Integrali esame

$$\int fg = F{*}g - \int g'F$$

$$\frac{1}{x^2},\ \ln\left(1+4x^4\right)\ \ per\ x \to 0$$
$$x = 2\ \to 0.25,\ 4.17$$
$$x = 1 \to 1,\ 1.6$$
$$x = \frac{1}{2} \to 4,\ 0.22$$
$$x = \frac{1}{4} \to 16,\ 0.0145$$

$$\int_0^1 \frac{\ln\left(1+4x^4\right)}{x^3} dx = \left[-\frac{1}{2x^2} * \ln\left(1+4x^4\right) \right]_0^1 + 2\int_0^1 \frac{4x}{1+\left(2x^2\right)^2}\, dx =$$

$$= -\frac{1}{2}\ln(5) + \left[2\arctan\left(2x^2\right)\right]_0^1 = -\frac{1}{2}\ln(5) + 2\arctan(2) - 2\arctan(0)$$

$$(Digressione\ 1)$$
$$g = \ln\left(1+4x^4\right),\ g' = \frac{16x^3}{1+4x^4}$$
$$f = x^{-3},\ F = -\frac{1}{2}x^{-2}$$

$$(Digressione\ 2)$$
$$2\int_0^1 \frac{4x}{1+\left(2x^2\right)^2}\, dx$$

$$f = \frac{1}{1+\varphi^2},\ F = \arctan(\varphi)$$

$$\varphi = 2x^2,\ \varphi' = 4x$$

Integrale

$$\int_0^{+\infty} \frac{1}{e^{2x}+2e^x+2} dx$$

Studio convergenza

$$\frac{1}{e^{2x}+2e^x+2} < \frac{1}{x^2+2x+2} \approx \frac{1}{x^2}$$

$$Siccome\ la\ funzione\ g = \frac{1}{x^2}\ e'\ integrabile\ impropiamente\ in\ [0,+\infty[$$
$$allora\ la\ funzione\ \frac{1}{e^{2x}+2e^x+2}\ e'\ integrabile\ in\ senso\ improprio\ su\ [0,+\infty[$$
$$ne\ consegue\ che\ l'integrale\ iniziale\ e'\ convergente$$

$$\int_0^{+\infty} \frac{1}{e^{2x}+2e^x+2} dx$$

$$e^x = t,\ e^0 = 1,\ e^{+\infty} = +\infty$$

$$\frac{d}{dx}e^x = \frac{dt}{dx} \to e^x = \frac{dt}{dx} \to t = \frac{dt}{dx} \to dx = \frac{dt}{t}$$

$$\int_1^{+\infty} \frac{1}{t\left(t^2+2t+2\right)} dt =$$

$$\frac{1}{t\left(t^2+2t+2\right)} = \frac{A}{t} + \frac{Bt+C}{t^2+2t+2}$$

$$1 = A\left(t^2+2t+2\right) + Bt^2 + Ct$$

$$1 = t^2(A+B) + t(2A+C) + 2A$$

$$A+B = 0$$
$$2A+C = 0$$
$$2A = 1$$

$$B = -\frac{1}{2}$$
$$C = -1$$
$$A = \frac{1}{2}$$

$$\int_1^{+\infty} \frac{1}{2t} + \frac{1}{2}\frac{t-2}{t^2+2t+2}dt =$$

$$\int_1^{+\infty} \frac{1}{2t} + \frac{1}{2}\frac{t+2-4}{t^2+2t+2}dt =$$

$$\int_1^{+\infty} \frac{1}{2t}dt + \frac{1}{2}\int_1^{+\infty} \frac{t+2}{t^2+2t+2} + \frac{1}{2}\int_1^{+\infty} \frac{-4}{t^2+2t+2}dt =$$

$$\int_1^{+\infty} \frac{1}{2t}dt + \frac{1}{2}\int_1^{+\infty} \frac{t+2}{t^2+2t+2} + \frac{1}{2}\int_1^{+\infty} \frac{-4}{t^2+2t+1+1}dt = ...$$

$$-2\int_1^{+\infty} \frac{1}{(t+1)^2+1}dt =$$

$$\int_1^{+\infty} \frac{1}{2t}dt + \frac{1}{2}\int_1^{+\infty} \frac{t+2}{t^2+2t+2} - 2\int_1^{+\infty} \frac{1}{(t+1)^2+1}dt =$$

$$\left[\frac{1}{2}\ln(t)\right]_1^{+\infty} + \left[\frac{1}{2}\ln\left(t^2+2t+2\right)\right] - [2\arctan(t+1)] =$$

$$e^x = t$$

$$\left[\frac{1}{2}x\right]_0^{+\infty} + \left[\frac{1}{2}x^2 * x * \frac{1}{2}\ln(2)\right] - \left[2\arctan\left(e^x+1\right)\right]$$

Se nella fattorizzazione appare	Associamo
$x - a_i$	$\frac{A_i}{x - a_i}$
$(x - a_i)^n$	$\frac{A_{i,1}}{(x - a_i)} + \frac{A_{i,2}}{(x - a_i)^2} + \ldots + \frac{A_{i,n}}{(x - a_i)^n}$
$x^2 + a_i x + b_i$ con $\Delta < 0$	$\frac{A_i x + B_i}{x^2 + a_i x + b_i}$
$(x^2 + a_i x + b_i)^n$ con $\Delta < 0$	$\frac{A_{i,1} x + B_{i,1}}{x^2 + a_i x + b_i} + \frac{A_{i,2} x + B_{i,2}}{(x^2 + a_i x + b_i)^2} + \ldots + \frac{A_{i,n} x + B_{i,n}}{(x^2 + a_i x + b_i)^n}$