Question **2**

Correct

Marked out of 35

(*Bisection Method*). All numerical answers should be rounded to 6-digit floating-point numbers.

(i) Consider the polynomial

$$f(x) = x^3 + 9x^2 - 10x - 158.$$

Please accept as a given that the polynomial $f(x)$ has three real roots in $[-10, 10]$.

(a) Let then r_1, r_2, r_3 be the roots of $f(x)$ written in increasing order. For each of the roots r_i, find a pair of integer numbers $m, m+1$ that bracket the root r_i:

1) r_1 is between -8 ✔ and -7 ✔

2) r_2 is between -6 ✔ and -5 ✔

3) r_3 is between 3 ✔ and 4 ✔

(b) Now, according to (a), is it true that the polynomial $f(x)$ has a unique root in the closed interval $[-8, -7]$?

- (•) Yes ✔
- ( ) No

(ii) Use the Bisection to find an approximation p_N of the unique root of the function $f(x)$ in $[-8, -7]$ satisfying

$$\mathrm{RE}(\tilde{p}_N \approx \tilde{p}_{N-1}) < 10^{-3},$$

where $\tilde{z}$ denotes the result $\mathrm{fl}(z)$ of rounding of a real number z to 6-digit floating-point number.

(iii) Show then your work by filling in the table that follows. In each input field in the column labelled by

$$f(a_n)f(p_n),$$

please enter either a plus sign + (if $f(a_n)f(p_n) > 0$), or a minus sign - (if $f(a_n)f(p_n) < 0$). If a particular row of the table is not necessary, enter an asterisk $*$ in each input field in the row. In order to calculate the relative error

$$\mathrm{RE}(\tilde{p}_1 \approx \tilde{p}_0)$$

in the first row, assume formally that $p_0 = -8$.

n	a_n	p_n	b_n	$f(a_n)f(p_n)$	$\mathrm{RE}(\tilde{p}_n \approx \tilde{p}_{n-1})$
1	-8 ✔	-7.5 ✔	-7 ✔	- ✔	0.0666667 ✔
2	-8 ✔	-7.75 ✔	-7.5 ✔	+ ✔	0.0322581 ✔
3	-7.75 ✔	-7.625 ✔	-7.5 ✔	+ ✔	0.0163934 ✔
4	-7.625 ✔	-7.5625 ✔	-7.5 ✔	+ ✔	0.00826446 ✔
5	-7.5625 ✔	-7.53125 ✔	-7.5 ✔	- ✔	0.00414938 ✔
6	-7.5625 ✔	-7.54688 ✔	-7.53125 ✔	- ✔	0.00207105 ✔
7	-7.5625 ✔	-7.55469 ✔	-7.54688 ✔	- ✔	0.00103379 ✔
8	-7.5625 ✔	-7.55859 ✔	-7.55469 ✔	+ ✔	0.000515969 ✔
9	* ✔	* ✔	* ✔	* ✔	* ✔
10	* ✔	* ✔	* ✔	* ✔	* ✔
11	* ✔	* ✔	* ✔	* ✔	* ✔

1) If you are going to use a scientific calculator, you can create an OpenOffice (or Excel) worksheet with a copy of the table given below in order to smooth up the calculations.

2) In the process of calculations, enter in the rows of your table in the worksheet the terms

$$a_n, p_n, b_n,$$

followed by the sign of the number $f(a_n)f(p_n)$ as described above.

3) If you'll feel that the terms p_n may have become close enough to satisfy the stopping criterion, fill in the last column of the table in your worksheet by calculating the required relative errors

$$\mathrm{RE}(\tilde{p}_n \approx \tilde{p}_{n-1}).$$

Note the step N at which the stopping criterion became true; if all your relative errors are still greater than the tolerance, continue generating the terms $a_n, p_n, b_n,$ and so on.

4) Once the table in your worksheet is ready, check your answers, redo the table if necessary, and then copy-paste your answers to the table in this page.

(iii) Accordingly, by (i) and (ii),

$p_N \doteq$ -7.5586 ✔ .

Check