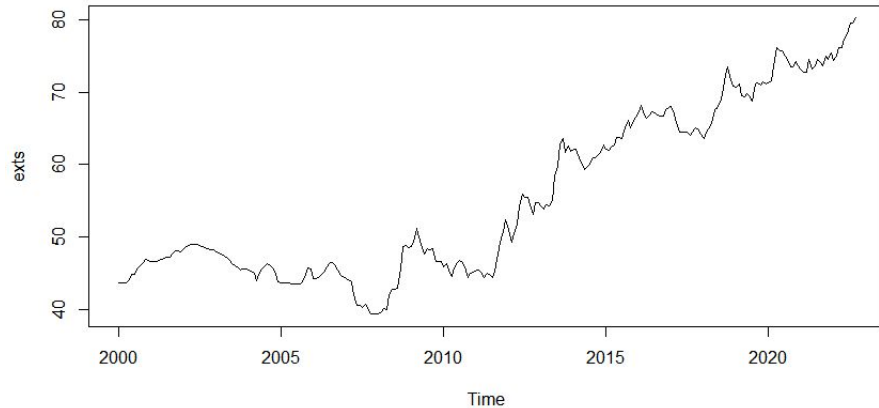


Time Series Definition and Source

Summary(exts):

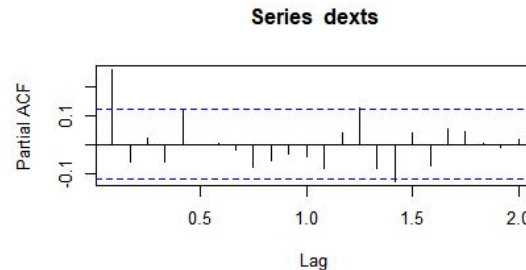
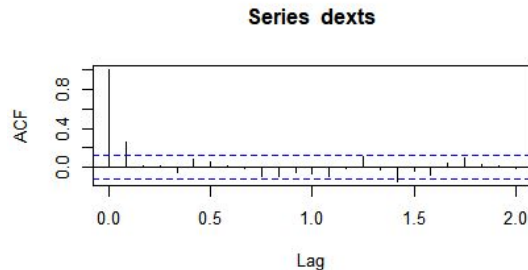
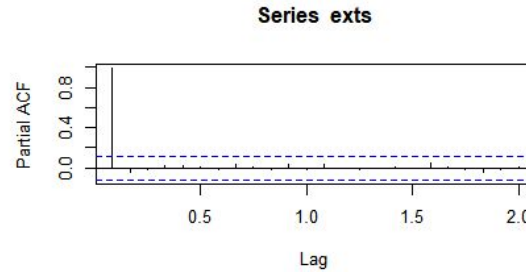
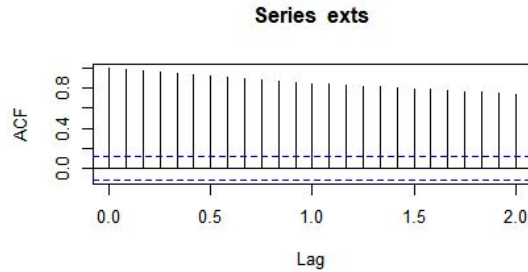
Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
39.27	45.55	48.98	55.44	66.17	80.25



- Indian Rupees to US Dollars Exchange Rate
- Monthly data from Jan, 2000 to Sep, 2022
- Source: Federal Reserve Economic Data
- Link to the data:
<https://fred.stlouisfed.org/series/exinus>

Looking at the time series plot, we can see that the exchange rate is increasing, and is not stationary.

Univariate Analysis

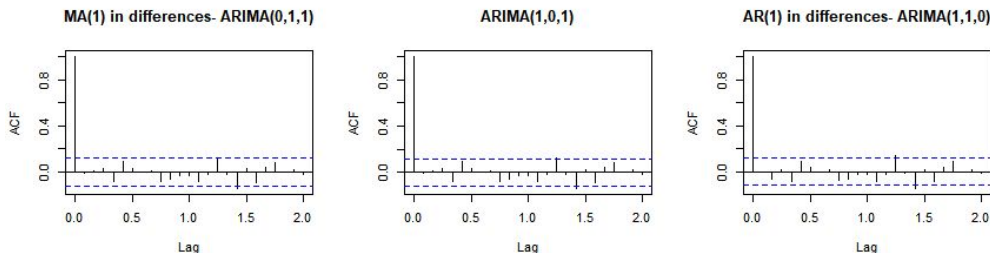


`CADFtest(dexts, type= "drift", criterion= "BIC", max.lag.y=max.lag)` ADF test
ADF(0) = -12.226, p-value < 2.2e-16

- The ACF plot of series in differences(dexts) suggests a Moving Average model of order 1.
- The ADF test for the series in levels has the p-value 0.9811, whereas for the series in differences has p-value < 2.2e-16 which implies the series in differences is stationary.
- We reject the box test for the both the series, thus no white noise.



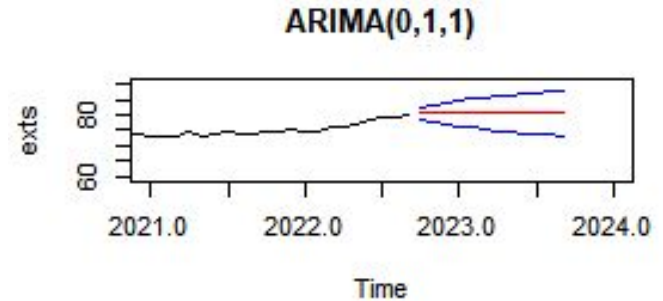
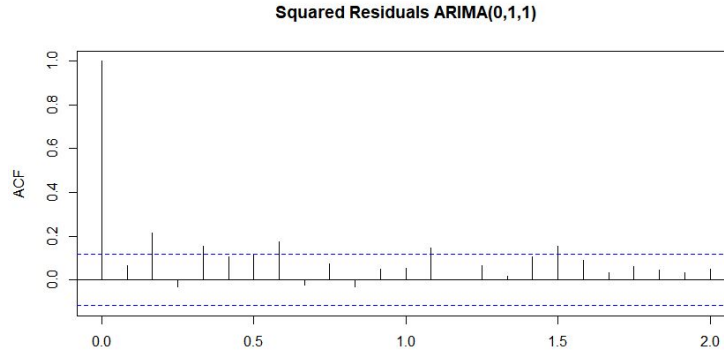
Comparison of Models



	Model Validation			Model Performance			
Model	Highest Order Term Significant	Box Test	Residual Correlogram	AIC	BIC	MAE	Forecasting
ARIMA(0,1,1)	Yes	Yes 0.553	Looks like white noise	691.8572	699.0688	0.8062	
ARIMA(1,0,1)	Yes	Yes 0.585	Looks like white noise	704.1328	718.556	0.8081	
ARIMA(1,1,0)	Yes	Yes 0.395	Looks like white noise	692.7521	699.9637	0.7926	

Comparison of Models, ARIMA(0,1,1) Forecast

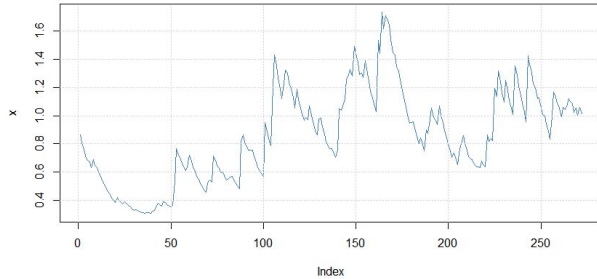
- All the models have highest order terms significant, are validated by the box test and ACF of residuals.
- According to Diebold-Mariano Test at horizon=1, we do not reject H_0 and conclude that the forecast performance of the three models, using the absolute value loss, is not significantly different.
- We suggest model 1, that is ARIMA(0,1,1) on the basis of least BIC and the 12 step ahead forecast with 95% CI is as below



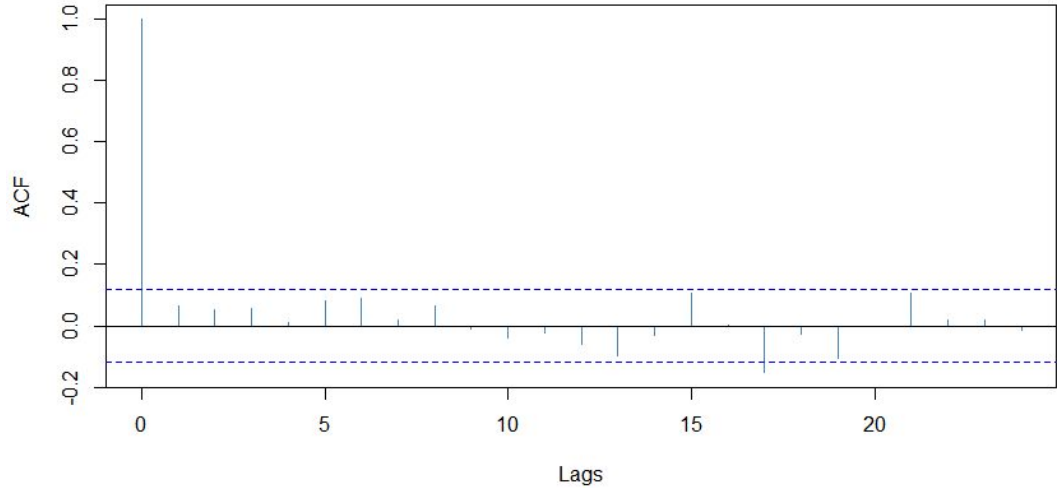
- We can see a few significant autocorrelations in the squared residuals in the selected model which suggests presence of heteroskedasticity.

Garch Effects

Conditional SD



ACF of Standardized Residuals

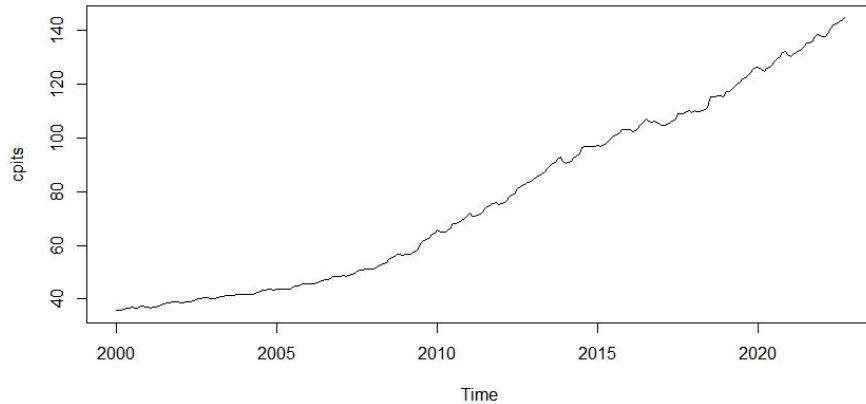


- We fit $ARMA(0,1)$ $GARCH(1,1)$ model to the series in differences, and found significant values of $ma1$, α_1 , and β_1 .
- The QQ-plot of the standardized residuals deviates from normality, we use QMLE. In both cases, we reject H_0 and conclude that the standardized residuals are not normal.
- We observe not the conditional standard deviation is not constant over time and that there are clusters of high volatility.

Multivariate Analysis- Second Series

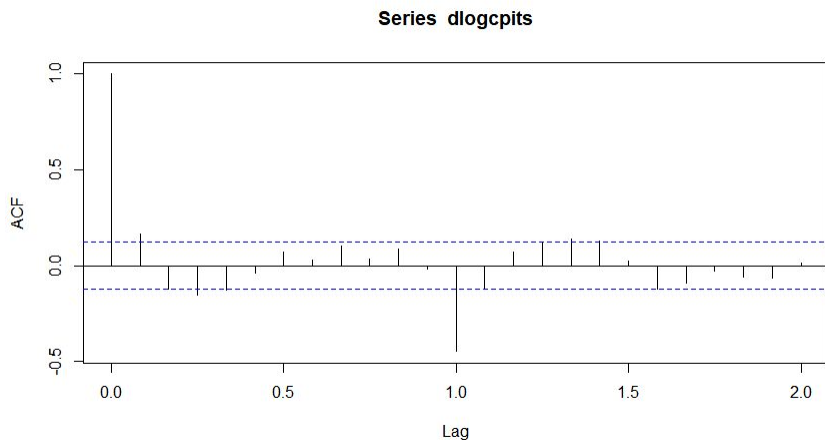
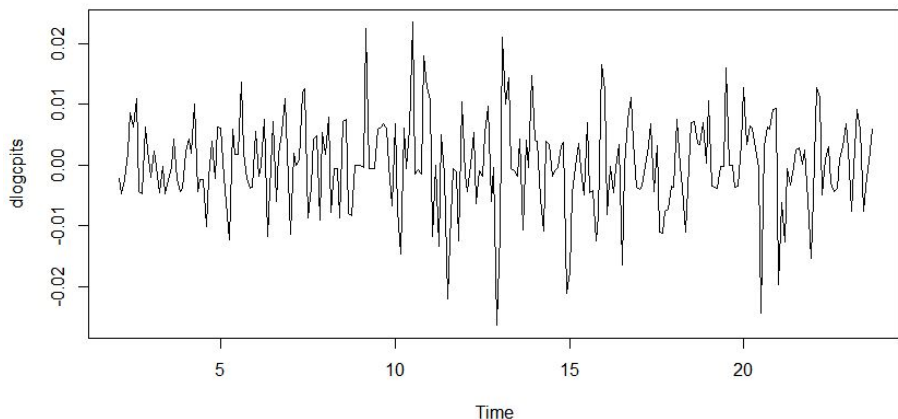
Summary(exts):

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
35.65	44.94	71.92	77.92	105.96	144.66



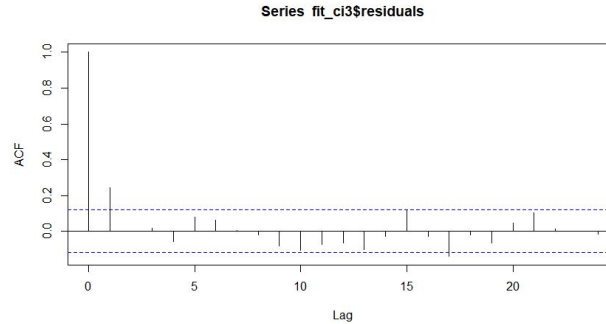
- **Consumer Price Index: All Items for India**
- Monthly data from Jan, 2000 to Sep, 2022
- Source: Federal Reserve Economic Data
- Link to the data:
<https://fred.stlouisfed.org/series/INDCPIALLMINMEI>

Looking at the time series plot, we can see that the CPI is increasing, and is not stationary.



- We might be interested in knowing whether the percentage change in CPI affects the exchange rate of India
- We take the log differences of the series `cpi`, as we are interested in the percentage change and `log(cpi)` is an `I[1]` series.
`CADFtest(dlogcpits, type= "drift", criterion= "BIC", max.lag.y=max.lag)`
`ADF(11) = -6.3485, p-value = 5.501e-08`
- We also reject the Box Test for both the series, which implies no white noise.
- An Engle-Granger test finds no cointegration relationship between the two series(`exts`, `cpits`).
`CADFtest(res_fit_ci2,type="drift",criterion="BIC",max.lag.y=max.lag)`
`ADF(1) = -1.7746, p-value = 0.3926`
 We do not reject the hypothesis of no cointegration.
- Now we test for Granger causality, we predict the exchange rate with an `ADLM(1)` as the correlogram of a linear regression fit of `dlogcpits` on `dexts` has significant autocorrelation at lag 1(graph on next slide).

Granger Causality



- Then we test for Granger causality by comparing the ADLM(1) with the model without lagged explanatory variables, and we did not find significant difference between the two models. Thus $\log(\text{cpi})$ has **no incremental explanatory power** in predicting the exchange rate(exts). (p-value of anova test between the two models = 0.6665)

VAR Model

```
VARselect(multivardata, lag.max = max.lag, type = "const")
```

```
$selection
```

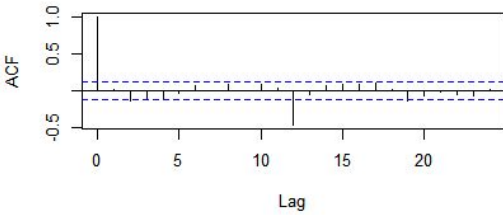
```
AIC(n) HQ(n) SC(n) FPE(n)
```

```
12    1    1    12
```

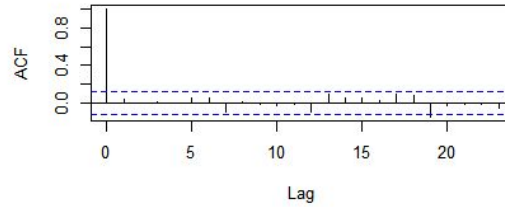
- First we choose VAR(1) as suggested by SC. However, this specification leaves several significant autocorrelations and cross-correlation at lag 12 in the residuals(as seen in the graph on the next slide).
- So, we choose VAR(12) based on AIC.

VAR Model and impulse response

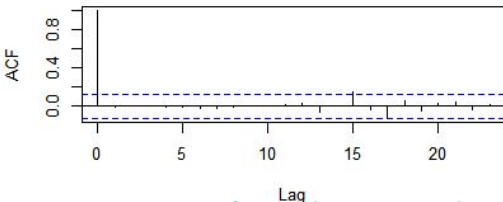
Series var1_residuals[, 1]



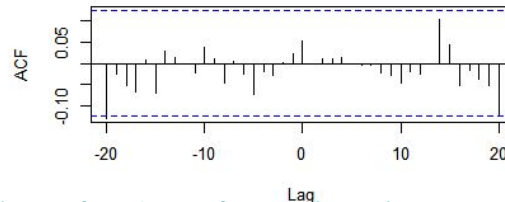
Series var12_residuals[, 1]



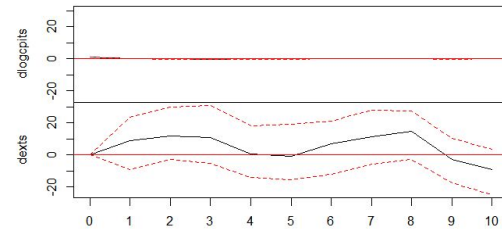
Series var12_residuals[, 2]



var12_residuals[, 1] & var12_residuals[, 2]



Impulse Response from dlogcpi



95 % Bootstrap CI, 100 runs

- VAR(12) has no notable correlation structure or cross-correlation structure left in the residuals, they look like multivariate white noise (we do not reject H_0 for the box test for both the series).
- The VAR(1) is significant with lower BIC, less parameters, and higher multiple R squared, whereas VAR(12) has multivariate white noise structure.
- We continue with the VAR(12) model.

- We see that given a unitary impulse from log cpi at time t , we see a significant positive response in exchange rate at time $t+2$, $t+8$.

VECM and Forecast

- The figure in the top right is the 12 step ahead forecast using VAR
- The Schwarz criterion selects the order 2 for the VAR on the time series in levels, but the AIC and FPE suggests the order 16.
- According to Johansen's trace test statistic, there is at least one cointegrating relation. Hence, exts and $\log(\text{CPI})$ are cointegrated. $(30.33128 + \text{exts} - 24.2574 \log(\text{CPI}) = \delta t)$
- We estimate a VECM(15) on the basis of AIC
- The figures in the bottom right plots the 12 step-ahead forecast of exts and $\log(\text{CPI})$ based on the VECM(15).

