

Question **4**

Correct

Marked out of 27

(*Babylonian Method*). All numerical answers should be rounded to 6-digit floating-point numbers.

(0) If $A > 0$ is a positive number, then given a real number x_0 , the sequence (x_n) defined recursively by

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{A}{x_n} \right), \quad (n \geq 0),$$

where $n \geq 0$ is a natural number, converges to \sqrt{A} , that is,

$$\lim_{n \rightarrow \infty} x_n = \sqrt{A}$$

provided that the initial term x_0 is chosen 'not too badly'. In practice, calculation of terms x_n is carried out till this or that stopping criterion is triggered.

(i) Let (x_n) be the sequence defined recursively by

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{710}{x_n} \right), \quad (n \geq 0),$$

and $x_0 = 71$ (as it could be guessed from part (0), the sequence (x_n) converges to $\sqrt{710}$).

(ii) Generate the terms

$$x_1, x_2, \dots$$

of the sequence (x_n) till you find the term x_N satisfying the condition

$$\text{fl}(x_N) = \text{fl}(x_{N-1})$$

(our stopping criterion), where $\text{fl}(z)$ denotes the result of rounding of a real number z to a 6-digit floating-point number.

Along the way, once the k -th term x_k is generated, enter in the corresponding input field the result $\text{fl}(x_k)$ of rounding of x_k to a 6-digit floating-point number. As instructed, stop generation of the terms if the stopping criterion is triggered at some step N . Accordingly, the number $\text{fl}(x_N)$ must be the *last number* you need to enter. Enter an asterisk $*$ in each of the remaining input fields, if any.

$x_0 \doteq$

✓

$x_1 \doteq$

✓

$x_2 \doteq$

✓

$x_3 \doteq$

✓

$x_4 \doteq$

✓

$x_5 \doteq$

✓

$x_6 \doteq$

✓

$x_7 \doteq$

✓

$x_8 \doteq$

✓

$x_9 \doteq$

✓

(ii) To see how well the term x_N you found in the previous part approximates $\sqrt{710}$, find $(x_N)^2$ and round the result to a 6-digit floating-point number:

$(x_N)^2 \doteq$

✓

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