Question **3**

Correct

Marked out of 22

(*Taylor Series/Polynomials, Natural Logarithm*). All numerical answers, unless otherwise required, should be rounded to 6-digit floating-point numbers.

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(0) It is known that

$$\ln\frac{1+x}{1-x} = 2\left(\sum_{k=0}^{\infty}\frac{x^{2k+1}}{2k+1}\right) \tag{2.1}$$

for all $x \in (-1, 1)$. In fact, the series in the right-hand side is the Taylor series at zero of the function

$$f(x) = \ln\frac{1+x}{1-x};$$

in effect, any partial sum of the series is a suitable Taylor polynomial of the function $f(x)$ at zero.

The formula (2.1) is often used to approximate the values of the natural logarithm. Say, setting $x = 1/3$, we obtain from (2.1) the infinite series representing $\ln(2)$, due to

$$\frac{1+1/3}{1-1/3} = 2.$$

(i) Find a *rational* number $x = p/q$ which is the solution of the equation

$$\frac{1+x}{1-x} = 1.49$$

and enter it (in reduced form) in the input field below:

$x =$ 49/249 ✔ .

We then have $\ln(1.49)$ is equal to the sum of the series in (2.1) for $x = p/q$.

(ii) Consider the first five terms of the series in (2.1), that is, the ninth Taylor polynomial of the function $f(x)$ at zero,

$$T(x) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9}\right).$$

Find an approximation y^* of $y = \ln(1.49)$ as

$$y^* = T(p/q),$$

where p/q is the rational number you have found in (i), and enter your result, rounded to a 6-digit floating-number, in the input field below:

$y^* \doteq$ 0.398776 ✔ .

(iii) Find the absolute, the relative error, and the number of significant digits in the approximation of $y = \ln(1.49)$ by y^*:

$\mathrm{AE}(y \approx y^*) \doteq$ 3.221683e-9 ✔ ;

$\mathrm{RE}(y \approx y^*) \doteq$ 8.078920e-9 ✔ ;

$\mathrm{SD}(y \approx y^*) =$ 8 ✔ .

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