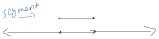
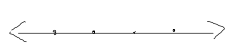
Quadrilaterals

point → A point is that which determines location

→ line segment



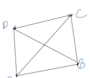
→ line.



Collinear points – Points lying on same line.

Quadrilaterals

Any enclosed figure made with 4 non-collinear points is known as quadrilateral



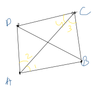
Vertices – A, B, C, D

Sides – AB, BC, CD, DA.

Angles – $\angle A, \angle B, \angle C, \angle D$

Digonals – AC and BD

Thm **8.2 Angle Sum Property of a Quadrilateral**



Given: ABCD is a quadrilateral

To prove: $\angle A + \angle B + \angle C + \angle D = 360^\circ$

proof: In ΔABC

$\angle 1 + \angle B + \angle 3 = 180^\circ$ (Angle sum Property of a Δ) – (i)

In ΔADC

$\angle 2 + \angle D + \angle 4 = 180^\circ$ (") – (ii)

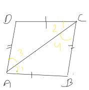
$(\angle 1 + \angle B + \angle 3) + (\angle 2 + \angle D + \angle 4) = 360^\circ$

$(\angle 1 + \angle 2) + \angle B + (\angle 3 + \angle 4) + \angle D = 360^\circ$

$\boxed{\angle A + \angle B + \angle C + \angle D = 360^\circ}$

Theorem 8.1 : *A diagonal of a parallelogram divides it into two congruent triangles.*

||gm – Quadrilateral whose opposite sides are ||.



Given. ||gm ABCD.

To prove: $\Delta ABC \cong \Delta ADC$

proof: In ΔABC and ΔADC

AB || DC, AD || BC, AC is a transversal.

$\angle 1 = \angle 2$ (alter interior angles)

$\angle 4 = \angle 3$ (")

AC = AC (common)

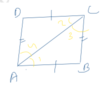
$\therefore \Delta ABC \cong \Delta ADC$ (ASA rule).

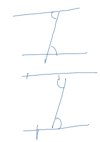
Theorem 8.2 : *In a parallelogram, opposite sides are equal.*

$\Delta ABC \cong \Delta DCA$

AB = DC
AD = BC (CPCT)

Theorem 8.3 : *If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.*





Given: quadrilateral ABCD,
AB = DC,
AD = BC

To prove: ABCD is a ||gm

Constr: Join A to C

proof: In ΔABC and ΔCDA

AB = DC (given)

BC = AD (given)

AC = AC (common)

$\therefore \Delta ABC \cong \Delta CDA$ (SSS rule)

$\angle 1 = \angle 2$ (CPCT)

$\angle BAC = \angle DCA$

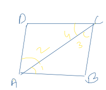
$\angle 3 = \angle 4$ (CPCT)

but these are alternate interior angles

$\therefore$ AB || DC
AD || BC

Which implies ABCD is a ||gm.

Theorem 8.4 : *In a parallelogram, opposite angles are equal.*



Given: ABCD is a ||gm, AB || DC
& AD || BC.

To proof: $\angle A = \angle C$ and
$\angle B = \angle D$

proof: AB || CD and AC is transversal.

$\angle 1 = \angle 4$ – (i) (alt interior angles)

AD || BC and AC is transversal

$\angle 2 = \angle 3$ – (ii) (alt interior angles).

Add (i) and (ii)

$\angle 1 + \angle 2 = \angle 4 + \angle 3$

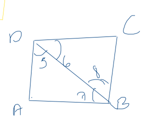
$\boxed{\angle A = \angle C}$

$\angle 5 = \angle 8$ (alt interior)

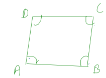
$\angle 6 = \angle 7$ (")

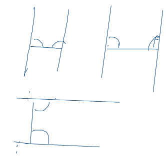
add

$\boxed{\angle D = \angle B}$



Theorem 8.5 : *If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.*





Given: $\angle A = \angle C$ and
$\angle B = \angle D$

To proof: ABCD is a ||gm

proof: $\angle A + \angle B + \angle C + \angle D = 360^\circ$ (Angle sum property of a quadrilateral)

$\angle A + \angle B + \angle A + \angle B = 360^\circ$

$2\angle A + 2\angle B = 360^\circ$

$\angle A + \angle B = 180^\circ$

but these are co-interior angles

$\therefore$ AD || BC – (i)

Similarly

$\angle A + \angle B = 180^\circ$

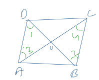
$\angle A + \angle D = 180^\circ$

$\therefore$ AB || CD – (ii)

$\therefore$ From (i) and (ii)

ABCD is a ||gm

Theorem 8.6 : *The diagonals of a parallelogram bisect each other.*



Given: ||gm ABCD.

To prove: AO = CO, OB = OD

proof: In ΔAOD and ΔCOB

AD = BC (opposite sides of ||gm)

$\angle 1 = \angle 2$ (alt interior angles)

$\angle 3 = \angle 4$ (")

$\therefore \Delta AOD \cong \Delta COB$ (ASA rule)

AO = CO (CPCT)

OD = OB (CPCT)