



Modelling and Control of Underwater Vehicle (Report)

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Underwater Robotics, Modelling and Control

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1. Introduction

To initiate the modelling process for the "SPARUS" submarine, it's essential to first gather its basic characteristics. The assignment's specifications do not provide all the dimensions of the craft, necessitating their measurement.

1.1 Dimensions of Body:

All the dimensions are calculated in AutoCAD "Sparus_AUV_Dimension" file can be found in attachment.

1.2 Approximated Dimensions

All the dimensions are drafted in AutoCAD model which is attached however we made some approximation to proceed our calculations which are summarised in table only those dimensions are mentioned which we use in calculation part, rest can be found in CAD document attached. All the approximated dimensions chosen after clear intuition.

Table 1: Table of dimensions of Sparus parts in meters

Part	Dimension (m)
Antenna -Height	0.255
Antenna -width	0.66
Antenna -thickness	0.03
Thruster -Length	0.285
Thruster-Radius	0.045
Thruster-Width	0.098

All other dimension like radius and length of the Sparus is given in problem.

It is also important to know the position of the CG of everybody when measured from the submarine CG to the body CG. These vectors are given in the following table.

Table 2: Vectors of Sparus in meters

Part	X(m)	Y(m)	Z(m)
Sub main body	0.055	0	0
Antenna	-0.412	0	-0.242
Thruster_RT	-0.532	0.148	0
Thruster_LT	-0.532	-0.148	0

2. Global Mass Matrix:

Global mass matrix of the submarine consists of two matrices: Body mass matrix and Added mass matrix. It is calculated using the following formula:

$$M_G^b = M_B^b + M_A^b$$

2.1 Global Real Mass:

$$M_{RB}^{CO} = \begin{bmatrix} 52 & 0 & 0 & 0 & -0.1 & 0 \\ 0 & 52 & 0 & 0.1 & 0 & -1.3 \\ 0 & 0 & 52 & 0 & 1.3 & 0 \\ 0 & 0.1 & 0 & 0.5 & 0 & 0 \\ -0.1 & 0 & 1.3 & 0 & 9.4 & 0 \\ 0 & -1.3 & 0 & 0 & 0 & 9.5 \end{bmatrix}$$

Given real mass matrix can be compared with the matrix given below to explain all the terms in matrix as this matrix has been first calculated at centre of gravity then transformed to centre of origin (CO).

This real mass matrix is the sum of all the sub mass matrices of the Sparus and then transformed to centre of origin/centre of gravity of Sparus.

$$M_{RB} = \begin{bmatrix} mI_{3 \times 3} & -mS(\vec{r}_g^b) \\ mS(\vec{r}_g^b)^T & I_b \end{bmatrix} = \begin{bmatrix} m & 0 & 0 & 0 & mz_g & -my_g \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & my_g & -mx_g & 0 \\ 0 & -mz_g & my_g & I_x & -I_{xy} & -I_{xz} \\ mz_g & 0 & -mx_g & -I_{yx} & I_y & -I_{yz} \\ -my_g & mx_g & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix}$$

First part of the matrix top left corner is real mass matrix with having the only diagonal values having the real mass values of model (52kg)

Second part of the matrix bottom right corner is inertia matrix having inertia values with only diagonal values and having zero non diagonal values.

Third part and fourth of matrix top right corner and bottom left corner are skew symmetric matrix which comes in effect of transformation product of mass with vector distance (rg) when it is transformed to centre of origin/centre of origin.

The diagonal terms (52, 52, 52, 0.5, 9.4, 9.5) represent how the mass of the system is distributed along its main axes. These values are the mass components and mass moments of inertia for different types of motion.

The off-diagonal terms (-0.1, 0.1, 1.3, 0.1, -1.3, 1.3) represent how the motion in one direction is related to the motion in another direction. These values show how the motion in one direction affects or is affected by the motion in another direction. The negative sign indicates a sense of opposition or anti-alignment in the corresponding motions. Now, let's analyse the orders of magnitude:

The diagonal terms (mass components) are of the order of 9 to 50, indicating significant mass distribution along the main axes. The diagonal terms are much larger than the off-diagonal terms,

which means that the system is mainly dominated by its mass components and that the coupling effects are negligible.

The off-diagonal terms (coupling terms) are generally smaller, ranging from -1.3 to 1.3. These terms are relatively smaller compared to the diagonal terms, suggesting that the relation between different types of motion is weaker than the properties of each type of motion. Negative sign means that there is a negative coupling or anti-alignment between the motions in the corresponding directions. The off-diagonal terms are usually small. In conclusion, the given global real mass matrix describes the mass distribution and relation between different types of motion in a physical system. The diagonal terms represent the mass components along the main axes, while the off-diagonal terms represent the relation between different types of motion. The orders of magnitude indicate the relative importance of mass components compared to coupling terms.

3. Added Mass Matrices

Added mass for everybody that we named is calculated a bit differently, but they all have one thing in common. They are all calculated in the CG of the body in question, so we need to move them to the CG of the entire craft. We can do this using the following formula:

$$M_{rb/a,i}^A = H^T(\vec{AB})M_{rb/a,i}^B H(\vec{AB})$$

Once we get all the Added mass matrices in the craft CG, we can sum them and get the Added mass matrix of the craft. In the following sections a brief explanation of Added mass matrices calculation for each body is given, but we can get a more detailed insight into the calculations by looking at the python code. Furthermore, some coefficients are omitted, and will be explained in further section.

3.1 Submarine Body Added Mass.

To determine the Added mass matrix for the body in question, two distinct theoretical approaches are employed. The added mass coefficient m11, which pertains to movement along the x-axis, is derived using the Prolate Spheroid theory. For calculating the remaining coefficients, the Slender Body theory is utilized, except for m44 (which represents rotational movement around the x-axis), that is assigned a value of zero. In the Added mass matrix for this body, the coefficient m44 (rotation around x-axis) is 0 because the body is cylindrical and thus will not displace any water while rotating along its longitudinal axis (x-axis). We considered it as a cylinder and having two half spheres at two ends with same radius as cylinder, which makes our calculation easy.

$$\begin{bmatrix} 1.323 & 0 & 0 & 0 & 0 & 0 \\ 0 & 63.29 & 0 & 0 & 0 & 0 \\ 0 & 0 & 63.29 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 12.28 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12.28 \end{bmatrix}$$

The Added mass matrix we obtained is in the CG of the Submarine body and now it is necessary to move it to the CG of the entire craft. Finally, the Added mass matrix of the Submarine body in the CG of the entire craft is:

$$\begin{bmatrix} 1.323 & 0 & 0 & 0 & 0 & 0 \\ 0 & 63.29 & 0 & 0 & 0 & 3.512 \\ 0 & 0 & 63.29 & 0 & -3.512 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.512 & 0 & 12.28 & 0 \\ 0 & 3.512 & 0 & 0 & 0 & 12.28 \end{bmatrix}$$

3.2 Antenna Added Mass.

To calculate the Added mass matrix for this body we are using slender body theory. In the added mass matrix for this body, the coefficient m33, which corresponds to motion along the z-axis, can be taken as zero. This is due to the relatively small cross-sectional area of the body in the vertical (heave) direction compared to the overall cross section of the submarine. Consequently, this smaller cross section has a negligible effect on the overall added mass of the craft when it moves along the z-axis. But I have added it here.

$$\begin{bmatrix} 0.437 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.322 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.38 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0074 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.002 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.00017 \end{bmatrix}$$

The Added mass matrix we obtain is at the CG of the Antenna and now it is necessary to move it to the CG of the entire craft. Finally, the Added mass matrix of the Antenna in the CG of the entire craft is:

$$\begin{bmatrix} 0.306 & 0 & 0 & 0 & -0.074 & 0 \\ 0 & 1.18 & 0 & 0.287 & 0 & -0.48 \\ 0 & 0 & 0.301 & 0 & 0.12 & 0 \\ 0 & 0.287 & 0 & 0.076 & 0 & -0.118 \\ -0.074 & 0 & 0.12 & 0 & 0.0708 & 0 \\ 0 & -0.48 & 0 & -0.118 & 0 & 0.201 \end{bmatrix}$$

3.3 Thrusters Added Mass

To determine the Added mass matrix of this structure, two distinct methodologies are utilized. The added mass coefficient m11, which corresponds to movement along the x-axis, is calculated using the Prolate Spheroid theory. For the calculation of the remaining coefficients, the Slender Body theory is applied. However, it's important to note that the coefficients m22 (associated with movement along the y-axis) and m66 (related to rotation around the z-axis) are assigned a value of zero. In the added mass matrix for this body, coefficients m22 (movement along the y-axis) and m66 (rotation around the z-axis) are set to zero. This is due to the body's small cross-sectional area in the sway direction compared to the overall submarine body, which renders its impact on the added mass in sway (y-axis movement) and yaw (z-axis rotation) motions negligible.

$$\begin{bmatrix} 0.121 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.71 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.004 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0018 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The Added mass matrix we obtained is in the CG of the Thruster and now it is necessary to move it to the CG of the entire craft for both thrusters, left and right. Finally, the Added mass matrix of the Thrusters (both of them - after summing the left and the right thruster matrices) at the CG of the entire craft is:

$$\begin{bmatrix} 0.242 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.85 & 0 & 3.11 & 0 \\ 0 & 0 & 0 & 0.137 & 0 & 0 \\ 0 & 0 & 3.11 & 0 & 1.66 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0053 \end{bmatrix}$$

3.4 Final Added Mass Matrix at CG:

After calculating all the individual Added mass matrices in the CG of the entire craft and summing them together we get the following Added mass matrix for the entire submarine.

$$\begin{bmatrix} 2.004 & 0 & 0 & 0 & -0.106 & 0 \\ 0 & 64.61 & 0 & 0.32 & 0 & 2.96 \\ 0 & 0 & 69.52 & 0 & -0.241 & 0 \\ 0 & 0.32 & 0 & 0.227 & 0 & -0.132 \\ -0.106 & 0 & -0.241 & 0 & 14.23 & 0 \\ 0 & 2.96 & 0 & -0.132 & 0 & 12.71 \end{bmatrix}$$

3.6 Added Mass Matrix at CB

The Final added mass matrix we obtain at the CG and now to move it to the CB Finally, the Added mass matrix of the of Craft at the CB of the entire craft is:

$$\begin{bmatrix} 1.87 & 0 & 0 & 0 & -0.111 & 0 \\ 0 & 64.47 & 0 & 1.57 & 0 & 3.02 \\ 0 & 0 & 69.44 & 0 & -0.2744 & 0 \\ 0 & 1.57 & 0 & 0.251 & 0 & -0.058 \\ -0.111 & 0 & -0.2744 & 0 & 14.21 & 0 \\ 0 & 3.02 & 0 & -0.058 & 0 & 12.68 \end{bmatrix}$$

These coefficients represent the forces in six different degrees of freedom due to acceleration in each combination of degrees of freedom. Force in the i th direction due to an acceleration in the j th direction. The diagonal elements of the matrix are the primary coefficients, relating movement in one direction to the force or moment in that same direction. The non-diagonal coefficients are the coupled or secondary coefficients. All added mass coefficients depend entirely on the geometry of the vehicle, together with the density of the surrounding fluid.

3.7 Observations

The main solid, thrusters, and antenna significantly contribute to the diagonal terms. The antenna plays a crucial role in the cross terms, contributing over to most instances, significantly affecting Sparus' dynamics. Therefore, it cannot be overlooked.

4. Comparison of Different Matrix mass.

4.1 Compare Added mass at CB vs CG.

If we compare the added mass matrix at centre of gravity with the added mass at centre of buoyancy, there is not much difference however the major difference can be seen on off diagonal elements with minimal changes in diagonal elements. Still more contribution is from the diagonal elements which is obvious. The major differences can be seen in m_{46} , m_{64} , m_{24} , m_{42} and these effects are due to the antenna off diagonals. All other values have small difference may be because of the XZ symmetry we have in Sparus.

4.2 Compare Added mass matrix of sub main vs Antenna.

If we compare the added mass matrix of the different bodies with the total mass matrix the contribution of the submain body has highest contribution along the diagonal elements which to close to 97%.

The maximum value of the elements of submain body is 63.29, which is quite large. This means that the three matrices are not very similar. The most significant differences are in the diagonal elements, which correspond to the added mass in the same direction as the acceleration. The submain body has much larger added mass coefficients than the antenna and thruster in the surge, sway, and heave directions, which means that it experiences more fluid inertia forces when accelerating in those directions. The antenna has a slightly larger added mass coefficient in the roll direction, which means that it experiences more fluid inertia torque when rotating about the longitudinal axis. The other off diagonal elements of matrices are relatively small, which means that the cross-coupling effects between different directions are not very different for the submarine and the Thrusters.

4.3 Compare Added mass matrix of Thruster vs Antenna.

The maximum absolute value of the elements of thrusters' matrix is 5.85, which is quite large. This means that the two matrices are not very similar. The most significant difference is in the m_{33} element, which corresponds to the added mass in the heave direction due to a unit angular acceleration in the heave direction. The thruster has a much larger added mass coefficient than the antenna in this direction, which means that it experiences more fluid inertia torque when moving about the Z-axis. The difference other elements are relatively smaller, but still indicate some differences in the fluid inertia effects for the thruster and the antenna. The antenna has a larger added mass coefficient in the surge direction (m_{22}), which means that it experiences more fluid inertia force when accelerating in the Y-direction. The off-diagonal elements of matrix represent the cross-coupling effects between different directions, such as the added mass in the pitch direction due to a unit acceleration in the sway direction (m_{25}). These effects are generally smaller than the diagonal elements, but they can still influence the dynamics of the body.

4.4 Comparison of Added mass with Real mass.

We can see that the added mass matrix has larger values than the real mass matrix, especially along the diagonal elements. This means that the fluid adds more inertia to the submarine than its own mass. The added mass matrix also has some off-diagonal elements, which indicate that the fluid causes some coupling effects between different degrees of freedom. For example, the element m_{12} represents the added mass in the x-direction due to an acceleration in the y-direction, and vice versa. This means that

the fluid will induce some sway motion when the submarine surges, and some surge motion when the submarine sways.

The effects of the added mass matrix and the real mass matrix on the dynamics of the body can be seen by looking at the equation of motion:

$$F = M_G * A$$

where F is the force vector, M_G is the general mass matrix, and A is the acceleration vector. The general mass matrix is the sum of the added mass matrix and the real mass matrix. The equation of motion shows that the fluid forces depend on both the added mass and the real mass of the body, as well as the acceleration of the body. The added mass matrix will increase the magnitude of the fluid forces and introduce some cross-coupling effects between different directions of motion. The real mass matrix will also affect the fluid forces, but in a more straightforward way.

To conclude, the added mass matrix and the real mass matrix are two different ways of representing the inertial effects of a fluid on a moving body. The added mass matrix accounts for the extra mass that the body has to accelerate or decelerate due to the fluid surrounding it, while the real mass matrix is the actual mass of the body itself. The added mass matrix depends on the shape, size, and orientation of the body, as well as the fluid properties, while the real mass matrix is constant. The added mass matrix and the real mass matrix have different values and different effects on the dynamics of the body. The added mass matrix will increase the magnitude of the fluid forces and introduce some cross-coupling effects between different directions of motion. The real mass matrix will also affect the fluid forces, but in a more straightforward way. The most significant difference is in the m_{11} element, which corresponds to the inertia in the surge direction due to a unit acceleration in the surge direction. The global real mass matrix has a much larger coefficient than the added mass matrix in this direction, which means that the body has more inertia than the fluid when accelerating in the horizontal direction.

5 Drag Matrices

For the simplification of calculations, we will consider three plane symmetry, or we simply neglect the non-diagonal element as it not easy to calculate all the values. so, we will be calculating only the diagonal values.

To calculate the drag coefficients, we use two things: the projected surfaces, for K_{11} , K_{22} , K_{33} , K_{44} , K_{55} and K_{66} and the coefficients of drag. They are given in the next sections.

5.1 Projected Surfaces

The projected surfaces are the surfaces that we see if we project the shape of the part in the direction of its movement. We have three movements, surge, sway, and heave. The results of the study are given in the table 1 of the projected surfaces below. We use the projected surfaces for the coefficients K_{22} and K_{33} as well instead of the integrals because the C_d and the ρ are constants and that means we are studying a surface in the end. It would have been different if we had a variation of ρ for example. The next table contains our results.

Table 3: Projected Surface results with formulas

Part	Direction	Surface	Formula
Submain Body	Surge	Circle	$\pi \times R_{Body}^2$
	Sway	Rectangle	$2 \times R \times Length$
	Heave	Rectangle	$2 \times R \times Length$
Antenna	Surge	Rectangle	$Height \times Width$
	Sway	Rectangle	$Height \times Thickness$
	Heave	Rectangle	$Thickness \times Width$
Thrusters	Surge	Circle	$\pi \times R_{Th}^2$
	Sway	Rectangle	$2 \times R_{th} \times L_{th}$
	Heave	Rectangle	$2 \times R_{th} \times L_{th}$

5.2 Drag Coefficient:

These drag coefficient are chosen on the bases of the ratio of two characterises length as discussed / mentioned the lecture notes.

Table 4: Drag Coefficient results w.r.t Sparus parts.

Part	Direction	Surface	C_d
Submain Body	Surge	Elliptical rod	0.1125
	Sway	Cylindrical rod	0.8
	Heave	Cylindrical rod	0.8
Antenna	Surge	Rectangle	2
	Sway	Rectangle	1.9
	Heave	Rectangle	2.5
Thrusters	Surge	Elliptical rod	0.7
	Sway	Cylindrical rod	1.9
	Heave	Cylindrical rod	1.9

We obtained matrices with some removable coefficients. As discussed above we will be calculating only diagonal matrix.

5.3 Sub Main Body:

Here in submain body, we have taken the m44 zero the explanation for this is same as in added mass in the above section of added mass calculation.

$$\begin{bmatrix} 2.337 & 0 & 0 & 0 & 0 & 0 \\ 0 & 73.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 73.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9.4 \end{bmatrix}$$

5.4 Antenna

$$\begin{bmatrix} 7.65 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.38 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8.5 \times 10^{-6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.7 \times 10^{-6} \end{bmatrix}$$

5.4 Thrusters

$$\begin{bmatrix} 2.32 & 0 & 0 & 0 & 0 & 0 \\ 0 & 24.9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 24.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9.4 \times 10^{-3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 9.4 \times 10^{-3} \end{bmatrix}$$

6 Coriolis Matrix Modelling:

The Coriolis force is dependent on the velocity of the Sparus.

The Coriolis matrix C is given in Equation:

$$C = \begin{bmatrix} 0 & -S(M_{11}v_1 + M_{12}v_2) \\ -S(M_{11}v_1 + M_{11}v_2) & -S(M_{21}v_1 + M_{22}v_2) \end{bmatrix}$$

The values of M11, M12 and M22 are obtained from the global mass matrices. The values of v1 and v2 are gotten from the DVL (is implemented in MATLAB code).

6.1 Thruster Mapping:

As we have three thrusters one in vertical and two in horizontal direction, two thrusters in horizontal direction create moment in along the Y and Z direction which in last part of below matrix.

$$E_{6 \times 3}^b = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -0.17 & 0.17 \end{bmatrix}$$

Note: For the calculation, please find the attached python file
“Under_water_modelling_Calculation.ipynb”.

7 Simulations Results

Three simulations are tested to validate the Simulation and check if the outputs are consistent with what was expected from the Mass Matrices.

7.1 Checking Effect of Buoyancy

If we turn off all three thrusters to see the effect of buoyancy and it is supposed to come up towards the surface due to the Buoyancy.

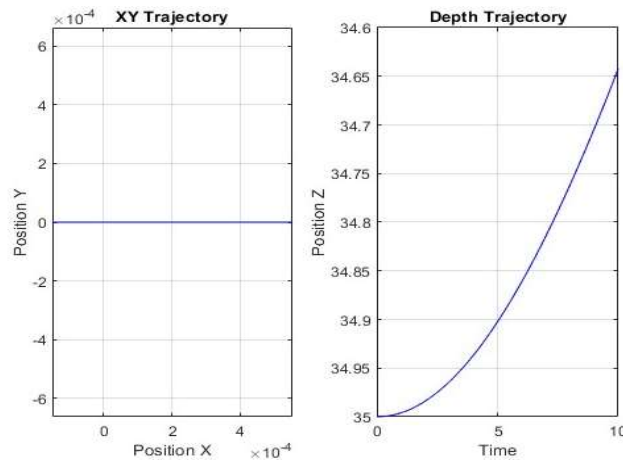


Figure 1: XY-trajectory and Depth Trajectory of the Sparus at Thrusters [0%, 0%, 0%].

Summary

As expected if the three thrusters are off, sparus moves towards to the surface.

7.2 Thrusters [100%, 0%, 0%]

Apply in the thrust in the downward direction.

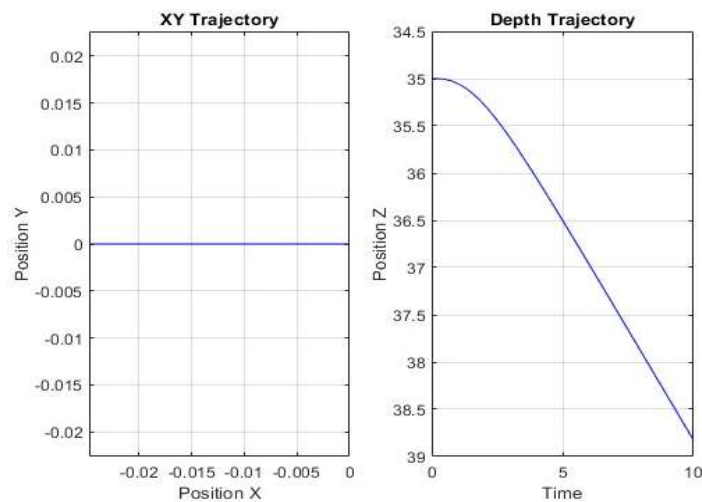


Figure 2: XY-trajectory and Depth Trajectory of the Sparus at Thrusters [100%, 0%, 0%].

Having acceleration in Z direction, we see the effects in pitch as expected as in the global mass matrix as we have component in that direction.

Conversely, an angular rate in Y will cause an acceleration in X and Z according to the global mass matrix. Hence, this also explains the change in acceleration and velocities in X in Figure 3.

It is also suspected that the Coriolis could have effect on the velocity in X. Since the body has a velocity in Z and it is also rotating in Y but its effect is small.

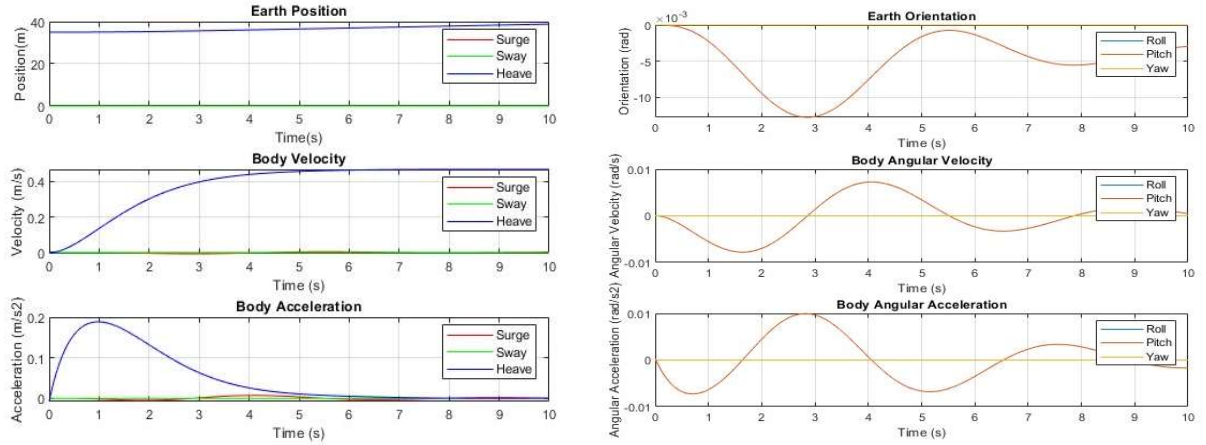


Figure 3: Position, Velocity and Acceleration of the Sparus at Thrusters [100%, 0%, 0%].

7.3 Thrusters [0%, 0%, 25%]

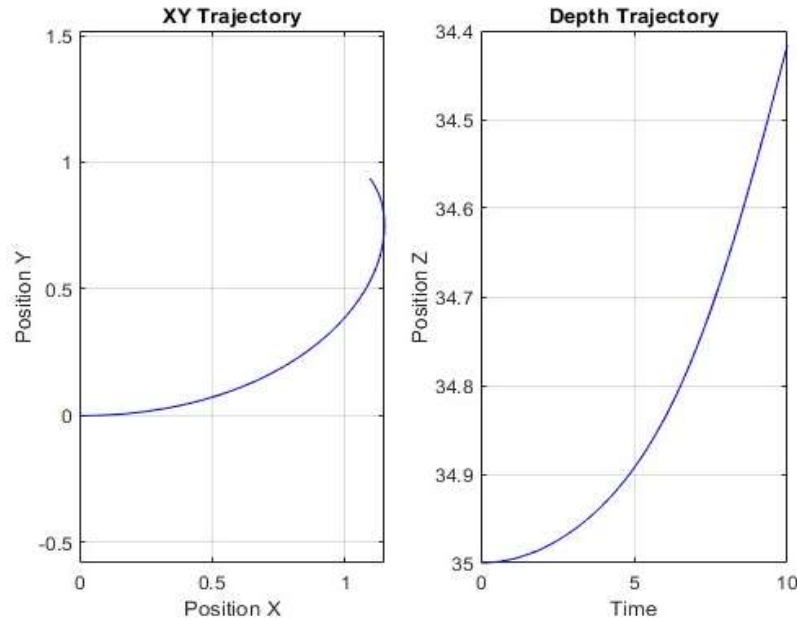


Figure 4: XY-trajectory and Depth Trajectory of the Sparus at Thrusters [0%, 0%, 25%].

If we activate the one of the thrusters in horizontal direction with 25% power. This will create moment with respect to the sway and we will have curved path or spiral path. And the Coriolis force comes into effect as we have the acceleration and because of buoyancy it moves towards the surface.

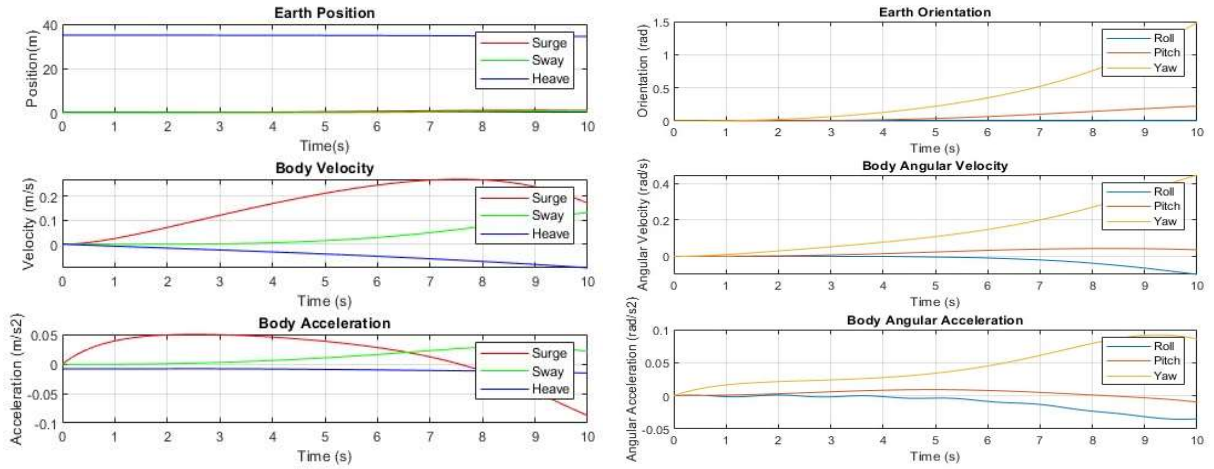


Figure 5: Position, Velocity and Acceleration of the Sparus at Thrusters [0%, 0%, 25%].

Summary:

The Sparus, a watercraft, exhibited a curved movement in the XY plane because of the action of its single thruster. Its upward motion towards the surface was caused by buoyancy, as its vertical thruster was not in use. Additionally, a significant angular rate around the Z-axis was observed, attributed to the moment generated by the horizontal forces of the thrusters, which act at a distance from the Sparus's centre.

7.4 Imposing Linear Speed

7.4.1 Taking in Account only Added Mass:

Implementing the constant linear speed along the X direction and check the effect of different drag matrix on model. First, we neglected all the drag matrix and plot the effect on model motion. Our model behaves very weirdly which is obvious and shows the importance of drag in model design.

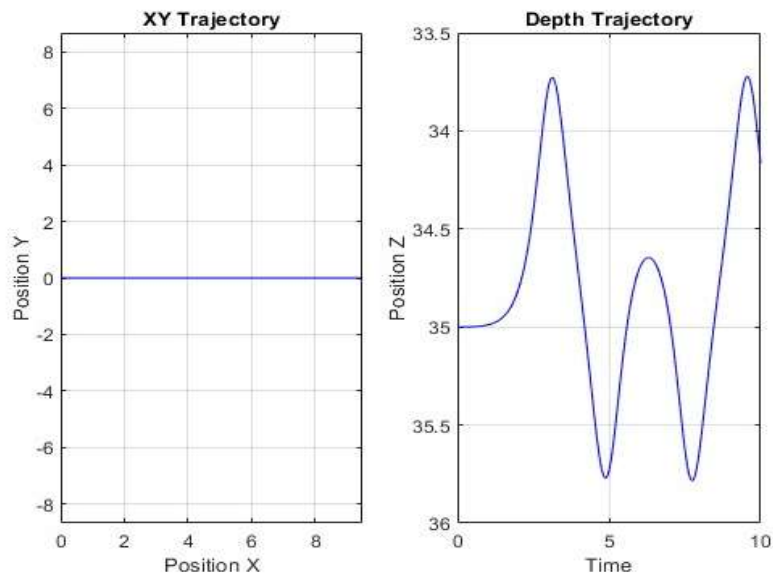


Figure 6: XY and Depth Trajectory while linear speed applied along the X direction.

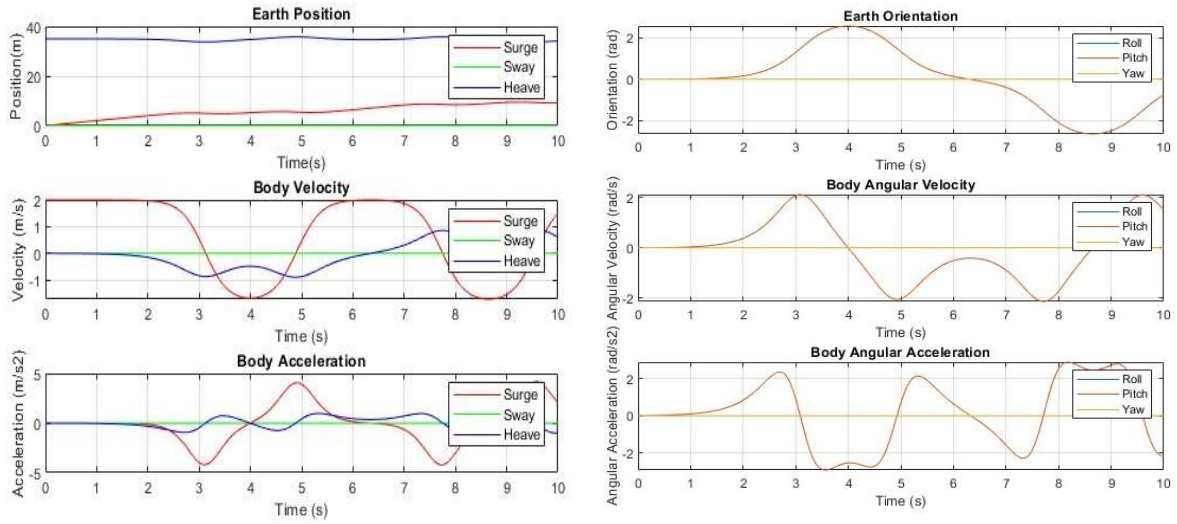


Figure 7: Position, Velocity and Acceleration of Sparus when linear speed applied on X direction.

7.4.2 Taking in Account Only Sub Main Body Drag:

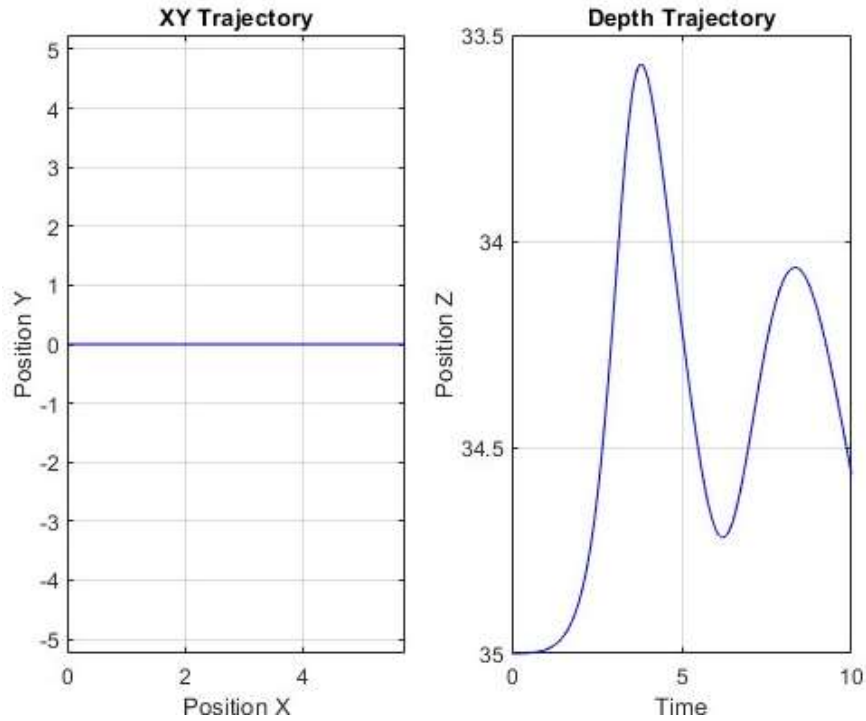


Figure 8: XY and Depth Trajectory while linear speed applied along the X direction.

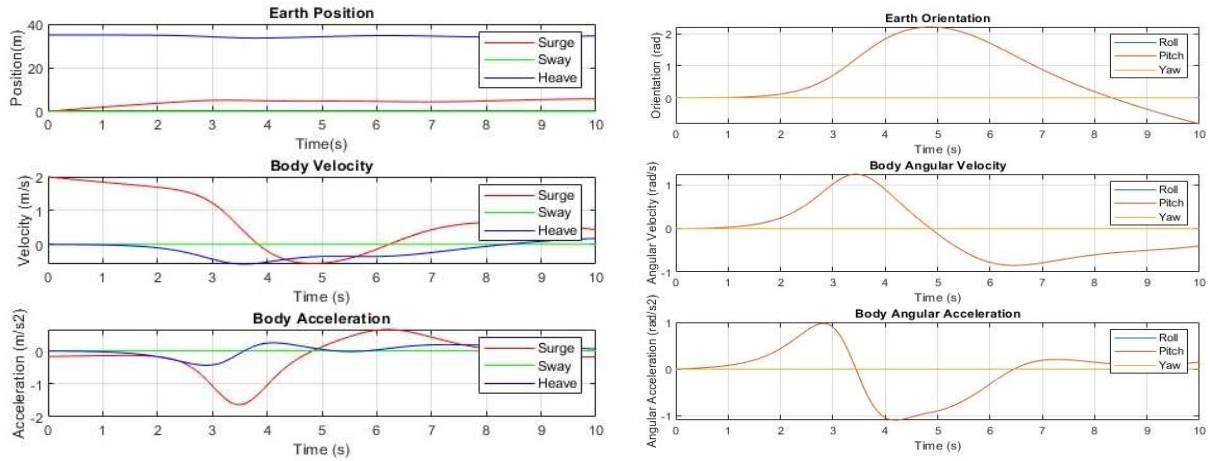


Figure 9: Position, Velocity and Acceleration while linear speed applied on X direction.

8.Conclusion:

The report emphasizes the essential role of the drag matrix in the design of underwater vehicles, demonstrating that every subcomponent, or sub-body, significantly influences hydrodynamic performance and stability. The interplay of drag forces on parts like antennas, thrusters, and the hull is critical to the vehicle's responsiveness. Therefore, the design must consider these forces collectively to optimize each element and achieve superior vehicular performance.

References:

- [1] J. Severholt, "Generic 6-DOF Added Mass Formulation for Arbitrary Underwater Vehicles based on Existing Semi-Empirical Methods," *R. Institue Technol. , Sweden*, p. 51, 2017,
- [2] T. I. Fossen, *Handbook of Marine Craft Hydrodynamics and Motion Control*. Wiley, 2011.
- [3] "IQUA Robotics." <https://iquarobotic5s.com/sparus-ii-auv> (accessed Dec. 20, 2023).
- [4] Modeling and robust control approach for autonomous underwatervehicles(Rui Yang)
- [5] Lecture notes .