Question 4

Correct

Marked out of 20

(Fixed-Point Iteration). Unless otherwise required, all numerical answers should be rounded to 7-digit floating-point numbers. Given a real number z, the symbol  $\tilde{z}$  denotes the result of rounding of z to a 7-digit floating point number.

Consider the polynomial

$$f(x) = 0.48x^3 + 0.52x^2 - 6.24x + 1.56$$

In what follows, we will apply the Fixed-Point Iteration (FPI) method to approximate a unique root of the function f(x) in [0, 1].

(i) (a) Demonstrate that

$$0.48x^3 + 0.52x^2 - 6.24x + 1.56 = 0 \iff x = \frac{x^3}{13} + \frac{x^2}{12} + \frac{1}{4}$$

on [0,1], thereby obtaining a reduction to a fixed-point problem for the (iteration) function

$$g(x) = \frac{x^3}{13} + \frac{x^2}{12} + \frac{1}{4}$$

on [0, 1].

(b) Clearly, the function g(x) is strictly increasing  $\bullet$  on [0,1]. Accordingly, for every  $x \in [0,1]$ ,

$$g(?_1) \leq g(x) \leq g(?_2),$$

where

$$?_1 = \boxed{ 0 }$$
 and  $?_2 = \boxed{ 1 }$ 

(please enter suitable points/numbers of the interval [0, 1]) and

$$g(?_1) = \boxed{1/4}$$
 and  $g(?_2) = \boxed{16/39}$ 

(please enter suitable rational numbers).

(c) Does the argument in (b) imply that the function g(x) takes the interval [0, 1] into itself?



(d) Next, the derivative g'(x) of the function g(x) is also strictly increasing on [0, 1]. Consequently, for every  $x \in [0, 1]$ ,

$$|g'(x)| \le |g'(?_3)| = k$$

where

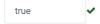
(please enter a suitable point/number of the interval [0,1]) and

$$k = |g'(?_3)| = \boxed{31/78}$$

(please enter a suitable *rational* number), and *k* is evidently less than 

✓ (please enter a *relevant* number).

(e) Now we see that both conditions from the main statement on convergence of the FPI in the lecture notes are ... (here and in the next part, please enter a suitable word)



for the function g(x) on [0, 1], and hence the FPI for the function g(x) on [0, 1] ...



(ii) Use the Fixed-Point Iteration method to find an approximation  $p_N$  of the fixed-point p of g(x) in [0, 1], the root of the polynomial f(x) in [0, 1], satisfying

$$RE(\tilde{p}_N \approx \tilde{p}_{N-1}) \le 10^{-5}$$

by taking  $p_0 = 1$  as the initial approximation.

(iii) Show your work by filling in the following table (if a particular row of the table is not necessary, please enter an asterisk \* in each input field in this row):

To repeat the advice given in Problem 2 once again, if you are going to use a scientific calculator, first create a similar table in an OpenOffice (or Excel) worksheet, and then copy-paste your answers.

n	$p_{n-1}$	$p_n$	$RE(\tilde{p}_n \approx \tilde{p}_{n-1})$
1	1	0.4102564	1.4375
2	0.4102564	0.2693374	0.5232062
3	0.2693374	0.2575482	0.04577473
4	0.2575482	0.2568417	0.002750722
5	0.2568417	0.2568006	0.0001600464
6	0.2568006	0.2567982	9.34586e-06
7	*	*	*
8	*	*	*

(iii) Acco	ordingly, by (i)	and (ii),
$p_N \doteq$	0.2567981	<b>~</b> .
Check		

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