Question **3**

Correct

Marked out of 30

(*Bisection Method*). Prior to entering in the corresponding input fields, all numerical answers should be rounded to 6-digit floating-point numbers. Given a real number z, the symbol $\tilde{z}$ denotes the result of rounding of z to a 6-digit floating-point number.

(i) Use the Bisection method to find an approximation p_N of the unique solution p the equation

$$3x(1-x^2+x)\ln(x) = x^2 - 1$$

in $[a, b] = [0.05, 0.5]$ such that

$$\mathrm{RE}(\tilde{p}_N \approx \tilde{p}_{N-1}) < 10^{-3}.$$

(iii) Show then your work by filling in the table that follows. In each input field in the column labelled by

$$f(a_n)f(p_n),$$

please enter either a plus sign + (if $f(a_n)f(p_n) > 0$), or a minus sign - (if $f(a_n)f(p_n) < 0$). If a particular row of the table is not necessary, enter an asterisk $*$ in each input field in the row. In order to calculate the relative error

$$\mathrm{RE}(\tilde{p}_1 \approx \tilde{p}_0)$$

in the first row, assume formally that $p_0 = 0.05$.

n	a_n	p_n	b_n	$f(a_n)f(p_n)$	$\mathrm{RE}(\tilde{p}_n \approx \tilde{p}_{n-1})$
1	0.05 ✔	0.275 ✔	0.5 ✔	- ✔	0.818182 ✔
2	0.05 ✔	0.1625 ✔	0.275 ✔	- ✔	0.692308 ✔
3	0.05 ✔	0.10625 ✔	0.1625 ✔	+ ✔	0.529412 ✔
4	0.10625 ✔	0.134375 ✔	0.1625 ✔	+ ✔	0.209302 ✔
5	0.134375 ✔	0.148438 ✔	0.1625 ✔	+ ✔	0.0947399 ✔
6	0.148438 ✔	0.155469 ✔	0.1625 ✔	- ✔	0.0452244 ✔
7	0.148438 ✔	0.151953 ✔	0.155469 ✔	+ ✔	0.0231387 ✔
8	0.151953 ✔	0.153711 ✔	0.155469 ✔	+ ✔	0.011437 ✔
9	0.153711 ✔	0.15459 ✔	0.155469 ✔	- ✔	0.00568601 ✔
10	0.153711 ✔	0.15415 ✔	0.15459 ✔	- ✔	0.00285436 ✔
11	0.153711 ✔	0.153931 ✔	0.15415 ✔	- ✔	0.00142272 ✔
12	0.153711 ✔	0.153821 ✔	0.153931 ✔	+ ✔	0.000715117 ✔
13	* ✔	* ✔	* ✔	* ✔	* ✔

As suggested in the previous problem, users of scientific calculators may first create a copy of the above table in an OpenOffice (or Excel) worksheet, and then copy-paste their answers.

(ii) According to your results in (i) and (ii),

$p_N \doteq$ 0.153822 ✔ .

Check