Image Denosing Using Discrete Wavelet transform And Thresholding

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***Abstract* - This project proposes an adaptive threshold estimation method for image denoising in the wavelet domain based on the generalized Gaussian distribution modeling of subband coefficients. Wavelet provides an appropriate basis for separating noisy signal from the image signal. When we decompose data using the wavelet transform, we use filters that act as averaging filters, and others that produce details. Some of the resulting wavelet coefficients correspond to details in the data set (high frequency sub-bands). If the details are small, they might be omitted without substantially affecting the main features of the data set. The idea of thresholding is to set all high frequency sub-band coefficients that are less than a particular threshold to zero. These coefficients are used in an inverse wavelet transformation to reconstruct the data set.**

***Keywords* - Wavelet Thresholding, Soft thresholding, Image Denoising, Discrete Wavelet Transform, Adaptive Filtering**

I. Introduction

An image is often corrupted by noise in its acquisition and transmission. Image denoising is used to remove the additive noise while retaining as much as possible the important signal features. Wavelet provides an appropriate basis for separating noisy signal from the image signal. The motivation is that as the wavelet transform is good at energy compaction, the small coefficient are more likely due to noise and large coefficient due to important signal features. It is reasonable to obtain a fast denoising of a given image if we perform two basic operations:

- Eliminate in the wavelet representation those elements with small coefficients, and
- Decrease the impact of elements with large coefficients.

These small coefficients can be thresholded without affecting the significant features of the image. Thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its most basic form, each coefficient is thresholded by comparing against threshold, if the coefficient is smaller than threshold, set to zero; otherwise it is kept or modified. Replacing the small noisy coefficients by zero and inverse wavelet transform on the result may lead to reconstruction with the essential signal characteristics and with less noise.

The flowchart for our approach towards denoising image is:

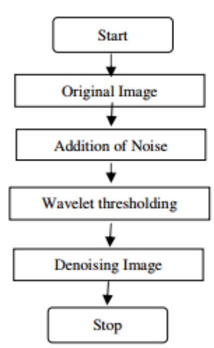

Fig. 1: Flow Chart

In this paper, Section 2 describes theoretical details about 2-D Discrete wavelet transform [2], Section 3 describes general approach of soft wavelet thresholding [1], Section 4 gives a approach for how to mathematically calculate threshold using adaptive filtering [3], Section 5 gives the implementation results.

II. Discrete wavelet transform

2-Channel Perfect Reconstruction Filter Bank: The analysis filter bank decomposes the input signal x(n) into two sub-band signals, c(n) and d(n). The signal c(n) represents the low frequency (or coarse) part of x(n), while the signal d(n) represents the high frequency (or detail) part of x(n). The analysis filter bank first filters x(n) using a low-pass and a high-pass filter. We denote the low-pass filter by af1 (analysis filter 1) and the high-pass filter by af2 (analysis filter 2). As shown in the figure, the output of each filter is then down-sampled by 2 to obtain the two sub-band signals, c(n) and d(n).

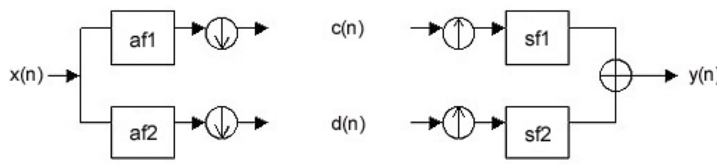

Fig. 2: Channel Reconstruction Filter Bank

The synthesis filter bank combines the two subband signals c(n) and d(n) to obtain a single signal y(n). The synthesis filter bank first up-samples each of the two subband signals. The signals are then filtered using a lowpass and a highpass filter. We denote the lowpass filter by sf1 (synthesis filter 1) and the highpass filter by sf2 (synthesis filter 2). The signals are then added together to obtain the signal y(n). If the four filters are designed so as to guarantee that the output signal y(n) equals the input signal x(n), then the filters are said to satisfy the perfect reconstruction condition.

The discrete wavelet transform (DWT) gives a multiscale representation of a signal x(n). The DWT is implemented by iterating the 2-channel analysis filter bank described above. Specifically, the DWT of a signal is obtained by recursively applying the lowpass/highpass frequency decomposition to the lowpass output as illustrated in the diagram. The diagram illustrates a 3-scale DWT. The DWT of the signal x is the collection of subband signals. The inverse DWT is obtained by iteratively applying the synthesis filter bank.

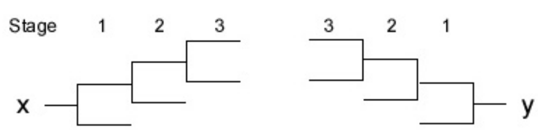

Fig. 3: Multiscale Representation of wavelet transform

In the 2D case, the 1D analysis filter bank is first applied to the columns of the image and then applied to the rows. If the image has N1 rows and N2 columns, then after applying the 1D analysis filter bank to each column we have two subband images, each having N1/2 rows and N2 columns; after applying the 1D analysis filter bank to each row of both of the two subband images, we have four subband images, each having N1/2 rows and N2/2 columns. After implementing discrete wavelet transform, we perform soft thresholding method.

III. Wavelet Thresholding

Let $f = f_{ij}, i, j = 1, 2, ...M$ denote the $M \times M$ matrix of the original image to be recovered and M is some integer power of 2. During transmission the signal f is corrupted by independent and identically distributed (i.i.d) zero mean, white Gaussian Noise n_{ij} with standard deviation σ i.e. $n_{ij} - N(0, \sigma^2)$ and at the receiver end, the noisy observations $g_{ij} = f_{ij} + \sigma n_{ij}$ is obtained. The goal is to estimate the signal f from noisy observations g_{ij} such that Mean Squared error (MSE) is minimum. Let W and W-1 denote the two dimensional orthogonal discrete wavelet transform (DWT) matrix and its inverse respectively.

Then $Y = Wg$ represents the matrix of wavelet coefficients of g having four subbands (LL, LH, HL, HH) . The sub-bands HH_k, HL_k, LH_k are called details, where k is the scale varying from $1, 2, ...J$ and J is the total number of decompositions. The size of the subband at scale k is $N/2^k \times N/2^k$. The subband LL_J is the low-resolution residue.

The wavelet thresholding denoising method processes each coefficient of Y from the detail subbands with a soft threshold function to obtain X . The denoised estimate is inverse transformed to $f = W^{-1}X$. In the experiments, soft thresholding has been used over hard thresholding because it gives more visually pleasant images as compared to hard thresholding; reason being the latter is discontinuous and yields abrupt artifacts in the recovered images especially when the noise energy is significant.

IV. Determination of parameters

This section focuses on the estimation of the GGD parameters, which in turn yields a data-driven estimate of that is adaptive to different sub band characteristics. The noise variance need to be estimate first. In some situations, it may be possible to measure based on information other than the corrupted image. If such is not the case, it is estimated from the sub band by the healthy median estimator - [4],[5]. We calculate the threshold value (T_N), which is adaptive to different subband characteristics.

$$T_N = \frac{\beta\sigma^2}{\sigma_y}$$

where, the scale parameter β is computed once for each scale using the following equation

$$\beta = log\frac{L_k}{v},$$

L_k is the length of the subband at k^{th} scale. σ^2 is the noise variance, which is estimated from the subband HH_1, using the formula:

$$\sigma^2 = \frac{median(Y_{ij})}{0.6745}$$

where $Y_{ij} \in$ subband HH_1 and σ_y is the standard deviation of the subband under consideration computed by using the standard MATLAB command.

V. Implementation Results

Our main discrete wavelet transform function has two parameters, one for noise signal and the other for threshold point. A sample noise signal is taken whose dimension is 512 x 512. We first take the forward DWT over 4 scales (J=4). Then a denoising method called soft thresholding is applied to wavelet coefficients through all scales and subbands. A Function implementing soft thresholding sets coefficients with values less than the threshold(T) to 0, then subtracts T from the non-zero coefficients.

After soft thresholding, we take inverse wavelet transform. We have used a threshold value of 35, which is the optimal threshold point for this case, which is trial and error value for now using the plot of rms error vs threshold, but could otherwise be calculated using algorithm stated in section 4. Image de-noising results are shown below:

VI. Conclusion

In this project, a simple and subband adaptive threshold is proposed to address the issue of image recovery from its noisy counterpart. It is based on the generalized Gaussian distribution modeling of subband coefficients. The image denoising algorithm uses soft thresholding to provide smoothness and better edge preservation at the same time.

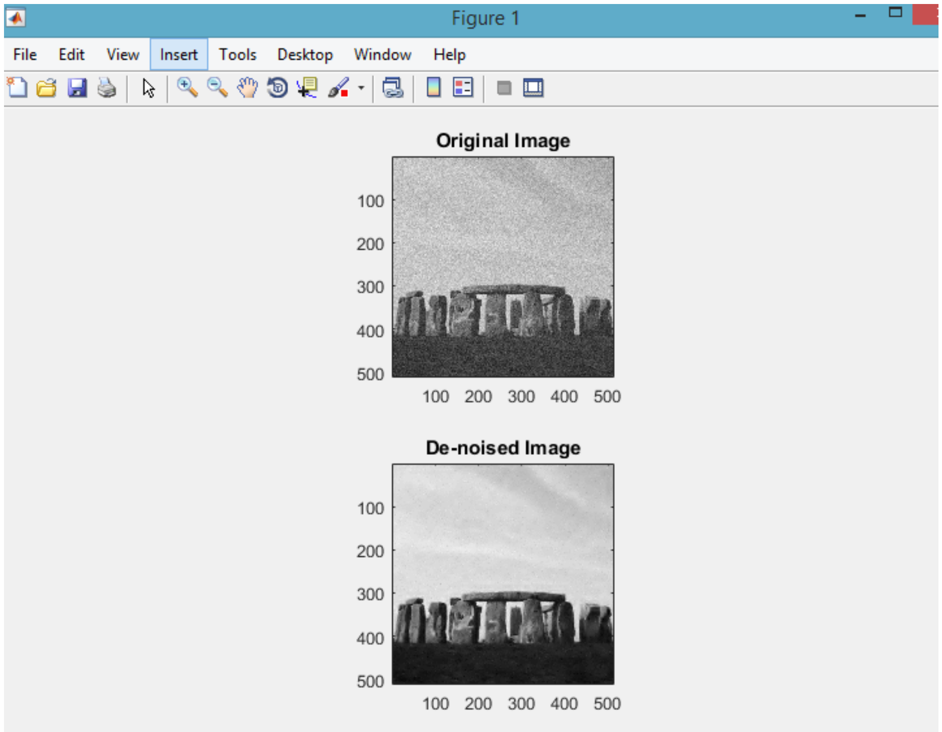

Fig. 4: Noised Image And Denoised image

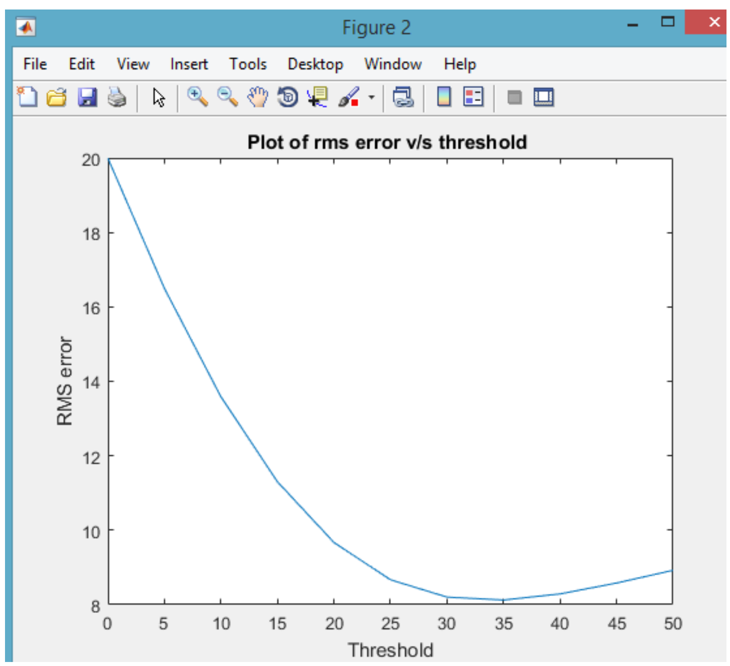

Fig. 5: RMS plot v/s threshold

VII. References

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