"Number System"

Face value: 6783, Face value of 6 = 6

Place value: 6783, Place value of 6 = 6000

Classification of Numbers: All numbers in this world fall under the category of complex numbers.

Complex Number ($a+ib$)

→ Real Number ($-\infty$ to $+\infty$)

→ Imaginary No. $\sqrt{-1} = i$, $i^2 = -1$

Real Number → Decimal, Integer

Decimal
- Rational: (i) Terminating
- Rational: (ii) Non terminating, Repeating
- Irrational: (iii) Non terminating, Non Repeating

Integer
- → Positive ($+1, +2 \ldots +\infty$)
- → Zero
- → Negative ($-1, -2, \ldots -\infty$)

$0.5 = 5/10$ (Terminating Decimal) → Rational

$0.3333333 = 0.\bar{3}$ (Non terminating, Repeating) $= 3/9$ → Rational

$\pi, \sqrt{2}, \sqrt{3}$ all are Irrational numbers. $\sqrt{5}, \sqrt{7}$

Integers → Even (no. divisible by 2) $(-\infty \ldots -4, -2, 0, 2, 4)$
→ Odd (no. not divisible by 2)

Positive Integers

- Prime no (No. having only two factors 1 & itself)
- Composite (No. having more than two factors) e.g. 4, 6, 8
- Neither prime nor composite (e.g. = 1)

** Every prime no. is written in the form $6N+1$, $6N-1$

- 2 is the smallest prime no.
- 2 is the only even prime no.
- 101 is the smallest 3 digit prime no.

Relatively Prime : Set of any two no. having HCF = 1
e.g. (1, 3), {9, 25}

Co-prime No: Set of two prime No. having HCF = 1
e.g. {2, 3}, {5, 7}

- Every co-prime is relatively prime no.

Twin-Prime No. Set of two prime no. having difference is 2
e.g. (5, 7), (17, 19), (11, 13)

** Perfect No: If sum of all factors of a number (excluding that no.) is equal to that number.

e.g. 6 = 1, 2, 3, 6 = 1+2+3 = 6
28 = 1, 2, 4, 7, 14, 28 = 1+2+4+7+14 = 28

Extra Numerical

Q. Rational numbers b/w a & B = $\boxed{\dfrac{aK+B}{K+1}}$ wher K = 1, 2, 3 -- ∞

Q. If P and q are two relatively prime no. such that $P+q = 10$
P < q. How many pairs are possible for P, q

ans: $P + q = 10$

$1+9 = 10$ ✓

$2+8 = 10$ ✗

$3+7 = 10$ ✓

$4+6 = 10$ ✗

{1, 9}, {3, 7}

$\boxed{\text{2 pairs}}$ are possible.

DIVISIBILITY Rule:

→ 0 is divisible by all numbers except 0, but no number is divisible by 0.

→ 1 is not divisible by any number except 1, but all numbers are divisible by 1.

→ $\boxed{2}$ → (A no. is divisible by 2, if its last digit is 0 or Even)

→ $\boxed{3}$ → (A no. is divisible by 3, if sum of all digits is divisible by 3 for e.g. 4689)

→ $\boxed{4}$ → (A no. is divisible by 4 if the no. formed by the last 2 digits is divisible by 4)

→ $\boxed{5}$ → (A no. is divisible by 5 if the last digit is 0 or 5)

→ $\boxed{6}$ → (A no. is divisible by 6 if it is divisible by both 2 and 3)

→ $\boxed{7}$ (Take the unit's digit of the number, multiply it by 2 and subtract from the remaining number, if this is divisible by 7, the original number is also divisible by 7)

→ $\boxed{8}$ (A no. is divisible by 8 if the no. formed by Last three digits is divisible by 8)

→ $\boxed{9}$ (sum of digits should by divisible by 9)

→ $\boxed{10}$ (A no. is divisible by 10 if the Last digit is Zero)

→ $\boxed{11}$ (If the difference b/w sum of all digits at odd places & sum of all digits at even places is divisible by 11 or becomes zero)

→ $\boxed{12}$ (A no. is divisible by 12, if it is divisible by both 3 and 4)

Divisibility Rule of 3 & 9 can't be expanded for 27 as well.

e.g. Let take 54, sum of digit = 9, which is not divisible by 27.

but 54 is divisible by 27.

How the Rule goes for 3, & 9

$$xy = 10x + y = \underbrace{\boxed{9x}}_{\text{div. by } 3 \& 9} + x + y$$

Now we have to check $x+y$

if $(x+y)$ is divisible by 3 then it will be divisible by 3
if $(x+y)$ is divisible by 9 then it is div. by 9.

Similarly $xyz = 100x + 10y + z$

$$= \underbrace{99x + 9y}, + x + y + z$$

we have to Check for $x+y+z$

Questions: Find remainder when $857873\boxed{025}$ divided by 8.

ans: Rem = 1

Ques: Find Rem. when no. 53762 is divided by 11.

ans: Odd $= (5+7+2) = 14$, even $= (3+6) = 9$

Difference $= 14-9 = 5$

So remainder will be 5.

Q. If a no. $467x893$ is divisible by 3. Find sum of all possible values of x.

ans: $4+x$ so, $x = 2, 5, 8$

so, Sum $= 2+5+8 = \boxed{15}$

Q. If a no. $875x321$ is divisible by 9, then find value of x.

ans: $8+x$ $\boxed{x=1}$

** Q. If 876A43B is divisible by 9. & B is an even no. Find the sum of all values of A.

ans: $1+A+B$ it should be divisible by 9.

$1+6+2$

$1+4+4$

$1+2+6$

$1+\boxed{0,9}+8$

Sum of all values of A $= 6+4+2+0+9$

$= \boxed{21}$

Question: If a no. $857x32$ is divisible by 11. Find value of x.

ans: $857x32$ $\quad 3+7-x+1 =$ difference

$11-x$ so, $\boxed{x=0}$

Combined divisibility Rule of 7, 11, 13

$\underbrace{659}_{odd}\underbrace{624}_{even}$ difference $= 35 \rightarrow$ $\boxed{\text{divisible by } 7}$
not divisible by 11
not divisible by 13

so, given no. also divisible by 7
not divisible by 11
not divisible by 13.

If difference $= 77$, $\rightarrow$ No. will be divisible by 7 & 11 but not by 13.

Question: If a six digit no. is formed by repeating a three digit no. for e.g. 214214, 656656
so, this resultant no. will be divisible by
(a) Only 7 (b) Only 11 (c) only 13 (d) ~~Only~~ 1001

ans: $\underbrace{656}\underbrace{656}$ diff $= 0$ $\rightarrow$ div by 7
$\rightarrow$ div by 11
$\rightarrow$ div by 13

so, no. also divisible by $= 7 \times 11 \times 13$
$= \boxed{1001}$

$\boxed{\text{OR}}$ $656656 = 656000 + 656$
$= 656(1000+1) = 656 \times 1001$

It will be divisible by 1001.

<u>Question</u> If A, B, C, D, E are digits and ABC × DEED = ABCABC
Find DEED <u>ans</u> : $\boxed{1001}$

∴ AB × 101 = ABAB

∴ ABC × 1001 = ABCABC

∴ AB × 10101 = ABABAB

How to make divisibility Rule :

Let's take 13

(Last digit) x + 1
↓
any no.
(Last digit) any no. + Rem

for $\boxed{x = 4}$ it will be divisible by 13
↓ odd factor

<u>Q.</u>

$$\begin{array}{r} 104 \\ +16 \\ \hline \boxed{26} \end{array} \longrightarrow \times 4 \qquad \begin{array}{r} 169 \\ 36 \\ \hline \boxed{52} \end{array} \longrightarrow 4$$

<u>Odd factors</u> :

for 11 ⟶ −1
for 17 ⟶ −5
for 29 ⟶ 3
for 7 ⟶ −2

Unit digit concept:

$2^1 = \boxed{2}$ $\quad 2^5 = 3\boxed{2}$

$2^2 = \boxed{4}$ $\quad 2^6 = 6\boxed{4}$

$2^3 = \boxed{8}$ $\quad 2^7 = 12\boxed{8}$

$2^4 = 1\boxed{6}$ $\quad 2^8 = 25\boxed{6}$

cyclicity = 4

Cyclicity 1: $\{0, 1, 5, 6\}^n = \{0, 1, 5, 6\}$

Question: Find unit digit of $5^{444} + 6^{666} = 5 + 6 = \boxed{1}$

Cyclicity 2: $\{4, 9\}$

$4^1 = \boxed{4}$ | $4^3 = 6\boxed{4}$

$4^2 = 1\boxed{6}$ | $4^4 = 25\boxed{6}$

it means $\boxed{4^{odd} = 4, \; 4^{even} = 6}$

Similarly for 9, $9^1 = \boxed{9}$ | $9^3 = 72\boxed{9}$

$9^2 = 8\boxed{1}$ | $9^4 = 656\boxed{1}$

it means $\boxed{9^{odd} = 9, \; 9^{even} = 1}$

Question: Find unit digit $9^{124} + 4^{123}$

$= 1 + 4 = \boxed{5}$

Cyclicity 4: $\{2, 3, 7, 8\}$

$3^1 = \boxed{3}$ | $3^5 = 24\boxed{3}$ $\quad 7^1 = 7$ $\quad 8^1 = 8$

$3^2 = \boxed{9}$ | $3^6 = 72\boxed{9}$ $\quad 7^2 = 9$ $\quad 8^2 = 4$

$3^3 = 2\boxed{7}$ | $3^7 = 218\boxed{7}$ $\quad 7^3 = 3$ $\quad 8^3 = 2$

$3^4 = 8\boxed{1}$ | $3^8 = 656\boxed{1}$ $\quad 7^4 = 1$ $\quad 8^4 = 6$

Question: $2^{1037} = 2^1 = \boxed{2}$

Question $7^{5398} = 7^2 = \boxed{9}$

Question $8^{1000} = 8^4 = 64 \times 64 = \boxed{6}$

Question $3^{41} \times 7^{42} \times 8^{43}$

$3^1 \times 7^2 \times 8^3$

$= 3 \times 9 \times 2$

$= \boxed{4}$ unit digit

Q. Find unit place of $7^{44^{43}}$

$\frac{44^{43}}{4} = 4K$ completely div. by 4.

$\therefore$ ans $= \boxed{1}$

Q. Find unit place of $3^{42^{44}}$

$\frac{42^{44}}{4} = \frac{2^{44} \times 21^{44}}{4} \rightarrow$ Rem = 0

$\therefore$ ans $= \boxed{1}$

Question $(121)^{111} \times (1287)^{53}$

$= 1 \times 7^1 = \boxed{7}$

Question: $(259)^{148} - (123)^{43}$ Find unit digit

ans $9^{148} - 3^{43}$

$= 1 - 7 = 4$ (unit digit can't be negative)

Question Find unit digit of

$1! + 2! + 3! + 4! + 5! + ---- + 100!$

Unit digit of 5! & above that are having 0 at its unit place.

ans $1! + 2! + 3! + 4!$

$= 1 + 2 + 6 + 24$

$= \boxed{3}$ unit digit

Q. Find unit place $8^{43^{43}}$

$\frac{43^{43}}{4} = \frac{(-1)^{43}}{4} \Rightarrow \boxed{Rem = 3}$

$\therefore$ ans $= \boxed{2}$

Q. Find Unit digit of $2^{11!}$

$11! = 1\times2\times3\times4 --- \times 11$

$\Rightarrow$ Any factorial greater than 3! is divisible by 4 completely.

$2^{11!} = 2^4 = \boxed{6}$ unit digit

Q. Find unit digit of $(823)^{123!} \times (237)^{234!}$

$= 3^4 \times 7^4$

$= 1 \times 1 = \boxed{1}$ unit digit

Concept: $\underbrace{1, 1, 1, 1}, 1, 1, 1, 1 \rightarrow$ cyclicity 1

so, cyclicity of 1 can be 4

$\{0, 1, 5, 6\} \rightarrow$ can be 4

Similarly $4, 6, 4, 6, 4, 6, 4, 6$ $\{4, 9\}$ cyclicity 4

$\{2, 3, 7, 8\}$ cyclicity 4

All no. has cyclicity $= \boxed{4}$

Question: If x is a real no. $x^7 - x^3 = 1232$, how many value are possible for x?

ans: $x^7 - x^3 = 1232$

unit same $x^3 - x^3 = 123(2) \rightarrow$ Zero

it means there is no such value of x is possible.

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Conversion of a Number from Non-terminating & Repeating decimal to the form p/q

Question Convert 0·233333 --- into p/q.

ans: $N = 0.2\overline{3}$

$10N = 2.3333 ---$ —— (i)

$100N = 23.333 ---$ —— (ii)

eq. (2) - (i) = $90N = 21$

$N = \frac{21}{90} = \boxed{\frac{7}{30}}$

Short cut: $N = 0.2\overline{3}$

Fraction = $\frac{23-2}{90}$

2 → Non Repeating part

90 → Repeating part, Non Repeating

$= \frac{21}{90} = \frac{7}{30}$

Question: Convert 37·565656 --- into the form p/q

ans: Let, $N = 37.\overline{56}$ —— (1)

$100N = 3756.\overline{56}$ —— (2)

eq (2) - (1) $99N = 3756 - 37$

$\boxed{N = \frac{3719}{99}}$

Factors: A number which is able to completely divide a number greater than or equal to it.

Total Number of factors of a number:

If a number can be expressed as a power of its prime components as $\boxed{N = a^p \times b^q \times c^r}$ & so on

where a, b, c etc. are prime numbers and p, q, r etc are positive integers, then, number of factors of a number $= \boxed{(p+1) \times (q+1) \times (r+1)}$ **

Ex. Find no. of factors of 24

ans $24 = 2^3 \times 3^1$

No. of factors $= (3+1) \times (1+1) = 4 \times 2 = \boxed{8}$

We know those are, 1, 2, 3, 4, 6, 8, 12, 24

Ex. Find no. of factors of 3600

ans $3600 = 6^2 \times 10^2$

$= 2^2 \times 3^2 \times 2^2 \times 5^2$

$= 2^4 \times 3^2 \times 5^2$

$\therefore$ Total no. of factors $= (4+1) \times (3) \times (2+1)$

$= 5 \times 3 \times 3$

$= \boxed{45}$

Sum of factors

Ex. 45

$$45 = 3^2 \times 5^1$$

Now, split each prime factor as sum of every distinct factors.

$$= (3^0 + 3^1 + 3^2) \times (5^0 + 5^1)$$

$$= (1+3+9) \times (1+5) = 13 \times 6 = \boxed{78}$$

Non perfect squares will have even number of factors.
Every perfect square will have odd number of factors because its square root number will pair with itself.

Product of factors: As we know, factors will occur in pairs for the numbers except perfect squares.

So, product of factors $= \boxed{\frac{\text{Total factors}}{2}}$

for eg. 24 has 8 factors. So, product $= \boxed{4}$
which are (1, 24), (2×12), (3×8), (4, 6)

Even factors: Let $N = 2^3 \times 3^2 \times 5^1$

even factors $= 3 \times (2+1) \times (1+1) =$ 18 factors

Odd factors: $\boxed{\text{Total factor} - \text{even factors}}$

Conceptually, For finding out the number of odd factors, the contribution of the even prime number, that is, 2 will not be considered except for its power 0, that is, for an odd factor, only 2^0 will be taken into considerati--on.

Number of zeros : How to find no. of zeros in $n!$

For 1 zero we need a $10 \rightarrow 2, 5$

So, we will try to find no. of 5 in $n!$

$$= \left[\frac{n}{5}\right] + \left[\frac{n}{5^2}\right] + \left[\frac{n}{5^3}\right] + ---$$

Question Find no. of zeros in $100!$

ans

$$\begin{array}{c|l} 5 & 100 \\ \hline 5 & 20 \\ \hline & 4 \end{array}$$

$= 20+4 = 24$ ans

Question Find no. of zeros in $154!$

ans

$$\begin{array}{c|l} 5 & 154 \\ \hline 5 & 30 \\ \hline 5 & 6 \\ \hline & 1 \end{array}$$

$30+6+1 = 37$ ans

Question Find trailing zeros in $56!$

ans :

$$\begin{array}{c|l} 5 & 56 \\ \hline 5 & 11 \\ \hline & 2 \end{array}$$

$11+2 = 13$ ans

Question: $2 \times 4 \times 6 \times 8 \times --- \times 250$. Find no. of zeros.

ans: $2^{125}(1 \times 2 \times 3 \times 4 \times --- \times 125)$

$= 2^{125} \times 125!$

Now we will find how many 5 in 125!

5	125
5	25
5	5
	1

Total = 31

Question: $5 \times 10 \times 15 \times 20 \times --- \times 500$

$5^{100}(1 \times 2 \times 3 --- \times 100)$

$5^{100} \times 100!$

2	100
2	50
2	25
2	12
2	6
2	3
	1

Total zeros $= 50+25+12+6+3+1$

$= 97$

Question $10 \times 20 \times 30 \times 40 \times ---- \times 1000$ Find no. of zeros

ans: $10^{100}(1 \times 2 \times 3 \times 4 \times --- 100)$

$= 10^{100} \times 100!$ we know in $100! = 24$

No. of zeros $= 100 + 24 = 124$

Question: Find no. of zeros $126! - 125!$. Find zeros.

ans: $126 \times 125! - 125!$

$125! \times 125$

$= 31 + 3 = \boxed{34}$ ans

5	125
5	25
5	5
	1

$= 31$

Question: Find no. of zeros in $18! + 19! = 18! \times (20)$

$5 \,|\, 18$, quotient 3

$3 + 1 = \boxed{4}$ zeros.

Question: Find no. of zeros in

$(3^{123} - 3^{122} - 3^{121})\,(2^{122} - 2^{121} - 2^{120})$

$= 3^{121}(3^2 - 3^1 - 1)\; 2^{120}(2^2 - 2 - 1)$

$= 3^{121} \times 5 \times 2^{120} \times 1$

$=$ 1 zero.

Question: Find no. of zeros in

$1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times \text{---} \; 100^{100}$

ans: Try to find 5 here,

$5^5 = 1$ five, $10^{10} = 10$ fives, $15^{15} = 15$ fives, $20^{20} = 20$ fives $25^{25} = 50$ fives

we will not disturb A.P. & write double terms alone.

$5 + 10 + 15 + 20 + 25 + \text{---} + 100$

$= \frac{n}{2}(a + l) = (100 + 5) \times \frac{20}{2} = 10 \times 105$

$= \boxed{1050}$ zeros

Now for double terms $25 + 50 + 75 + 100 = \boxed{250}$ zeros

so, Total zeros $= 1050 + 250$

$= \boxed{1300}$

Remainder Theorem :

$$\begin{array}{r|l} 5 & 17 \\ & 15 \\ \hline & 2 \end{array} \quad 3$$

3 → Quotient

17 → divident

divisor

2 → Remainder

$$\boxed{\text{Divident} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}}$$

Question 1 : In a division process divisor is 5 times of quotient & Remainder is 3 times of quotient. If remainder is 15. Find divident

ans Divident $= (5 \times 25) + 15 = 125 + 15 = \boxed{140}$

$$\begin{array}{r|l} 25 & 140 \\ \hline & 15 \end{array} \quad 5$$

Question : When a no. is divided by 102 leaves remainder 87. If same no. is divided by 17. Find Remainder

ans

$$\begin{array}{r|l} 102 & m \\ \hline & 87 \end{array} \quad x$$

$m = 102x + 87$

Now when divided by 17

$$\frac{102x + 87}{17}$$

$= \boxed{2}$ ans

102 ⟶ 87

17 ⟶ 85

$\boxed{2}$ Remainder

Q. When a no. is divided by 361 leaves remainder 47 if the same no. is divided by 19. Find remainder.

ans: $361 \longrightarrow 47$

$19 \longrightarrow 38$

Rem. $\boxed{9}$ ans

Note →

$5\overline{)26}(5$, 25, $\boxed{R_1 = 1}$

$5\overline{)27}(5$, 25, $\boxed{R_2 = 2}$

Now $26+27 = 53$

$5\overline{)53}(10$, 50, $\boxed{3}$

$5\overline{)28}(5$, 25, $R = 3$ + $5\overline{)29}(5$, 25, $R = 4$ = $5\overline{)57}(11$, 55, $\boxed{R = 2}$

it means, $\boxed{\text{divisor} = R_1 + R_2 - R_3}$

Question: When a certain no. is divided by a certain divisor leaves remainder 43 and another no. is divided by the same divisor leaves remainder 37. If the sum of both no. is divided by same divisor leaves remainder 13. Find divisor.

ans Divisor $= 43 + 37 - 13$

$= \boxed{67}$

NOTES

Important concept: $\frac{x}{D} \rightarrow \boxed{Rem = R}$, $\frac{2x}{D} \rightarrow Rem = \boxed{\frac{2R}{D}}$

e.g. $x/13 \rightarrow$ Remainder $= 10$

$2x/13 \rightarrow Rem = 20/13 = \boxed{7}$

Question: x/m Rem $\rightarrow 20$, $2x/m \rightarrow 9$

divisor $m = 9$

ans $m = 40 - 9 = \boxed{31}$

* उ $x/D \rightarrow Rem = R$, $x^2/D \rightarrow Rem = R^2/D$

$x^3/D \rightarrow Rem = R^3/D$

e.g. $x/11 \rightarrow Rem = 7$, $x^2/11 \rightarrow Rem = 49/11$ $\boxed{R = 5}$

उ $\frac{A \times B}{m}$ $\boxed{R = A_R \times B_R}$

उ $\frac{A + B}{m}$ $\boxed{R = A_R + B_R}$

Negative Remainder

$11 \overline{)98}\ 9$

99

$R = -1 + 11 = \boxed{10}$

Concept of modularity:

$\frac{14}{4}$ $\boxed{R = 2}$

we can't cut down the ratios

$\frac{7}{2}$ $\boxed{R = 1}$ if done then multiply New R with 2, $1 \times 2 = \boxed{2}$ R

$\Rightarrow$ $\boxed{\begin{array}{l} \dfrac{(x+a)^n}{x} \Rightarrow R = a^n \\ \dfrac{(\pm x+a)^n}{x} \Rightarrow R = a^n \end{array}}$

$\Rightarrow$ $\dfrac{(x+1)^n}{x} \Rightarrow \text{Rem} = 1^n = 1$

$\dfrac{(x-1)^n}{x} \Rightarrow \text{Rem} = (-1)^n$ → 1 when n is even; → -1 when n is odd → $\boxed{-1+x}$

Q. $\dfrac{25^{26}}{26} = \dfrac{(26-1)^{26}}{26} = (-1)^{26} = 1$

Q. $\dfrac{27^{29}}{28} = \dfrac{(28-1)^{29}}{28} = (-1)^{29} = -1+28 \Rightarrow \boxed{\text{Rem} = 27}$

Q. $\dfrac{37^{39}}{36} = \dfrac{(36+1)^{39}}{36} = (1)^{39} = 1$

Q. $\dfrac{4^{5^{6^{7^{\cdots 12}}}}}{6}$

$\boxed{\text{Rem} = 4}$

Every power of 4 when divided by 6 will give us $\boxed{4}$ as remainder.

$\dfrac{4^1}{6}$ Rem = 4

$\dfrac{4^2}{6}$ Rem = 4

$\dfrac{4^3}{6}$ Rem = 4

**

Question: $\frac{2^{111}}{5}$ Find remainder.

ans: $\frac{2 \cdot 2^{110}}{5} = \frac{2 \cdot 4^{55}}{5} = \frac{2(5-1)^{55}}{5} = -2 + 5$

$= \boxed{3}$ final remainder

Question: $\frac{3^{162}}{162} \Rightarrow$ Find remainder.

ans $\frac{3^{162}}{162} = \frac{\cancel{3^4} \cdot 3^{158}}{2 \times \cancel{81}} = \frac{3^{158}}{2} = \frac{(2+1)^{158}}{2}$

Rem $= (1)^{158} = 1$ (which is not correct by concept of modularity)

$\boxed{Rem = 81}$

Question: $\frac{65 \times 78}{9} = \frac{2 \times 6}{9} = 3$ Remainder

Question: $\frac{1! + 2! + 3! + 4! + 5! + 6! + \cdots + 1000!}{10}$

When any no. is divided by 10, the remainder will be last digit of that no.

ans $\frac{1 + 2 + 6 + 24 + 120 + \cdots}{10} = \frac{13}{10}$

$\boxed{Rem = 3}$

Similarly, when you divide by 100 then the remainder is the last two digits.

$$\# \quad \frac{\overset{+3}{103} \times \overset{-2}{1298} \times \overset{+2}{13702} \times \overset{-3}{1197}}{100} = \frac{36}{100} \quad Rem = \boxed{36}$$

Q. $\frac{(37)^{27}}{9} = \frac{(9\times4+1)^{27}}{9} = \frac{(+1)^{27}}{9} \Rightarrow \boxed{Rem = 1}$

Q. $\frac{2^{63}}{9} = \frac{(2^3)^{21}}{9} = \frac{(9-1)^{21}}{9} = \frac{(-1)^{21}}{9} \Rightarrow Rem = -1+9$
$= \boxed{8}$

Q. $\frac{2^{64}}{9} = \frac{(2^3)^{21}\times2}{9} = \frac{(9-1)^{21}\times2}{9} \Rightarrow Rem = -2+9 = \boxed{+7}$

Q. $\frac{3^{68}}{82} = \frac{(3^4)^{17}}{82} = \frac{(82-1)^{17}}{82} = \frac{(-1)^{17}}{82} = -1+82 \Rightarrow \boxed{81}$

Q. $\frac{7^{400}}{400} = \frac{(7^4)^{100}}{400} = \frac{(2401)^{100}}{400} = \frac{(1)^{100}}{400} \quad \boxed{Rem = 1}$

Q. $\frac{3^{4321}}{80} = \frac{(3^4)^{1080}\times3}{80} = \frac{1\times3}{80} = \boxed{3}$ Rem

Q. $\frac{2^{76}}{96} = \frac{2^5\times2^{71}}{2^5\times3} = \frac{2^{71}}{3} = \frac{(-1)^{71}}{3} = +2$

Final remainder $= 2\times2^5 = 64$

Q. $\frac{5^{500}}{500} = \frac{5^{500}}{5^3\times4} = \frac{5^{497}}{4} = \frac{(1)^{497}}{4}$ Rem $= 1$

Final remainder $= 1\times125 = \boxed{125}$

Q. $\frac{2^{35}}{10} = \frac{2^{35}}{2\times5} = \frac{2^{34}}{5} = \frac{(4)^{17}}{5} = \frac{(-1)^{17}}{5} \Rightarrow Rem = -1+5$
$= 4$

Final remainder $= 4\times2 = \boxed{8}$

Q. $\frac{15^{40}}{100} = \frac{5^{40}\times 3^{40}}{5^2 \times 4} = \frac{5^{38}\times 3^{40}}{4} = \frac{1\times 1}{4} = \boxed{1}$

Rem = $1 \times 25 = 25$

Q. Euler's Theorem $\boxed{\frac{n^{\theta(p)}}{p} = 1}$ where n & p are co-prime no.

where, $\theta(p)$ = Euler's toient for p (cyclicity)

if $p = a^b \times c^d \times e^f$ where a, c, e are prime factors.

then $\theta(p) = p\left(1-\frac{1}{a}\right)\times\left(1-\frac{1}{c}\right)\times\left(1-\frac{1}{e}\right)$

Q. $\frac{2^{111}}{5} = \frac{2^3}{5} = \frac{8}{5} = \boxed{Rem = 3}$

How to find totient of $5 = 5\times\left(1-\frac{1}{5}\right) = 4$

→ for any prime no. Euler's toient is 1 less than the number.

Q. $\frac{3^{2146}}{17 \rightarrow \text{toient} = 16} = \frac{3^2}{17} = \frac{9}{17} \Rightarrow \boxed{Rem = 9}$

Q. $\frac{2^{5557}}{13 \rightarrow \text{toient} = 12} = \frac{2^1}{13} \Rightarrow \boxed{Rem = 2}$

Q. $\frac{32^{32^{32}}}{7 \rightarrow \text{toient} = 6}$ Find remainder $\frac{32^{32}}{6} = \frac{(36-4)^{32}}{6}$ Rem=4

$\frac{32^4}{7} = \frac{4^4}{7} = \frac{16\times 16}{7} = 4$ Rem

Fermat Theorem $\boxed{\frac{a^{p-1}}{p} = 1}$ Rem. where p is prime

Q. $\frac{2^{100}}{101} = 1$

LCM & HCF :

LCM : (Least common multiple)

e.g. 12 = 12, 24, 36, 48, 60, 72, 84, 96 ___

16 = 16, 32, 48, 64, 80, 96 ____

common multiple = 48, 96

Least common multiple = $\boxed{48}$

> LCM of x, y, z is the smallest no. which is exactly divisible by x, y, z.

How to Find LCM : (i) Prime Factorization method

$$12 = 2 \times 2 \times 3$$

$$16 = 2 \times 2 \times 2 \times 2$$

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 = \boxed{48}$$

Division method :

2	12	16
2	6	8
2	3	4
2	3	2
3	3	1
	1	1

ans $= 2 \times 2 \times 2 \times 2 \times 3$

$= \boxed{48}$

Third method : Let us suppose 16 is LCM.

$\left.\begin{matrix}3\\4\end{matrix}\right\rangle 12$

$16 \times 3 = \boxed{48}$

Q. LCM of 9, 12, 15

ans $15 \times 3 \times 4 = \boxed{180}$

NOTES

Question	Answer
Q. Find smallest no. which is exactly divisible by x, y, z	Lcm of x, y, z
Q. Find the smallest no. which when divided by x, y, z and leaves remainder R in each case	$\boxed{\text{Lcm of } x,y,z} + R$ ↙ exactly div. by x, y, z
Q. Find the smallest no. which when divided by x, y, z Leaves remainder a, b, c respectively.	$\boxed{\text{Lcm of } x,y,z \ -K}$ where $K = (x-a) = (y-b) = (z-c)$

Question: Find the smallest no. which when divided by 25, 15, 30 and leaves remainder 21, 11, 26 respectively.

ans: $25-21 = 15-11 = 30-26 = 4$

Now, Lcm of 25, 15, 30 $= 30 \times 5 = \boxed{150}$

ans will be, $150 - 4 = \boxed{146}$

Lcm =
$150 = 125+25 = 125+21+\cancel{4}$
$150 = 135+15 = 135+11+\cancel{4}$
$150 = 120+30 = 120+26+\cancel{4}$

146

Question: Find the smallest no. which when divided by 2, 3, 4, 5, 6 and leaves remainder 1 in each case & no. is completely divisible by 7.

ans: LCM of $(2,3,4,5,6)+1$

$= 60+1 = 61$ but it is not divisible by 7.

So, we will take $60K+1 = 56K + (4K+1)$

for $K = 5$

$\therefore$ No. will be $(60 \times 5 + 1)$

$= 300 + 1$

$= \boxed{301}$

Question: Find the sum of digit of a smallest no. which is divided by 5, 6, 7, 8 leaves remainder 3 but when divided by 9 leaves no remainder.

(a) 12 (b) 16 (c) 18 (d) 21

ans LCM of $(5,6,7,8) = 840$

$\therefore$ No. can be $840K+1 = 840K+3$

$= 810K + 30K + 3$

$= 837K + \underbrace{3K+3}$ for $\boxed{K=2}$

it is divisible by 9.

$\therefore$ No. will be $= 840 \times 2 + 3$

$= 1680 + 3 = 1683$

$\therefore$ sum of digits $= 1+6+8+3$

$= 18$ ans

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Question: Let x be a smallest no. which when added to 2000 and resultant no. is exactly divisible by 12, 16, 18, 21. Find the sum of x's digits.

ans LCM of $(12, 16, 18, 21) = 21 \times 6 \times 2 \times 2 \times 2 = 1008$

Now as no. is added to 2000 the LCM can be doubled $2016 = 2000 + 16$

$\downarrow$ added no.

and sum of digits of $16 = 1 + 6 = \boxed{7}$

(a) 6 (b) 7 (c) 8 (d) 4

we can go with options. As no. divisible by 18 it is also divisible by 9.

$2000 + x$

$\underset{\text{(sum of 2000)}}{2 + x} \Rightarrow 2 + 7 = 9$

Question: Find the largest four digit no. which is completely divisible by 12, 15, 18, 27

(a) 9690 (b) 9930 (c) 9960 (d) 9720

$$\begin{array}{r|l|l} 540 & 9999 & 18 \\ & 540 & \\ \hline & 4599 & \\ & 4320 & \\ \hline \end{array}$$

Rem = 279

$\therefore$ Largest No. $= 9999 - 279$

$= \boxed{9720}$

HCF (Highest common factor)

12 = 1, 2, 3, 4, 6, 12 common factors = 1, 2, 4

16 = 1, 2, 4, 8, 16 Highest common factor = $\boxed{4}$

HCF of x, y, z is the largest no. which can divide x, y, z exactly.

How to Find HCF: ① Prime factorization method:

$$12 = \boxed{2} \times \boxed{2} \times 3$$
$$16 = \boxed{2} \times \boxed{2} \times 2 \times 2$$
$$HCF = 2 \times 2 = 4$$

② **Division method:** For e.g. 12 & 16

```
12 ) 16 ( 1
     12
     ----
      4 ) 12 ( 3
          12
          ----
          [0]
```

when remainder becomes zero then final divisor is called HCF.

③ **Third method:**

difference never less than 15

a b

HCF = 15

a is also divisible by 15.

b is also divisible by 15.

HCF can never be greater than difference of two numbers. HCF may be difference or factor of difference.

e.g. 12, 16 (4)

$\boxed{HCF = 4}$

"HCF"

Q. Find the largest no. which can divide x, y, z exactly	HCF of (x, y, z)
Q. Find the largest no. which can divide x, y, z and leaves remainder R in each case	HCF of $(x-R), (y-R), (z-R)$ "OR" HCF of $\lvert x-y\rvert, \lvert y-z\rvert, \lvert z-x\rvert$
Q. Find the largest no. which can divide x, y, z & leaves remainder a, b, c respectively.	HCF of $(x-a), (y-b), (z-c)$

Q. Find the largest no. which can divide 306, 340, 187 exactly.

ans: HCF of 306, 340, 187

$$\underbrace{306, 340}_{34}, 187$$

$34 \rightarrow 17, 2$

ans = 17

Q. Find the largest no. which can divide 1305, 4665, 6905 & leaves remainder equal in each case.

ans hcf of $\lvert 6905-1305\rvert, \lvert 6905-4665\rvert, \lvert 4665-1305\rvert$

hcf of 5600, 2240, 3360

HCF = $\boxed{1120}$ ans

we can divide the values to get HCF.

2240, 3360, 5600

$$3360 \overline{)5600}(1$$
$$\underline{3360}$$
$$2240 \overline{)3360}(1$$
$$\underline{2240}$$
$$1120 \overline{)2240}(2$$
$$\underline{2240}$$
$$\times$$

$\therefore$ HCF = $\boxed{1120}$

** If two no. are divided by their difference or factor of difference then leaves same remainder.

29 39

10 → 2, 5, 10

Question: Two no. 225 and 147 are divided by A two digit no. then leaves same remainder. How many two digit no. are possible?

ans:

225 147

78 → 39, 26, 13

ans is 4 no. are possible.

Question: Two no. 875 and 2272 are divided by a 3 digi- no. & leaves same remainder in each case. Find the sum of digit of that no.

ans:

875 2272

1397 = 11 × 127

sum of digit = 1+2+7 = 10

Relation b/w HCF & LCM : Product of two no. = HCF x LCM

$$\boxed{a \times b = HCF \times LCM}$$

Q. HCF & LCM of two no. are 144 & 864, if one no. is 288. Find second no.

ans:

$$\cancel{288}_{2} \times b = \cancel{144} \times 864$$

$$\boxed{b = 432}$$

Q. If LCM of two no. is twice of bigger no. & difference of HCF & smaller no. is 4. Find smaller no.

ans: Let Larger no. = a, Smaller = b

$$\underset{LCM}{2a} \times (b-4) = a \times b$$

$$2b - 8 = b \Rightarrow \boxed{b = 8}$$

20 (5×4), 25 (5×5) → $\boxed{HCF = 5}$

Nothing common in 4 & 5

first no = 5a

Second no. = 5b

If HCF of two no. is H then no. can be written as Hx, Hy where (x & y) are relatively prime no.

Question: If hcf & LCM of two no. are 13 & 455 if one no. is b/w 75 & 125. Find other no.

ans: No. will be 13x, 13y

$$13x \times 13y = 13 \times 455$$

$$13xy = 455 \Rightarrow \boxed{xy = 35}$$

x & y can't be 1, 35

7, 5 $\Rightarrow 13x = 13 \times 7 = 91$

$13y = 13 \times 5 = \boxed{65}$ ans

Question If HCF & Lcm of two no. are 17 & 408
How many numbers are possible for such type.

ans first $= 17x$, $17y$ = second

$17xy = 408 \Rightarrow \boxed{xy = 24}$

(1, 24) ✓
(3, 8) ✓
(4, 6) ✗
(12, 2) ✗

Only 2 pair are possible.

Question If HCF & Lcm of two no. are 29 & 4147. Find the sum of both no., if both numbers are greater than 29.

ans Let $29x$, $29y$ $\Rightarrow 29xy = 4147$

$xy = 143$

11, 13

$29x = 29 \times 11$

$29y = 29 \times 13$

$\therefore$ sum $= 29(11+13) = 29 \times 24$

$= \boxed{696}$

Last 2 places :

1. $2^{10 \times Even} = ----76$
2. $2^{10 \times odd} = -----24$
3. $(---A1)^{---B} = ----(AB)1$
4. $(---5)^{odd} = -----75$

   $-----25$, otherwise

Q. $2^{240} = 2^{24 \times 10} = ---76$

Q. $2^{170} = 2^{17 \times 10} = ---24$

Q. $(71)^{123} = -----11$

Q. $(65)^{125} = ----25$

Even

Q. Find last two places of $(88)^{76}$

$88^{76} = (8 \times 11)^{76} = 8^{76} \times 11^{76} = 2^{3 \times 76} \times 11^{76}$

$= 2^{228} \times 11^{76}$

$= 2^{220} \times 2^{8} \times 11^{76}$

$= ---76 \times ---56 \times ---61$

$= \boxed{16 \text{ ans}}$

Q. Find last two places of $(59)^{27}$.

ans $59^{27} = (59)^1 \times (59)^{26}$

$= 59^1 \times (59^2)^{13}$

$= 59 \times (81)^{13}$

$= 59 \times 41$

$= \boxed{19}$ ans

Q. Find last two places of $(75)^{24}$

ans $\boxed{25}$

Q. Find last two places of $2^{2^{2222}}$

ans $2^{2^{2222}}$ $\quad 2^{2222} = 2^{2220} \times 2^2$

$= 2^{222 \times 10} \times 2^2$

$= ---76 \times 4$

$= ---04$

$2^{2^{2222}} = 2^{---04} = 2^{A04}$ even

$= 2^{A00} \times 2^4 = 2^{A0 \times 10} \times 16$

$= 76 \times 16$

$= \boxed{16}$ ans

Q. Find last two places of $21^{50} - 8$

ans $21^{50} = -----01$

-08

$\boxed{93}$ ans

Q. Find unit place of 89^{43} ans 9

Q. Find unit place of $2^{88^{18}}$ ans 6

Q. Find last two places of 13^{48} ans 21

Q. Find last two places of $2^{3^{100}}$ ans 52