Dashboard / My courses / Numerical Analysis (CEN), 23s / Mar 20 - Mar 26 (Week 5) / HW #2 (due Mar 31, 18:00)

Question 4

(Babylonian Method). All numerical answers should be rounded to 6-digit floating-point numbers.

Correct

Marked out of 27

(0) If A>0 is a positive number, then given a real number $x_0,$ the sequence (x_n) defined recursively by

$$x_{n+1}=rac{1}{2}igg(x_n+rac{A}{x_n}igg)\,,\quad (n\geqslant 0),$$

where $n\geqslant 0$ is a natural number, converges to $\sqrt{A},$ that is,

$$\lim_{n o\infty}x_n=\sqrt{A}$$

provided that the initial term x_0 is chosen 'not too badly'. In practice, calculation of terms x_n is carried out till this or that stopping criterion is triggered.

(i) Let (x_n) be the sequence defined recursively by

$$x_{n+1}=rac{1}{2}igg(x_n+rac{710}{x_n}igg)\,,\quad (n\geqslant 0),$$

and $x_0=71$ (as it could be guessed from part (0), the sequence (x_n) converges to $\sqrt{710}$).

(ii) Generate the terms

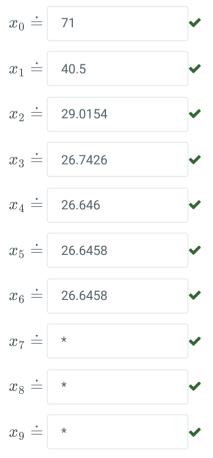
$$x_1, x_2, \dots$$

of the sequence (x_n) till you find the term x_N satisfying the condition

$$fl(x_N) = fl(x_{N-1})$$

(our stopping criterion), where fl(z) denotes the result of rounding of a real number z to a 6-digit floating-point number.

Along the way, once the k-th term x_k is generated, enter in the corresponding input field the result $\mathrm{fl}(x_k)$ of rounding of x_k to a 6-digit floating-point number. As instructed, stop generation of the terms if the stopping criterion is triggered at some step N. Accordingly, the number $\mathrm{fl}(x_N)$ must be the last number you need to enter. Enter an asterisk * in each of the remaining input fields, if any.



(ii) To see how well the term x_N you found in the previous part approximates $\sqrt{710}$, find $(x_N)^2$ and round the result to a 6-digit floating-point number:

$$(x_N)^2 \doteq$$
 710

Check

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