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Q1. Let $(x_1, x_2 \dots x_n)$ be random sample of size n from Normal with θ_1 = mean & variance = θ_2. Find MLE

Sol:- Let mean = μ and variance = σ^2

Probability density function:

$$f(x_i / \theta_1, \theta_2) = \frac{1}{\sqrt{\theta_2}\sqrt{2\pi}} \exp\left[-\frac{(x_i-\theta_1)^2}{2\theta_2}\right]$$

For $-\infty < \theta_1 < \infty$ & $0 < \theta_2 < \infty$, we get

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i / \theta_1, \theta_2) = \theta_2^{-n/2} (2\pi)^{-n/2} \exp\left[-\frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i-\theta_1)^2\right]$$

Taking log,

$$\log L(\theta_1, \theta_2) = -\frac{n}{2} \log \theta_2 - \frac{n}{2} \log(2\pi) - \frac{\Sigma (x_i - \theta_1)^2}{2\theta_2}$$

now,
$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_1} = \frac{-2\Sigma(x_i - \theta_1)(-1)}{2\theta_2} = 0$$

$$\Rightarrow \quad \Sigma x_i - n\theta_1 = 0$$

$$\therefore \hat{\theta}_1 = \frac{\Sigma x_i}{n} = \bar{x}$$

Now
$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{\Sigma(x_i - \theta_1)^2}{2\theta_2^2} = 0$$

$$\hat{\theta}_2 = \frac{\Sigma(x_i - \hat{\theta}_1)^2}{n} = \frac{\Sigma(x_i - \bar{x})^2}{n} = \hat{\sigma}^2$$

$\therefore$ maximum likelihood estimators

$$\theta_1 = \frac{\Sigma x_i}{n} = \bar{X} = \text{mean}$$

$$\theta_2 = \frac{\Sigma(x_i - \bar{x})}{n} = \sigma^2 = \text{Variance.}$$

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Q: Let $x_1, x_2 ...$ be a random sample from $B(m, \theta)$ where $\theta \in (0,1)$
Compute θ using MLE

Probability mass func. of binomial = (PMF)
Let y be the number of successes resulting from m independent trials with unknown success prob. θ

$$\therefore PMF = {}^{m}C_{y}\, \theta^{y} (1-\theta)^{m-y}$$

Taking log, we get $\log {}^{m}C_{y} + y\log\theta + (m-y)\log(1-\theta)$

now $\frac{\partial(\log {}^{m}C_{y})}{\partial\theta} + \frac{\partial}{\partial\theta}(y\log\theta) + \frac{\partial}{\partial\theta}(m-y)\log(1-\theta) = 0$

$$\Rightarrow \frac{y}{\theta} - \frac{m-y}{1-\theta} = 0$$

$$y - \cancel{y\theta} - m\theta + \cancel{y\theta} = 0$$

$$y = m\theta$$

$$\hat{\theta} = \frac{y}{m}$$

$\therefore$ The maximum likelihood estimator

$$\hat{\theta} = \frac{y}{m}.$$