Question **3**

Correct

Marked out of 30

(*Bisection Method*). Prior to entering in the corresponding input fields, all numerical answers should be rounded to 6-digit floating-point numbers. Given a real number \(z,\) the symbol \(\widetilde z\) denotes the result of rounding of \(z\) to a 6-digit floating-point number.

(i) Use the Bisection method to find an approximation \(p_N\) of the unique solution \(p\) the equation

$$ 3.38 x (1-x^2+x) \ln (x) = x^2-1 $$

in \([a,b]=[0.05,0.5]\) such that

$$ \mathrm {RE}(\widetilde p_N \approx \widetilde p_{N-1}) < 10^{-3}. $$

(iii) Show then your work by filling in the table that follows. In each input field in the column labelled by

$$ f(a_n) f(p_n), $$

please enter either a plus sign \(\texttt {+}\) (if \(f(a_n) f(p_n) > 0)\), or a minus sign \(\texttt {-}\) (if \(f(a_n) f(p_n) < 0\)). If a particular row of the table is not necessary, enter an asterisk \(*\) in each input field in the row. In order to calculate the relative error

$$ \mathrm {RE}(\widetilde p_1 \approx \widetilde p_0) $$

in the first row, assume formally that \(p_0=0.05.\)

\(\quad n \quad \)	\(\quad a_n\quad \)	\(\quad p_n\quad \)	\(\quad b_n\quad \)	\(\quad f(a_n) f(p_n)\quad \)	\(\quad \mathrm {RE}(\widetilde p_n \approx \widetilde p_{n-1}) \quad \)
\(\quad 1 \quad \) ✔	0.05 ✔	0.275 ✔	0.5 ✔	- ✔	0.818182 ✔
\(\quad 2 \quad \) ✔	0.05 ✔	0.1625 ✔	0.275 ✔	- ✔	0.692308 ✔
\(\quad 3 \quad \) ✔	0.05 ✔	0.10625 ✔	0.1625 ✔	+ ✔	0.529412 ✔
\(\quad 4 \quad \) ✔	0.10625 ✔	0.134375 ✔	0.1625 ✔	- ✔	0.209302 ✔
\(\quad 5 \quad \) ✔	0.10625 ✔	0.120313 ✔	0.134375 ✔	+ ✔	0.116878 ✔
\(\quad 6 \quad \) ✔	0.120313 ✔	0.127344 ✔	0.134375 ✔	- ✔	0.0552127 ✔
\(\quad 7 \quad \) ✔	0.120313 ✔	0.123828 ✔	0.127344 ✔	+ ✔	0.0283942 ✔
\(\quad 8 \quad \) ✔	0.123828 ✔	0.125586 ✔	0.127344 ✔	+ ✔	0.0139984 ✔
\(\quad 9 \quad \) ✔	0.125586 ✔	0.126465 ✔	0.127344 ✔	+ ✔	0.00695054 ✔

\(\quad 10 \quad \)	0.126465 ✔	0.126904 ✔	0.127344 ✔	+ ✔	0.00345931 ✔
\(\quad 11 \quad \)	0.126904 ✔	0.127124 ✔	0.127344 ✔	- ✔	0.00173059 ✔
\(\quad 12 \quad \)	0.126904 ✔	0.127014 ✔	0.127124 ✔	- ✔	0.000866046 ✔
\(\quad 13 \quad \)	* ✔	* ✔	* ✔	* ✔	* ✔

As suggested in the previous problem, users of scientific calculators may first create a copy of the above table in an OpenOffice (or Excel) worksheet, and then copy-paste their answers.

(ii) According to your results in (i) and (ii),

\(p_N \doteq \) 0.127014 ✔ .

Check

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