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Que 1:- Integrate $(1+x^2)(dy/dx)+2xy-4x^2=0$ to obtain the equation of the curve satisfying this equation and passing through origin.

$(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$

$1+x^2=t$

$2x\,dx=dt$

$\frac{dy}{dx}+\frac{2x}{(1+x^2)}y=\frac{4x^2}{1+x^2}$

$IF = e^{\int P dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\int \frac{dt}{t}} = e^{\ln|t|} = 1+x^2$

$(1+x^2)y = \int \frac{4x^2}{(1+x^2)}(1+x^2)\,dx + C$

$(1+x^2)y = \frac{4x^3}{3}+C$

Since the curve passing through origin (0,0)

$C=0$

$(1+x^2)y = 2x^2$.

$y = \frac{2x^2}{1+x^2}$.

Que 2:- Solve $x\left(\frac{dy}{dx}\right)+y\log y = xye^x$

$\frac{1}{y}\frac{dy}{dx}+\frac{1}{x}\log y = e^x$

$\log y = z$

$\frac{1}{y}\frac{dy}{dx}=\frac{dz}{dx}$

$\frac{dz}{dx}+\frac{1}{x}\cdot z = e^x$

$IF = e^{\int \frac{1}{x}dx} = e^{\log|x|} = x$

$xz = \int e^x x\,dx + C$

$= xe^x - \int e^x dx + C$

$= xe^x - e^x + C$

$= e^x(x-1)+C$

Qu 3:- Solve $(D^3-5D^2+7D-3)y = e^{2x}\cosh x$

$$D^3-5D^2+7D-3 = \frac{e^{2x}(e^x+e^{-x})}{2} \qquad \cosh x = \frac{e^x+e^{-x}}{2}$$

$$D^3-5D^2+7D-3 = \frac{e^{3x}+e^x}{2}$$

AE

$$m^3-5m^2+7m-3=0$$

$$m^2(m-1)-4m(m-1)+3(m-1)=0$$

$$(m-1)(m^2-4m+3)=0$$

$$(m-1)(m^2-3m-m+3)=0$$

$$(m-1)[m(m-3)-(m-3)]=0$$

$$(m-1)(m-1)(m-3)=0$$

$$y=(C_1+C_2x)e^x+C_3e^{3x} \to CF$$

PI

$$y=\frac{e^{3x}+e^x}{2(D^3-5D^2+7D-3)}$$

$$=\frac{1}{2}\frac{e^{3x}\cdot x}{(3D^2-10D+7)}+\frac{1}{2}\frac{xe^x}{3D^2-10D+7}$$

$$=\frac{1}{2}\frac{e^{3x}x}{27-30+7}+\frac{1}{2}\frac{x^2e^x}{6D-10}$$

$$=\frac{e^{3x}x}{8}-\frac{x^2e^x}{8}$$

$$=\frac{x}{8}(e^{3x}-xe^x)$$

$$y = CF + PI$$

$$=(C_1+C_2x)e^x+C_3e^{3x}+\frac{x}{8}(e^{3x}-xe^x)$$

4:- Solve $(D^2-1)y = xe^x + \cos^2 x$

Auxiliary equation

$$m^2-1=0$$

$$m=\pm 1$$

$$y = (c_1 e^x + c_2 e^{-x})$$

PI

$$y = \frac{xe^x + \cos^2 x}{D^2-1}$$

$$= e^x \frac{1}{[(D+1)^2-1]} x + \frac{1}{2}\left[\frac{\cos 2x}{D^2-1} + \frac{1}{D^2-1}\right]$$

$$= e^x \frac{1}{(D^2+1+2D-1)} x + \frac{1}{2}\left[\frac{\cos 2x}{-4-1} + \frac{e^{0x}}{-1}\right]$$

$$= e^x \frac{1}{D^2+2D} x + \frac{1}{2}\left[\frac{\cos 2x}{-5}\right] - \frac{1}{2}$$

$$= e^x \frac{x}{2D\left(\frac{D}{2}+1\right)} - \frac{1}{10}\cos 2x - \frac{1}{2}$$

$$= e^x \frac{\left(1+\frac{D}{2}\right)^{-1} x}{2D} - \frac{1}{10}\cos 2x - \frac{1}{2}$$

$$= e^x \frac{\left[1-\frac{D}{2}+\frac{D^2}{2}\right]}{2D} x - \frac{1}{10}\cos 2x - \frac{1}{2}$$

$$= e^x \frac{\left[1-\frac{D}{2}\right]}{2D} x - \frac{1}{10}\cos 2x - \frac{1}{2}$$

$$= \frac{e^x}{2D}\left[\frac{1}{D} - \frac{D}{2}\right] x - \frac{1}{10}\cos 2x - \frac{1}{2}$$

$$= \frac{e^x}{2}\left[x^2 - 1\right] - \frac{1}{10}\cos 2x - \frac{1}{2}.$$

$$= \frac{e^x}{2D}\left[1-\frac{D}{2}\right] x$$

$$= \frac{1}{2}\left[\int x e^x\,dx - \frac{1}{2}\int e^x\,dx\right]$$

$$= +\frac{1}{2}\left[e^x(x-1) - \frac{e^x}{2}\right] - \frac{1}{2} - \frac{\cos 2x}{10}$$

$$y = CF + PI$$

$$= (c_1 e^x + c_2 e^{-x}) + \frac{e^x}{4}[x^2 - 1] - \frac{1}{10}\cos 2x - \frac{1}{2}.$$

Ques:- Solve $\frac{d^2y}{dx^2} - 9y = x + e^{2x} - \sin 2x$ using method of undetermined coefficient for finding particular integral.

CF

Auxiliary eq:-

$m^2 - 9 = 0$

$m = \pm 3$

$y = (C_1 e^{3x} + C_2 e^{-3x})$

For PI ie the trial solution

$y = C_3 x + C_4 e^{2x} + (C_5 \sin 2x + C_6 \cos 2x)$

$\frac{dy}{dx} = C_3 + 2C_4 e^{2x} + (2C_5 \cos 2x - 2C_6 \sin 2x)$

$\frac{d^2y}{dx^2} = 4C_4 e^{2x} + (-4C_5 \sin 2x - 4C_6 \cos 2x)$

On comparing

$4C_4 e^{2x} - 4C_5 \sin 2x - 4C_6 \cos 2x - 9C_3 x - 9C_4 e^{2x} - 9C_5 \sin 2x$
$- 9C_6 \cos 2x = x + e^{2x} - \sin 2x$

$4C_4 - 9C_4 = 1$

$-5C_4 = 1$

$\boxed{C_4 = -\frac{1}{5}}$

$-9C_3 = 1$

$\boxed{C_3 = -\frac{1}{9}}$

$-4C_5 - 9C_5 = -1$

$-13C_5 = -1$

$\boxed{C_5 = \frac{1}{13}}$

$-13C_6 \cos 2x = 0$

$C_6 = 0$

assumed PI

$= -\frac{1}{9}x - \frac{1}{5}e^{2x} + (\frac{1}{13}\sin 2x + 0)$

$= -\frac{1}{9}x - \frac{1}{5}e^{2x} + \frac{1}{13}\sin 2x$

$y = (C_1 e^{3x} + C_2 e^{-3x}) - \frac{1}{9}x - \frac{1}{5}e^{2x} + \frac{1}{13}\sin 2x$

Q6:- Solve $y''' + y' = 2x^2 + 4\sin x$

$$\frac{d^3y}{dx^3} + \frac{dy}{dx} = 2x^2 + 4\sin x$$

CF $= C_1 + C_2\cos x + C_3 \sin x$

PI

$$y = \frac{2x^2 + 4\sin x}{D^3 + D}$$

$$= \frac{2x^2}{D^3+D} + 4 \cdot \frac{\sin x}{D^2 \cdot D + 1}$$

$$= \frac{2(1+D^2)^{-1}x^2}{D} + 4\,\frac{\sin x}{3D^2+1}$$

$$= \frac{2}{D}(1+D^2)^{-1}x^2 + \frac{4\sin x}{-2}$$

$$= \frac{2}{D}\left(1 - D^2 + D^4 \ldots\right]x^2 + \frac{4\sin x}{-2}$$

$$= \frac{2}{D}(x^2 - 2) - 2\sin x$$

$$= 2\left(\frac{x^3}{3} - 2x\right) - 2\sin x$$

$$= \frac{2x^3}{3} - 4x - 2\sin x$$

$$y = C_1 + (C_2\cos x + C_3 \sin x) + \frac{2x^3}{3} - 4x - 2\sin x$$

Que7:- Reduce the equation

$$2x^2y\frac{d^2y}{dx^2} + 4y^2 = x^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$$

to homogeneous form by making the substitution $y = z^2$ and hence solve it

$y = z^2$ $\qquad \frac{dy}{dx} = \frac{2z}{dx}$

$$\frac{d^2y}{dx^2} = \left[\frac{2dz}{dx} + 2z\frac{d^2z}{dx^2}\right]$$

$$\frac{d^2y}{dx^2} = 2x^2z^2\left[2\left(\frac{dz}{dx}\right)^2 + 2z\frac{d^2z}{dx^2}\right] + 4z^4$$

$$= 4x^2z^2\left(\frac{dz}{dx}\right)^2 + 4xz^3\frac{dz}{dx}$$

$$4x^2z^2\left(\frac{dz}{dx}\right)^2 + 4xz^3\frac{d^2z}{dx^3} + 4z^4 =$$

$$4x^2 z^2\left(\frac{dz}{dx}\right)^2 + 4xz^2\frac{dz}{dx}$$

$$4x^2\left(\frac{dz}{dx}\right)^2 + 4x^2z\frac{d^2z}{dx^2} + 4z^2 = 4x^2\left(\frac{dz}{dx}\right)^2 + 4xz\frac{dz}{dx}$$

$$x^2\frac{d^2z}{dx^2} - x\frac{dz}{dx} + z = 0$$ Homogenous Form.

$$[(5+2x)^2D^2-6(5+2x)D+8]y=x^2+1$$

$$5+2x=t$$

$$2dx=dt$$

$$t^2\frac{d^2y}{dt^2}-6t\frac{dy}{dt}+8y=\frac{(t-5)^2}{4}+1$$

$$\frac{d}{dt}=D$$

$$[t^2D^2-6tD+8]y=\frac{(t-5)^2}{4}+1$$

$$z=\log t \qquad t=e^z$$

$$D(D-1)y-6Dy+8y=\frac{(e^z-5)^2}{4}+1$$

$$[D^2-D-6D+8]y=\frac{e^{2z}}{4}-\frac{5}{2}e^z+\frac{29}{4}$$

auxiliary eq

$$m^2-7m+8=0$$

$$m=\frac{7\pm\sqrt{49-32}}{2}=\frac{7\pm\sqrt{17}}{2}=\frac{7}{2}\pm\frac{\sqrt{17}}{2}$$

$$y=e^{7/2z}\left(C_1\cos\frac{\sqrt{17}}{2}z+C_2\sin\frac{\sqrt{17}}{2}z\right)=e^{\log(5x+2)^{7/2}}\left(C_1\cos\frac{\sqrt{17}}{2}\log(5x+2)+C_2\sin\frac{\sqrt{17}}{2}\log(5x+2)\right)$$

$$=(5x+2)^{7/2}\left(C_1\cos\frac{\sqrt{17}}{2}\log(5x+2)+C_2\sin\frac{\sqrt{17}}{2}\log(5x+2)\right)$$

$$PI=\frac{\frac{e^{2z}}{4}-\frac{5}{2}e^z+\frac{29}{4}}{D^2-7D+8}$$

$$=\frac{1}{4}\frac{e^{2z}}{(4-14+8)}-\frac{5}{2}\frac{e^z}{1-7+8}+\frac{29}{4\times8}$$

$$=\frac{1}{4}\frac{e^{2z}}{(-2)}-\frac{5}{2}\frac{e^z}{(2)}+\frac{29}{32}$$

$$=\frac{e^{2z}}{-8}-\frac{5e^z}{4}+\frac{29}{32}$$

$$=-\frac{t^2}{8}-\frac{5t}{4}+\frac{29}{32}$$

$$=-\frac{(5+2x)^2}{8}-\frac{5}{4}(5+2x)+\frac{29}{32}$$

$$y=(5x+2)^{7/2}\left(C_1\cos\frac{\sqrt{17}}{2}\log(5x+2)+C_2\sin\frac{\sqrt{17}}{2}\log(5x+2)\right)-\frac{(5+2x)^2}{8}-\frac{5}{4}(5+2x)+\frac{29}{32}.$$

Que 9:- Solve $tDx + 2(x-y) = t$

$tDy + x + 5y = t^2$

where $D = d/dt$

$[(tD+2)x - 2y = t](tD+5)$

$[(tD+5)y + x = t^2]\,2$

$$(tD+2)(tD+5)x - 2(tD+5)y = t(tD+5)$$

$$+ \quad 2x \quad + 2(tD+5)y = 2t^2$$

$$[(tD+2)(tD+5)+2]x = t^2D + 5t + 2t^2$$

$$(t^2D^2 + 7tD + 12)x = 7t + 2t^2$$

Let $t = e^z \qquad z = \log t$

$$\frac{dz}{dt} = \frac{1}{t} \quad ; \quad \frac{d^2z}{dt^2} = -\frac{1}{t^2}$$

$$\frac{dx}{dt} = \left(\frac{dx}{dz}\right)\left(\frac{dz}{dt}\right)$$

$$\frac{dx}{dt} = \frac{dx}{dz}\left(\frac{1}{t}\right)$$

$$\frac{d^2x}{dt^2} = \frac{dx}{dz}\left(-\frac{1}{t^2}\right) + \frac{1}{t}\left(\frac{d^2x}{dz^2}\right)\left(\frac{dz}{dt}\right)$$

$$\frac{d^2x}{dt^2} = \frac{1}{t^2}\left[\left(\frac{d^2x}{dz^2}\right) - \frac{dx}{dz}\right] = \frac{1}{t^2}[D^2 - D]$$

Note $D = \frac{d}{dz}$

$$t^2\frac{d^2x}{dt^2} = D^2 - D$$

$$t^2\frac{d^2x}{dt^2} + 7t\frac{dx}{dt} + 12x = 7x + 2t^2$$

$$[(D^2-D) + 7D + 12]x = 7e^z + 2e^{2z}$$

CF

Auxiliary eq

$$(D^2 + 6D + 12) = 0$$

$$m^2 + 6m + 12 = 0$$

$$m = \frac{-6 \pm \sqrt{36-48}}{2} = \frac{-6 \pm \sqrt{-12}}{2} = \frac{-6 \pm 2i\sqrt{3}}{2} = -3 \pm \sqrt{3}i$$

$$x = e^{-3x}(C_1 \cos\sqrt{3}x + C_2 \sin\sqrt{3}x)$$

$= e^{-3z}(c_1\cos\sqrt{3}z + c_2\sin\sqrt{3}z) = \frac{1}{t^3}(c_1\cos\sqrt{3}\log t + c_2\sin\sqrt{3}\log t)$

PI

$$x = \frac{7e^{z} + 2e^{2z}}{D^2+6D+12} = \frac{7e^{z}}{19} + \frac{2e^{2z}}{4+12+12}$$

$$= \frac{7}{19}e^{z} + \frac{2e^{2z}}{28}$$

$$= \frac{7}{19}e^{z} + \frac{1}{14}e^{2z}$$

$$= \frac{7}{19}e^{z} + \frac{1}{14}e^{2z}$$

$$= \frac{7}{19}t + \frac{1}{14}t^2$$

$$x = \frac{1}{t^3}(c_1\cos(\sqrt{3}\log t) + c_2\sin(\sqrt{3}\log t)) + \frac{7}{19}t + \frac{1}{14}t^2$$

$$\frac{dx}{dt} = \left[-c_1\sin(\sqrt{3}\log t)\cdot\frac{\sqrt{3}}{t} + c_2\cos(\sqrt{3}\log t)\cdot\frac{\sqrt{3}}{t}\right]\frac{1}{t^3} + [c_1\cos\sqrt{3}\log t$$

$$+ c_2\sin\sqrt{3}\log t]\left(\frac{-3t^2}{t^6}\right) + \frac{7}{19} + \frac{1}{7}t$$

$$= \frac{\sqrt{3}}{t^4}[-c_1\sin\sqrt{3}\log t + c_2\cos\sqrt{3}\log t] - \frac{3}{t^4}[c_1\cos\sqrt{3}\log t + c_2\sin\sqrt{3}\log t]$$

$$+ \frac{7}{19} + \frac{1}{7}t$$

$$tDx + 2x - t = 2y$$

$$y = \frac{tDx + 2x - t}{2}$$

$$= \frac{t\left\{\frac{\sqrt{3}}{t^4}[-c_1\sin\sqrt{3}\log t + c_2\cos\sqrt{3}\log t] - \frac{3}{t^4}[c_1\cos\sqrt{3}\log t + c_2\sin\sqrt{3}\log t]\right\} + \frac{7}{19} + \frac{1}{7}t + 2\left\{\frac{1}{t^3}(c_1\cos\sqrt{3}\log t + c_2\sin\sqrt{3}\log t) + \frac{7}{19}t + \frac{1}{14}t^2\right\} - t}{2}$$

Ques 10 - Solve $\frac{dx}{dt} - y = e^t$ $\frac{dy}{dt} + x = \sin t$; $x(0) = 1$,,

$D(Dx - y = e^t) \rightarrow D^2x - Dy = e^t$

$x + Dy = \sin t$

$+$

$(D^2+1)x = e^t + \sin t$

auxiliary eq

$m^2+1=0$

$m^2 = -1$

$m = \pm i$

$x = e^{0x}(c_1\cos t + c_2\sin t)$

$x = (c_1\cos t + c_2\sin t)$

PI

$x = \frac{e^t}{D^2+1} + \frac{\sin t}{D^2+1}$

$= \frac{e^t}{2} + \frac{t\sin t}{2D}$

$= \frac{e^t}{2} + \frac{t(-\cos t)}{2} = \frac{e^t}{2} - \frac{t\cos t}{2}$

On putting $x(0)=1$ $y(0)=$
we get
$C_1 = 1$

$C_2 = \frac{1}{2}$

$\frac{dx}{dt} = \frac{e^t + \sin t}{2} - C_1\sin t + C_2\cos t$

$y = \frac{e^t + \sin t}{2} - C_1\sin t + C_2\cos t - e^t$

$= \frac{e^t}{2} + \frac{\sin t}{2} - \sin t + \frac{1}{2}\cos t$

Ques 11 Solve $x^2y'' + xy' - y = 0$ given that $\frac{x+1}{x}$ is one integra

$y = uv$

$y = (x + \frac{1}{x})\, v$

$\frac{dv}{dx} = z$

$\frac{d^2v}{dx^2} = \frac{dz}{dx}$

$\frac{d^2v}{dx^2} + \left[\left\{ P + \frac{2}{u} \right\} \frac{du}{dx} \right] \frac{dv}{dx} = \frac{R}{u}$

$\frac{d^2v}{dx^2} + \left[\frac{1}{x} + \frac{2}{x+\frac{1}{x}} \left(\frac{x^2-1}{x^2} \right) \right] \frac{dv}{dx} = 0$

$\frac{d^2v}{dx^2} + \frac{1}{x} \left[1 + 2\left(\frac{x^2-1}{x^2+1} \right) \right] \frac{dv}{dx} = 0$

$\frac{d^2z}{dx} + \left[\frac{1}{x} + \frac{2}{x} \left(\frac{x^2-1}{x^2+1} \right) \right] \frac{dv}{dx} = 0$

$= e^{\int P\,dx}$ $\quad = e^{\int \left[\frac{1}{x} \times \frac{2}{x}\left(\frac{x^2-1}{x^2+1}\right)\right] dx}$

$\frac{x^2-1}{x}$

$= e^{-2\ln x + \ln x + 2\ln(x^2+1)}$

$= e^{\ln x^{-2} + \ln x + \ln(x^2+1)^2}$

$= x^{-2} * x * (x^2+1)^2$

$= \frac{(x^2+1)^2}{x}$

$\frac{dv}{dx}\left(\frac{(x^2+1)^2}{x}\right) = c_1$

$\frac{dv}{dx} = \frac{c_1 x}{(x^2+1)^2}$

$dv = c_1 \frac{x\,dx}{(x^2+1)^2}$

$v = \frac{c_1}{2}\int \frac{dt}{t^2}$

$= \frac{c_1}{2} \frac{t^{-2+1}}{(-2+1)}$

$= \frac{c_1 t^{-1}}{-2}$

$= -\frac{c_1}{2}\frac{1}{t}$

$v = -\frac{c_1}{2(x^2+1)}$

$y = uv$

$= -\left(x+\frac{1}{x}\right)\frac{1}{2(x^2+1)}$

$= -\frac{1}{2x}$

$y = -\frac{c_1}{2x} + c_2\left(x+\frac{1}{x}\right)$

Que 12:- Solve $x^2y''+xy'-9y=0$ given that $y=x^3$ is a solution.

$u = x^3$

$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{9}{x^2}y = 0$

$y = uv$

$y = x^3 v$

$\frac{d^2v}{dx^2} + \left[P + \frac{2}{u}\frac{du}{dx}\right]\frac{dv}{dx} = \frac{R}{u}$

$\frac{d^2v}{dx^2} + \left[\frac{1}{x} + \frac{2}{x^3}3x^2\right]\frac{dv}{dx} = 0$

$\frac{d^2v}{dx^2} + \left[\frac{1}{x} + \frac{6}{x}\right]\frac{dv}{dx} = 0$

$\frac{dz}{dx} + \left[\frac{1}{x}+\frac{6}{x}\right]z = 0$

$IF = e^{\int P\,dx}$

$= e^{\int\left(\frac{1}{x}+\frac{6}{x}\right)dx}$

$= e^{\int \frac{1}{x}dx}\, e^{6\int \frac{1}{x}dx}$

$= x \cdot x^6$

$= x^7$

$x^2 z = C_1$

$x^2 \frac{dv}{dx} = C_1$

$x^2 dv = C_1 dx$

$dv = \frac{C_1}{x^7} dx$

$dv = \frac{-x^{-6}}{6} C_1$

$v = \frac{-C_1 x^{-6}}{6}$

$dv = \frac{-x^{-6} C_1}{6}$

$= \frac{-C_1 x^{-6}}{6}$

$y = \frac{-C_1 x^{-6}}{6} + C_2 x^3$

$= \frac{-C_1 x^{-6}}{6} + C_2 x^3$

Ques Solve $x^2 y_2 - 2x(1+x) y + 2(1+x) y = x^3$

$x^2 y_2 - 2x(1+x) y_1 + 2(1+x) y = x^3$

$y_2 - \frac{2(1+x) y_1}{x} + \frac{2(1+x) y}{x^2} = x$

$P + Qx = 0$

$u = x$

$y = uv$

$y = xv$

$\frac{d^2v}{dx^2} + \left[\frac{-2(1+x)}{x} + \frac{2}{x} \right] \frac{dv}{dx} = x^2$

$\frac{dv}{dx} = z$

$\frac{d^2v}{dx^2} = \frac{dz}{dx}$

$\frac{d^2v}{dx^2} + -2 \frac{dv}{dx} = x^2$

$\frac{dz}{dx} - 2z = x^2$

$IF = e^{\int -2 dx} = e^{-2x}$

$e^{-2x} \cdot z = \int x^2 \cdot e^{-2x} dx + C_1$

$e^{-2x} \cdot z = x^2 \frac{e^{-2x}}{(-2)} - \int 2x \cdot \frac{e^{-2x}}{(-2)} dx + C_1$

$e^{-2x} \cdot z = \frac{x^2 e^{-2x}}{(-2)} + \frac{x e^{-2x}}{(-2)} - \int \frac{e^{-2x}}{(-2)} dx + C_1$

$e^{-2x} \cdot z = \frac{x^2 e^{-2x}}{-2} - \frac{x e^{-2x}}{2} - \frac{e^{-2x}}{4} + C_1$

$$\cancel{e^{-2x}} \cdot \frac{dv}{dx} = -\frac{\cancel{e^{-2x}}}{2}\left(x^2+x+\frac{1}{2}\right)+C_1$$

$$\int dv = \int\left[-\frac{1}{2}\left(x^2+x+\frac{1}{2}\right) \cancel{dx} + C_1\right] dx$$

$$V = -\frac{1}{2}\left(\frac{x^3}{3}+\frac{x^2}{2}+\frac{x}{2}\right)+C_1x+C_2$$

$$y = uv$$

$$= x\left[-\frac{1}{2}\left(\frac{x^3}{3}+\frac{x^2}{2}+\frac{x}{2}\right)+C_1x+C_2\right]$$

Que14:- Solve $(1-x^2)y''+xy'-y = x(1-x^2)^{3/2}$

$$y''+\frac{x}{(1-x^2)}y'-\frac{1}{(1-x^2)}y = x(1-x^2)^{1/2}$$

$$\frac{d^2y}{dx^2}+\frac{x}{(1-x^2)}\frac{dy}{dx}-\frac{1}{(1-x^2)}y = x(1-x^2)^{1/2}$$

$$P = \frac{x}{1-x^2} \qquad Q = \frac{-1}{1-x^2}$$

Since, $(P+Qx) = 0$ part of CF $u = x$

$$y = uv = xv$$

$$\frac{d^2v}{dx^2}+\left[P+\frac{2}{u}\frac{du}{dx}\right]\frac{dv}{dx} = \frac{R}{u}$$

$$\frac{d^2v}{dx^2}+\left[\frac{x}{1-x^2}+\frac{2}{x}\right]\frac{dv}{dx} = \frac{x(1-x^2)^{1/2}}{x}$$

$$\frac{d^2v}{dx^2}+\left[\frac{x^2+2-2x^2}{x(1-x^2)}\right]\frac{dv}{dx} = (1-x^2)^{1/2}$$

$$\frac{d^2v}{dx^2}+\left[\frac{2-x^2}{x(1-x^2)}\right]\frac{dv}{dx} = (1-x^2)^{1/2}$$

$$\frac{dz}{dx}+\left(\frac{2-x^2}{x(1-x^2)}\right)z = (1-x^2)^{1/2}$$

$$IF = e^{\int\frac{2-x^2}{x(1-x^2)}dx} = e^{\int\frac{1}{x}dx} \cdot e^{-\frac{1}{2}\int\frac{1}{(x-1)}dx} \cdot e^{-\frac{1}{2}\int\frac{1}{x+1}dx}$$

$$= x(x-1)^{-1/2}(x+1)^{-1/2}$$

$$= x(x^2-1)^{-1/2} = \frac{x}{(x^2-1)^{1/2}}$$

$$IF = \frac{}{x(x^2-1)^{1/2}}$$

$$\frac{x}{(x^2-1)^{1/2}}z = \int \frac{\cancel{(1-x^2)^{1/2}}}{\cancel{(x^2-1)^{1/2}}\,x}\,dx + C_1$$

$$\frac{x}{(x^2-1)^{1/2}}\frac{dv}{dx} = \ln x + C_1$$

$$\frac{dv}{dx} = \left[\ln x + C_1\right]\frac{(x^2-1)^{1/2}}{x}$$

$$dv = \int\left[\ln x + C_1\right]\frac{(x^2-1)^{1/2}}{x}dx$$

$$dv = \int \frac{\ln x\,(x^2-1)^{1/2}}{x}dx + C_1\int\frac{(x^2-1)^{1/2}}{x}dx$$

$$v = -\frac{1}{9}(1-x^2)^{3/2} + C_1\left[-\frac{\sqrt{1-x^2}}{x} - \sin^{-1}x\right] + C_2$$

$$y = vx = -x(1-x^2)^{3/2} - C_1\left[\sqrt{1-x^2} + x\sin^{-1}x\right] + C_2x$$

Que15:- Solve $\frac{d}{dx}\left(\cos^2x\frac{dy}{dx}\right) + y\cos^2x = 0$

$$\frac{d^2y}{dx^2}\cos^2x - 2\sin x\cos x\frac{dy}{dx} + (\cos^2x)y = 0$$

$$\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} + y = 0$$

$P = -2\tan x \qquad Q = 1 \qquad R = 0$

$$Q_1 = Q - \frac{1}{2}\frac{dP}{dx} - \frac{P^2}{4} = 1 - \frac{1}{2}(-2\sec^2x) - \frac{4\tan^2x}{4}$$

$$= 1 + \sec^2x - \tan^2x$$

$$= 1 + 1$$

$$= 2$$

$$R_1 = Re^{\frac{1}{2}\int P dx} = 0$$

$$V = e^{-1/2\int P dx} = e^{-\frac{1}{2}\int -2\tan x\,dx} = e^{\int \tan x\,dx}$$

$$= e^{\log \sec x}$$

$$= \sec x$$

Normal equation is

$$\frac{d^2u}{dx^2} + Q_1 u = R_1$$

$$\frac{d^2u}{dx^2} + 2u = 0$$

$$(D_2 + 2)u = 0$$

$$D = \pm i\sqrt{2}$$

$$= (C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x)\sec x .$$

Que 16 - solve $x y_2 - y_1 + 4x^3 y = x^5$

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4x^3 y = x^5$$

$$\frac{d^2y}{dx} - \frac{1}{x}\frac{dy}{dx} + 4x^2 y = x^4$$

$P = -1/x \quad Q = 4x^2 \quad R = x^4$ choose z such that

$$\left(\frac{dz}{dx}\right)^2 = 4x^2$$

$$\frac{dz}{dx} = 2x$$

$$z = x^2$$

$$\frac{d^2z}{dx^2} = 2$$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} = \frac{2 - \frac{1}{x}(2x)}{4x^2} = 0$$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} = \frac{4x^2}{4x^2} = 1$$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{x^4}{4x^2} = \frac{x^2}{4}$$

Reduce eq is

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

$$\frac{d^2y}{dz^2} + y = \frac{x^2}{4} = \frac{z}{4}$$

$AE = m^2 + 1 = 0 ; m = \pm i$

$CF = C_1 \cos z + C_2 \sin z$

$$PI = \frac{z}{4(D^2+1)} = \frac{1}{4}(D^2+1)^{-1} z$$

$$= \frac{1}{4}[1 - D^2]$$

$$= \frac{z}{4}$$

$$y = C_1 \cos z + C_2 \sin z + \frac{z}{4}$$

$$y_1 = C_1 \cos x^2 + C_2 \sin x^2 + \frac{x^2}{4}$$

Que 17:- $(1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + 4y = 0$

$$\frac{d^2y}{dx^2} + \frac{2x}{1+x^2}\frac{dy}{dx} + \frac{4}{(1+x^2)^2}y = 0$$

$P = \frac{2x}{1+x^2}$ $\quad Q = \frac{4}{(1+x^2)^2}$ $\quad R = 0$

choose z such that $\left(\frac{dz}{dx}\right)^2 = Q = \frac{4}{(1+x^2)^2}$

$$\frac{dz}{dx} = \frac{2}{1+x^2}$$

Integration yield $z = 2\tan^{-1}x$

From (2) $\frac{d^2z}{dx^2} = \frac{-4x}{(1+x^2)^2}$

$P_1 = \dfrac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}$; $Q_1 = \dfrac{Q}{\left(\frac{dz}{dx}\right)^2}$; $R_1 = \dfrac{R}{\left(\frac{dz}{dx}\right)^2}$

$$P_1 = \dfrac{\frac{-4x}{(1+x^2)^2} + \frac{2x}{(1+x^2)}\cdot\frac{2}{(1+x^2)}}{\frac{4}{(1+x^2)^2}} = 0$$

$Q_1 = \dfrac{\frac{4}{(1+x^2)^2}}{\frac{4}{(1+x^2)^2}} = 1$ $\qquad R_1 = \dfrac{0}{\{4/(1+x^2)^2\}} = 0$

Reduced eq^n is $\frac{d^2y}{dz^2} + y = 0$

Auxilliary eq is $m^2+1=0$; $m = \pm i$

$CF = C_1\cos z + C_2\sin z$

$PI = 0$

$y = C_1\cos z + C_2\sin z$

CS is $y = C_1\cos(2\tan^{-1}x) + C_2\sin(2\tan^{-1}x)$

Que 18:- verify that e^x and x solution of the homogenous eq corresponding to

$$(1-x)y_2+xy_1-y=2(x-1)^2e^{-x} \quad 0<x<1$$

Thus find its general solution

Let $y=c_1e^x+c_2x$

$y_1=c_1e^x+c_2$

$y_2=c_1e^x$

$LHS=(1-x)y_2+xy_1-y$

$(1-x)c_1e^x+x(c_1e^x+c_2)-c_1e^x-c_2x=0$

Hence $y=c_1e^x+c_2x$ is solution of given diff eq clearly x and e^x are also solution of given diff eq $(1-x)y_2+xy_1-y=2(x-1)^2e^{-x}$

$$y_2+\frac{x}{1-x}y_1-\frac{y}{1-x}=-2(x-1)^2e^{-x}$$

$$y_2+\frac{x}{1-x}y_1-\frac{y}{1-x}=-2(x-1)e^{-x}$$

$$P+Qx=\frac{x}{1-x}-\frac{x}{1-x}=0$$

Hence $u=x$

Solution

$y=uv \quad y=xv$

$$eq \to \frac{d^2v}{dx^2}+\left(\frac{2}{x}+\frac{x}{1-x}\right)\frac{dv}{dx}=\frac{-2(x-1)e^{-x}}{x}$$

$$\frac{dz}{dx}+\left(\frac{2}{x}-\frac{x}{x-1}\right)z=\frac{-2(x-1)e^{-x}}{x}$$

$$IF=e^{\int\left(\frac{2}{x}-\frac{x-1+1}{x-1}\right)dx}$$

$$IF=e^{\int\left(\frac{2}{x}-1-\frac{1}{x-1}\right)dx}$$

$$=e^{2\log x}e^{-x}$$

$$=e^{-\log|x-1|}$$

$$z\cdot\frac{x^2}{x-1}e^{-x}=\int\frac{x^2}{x-1}e^{-x}\times\frac{-2(x-1)e^{-x}}{x}dx$$

$$z\frac{x^2}{x-1}e^{-x}=-2\int xe^{-2x}dx$$

$$\frac{zx^2e^{-x}}{(x-1)}=-2\left[\frac{xe^{-2x}}{-2}-\int\frac{e^{-2x}}{-2}dx\right]$$

$$\frac{x^2e^{-x}}{x-1}=-2\left[\frac{xe^{-2x}}{-2}-\frac{e^{-2x}}{4}\right]+c$$

$$\frac{x^2}{x-1};\frac{x^2}{x-1}e^{-x}=xe^{-2x}+\frac{e^{-2x}}{2}+c$$

$$z\cdot\frac{x^2}{x-1}e^{-x}=\left(x+\frac{1}{2}\right)e^{-2x}+c$$

$$z=\left(x+\frac{1}{2}\right)\left(\frac{x-1}{x^2}\right)e^{-x}+c\frac{(x-1)e^x}{x^2}$$

$$=\frac{(2x+1)}{2}\left(\frac{x-1}{x^2}\right)e^{-x}+c\left(\frac{1}{x}-\frac{1}{x^2}\right)e^x$$

$$=\frac{(2x^2-x-1)e^{-x}}{2x^2}+c\left(\frac{1}{x}-\frac{1}{x^2}\right)e^x$$

$$=e^{-x}-\frac{e^{-x}}{2}\left(\frac{1}{x}+\frac{1}{x^2}\right)+c\left(\frac{1}{x}-\frac{1}{x^2}\right)e^x$$

$$v=\int e^{-x}dx+c\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx-\frac{1}{2}\int e^{-x}\left(\frac{1}{x}+\frac{1}{x^2}\right)$$

$$v=e^{-x}+\frac{ce^x}{x}+\frac{1}{2}\int e^t\left(-\frac{1}{t}+\frac{1}{t^2}\right)dt$$

$$v=\frac{ce^x}{x}-e^{-x}+\frac{e^{-x}}{2x}+c$$

$$y=c_1e^x+e^{-x}/2-xe^{-x}+c_1x$$

Ans 19:- Solve $(x^2+1)y_2 - 2xy_1 + 2y = 6(x^2+1)^2$ by method of variation of parameter.

$$y_2 - \frac{2x}{x^2+1}y_1 + \frac{2}{x^2+1} = 6(x^2+1)$$

$$P = \frac{-2x}{x^2+1} \qquad Q = \frac{2}{x^2+1}$$

$$P + Qx = 0$$

$$u = x$$

$$y = uv$$

$$\frac{d^2v}{dx^2} + \left[P + \frac{2}{u}\frac{du}{dx}\right]\frac{dv}{dx} = \frac{R}{u}$$

$$\frac{d^2v}{dx^2} + \left[\frac{-2x}{x^2+1} + \frac{2}{x}\right]\frac{dv}{dx} = \frac{6(x^2+1)}{x}$$

$$\frac{d^2v}{dx^2} + \left[\frac{\cancel{-2x^2} + \cancel{2x^2} + 2}{(x^2+1)(x)}\right]\frac{dv}{dx} = \frac{6(x^2+1)}{x}$$

$$\frac{d^2v}{dx^2} + \frac{2}{x(x^2+1)}\frac{dv}{dx} = \frac{6(x^2+1)}{x}$$

$$\frac{dz}{dx} + \frac{2}{x(x^2+1)}z = \frac{6(x^2+1)}{x}$$

$$IF = e^{\int \frac{2}{x(x^2+1)}dx}$$

$$= e^{\int \frac{2}{x}dx} \cdot e^{-\int \frac{2x}{(x^2+1)}dx}$$

$$= \frac{2\cancel{\ln}x}{(x^2+1)}.$$

$$= \frac{2x}{(x^2+1)}$$

$$\frac{2x\,z}{(x^2+1)} = \int \frac{2\cancel{x}}{\cancel{(x^2+1)}} \cdot \frac{6\cancel{(x^2+1)}}{\cancel{x}}dx + C_1$$

$$\frac{2x\,z}{(x^2+1)} = 12x + C_1$$

$$2xz = (12x + C_1)(x^2+1)$$

$$2xz = 12x^3 + C_1x^2 + 12x + C_1$$

$$z = 6x^2 + \frac{C_1x}{2} + 6 + \frac{C_1}{2x}$$

$$\int dv = \int\left(6x^2 + \frac{C_1}{2}x + 6 + \frac{C_1}{2x}\right)dx$$

$$v = 2x^3 + \frac{C_1x^2}{4} + 6x + \frac{C_1}{2}\ln x$$

$$y = uv$$

$$= x\left(2x^3 + \frac{C_1x^2}{4} + 6x + \frac{C_1}{2}\ln x\right)$$

Ques 20:- Solve an RL circuit has an emf of given (in volts) by 4sint, a resistance of 100 Ω, an inductance of 4 henries with no initial current find the current at any time.

Let i be the current flowing in the circuit containing resistance R and inductance L in Henries with voltage source E, at any time t.

By voltage law:- $Ri + L\frac{di}{dt} = E$ = $\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$

This is linear differential eq in it

$e^{\int \frac{R}{L}dt} = e^{R/L\, t}$

Its solution is $i e^{R/L\, t} = \int \frac{E}{L} e^{R/L\, t} dt + C$

$i e^{R/L\, t} = \frac{E}{L} \times \frac{L}{R} \times e^{R/L\, t} + A$

$i = \frac{E}{R} + A e^{-R/L\, t}$

At $t=0$; $i=0 \rightarrow A = -\frac{E}{R}$

$i = \frac{E}{R}[1 - e^{-Rt/L}]$

Ques 21:- The damped LCR circuit is governed by eq $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$; where L, C and R are +ve constant. Find the condition under which the circuit is over damped, underdamped and critically damped. Find also the critical resistance.

given eq is $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \left(\frac{1}{C}\right)q = 0$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \left(\frac{1}{LC}\right)q = 0$$

Let $\frac{R}{L} = 2p$ $\frac{1}{LC} = \omega^2$ Thus eq (1) becomes

$$\frac{d^2q}{dt^2} + 2p\frac{dq}{dt} + \omega^2 q = 0$$

$$\frac{d^2q}{dt^2} + 2p\frac{dq}{dt} + \omega^2 q = 0$$

Its AE is $m^2 + 2pm + \omega^2 = 0$

$$m = -p \pm \sqrt{p^2 - \omega^2}$$

Case I :- when $p > 2$ roots are real and distincts

solution of eq (i) is $q = Ae^{(-p+\sqrt{p^2-\omega^2})t} + Be^{(-p-\sqrt{p^2-\omega^2})t}$

In this case q is always +ve, this is condition of over damping

Thus if $p > \omega$

$$\frac{R}{2L} > \frac{1}{\sqrt{LC}} \quad ; \quad R > 2\sqrt{\frac{L}{C}}$$

Case 2 :- when $p < \omega$ roots are imaginary

$$q = e^{-pt}\left(A\cos\sqrt{\omega^2 - p^2} + B\sin\sqrt{\omega^2 - p^2}\right)$$

period of oscillation decreases and this condition is of under damping

Case 3 :- when $p = \omega$ root are equal $q = (A + Bt)e^{-pt}$.

This is condition of critically damped

critical resistance is given by $p = \omega$

$$\frac{R}{2L} = \frac{1}{\sqrt{LC}}$$

$$R = 2\sqrt{\frac{L}{C}}.$$