$.

(viii) $p(x)=2x+1,\quad x=-\frac{1}{2}$

We need to check whether $p(x)=2x+1$ at $x=-\frac{1}{2}$ is equal to zero or not.

$$p\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)+1=-1+1=0$$

Therefore, $x=-\frac{1}{2}$ is a zero of the polynomial $p(x)=2x+1$

3. Find the remainder when x^3+3x^2+3x+1 is divided by

(i) $x+1$

(ii) $x-\frac{1}{2}$

(iii) x

(iv) $x+\pi$

(v) $5+2x$

Ans.

(i) $x+1$

We need to find the zero of the polynomial $x+1$.

$x+1=0 \qquad \Rightarrow x=-1$

While applying the remainder theorem, we need to put the zero of the polynomial $x+1$ in the polynomial x^3+3x^2+3x+1, to get

$$p(x)=x^3+3x^2+3x+1$$

$$p(0)=(-1)^3+3(-1)^2+3(-1)+1$$

=-1+3-3+1

= 0

Therefore, we conclude that on dividing the polynomial x^3+3x^2+3x+1 by $x+1$, we will get the remainder as 0.

(ii) $x-\frac{1}{2}$

We need to find the zero of the polynomial $x-\frac{1}{2}$.

$$x-\frac{1}{2}=0$$
$$\Rightarrow x=\frac{1}{2}$$

While applying the remainder theorem, we need to put the zero of the polynomial $x-\frac{1}{2}$ in the polynomial x^3+3x^2+3x+1, to get

$$p(x)=x^3+3x^2+3x+1$$

$$p\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1$$

$$=\frac{1}{8}+3\left(\frac{1}{4}\right)+\frac{3}{2}+1$$

$$=\frac{1+6+12+8}{8}$$

$$=\frac{27}{8}$$

Therefore, we conclude that on dividing the polynomial x^3+3x^2+3x+1 by $x-\frac{1}{2}$, we will get the remainder as $\frac{27}{8}$.

(iii) x

We need to find the zero of the polynomial x .

$x = 0$

While applying the remainder theorem, we need to put the zero of the polynomial x in the polynomial $x^3 + 3x^2 + 3x + 1$, to get

$p(x) = x^3 + 3x^2 + 3x + 1$

$p(0) = (0)^3 + 3(0)^2 + 3(0) + 1$

= 0+0+0+1

= 1

Therefore, we conclude that on dividing the polynomial $x^3 + 3x^2 + 3x + 1$ by x , we will get the remainder as 1.

(iv) $x + \pi$

We need to find the zero of the polynomial $x + \pi$.

$x + \pi = 0$
$\Rightarrow x = -\pi$

While applying the remainder theorem, we need to put the zero of the polynomial $x + \pi$ in the polynomial $x^3 + 3x^2 + 3x + 1$, to get

$p(x) = x^3 + 3x^2 + 3x + 1$

$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1.$

Therefore, we conclude that on dividing the polynomial $x^3 + 3x^2 + 3x + 1$ by $x + \pi$, we will get the remainder as $-\pi^3 + 3\pi^2 - 3\pi + 1$.

(v) $5 + 2x$

We need to find the zero of the polynomial $5+2x$.

$$5+2x=0$$
$$\Rightarrow x=-\frac{5}{2}$$

While applying the remainder theorem, we need to put the zero of the polynomial $5+2x$ in the polynomial x^3+3x^2+3x+1, to get

$$p(x)=x^3+3x^2+3x+1$$

$$p\left(-\frac{5}{2}\right)=\left(-\frac{5}{2}\right)^3+3\left(-\frac{5}{2}\right)^2+3\left(-\frac{5}{2}\right)+1$$

$$=-\frac{125}{8}+3\left(\frac{25}{4}\right)-\frac{15}{2}+1$$

$$=-\frac{125}{8}+\frac{75}{4}-\frac{15}{2}+1$$

$$=\frac{-125+150-60+8}{8}$$

$$=-\frac{27}{4}.$$

4. Determine which of the following polynomials has $(x+1)$ a factor:

(i) x^3+x^2+x+1

(ii) $x^4+x^3+x^2+x+1$

(iii) $x^4+3x^3+3x^2+x+1$

(iv) $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$

Ans. (i) x^3+x^2+x+1

While applying the factor theorem, we get

$p(x)=x^3+x^2+x+1$

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

=-1+1-1+1

=0

We conclude that on dividing the polynomial x^3+x^2+x+1 by $(x+1)$, we get the remainder as0.

Therefore, we conclude that $(x+1)$ is a factor of x^3+x^2+x+1.

(ii) $x^4+x^3+x^2+x+1$

While applying the factor theorem, we get

$p(x)=x^4+x^3+x^2+x+1$

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$

=1-1+1-1+1

=1

We conclude that on dividing the polynomial $x^4+x^3+x^2+x+1$ by $(x+1)$, we will get the remainder as1, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^4+x^3+x^2+x+1$.

(iii) $x^4+3x^3+3x^2+x+1$

While applying the factor theorem, we get

$p(x)=x^4+3x^3+3x^2+x+1$

$$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$$

=1-3+3-1+1

=1

We conclude that on dividing the polynomial $x^4+3x^3+3x^2+x+1$ by $(x+1)$, we will get the remainder as 1, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^4+3x^3+3x^2+x+1$.

(iv) $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$

While applying the factor theorem, we get

$$p(x)=x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$$

$$p(-1)=(-1)^3-(-1)^2-\left(2+\sqrt{2}\right)(-1)+\sqrt{2}$$

$$=-1-1+2+\sqrt{2}+\sqrt{2}$$

$$=2\sqrt{2}.$$

We conclude that on dividing the polynomial $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$ by $(x+1)$, we will get the remainder as $2\sqrt{2}$, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$.

5. Expand each of the following, using suitable identities:

(i) $(2x-y+z)^2$

(ii) $(-2x+3y+2z)^2$

(iii) $(3a-7b-c)^2$

(iv) $(-2x+5y-3z)^2$

(v) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

Ans.

(i) $(2x-y+z)^2$

We know that $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(2x-y+z)^2$.

$$(2x-y+z)^2=[2x+(-y)+z]^2$$

$$=(2x)^2+(-y)^2+(z)^2+2\times 2x\times(-y)+2\times(-y)\times z+2\times z\times 2x$$

$$=4x^2+y^2+z^2-4xy-2yz+4zx$$

(ii) $(-2x+3y+2z)^2$

We know that $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(-2x+3y+2z)^2$.

$$(-2x+3y+2z)^2=[(-2x)+3y+2z]^2$$

$$=(-2x)^2+(3y)^2+(2z)^2+2\times(-2x)\times 3y+2\times 3y\times 2z+2\times 2z\times(-2x)$$

$$=4x^2+9y^2+4z^2-12xy+12yz-8zx$$

(iii) $(3a-7b-c)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(3a-7b-c)^2$.

$$(3a-7b-c)^2 = [3a+(-7b)+(-c)]^2$$

$$=(3a)^2+(-7b)^2+(-c)^2+2\times 3a\times(-7b)+2\times(-7b)\times(-c)+2\times(-c)\times 3a$$

$$=9a^2+49b^2+c^2-42ab+14bc-6ac.$$

(iv) $(-2x+5y-3z)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(-2x+5y-3z)^2$.

$$(-2x+5y-3z)^2 = [(-2x)+5y+(-3z)]^2$$

$$=(-2x)^2+(5y)^2+(-3z)^2+2\times(-2x)\times 5y+2\times 5y\times(-3z)+2\times(-3z)\times(-2x)$$

$$=4x^2+25y^2+9z^2-20xy-30yz+12zx.$$

(v) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

$$\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2 = \left[\frac{a}{4}+\left(-\frac{b}{2}\right)+1\right]^2$$

$$=\left(\frac{a}{4}\right)^2+\left(-\frac{b}{2}\right)^2+(1)^2+2\times\frac{a}{4}\times\left(-\frac{b}{2}\right)+2\times\left(-\frac{b}{2}\right)\times 1+2\times 1\times\frac{a}{4}$$

$$=\frac{a^2}{16}+\frac{b^2}{4}+1-\frac{ab}{4}-b+\frac{a}{2}.$$

6. Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Ans.

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

The expression $4x^2+9y^2+16z^2+12xy-24yz-16xz$ can also be written as

$$(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x.$$

We can observe that, we can apply the identity
$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$ with respect to the expression
$(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x$, to get

$$(2x+3y-4z)^2$$

Therefore, we conclude that after factorizing the expression
$4x^2+9y^2+16z^2+12xy-24yz-16xz$, we get $(2x+3y-4z)^2$.

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

We need to factorize the expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$.

The expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$ can also be written as

$$\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\times\left(-\sqrt{2}x\right)\times y+2\times y\times\left(2\sqrt{2}z\right)+2\times\left(2\sqrt{2}z\right)\times\left(-\sqrt{2}x\right).$$

We can observe that, we can apply the identity

$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$ with respect to the expression

$\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\times\left(-\sqrt{2}x\right)\times y+2\times y\times\left(2\sqrt{2}z\right)+2\times\left(2\sqrt{2}z\right)\times\left(-\sqrt{2}x\right)$,

to get

$\left(-\sqrt{2}x+y+2\sqrt{2}z\right)^2$

Therefore, we conclude that after factorizing the expression

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$, we get $\left(-\sqrt{2}x+y+2\sqrt{2}z\right)^2$.

CBSE Class 9 Mathemaics

Important Questions

Chapter 2

Polynomials

5 Marks Questions

1. Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$

(ii) $p(x) = x - 5$

(iii) $p(x) = 2x + 5$

(iv) $p(x) = 3x - 2$

(v) $p(x) = 3x$

(vi) $p(x) = ax,\ a \neq 0$

(vii) $p(x) = cx + d, c \neq 0,\ c, d$ are real numbers.

Ans.

(i) $p(x) = x + 5$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = x + 5$ equal to 0, we get

$x + 5 = 0$
$\Rightarrow\ x = -5$

Therefore, we conclude that the zero of the polynomial $p(x)=x+5$ **is** -5 **.**

(ii) $p(x)=x-5$

$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers , we need to find $p(x)=0$.

On putting $p(x)=x-5$ equal to 0, we get

$$x-5=0$$
$$\Rightarrow \ x=5$$

Therefore, we conclude that the zero of the polynomial $p(x)=x-5$ is5.

(iii) $p(x)=2x+5$

$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers , we need to find $p(x)=0$.

On putting $p(x)=2x+5$ equal to 0, we get

$$2x+5=0$$
$$\Rightarrow \ x=\frac{-5}{2}$$

Therefore, we conclude that the zero of the polynomial $p(x)=2x+5$ **is** $\frac{-5}{2}$ **.**

(iv) $p(x)=3x-2$

$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers , we need to find $p(x)=0$.

On putting $p(x)=3x-2$ equal to 0, we get

$$3x-2=0$$
$$\Rightarrow \ x=\frac{2}{3}$$

Therefore, we conclude that the zero of the polynomial $p\left(x\right)=3x-2$ **is** $\frac{2}{3}$ **.**

(v) $p\left(x\right)=3x$

$ax+b,$ where $a\neq 0$ and $b\neq 0,$ and a and b are real numbers, we need to find $p\left(x\right)=0$.

On putting $p\left(x\right)=3x$ equal to 0, we get

$$3x=0$$
$$\Rightarrow \ x=0$$

Therefore, we conclude that the zero of the polynomial $p\left(x\right)=3x$ is0.

(vi) $p\left(x\right)=ax, a\neq 0$

$ax+b,$ where $a\neq 0$ and $b\neq 0,$ and a and b are real numbers, we need to find $p\left(x\right)=0$.

On putting $p\left(x\right)=ax$ equal to 0, we get

$$ax=0$$
$$\Rightarrow \ x=0$$

Therefore, we conclude that the zero of the polynomial $p\left(x\right)=ax, a\neq 0$ is 0.

(vii) $p\left(x\right)=cx+d, c\neq 0, c, d$ are real numbers.

$ax+b,$ where $a\neq 0$ and $b\neq 0,$ and a and b are real numbers, we need to find $p\left(x\right)=0$.

On putting $p(x) = cx + d$ equal to 0, we get

$cx + d = 0$

$\Rightarrow x = -\frac{d}{c}$.

Therefore, we conclude that the zero of the polynomial

$p(x) = cx + d, c \neq 0, c, d$ are real numbers. is $-\frac{d}{c}$.

2. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Ans. We know that if the polynomial $7 + 3x$ is a factor of $3x^3 + 7x$, then on dividing the polynomial $3x^3 + 7x$ by $7 + 3x$, we must get the remainder as 0.

We need to find the zero of the polynomial $7 + 3x$.

$7 + 3x = 0$

$\Rightarrow x = -\frac{7}{3}$

While applying the remainder theorem, we need to put the zero of the polynomial $7 + 3x$ in the polynomial $3x^3 + 7x$, to get

$p(x) = 3x^3 + 7x$

$= 3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right) = 3\left(-\frac{343}{27}\right) - \frac{49}{3}$

$= -\frac{343}{9} - \frac{49}{3} = \frac{-343 - 147}{9}$

$= \frac{-490}{9}$.

We conclude that on dividing the polynomial $3x^3 + 7x$ by $7 + 3x$, we will get the remainder as $\frac{-490}{9}$, which is not 0.

Therefore, we conclude that $7+3x$ is not a factor of $3x^3+7x$.

3. Factorize:

(i) x^3-2x^2-x+2

(ii) x^3-3x^2-9x-5

(iii) $x^3+13x^2+32x+20$

(iv) $2y^3+y^2-2y-1$

Ans.

(i) x^3-2x^2-x+2

We need to consider the factors of 2, which are $\pm 1, \pm 2$.

Let us substitute 1 in the polynomial x^3-2x^2-x+2, to get

$$(1)^3-2(1)^2-(1)+2=1-2-1+2=0$$

Thus, according to factor theorem, we can conclude that $(x-1)$ is a factor of the polynomial x^3-2x^2-x+2.

Let us divide the polynomial x^3-2x^2-x+2 by $(x-1)$, to get

$$
\begin{array}{r|l}
 & x^2 - x - 2 \\
\hline
x-1 & x^3 - 2x^2 - x + 2 \\
 & x^3 - x^2 \\
 & \underline{- \quad +} \\
 & -x^2 - x \\
 & -x^2 + x \\
 & \underline{+ \quad -} \\
 & -2x + 2 \\
 & -2x + 2 \\
 & \underline{+ \quad -} \\
 & \underline{\quad 0 \quad}
\end{array}
$$

$x^3 - 2x^2 - x + 2 = (x-1)(x^2 - x - 2).$

$x^3 - 2x^2 - x + 2 = (x-1)(x^2 - x - 2)$

$= (x-1)(x^2 + x - 2x - 2)$

$= (x-1)[x(x+1) - 2(x+1)]$

$= (x-1)(x-2)(x+1).$

Therefore, we can conclude that on factorizing the polynomial $x^3 - 2x^2 - x + 2$, we get $(x-1)(x-2)(x+1)$.

(ii) $x^3 - 3x^2 - 9x - 5$

We need to consider the factors of -5, which are $\pm 1, \pm 5$.

Let us substitute 1 in the polynomial $x^3 - 3x^2 - 9x - 5$, to get

$(-1)^3 - 3(-1)^2 - 9(-1) - 5 = -1 - 3 + 9 - 5 = 0$

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial $x^3 - 3x^2 - 9x - 5$.

Let us divide the polynomial x^3-3x^2-9x-5 by $(x+1)$, to get

$$
\begin{array}{r|l}
 & x^2-4x-5 \\
\hline
x+1 & x^3-3x^2-9x-5 \\
 & \underline{\underset{-}{x^3}+\underset{-}{x^2}} \\
 & -4x^2-9x \\
 & \underline{\underset{+}{-4x^2}\underset{+}{-4x}} \\
 & -5x-5 \\
 & \underline{\underset{+}{-5x}\underset{+}{-5}} \\
 & \underline{\quad 0 \quad}
\end{array}
$$

$x^3-3x^2-9x-5=(x+1)(x^2-4x-5)$

$=(x+1)(x^2+x-5x-5)$

$=(x+1)[x(x+1)-5(x+1)]$

$=(x+1)(x-5)(x+1).$

Therefore, we can conclude that on factorizing the polynomial x^3-3x^2-9x-5, we get $(x+1)(x-5)(x+1)$.

(iii) $x^3+13x^2+32x+20$

We need to consider the factors of 20, which are $\pm5, \pm4, \pm2, \pm1$.

Let us substitute -1 in the polynomial $x^3+13x^2+32x+20$, to get

$(-1)^3+13(-1)^2+32(-1)+20=-1+13-32+20=-20+20=0$

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial $x^3+13x^2+32x+20$.

Let us divide the polynomial $x^3+13x^2+32x+20$ by $(x+1)$, to get

$$
\begin{array}{r@{}l}
 & \quad x^2+12x+20 \\
x+1\big) & \overline{x^3+13x^2+32x+20} \\
 & \underline{\begin{matrix} x^3+x^2 \\ - \quad - \end{matrix}} \\
 & 12x^2+32x \\
 & \underline{\begin{matrix} 12x^2+12x \\ - \quad - \end{matrix}} \\
 & 20x+20 \\
 & \underline{\begin{matrix} 20x+20 \\ - \quad - \end{matrix}} \\
 & \underline{\quad 0 \quad}
\end{array}
$$

$$x^3+13x^2+32x+20=(x+1)(x^2+12x+20)$$

$$=(x+1)(x^2+2x+10x+20)$$

$$=(x+1)[x(x+2)+10(x+2)]$$

$$=(x+1)(x+10)(x+2).$$

Therefore, we can conclude that on factorizing the polynomial $x^3+13x^2+32x+20$, we get $(x+1)(x-10)(x+2)$.

(iv) $2y^3+y^2-2y-1$

We need to consider the factors of -1, which are ± 1.

Let us substitute 1 in the polynomial $2y^3+y^2-2y-1$, to get

$$2(1)^3+(1)^2-2(1)-1=2+1-2-1=3-3=0$$

Thus, according to factor theorem, we can conclude that $(y-1)$ is a factor of the polynomial $2y^3+y^2-2y-1$.

Let us divide the polynomial $2y^3+y^2-2y-1$ by $(y-1)$, to get

$$
\begin{array}{r|l}
 & 2y^2+3y+1 \\
\hline
y-1 & 2y^3+y^2-2y-1 \\
 & \underline{\underset{-}{2y^3}\underset{+}{-2y^2}} \\
 & 3y^2-2y \\
 & \underline{\underset{-}{3y^2}\underset{+}{-3y}} \\
 & y-1 \\
 & \underline{\underset{-}{y}\underset{-}{-1}} \\
 & \underline{\quad 0}
\end{array}
$$

$2y^3+y^2-2y-1=(y-1)(2y^2+3y+1)$

$=(y-1)(2y^2+2y+y+1)$

$=(y-1)[2y(y+1)+1(y+1)]$

$=(y-1)(2y+1)(y+1).$

Therefore, we can conclude that on factorizing the polynomial $2y^3+y^2-2y-1$, we get $(y-1)(2y+1)(y+1)$.

4. If x^2 – bx + c = (x + p) (x – q) then factories x^2 – bxy + cy^2

Ans. We have $x2-bx+c=(x+p)(x-q)$

$x^2-bx+c=x^2+(p-q)x-pq$

Equating coefficient of x and constant

$-b=p-q$ and $c=-pq$

Substituting these values of b and c in $x^2-bxy+cy^2$, We get

$x^2+(p-q)xy-pqy^2$

$x^2+pxy-qxy-pqy^2$

$x(x+py)-qy(x+py)$

$(x+py)(x-qy)$

5. Factories $(2x-3y)^3+(3y-4z)^3+(4z-2x)^3$

Ans. Let $a=2x-3y,\quad b=3y-4z,\quad c=4z-2x$

$then\ a+b+c=\cancel{2x}-\cancel{3y}+\cancel{3y}-\cancel{4z}+\cancel{4z}-\cancel{2x}=0$

$\therefore a^3+b^3+c^3=3abc$

$(2x-3y)^3+(3y-4z)^3+(4z-2x)^3=3(2x-3y)(3y-4z)(4z-2x)$

$=3(2x-3y)(3y-4z)\times 2(2z-x)$

$=6(2x-3y)(3y-4z)(2z-x)$

6. Factories: $\mathbf{12\left(y^2+7y\right)^2-8\left(y^2+7y\right)(2y-1)-15(2y-1)^2}$

Ans. Let $a=y^2+7y,\ b=2y-1$

$Then\ 12(y^2+7y)^2-8(y^2+7y)(2y-1)-15(2y-1)^2$

$=12a^2-8ab-15b^2$

$=12a^2-18ab+10ab-15b^2$

$=6a(2a-3b)+5b(2a-3b)$

$=(2a-3b)(6a+5b)$

$Put\ a = y^2 + 7y\ and\ \ b = 2y - 1$

$= [2(y^2 + 7y) - 3(2y - 1)][6(y^2 + 7y) + 5(2y - 1)$

$= [2y^2 + 14y - 6y + 3][6y^2 + 42y + 10y - 5]$

$= (2y^2 + 8y + 3)(6y^2 + 52y - 5)$

7. Factories x^6 + $8y^6$ - z^6+$6x^2y^2z^2$

Ans. $x^6 + 8y^6 - z^6 + 6x^2y^2z^2$

$= (x^2)^3 + (2y^2)^3 + (-z^2)^3 - 3(x^2)\ (2y^2)\ (-z^2)$

$= \ [x^2 + y^2 - z^2]\ [(x^2)^2 + (2y^2)^2 + (-z^2)^2 - x^2 \times 2y^2 - 2y^2(-z^2) - x^2 \times (-z^2)^2$

$= \ (x^2 + 2y^2 - z^2)\ (x^4 + 4y^4 + z^4 - 2x^2y^2 + 2y^2z^2 + x^2z^2)$

8. Factories: $\frac{1}{27}(2x + 5y)^3 + \left(\frac{-5}{3}y + \frac{3}{4}z\right)^3 - \left(\frac{3}{4}z + \frac{2}{3}x\right)^3$

Ans. Given expression can be written as

$$\left[\frac{1}{3}(2x + 5y)\right]^3 + \left[-\frac{5}{3}y + \frac{3}{4}z\right]^3 + \left[-\frac{3}{4}z - \frac{2}{3}x\right]^3$$

$Let\ \frac{1}{3}(2x + 5y) = a, \frac{-5}{3}y + \frac{3}{4}z = b$

$and\ \ \frac{-3}{4}z - \frac{2}{3}x = C$

$a + b + c = \frac{2}{3}x + \frac{5}{3}y - \frac{5}{3}y + \frac{5}{4}z - \frac{3}{4}z - \frac{2}{3}x = 0$

$\therefore\ a^3 + b^3 + c^3 = 3abc$

Thus,

$$\frac{1}{27}(2x+5y)^3+\left(\frac{-5}{3}y+\frac{3}{4}z\right)^3-\left(\frac{3}{4}z+\frac{2}{3}x\right)^3$$

$$=3\left[\frac{1}{3}(2x+5y)\left(\frac{-5}{3}y+\frac{3}{4}z\right)\left(\frac{-3}{4}z-\frac{2}{3}x\right)\right]$$

$$=-(2x+5y)\left(\frac{-5}{3}y+\frac{3}{4}z\right)\left(\frac{3}{4}z+\frac{2}{3}x\right)$$

$$=-(2x+5y)\left(\frac{-20y+9z}{12}\right)\left(\frac{9z+8x}{12}\right)$$

$$=\frac{1}{144}(2x+5y)\ (20y-9z)\ (9z+8x)$$