Question **5**

Partially correct

Marked out of 25

(*Lagrange Polynomials*). All numerical answers should be rounded to 5-digit floating-point numbers.

The following table, based on the national census counts since 1927, represents the historical data on the population of Turkey, where t is the number of years since 1927 and $f(t)$ is the population of Turkey in the year $1927 + t$ (in millions of people):

t	0	8	13	18	23	28	33	38	43
$f(t)$	13.649	16.159	17.822	18.791	20.947	24.066	27.756	31.392	35.606
t	48	53	58	63	73	80	82	84	86
$f(t)$	40.349	44.737	50.665	56.474	67.805	70.586	72.561	74.724	76.668
t	88	90	92	94					
$f(t)$	78.741	80.811	83.155	84.680					

Thus, according to the table, the population of Turkey in the year 1955 was
$f(1955 - 1927) = f(28) \doteq 24.066$ million people.

Let $L(x)$ be the third Lagrange polynomial for the function $f(x)$ with the nodes

$$x_0 = 0, x_1 = 43, x_2 = 80, x_3 = 94$$

and let y_k denote $f(x_k)$, where $k = 0, 1, 2, 3$.

(i) (a) Find the value $L(58)$ of the Lagrange polynomial at $x = 58$ and the relative error in the approximation of $f(58)$ by $L(58)$.

(b) Show your work by filling in the standard table for the method showing the process of evaluation of the Lagrange polynomial at the given point:

x_k	0 ✔	43 ✔	80 ✔	94 ✔
y_k	13.649 ✔	35.606 ✔	70.586 ✔	84.68 ✔
$L_k(x)$	-0.036739 ✔	0.56613 ✔	0.75579 ✔	-0.28518 ✔
$y_k L_k(x)$	-0.50145 ✔	20.157 ✔	53.348 ✔	-24.149 ✔

(c) Accordingly,

$L(58) \doteq$ 48.855 ✔ ,

and the relative error in question is given by

$\mathrm{RE}(\ f(58) \approx L(58)\ ) \doteq$ 0.035725 ☑ .

(ii) Find the value $L(84)$ of the Lagrange polynomial at $x = 84$ and the relative error in the approximation of $f(84)$ by $L(84)$.

Proceed then as in the previous part by filling in the standard table

x_k	0 ✔	43 ✔	80 ✔	94 ✔
y_k	13.649 ✔	35.606 ✔	70.586 ✔	84.68 ✔
$L_k(x)$	0.0050717 ✔	-0.041409 ✔	0.83108 ✔	0.20526 ✔
$y_k L_k(x)$	0.069224 ✔	-1.4744 ✔	58.663 ✔	17.381 ✔

and by providing the required values:

$L(84) \doteq$ 74.639 ✔ ;

$\mathrm{RE}(\ f(84) \approx L(84)\ ) \doteq$ 0.0011375 ✖ .

(iii) Find the value $L(101) = L(2028 - 1927)$ to approximate/extrapolate the population of Turkey in the year 2028 (this time, we skip the standard table):

$L(101) \doteq$ 91.52 ✔ .

Check