Question **4**

Correct

Marked out of 27

(*Babylonian Method*). All numerical answers should be rounded to 6-digit floating-point numbers.

(0) If $A > 0$ is a positive number, then given a real number x_0, the sequence (x_n) defined recursively by

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{A}{x_n}\right), \quad (n \geqslant 0),$$

where $n \geqslant 0$ is a natural number, converges to $\sqrt{A}$, that is,

$$\lim_{n\to\infty} x_n = \sqrt{A}$$

provided that the initial term x_0 is chosen 'not too badly'. In practice, calculation of terms x_n is carried out till this or that stopping criterion is triggered.

(i) Let (x_n) be the sequence defined recursively by

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{694}{x_n}\right), \quad (n \geqslant 0),$$

and $x_0 = 69$ (as it could be guessed from part (0), the sequence (x_n) *converges* to $\sqrt{694}$).

(ii) Generate the terms

$$x_1, x_2, \ldots$$

of the sequence (x_n) till you find the term x_N satisfying the condition

$$\mathrm{fl}(x_N) = \mathrm{fl}(x_{N-1})$$

(our stopping criterion), where $\mathrm{fl}(z)$ denotes the result of rounding of a real number z to a 6-digit floating-point number.

Along the way, once the k-th term x_k is generated, enter in the corresponding input field the result $\mathrm{fl}(x_k)$ of rounding of x_k to a 6-digit floating-point number. As instructed, stop generation of the terms if the stopping criterion is triggered at some step N. Accordingly, the number $\mathrm{fl}(x_N)$ must be the *last number* you need to enter. Enter an asterisk $*$ in each of the remaining input fields, if any.

$x_0 \doteq$ 69

$x_1 \doteq$ 39.528986

$x_2 \doteq$ 28.542861

$x_3 \doteq$ 26.428586

$x_4 \doteq$ 26.344015

$x_5 \doteq$ 26.34388

$x_6 \doteq$ 26.34388

$x_7 \doteq$ *

$x_8 \doteq$ *

$x_9 \doteq$ *

(ii) To see how well the term x_N you found in the previous part approximates $\sqrt{694}$, find $(x_N)^2$ and round the result to a 6-digit floating-point number:

$(x_N)^2 \doteq$ 694 .

Check