Integrali funzione composta

Esempio 1

$$\left[\int f(\varphi(x))\varphi'(x)\,dx \;=\; F(\varphi(x))+c\right]$$

$$f = \frac{1}{\sqrt{1-\varphi(x)^2}}$$

$$\varphi = e^x$$

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}dx = \int \frac{e^x}{\sqrt{1-\left(e^x\right)^2}}dx = \arcsin\left(e^x\right)+c$$

$$\left[\int \frac{1}{1-f(x)^2}dx = \arcsin(f(x))+c\right]$$

Esercizio 2

$$\int \frac{x^2}{\sqrt{1-x^6}}dx \;=\frac{1}{3}\int \frac{3x^2}{\sqrt{1-\left(x^3\right)^2}}dx \;=\frac{1}{3}\arcsin\left(x^3\right)$$

$$f \;=\; \frac{1}{\sqrt{1-\varphi(x)^2}}$$

$$\varphi \;=\; x^3,\; \varphi' \;=\; 3x^2$$

Integrali per sostituzione

Es 1

$$\int_0^{\frac{\pi}{2}} \frac{\sin(x)+2}{\sin^2 x+1}cos(x)dx = \int_0^1 \frac{t+2}{t^2+1}dt =$$

$$\sin(x) = t$$

$$x = \arcsin(t)$$

$$sin\left(\frac{\pi}{2}\right) = 1$$
$$sin(0) = 0$$

$$\frac{d}{dx}sin(x) = \frac{d}{dx}t$$

$$cos(x) = \frac{dt}{dx} \longrightarrow dt = \cos(x)dx$$

$$\int_0^1 \frac{t+2}{t^2+1}dt = \int_0^1 \left(\frac{t}{t^2+1} + \frac{2}{t^2+1}\right)dt = \int_0^1 \frac{t}{t^2+1}dt + 2\int_0^1 \frac{1}{t^2+1}\, dt =$$
$$\frac{1}{2}\int_0^1 \frac{2t}{t^2+1}dt + 2\int_0^1 \frac{1}{t^2+1}\, dt = \left[\frac{1}{2}\ln\left(t^2+1\right)\right]_0^1 + [2\arctan(t)]_0^1$$

$$f = \frac{1}{\varphi(t)+1}$$

$$\varphi(t) = t^2$$

$$\left[\frac{1}{2}\ln\left(t^2+1\right)\right]_0^1 + [2\arctan(t)]_0^1 = \frac{1}{2}\ln(2) + \frac{\pi}{2}$$

$$\arctan(1) = 45^o =$$

$$\pi : 180 = x : 45^o$$

$$0^0 = 0$$
$$180^0 = \pi$$
$$90^0 \ldots.$$

$$sin\left(60^o\right) = \frac{\sqrt{2}}{2}$$

$$\tan(\alpha) = 1$$

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$$

Esercizio 2

$$\int_0^1 x\sqrt{x+1}\,dx = \int_0^1 x(x+1)^{\frac{1}{2}}dx$$

$$t = (x+1)^{\frac{1}{2}},\ x = t^2 - 1$$

$$\frac{dt}{dx} = \frac{1}{2}(x+1)^{-\frac{1}{2}}$$

$$dt \ = \frac{1}{2}(x+1)^{-\frac{1}{2}}\,dx$$

$$\frac{1}{2}(x+1)^{-\frac{1}{2}}\,dx = dt$$

$$dx = 2dt(x+1)^{\frac{1}{2}} = 2t\,dt$$

$$\int_1^{\sqrt{2}}\left(t^2-1\right)2t^2\,dt = \int_1^{\sqrt{2}}2t^4dt + \int_1^{\sqrt{2}} - 2t^2\,dt = \left[2\frac{t^5}{5}\right]_1^{\sqrt{2}} + \left[-2\frac{t^3}{3}\right]_1^{\sqrt{2}} =$$

$$= \frac{2}{5}2^{\frac{5}{2}} - \frac{2}{5} - \frac{2}{3}2^{\frac{3}{2}} + \frac{2}{3}$$

Es 3

$$\int_2^3 \frac{x^{\frac{1}{2}}}{1+x}\,dx$$

$$t = x^{\frac{1}{2}},\ x = t^2\ \left(\sqrt{2},\sqrt{3}\right)$$

$$\frac{dt}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$2dt\, x^{\frac{1}{2}} = dx$$

$$dx = 2t\, dt$$

$$2\int_{\sqrt{2}}^{\sqrt{3}} \frac{t^2+1-1}{1+t^2}\, dt = 2\int_{\sqrt{2}}^{\sqrt{3}} 1 - \frac{1}{1+t^2}\, dt = 2[t]_{\sqrt{2}}^{\sqrt{3}} - 2[\arctan(t)].....$$