Question **4**

Correct

Marked out of 27

(*Secant Method*). All numerical answers should be rounded to 7-digit floating-point numbers. Given a real number z, the symbol $\tilde{z}$ denotes the result of rounding of z to a 7-digit floating-point number.

(i) Apply the Secant method to find an approximation p_N of the solution of the equation

$$x\sin(0.34x) = 0.17$$

in $[0, 1]$ satisfying

$$\mathrm{RE}(\tilde{p}_n \approx \tilde{p}_{N-1}) < 10^{-6}$$

by taking $p_0 = 1$ and $p_1 = 0.8$ as the initial approximations.

(ii) Show your work by filling the following standard output table for the Secant method (if a particular row is not necessary, please type an asterisk $*$ in each input field of that row):

n	p_{n-2}	p_{n-1}	p_n	$\mathrm{RE}(\tilde{p}_n \approx \tilde{p}_{n-1})$
2	1	0.8	0.7242128	0.1046477
3	0.8	0.7242128	0.7113283	0.0181133
4	0.7242128	0.7113283	0.7105682	0.001069707
5	0.7113283	0.7105682	0.7105612	9.851368e-06
6	0.7105682	0.7105612	0.7105612	0
7	*	*	*	*
8	*	*	*	*
9	*	*	*	*

(ii) According to your results in (i) and (ii),

$p_N \doteq$ 0.7105612 .

Check

Previous Activity

Jump to...

Next Activity

Question **3**

Correct

Marked out of 30

(*Joint Use of the Bisection and Newton's Method*). All numerical answers should be rounded to 7-digit floating-point numbers. Given a real number z, the symbol $\tilde{z}$ denotes the result of rounding of z to a 7-digit floating-point number.

Consider the function

$$f(x) = 14x^3 - 12x^2 + 11x - 4.$$

We are going to approximate one of the roots of the function $f(x)$ by first applying the Bisection Method followed by application of Newton's Method.

(i) (a) Clearly, the product $f(0) \cdot f(1)$ is negative ✔ , and therefore $f(x)$ has a root in $[0, 1]$.

(b) Next, find the derivative $f'(x)$ of the function $f(x)$, and solve the equation

$$f'(x) = 0$$

to determine the critical points of the function $f(x)$, if any.

(c) We then see that, by (b), the function $f(x)$ has no ✔ critical points in the interval $[0, 1]$, hence the derivative $f'(x)$ is of constant nonzero sign on the interval $[0, 1]$, hence strictly monotone on the interval $[0, 1]$.

(d) Accordingly, by (a) and (c), the function $f(x)$ has a unique root ✔ in the interval $[0, 1]$.

(ii) Let (q_n) denote the sequence the Bisection method for the function $f(x)$ on $[a_1, b_1] = [0, 1]$ generates.

(a) Find the first *three* terms of the sequence (q_n).

(b) Show your work by filling in the following table (in each input field in the column labelled $f(a_n)f(q_n)$, please enter a suitable letter sign, i.e. either + or −; in order to evaluate the relative error $\mathrm{RE}(\tilde{q}_1 \approx \tilde{q}_0)$, assume formally that $q_0 = 0$):

n	a_n	q_n	b_n	$f(a_n)f(q_n)$	$\mathrm{RE}(\tilde{q}_n \approx \tilde{q}_{n-1})$
1	0 ✔	0.5 ✔	1 ✔	- ✔	1 ✔
2	0 ✔	0.25 ✔	0.5 ✔	+ ✔	1 ✔
3	0.25 ✔	0.375 ✔	0.5 ✔	+ ✔	0.3333333 ✔

(c) Accordingly,

$q_3 \doteq$ 0.375 ✔ .

(iii) Find the iteration function

$$g(x) = x - \frac{f(x)}{f'(x)}$$

for Newton's method (for your own use; this time, an analysis of convergence is not required).

(iv) (a) Use Newton's method to find an approximation p_N of the root p of $f(x)$ in $[0, 1]$ satisfying

$$\mathrm{RE}(\tilde{p}_N \approx \tilde{p}_{N-1}) < 10^{-7}$$

by taking $p_0 = q_3$ from (ii) as the initial approximation (so, we start with Newton's Method at the last approximation found by the Bisection method).

(b) Show your work by filling in the standard output table for Newton's method (please enter an asterisk $*$ in each input field of the unnecessary rows):

n	p_{n-1}	p_n	$\mathrm{RE}(\tilde{p}_n \approx \tilde{p}_{n-1})$
1	0.375 ✔	0.479249 ✔	0.2175258 ✔
2	0.479249 ✔	0.4730578 ✔	0.01308762 ✔
3	0.4730578 ✔	0.4730237 ✔	7.208941e-05 ✔
4	0.4730237 ✔	0.4730237 ✔	0 ✔
5	* ✔	* ✔	* ✔
6	* ✔	* ✔	* ✔

(c) Consequently, by (a) and (b),

$p_N \doteq$ 0.4730237 .

Check

Question **2**

Correct

Marked out of 20

(*Newton's Method*). All numerical answers should be rounded to 7-digit floating-point numbers. Given a real number z, the symbol $\tilde{z}$ denotes the result of rounding of z to a 7-digit floating-point number.

In this problem, we will work to approximate the root of the function

$$f(x) = x + e^{1.134x}.$$

in $[-1, 0]$ by means of Newton's method.

(i) (a) Find, for your own use, the iteration function

$$g(x) = x - \frac{f(x)}{f'(x)}$$

to apply with Newton's method (as a Fixed-Point Iteration technique).

(b) Work as instructed in Problem 1, that is, plot the graphs of $g(x)$ and $g'(x)$ on $[-1, 0]$, each time together with the corresponding horizontal lines, using the WolframAlpha website (or any other familiar online resource/math software).

(c) Now, according to your results in (b),

- Does the graph of the function $g(x)$ on $[-1, 0]$ lie between the lines $y = -1$ and $y = 0$?

(•) Yes ✔

( ) No

- Does the graph of the derivative $g'(x)$ on $[-1, 0]$ lie strictly between the lines $y = -1$ and $y = 1$?

(•) Yes ✔

( ) No

- Does the FPI for the function $g(x)$ on $[-1, 0]$, or equivalently Newton's method for the function $f(x)$ on $[-1, 0]$, converge?

(•) Yes ✔

( ) No

If your answer to the last question is 'no', try to continue anyway, as the initial approximation may be close enough to the root to result in convergence.

(ii) (a) Apply Newton's method to find an approximation p_N of the root of the function $f(x)$ in $[-1, 0]$ satisfying

$$\mathrm{RE}(\tilde{p}_N \approx \tilde{p}_{N-1}) < 10^{-6}$$

by taking $p_0 = -1$ as the initial approximation.

(b) Show your work by filling in the following standard output table for Newton's method (if a particular row is not necessary, please type an asterisk $*$ in each input field of that row): :

n	p_{n-1}	p_n	$\mathrm{RE}(\tilde{p}_n \approx \tilde{p}_{n-1})$
1	-1 ✔	-0.503057 ✔	0.9878463 ✔
2	-0.503057 ✔	-0.5409634 ✔	0.07007202 ✔
3	-0.5409634 ✔	-0.5412823 ✔	0.0005891565 ✔
4	-0.5412823 ✔	-0.5412824 ✔	1.847464e-07 ✔
5	* ✔	* ✔	* ✔
6	* ✔	* ✔	* ✔

(c) According to your results in (a,b),

$p_N \doteq$ -0.5412824 ✔ .

Check

Previous Activity Jump to... Next Activity

Question **1**

Not complete

Marked out of 23

(*Fixed-Point Iteration*). All numerical results should be rounded to 9-digit floating-point numbers. Given a real number z, the symbol $\tilde{z}$ denotes the result of rounding of z to a 9-digit floating-point number.

Consider the equation

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} = \frac{1}{2}. \tag{2.1}$$

Since we have the seventh Taylor polynomial of the sine function centered at zero in the left-hand side of the equation, it is reasonable to expect that the minimal positive solution of the equation can be a good enough approximation of $\pi/6$.

(i) (a) Rewrite the equation (2.1) in the form

$$x = \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{1}{2}$$

suitable for application of the Fixed-Point Iteration (FPI) method, thereby reducing the problem to a fixed-point problem for the function

$$g(x) = \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{1}{2}.$$

We shall consider the FPI for the function $g(x)$ for interval $[1/2, 11/21]$ (recall that $3 < \pi < 22/7$, and hence $1/2 < \pi/6 < 11/21$).

(b) Execute the command

```
Plot[ {x^3/3! - x^5/5! + x^7/7! + 1/2, 1/2, 11/21}, {x,1/2,11/21} ]
```

at the WolframAlpha website to plot the graph of the function $g(x)$ on $[1/2, 11/21]$ and the lines $y = 1/2$ and $y = 11/21$. Judging by the graphs you have obtained, is it true that the graph of the function $g(x)$ on $[1/2, 11/21]$ lies between the lines $y = 1/2$ and $y = 11/21$?

- (•) Yes ✔
- ( ) No

(c) Next, execute the command

```
Plot[ {D[x^3/3! - x^5/5! + x^7/7! + 1/2], -1, 1}, {x,1/2,11/21} ]
```

at the WolframAlpha website to plot the graph of the derivative $g'(x)$ of the function $g(x)$ on $[1/2, 11/21]$ and the lines $y = -1$ and $y = 1$. Judging by the graphs you have obtained, is it true that the graph of the function $g'(x)$ on $[1/2, 11/21]$ lies *strictly* between the lines $y = -1$ and $y = 1$?

- (•) Yes ✔
- ( ) No

(d) Now, do your results in (b,c) imply that the FPI for the function $g(x)$ on $[1/2, 11/21]$ converges?

- (•) Yes ✔
- ( ) No

(ii) (a) Use the Fixed-Point Iteration method to find an approximation p_N of the fixed-point p of the function $g(x)$ in $[1/2, 11/21]$ such that

$$\mathrm{RE}(\tilde{p}_N \approx \tilde{p}_{N-1}) < 10^{-7}$$

by taking $p_0 = 0.5071$, a randomly chosen number in $[1/2, 11/21]$, as the initial approximation.

(b) Show your work by filling in the following table (if a particular row is not necessary, please enter an asterisk $*$ in each input field of that row):

n	p_{n-1}	p_n	$\mathrm{RE}(\tilde{p}_n \approx \tilde{p}_{n-1})$
1	0.5071 ✔	0.521455768 ✔	0.0275301739 ✔
2	0.521455768 ✔	0.523312821 ✔	0.00354864801 ✔
3	0.523312821 ✔	0.523560493 ✔	0.000473053264 ✖
4	0.523560493 ✔	0.523593655 ✔	6.33353741e-05 ✖
5	0.523593655 ✔	0.523598098 ✔	8.48551593e-06 ✔
6	0.523598098 ✔	0.523598693 ✔	1.13636647e-06 ✔

7	0.523598693	0.523598773	1.52788746e-07
8	0.523598773	0.523598783	1.90985928e-08
9	*	*	*
10	*	*	*

(c) Accordingly, by (a) and (b),

$p_N \doteq$ 0.523598783 .

(iii) Finally, evaluate

$$y^* = 6p_N,$$

and then find the relative error and the number of significant digits in approximating of π by y^*:

$y^* \doteq$ 3.1415927 ;

$\mathrm{RE}(\pi \approx y^*) \doteq$ 1.47750463e-08 ;

$\mathrm{SD}(\pi \approx y^*) =$ 8 .

Check

Previous Activity

Jump to...

Next Activity