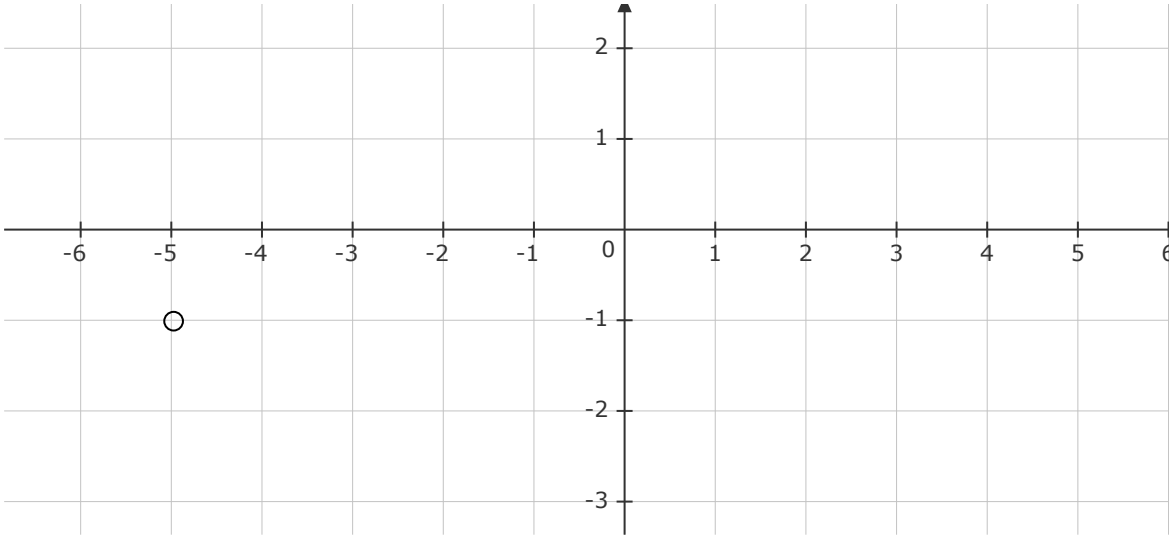
Sara NUMERI COMPLESSI



Definizione intuitiva numeri complessi

$$P(x, y) = (-5, -1)$$

$$P = (-5, 0) - (0, 1) = (-5, -1) = -5 - i$$

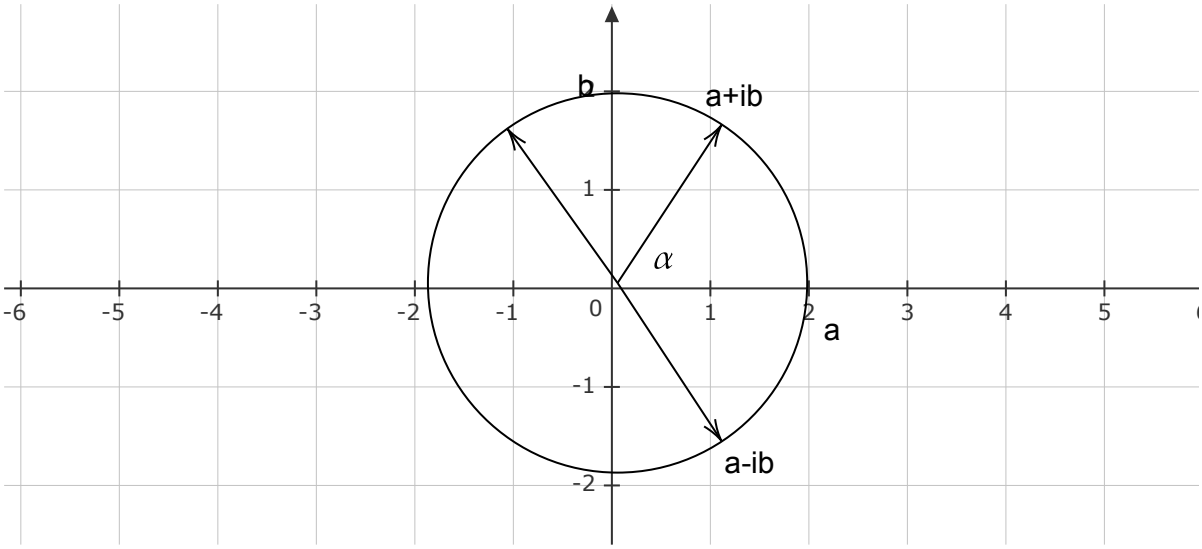
$$i = (0, 1)$$

$$\mathbf{i^2 = -1}$$

$$(0, 1) * (0, 1) = (0 - 1, 0 + 0) = (-1, 0) = -1$$

$$P_1(a, b)\ P_2(c, d)$$

$$P_1 * P_2 = (ac - bd,\ ad + bc)$$



Modulo numero complesso

$$I\,z\,I = \sqrt{a^2 + b^2}$$

"Sistimare" un numero complesso in frazione

$$\frac{1}{(1+i)}\frac{(1-i)}{(1-i)} = \frac{1-i}{1-i+i-i^2} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$$

Es numero complessi

$$\frac{3+i}{2+i}$$

$$\frac{(3+i)}{(2+i)}\frac{(2-i)}{(2-i)} = \frac{6-3i+2i-i^2}{5} = \frac{7-i}{5} = \frac{7}{5} - \frac{1}{5}i$$

$$\Re e\left\{\frac{3+i}{2+i}\right\} = \frac{7}{5}$$

$$\Im()...$$

$$modulo = \sqrt{\frac{49}{25} + \frac{1}{25}} = \sqrt{2}$$

Possibili scritture di un numero complesso

Argomento numero complesso

$$arg\ z = Arctg\left(\frac{y}{x}\right)\ caso\ generale,\ ma\ a\ seconda\ del\ quadrante\ in\ cui\ mi\ trovo\ cambia\ leggermente\ questa\ formula$$

$$\theta := Arg(z) = \begin{cases} \frac{\pi}{2} & \text{se } a = 0,\ b > 0 \\ -\frac{\pi}{2} & \text{se } a = 0,\ b < 0 \\ \text{non definito} & \text{se } a = 0,\ b = 0 \\ \arctan\left(\frac{b}{a}\right) & \text{se } a > 0,\ b \text{ qualsiasi} \\ \arctan\left(\frac{b}{a}\right) + \pi & \text{se } a < 0,\ b \geq 0 \\ \arctan\left(\frac{b}{a}\right) - \pi & \text{se } a < 0,\ b < 0 \end{cases}$$

Forma numero complesso

$$e^{it} = cos(t) + i\ sen(t)\ (Formula\ eulero)$$

FORMA ESPONENZIALE

$$z = !z!\ e^{it}$$

t e' l'argomento del numero complesso

FORMA ALGEBRICA

$$z = a + ib$$

FORMA TRIGONOMETRICA

$$z = !z!\ [cos(t) + i\ sen(t)]$$

t e' l'argomento del numero complesso

Esercizio compito (1) del 26/05/2015

Scrivere forma trigonometrica, esponenziale, algebrica del seguente numero complesso

$$z = \frac{2i+1}{3+i}$$

$$z = \frac{2i+1}{3+i}\frac{3-i}{3-i} = \frac{6i+2+3-i}{9+1} = \frac{5i+5}{10} = \frac{1}{2}+\frac{1}{2}i$$

$$z = \frac{1}{2}+\frac{1}{2}i \qquad (forma\ algebrica)$$

$$!z! = \frac{\sqrt{2}}{2}$$

$$arg\ z = \ arctg(1) = 45^0 \left(= \frac{\pi}{4}\right)$$

$$tg(\alpha) = 1$$

$$\frac{sen(\alpha)}{cos(\alpha)} = 1$$

$forma\ trigonometrica$:

$$z = \frac{\sqrt{2}}{2}\left[cos\left(\frac{\pi}{4}\right) + i\ sen\left(\frac{\pi}{4}\right)\right] = \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}(1+i) = \frac{1}{2}(1+i)$$

$forma\ esponenziale$:

$$z = \frac{\sqrt{2}}{2}\ e^{i\frac{\pi}{4}}$$

α (°)	α (rad)	sen α	cos α
$0°$	0	0	1
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$90°$	$\frac{\pi}{2}$	1	0
$180°$	π	0	-1