Question **4**

Partially correct

Marked out of 21

(*Regula Falsi Method as an FPI Technique*). All numerical answers should be rounded to 6-digit floating-point numbers.

Consider the problem of approximating the unique root p of the function

$$f(x) = x^2 - 0.17\sqrt{x} - 1.22$$

in $[a, b] = [1, 2]$ with the Regula Falsi method as an FPI technique.

(i) (a) Verify that $f''(x) > 0$ on $[a, b] = [1, 2]$.

(b) Evaluate $\sigma = f(a)f''(a)$,

$\sigma \doteq$ -0.796575 ✔ .

(c) Based on (b), find and simplify the iteration function given either by

$$g(x) = \frac{bf(x) - xf(b)}{f(x) - f(b)} \quad [\text{if } \sigma < 0], \text{ or by } g(x) = \frac{af(x) - xf(a)}{f(x) - f(a)} \quad [\text{if } \sigma > 0].$$

Set then $p_0 = a$ in the first case, or $p_0 = b$ in the second case.

(ii) Use the Fixed-Point Iteration with the iteration function $g(x)$ and the initial approximation p_0, you have determined in (i), to obtain an approximation p_N of p satisfying

$$\text{RE}(\tilde{p}_N \approx \tilde{p}_{N-1}) < 10^{-5}.$$

Show your work by filling in the following standard output table (enter asterisks in unnecessary fields):

n	p_{n-1}	p_n	$\text{RE}(\tilde{p}_n \approx \tilde{p}_{n-1})$
1	1 ✔	1.13312 ✔	0.117481 ✔
2	1.13312 ✔	1.1713 ✔	0.0325963 ✔
3	1.1713 ✔	1.18162 ✔	0.00873377 ✖
4	1.18162 ✔	1.18437 ✔	0.00232191 ✖
5	1.18437 ✔	1.1851 ✔	0.000615982 ✔
6	1.1851 ✔	1.18529 ✔	0.000160298 ✔
7	1.18529 ✔	1.18535 ✔	5.0618e-05 ✔
8	1.18535 ✔	1.18536 ✔	8.43626e-06 ✔
9	* ✔	* ✔	* ✔
10	* ✔	* ✔	* ✔

Accordingly,

$p_N \doteq$ 1.18536 ✔ .

Check