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Error

Absolute Error :- The diff. btw the exact value and the calculated value

$$E_A = |x - x_c|$$

x_c → Calculated value

x → Exact value

Relative Error :- The Ratio of Absolute Error to the Exact value is called Relative Error.

$$\boxed{E_R = \frac{E_A}{x}}$$

If a number is rounded upto n decimal then the absolute Error will be

$$\boxed{\Delta x = \frac{1}{2} \times 10^{-n}}$$

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Q Find best approximate value of $\frac{1}{3}$ such that it contains

0.30 & 0.33 & 0.34

$$\left|\frac{1}{3} - 0.3\right| = \frac{1}{30} = 0.0333$$

$$\left|\frac{1}{3} - 0.33\right| = \frac{1}{300} = 3.33 \times 10^{-3}$$

→ [least absolute Error]

$$\left|\frac{1}{3} - 0.34\right| = \frac{-1}{150} \Rightarrow$$

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Q Relative Error? if in 8.6 both digits are correct.

∵ the digit is round to 10 decimal place ~~so~~

$$\text{Relative Error} = \frac{E_A}{X}$$

$$\Rightarrow \quad E_A = \frac{1 \times 10^{-n}}{2} \quad [\text{10 decimal place}]$$

$$= \frac{1}{2} \times 10^{-1}$$

$$E_A = \frac{1}{2 \times 10} = \frac{1}{20} = 0.05$$

$$\boxed{E_R = \frac{E_A}{X}} \qquad E_R = \frac{0.05}{8.6} = 5.813 \times 10^{-3}$$

Q Find the sum of 0.1532, 15.45, 0.000354, 305.1, 8.12, 143.3, 0.0212, 0.643, 0.1734.

$$\begin{array}{r} 305.1 \\ 143.3 \\ 0.17 \\ 0.00 \\ 0.64 \\ 0.02 \\ 8.12 \\ 15.45 \\ 0.15 \\ \hline 472.95 \end{array}$$

472.9

$$E_A = \frac{1}{2}(10^{-1}) \times 2 + \frac{(1 \times 10^{-2})}{2} \times 7$$

$$= 0.05 \times 2 + 7 \times 0.005$$

$$= 0.1 + 0.035$$

$$= 0.135 = 0.14$$

$$E_A = 0.005 = 0.01$$

Final value = 472.95 ± 0.15

Q diff: $\sqrt{6.37} - \sqrt{6.36}$

$\sqrt{6.37} = 2.523885$
$\sqrt{6.36} = 2.521904$

$2.523885 - 2.521904$
$= 1.981 \times 10^{-3}$

3a+b
−76+2a+46
117

38−a−a+8
b−38−38+a
b+a−76+2a
41−2b−b−a+76
−3b

doubt

x	y	Δy	Δ²y	Δ³y	Δ⁴y
0	2	3	0	−2a−46	5a+b−30
1	5	3	46 −2a	3a+b−76	193 −4b −4a
2	8	a−8	b+a−76	117−3b−a	
3	a	38−a	41−2b		
4	38	b−38			
5	b	3−b			
6	3				

$5a + b = 30$

$117 - 3b - a - 3a - b + 76$
$-4b - 4a + 193$

~~−4a−4b~~
$-4a - 4b = -193$

$b = 30 - 5a$

$-4a - 4(30 - 5a) = -193$

General Error formula

$u = f(x, y, z)$

Let the Error in x be Δx
Let the Error in y be Δy
Let the Error in z be Δz

$(u + \Delta u) = f(x+\Delta x, y+\Delta y, z+\Delta z)$

Absolute Error $= \Delta u = \frac{du}{dx}\Delta x + \frac{du}{dy}\Delta y + \frac{du}{dz}\Delta z$

Relative Error $= \frac{\Delta u}{u}$

Q $u = \frac{5xy^2}{z^3}$ $\boxed{u=5}$

$x = y = z = 1$ $x = y = z = 0.001$

$\frac{du}{dx} = \frac{5y^2}{z^3}$ $\frac{du}{dz} = -3\times 5\, z^{-3+1} xy^2$

$= -\frac{15xy^2}{z^2}$

$\frac{du}{dy} = 10xy$

$\Delta u = 5 \times \Delta x + 10 \times \Delta y + (-15) \Delta y$

$\Delta u = 5 \times 0.001 + 10 \times 0.001 + (-15) \times 0.001$

$\Delta u = 0.005$

Relative Error $= \frac{\Delta u}{u}$

$= \frac{0.005}{5}$

Q Find the Absolute and Relative Error in the product of 2 No's

Let the no be a & b

The Error in a is E_a
The Error in b is E_b

Product of the no = ab
Product of the no with error = $(E_a + a)(E_b + b)$

$E_{ab} = ab - (a + E_a)(b + E_b)$

$= \cancel{ab} - \cancel{ab} - a E_b - E_a b - E_a E_b$

$=$

Determination of Roots of Algebric and Trancdaitional Eqn

(1) **Bisection method**

<1> $2x - 3\sin x = 5$

$x=2 \quad x=3$

$\boxed{x=2}$

$2\times 2 - 3\sin(2) - 5 = 0$

$4 - 3\times\sin(2) - 5 = -1.1046.$
(−ve)

$x = 3$

$2\times 3 - 3\sin(3) - 5 = 0.84$ (+ve)

$[2, 3]$

$x_1 = \dfrac{2+3}{2} = 2.5$

$2\times 2.5 - 3\sin(2.5) - 5 = f(2.5)$

$f(2.5) < 0$

$[2.5, 3]$

$x_2 = \dfrac{2.5+3}{2} = 2.75$

$2 \times 2.75 - 3\sin(2.75) - 5 = 0$

$\Rightarrow 0.35606$

$f(2.75) > 0$

$[2.75, 2.5]$

$$x_3 = \frac{2.75 + 2.5}{2}$$

$x_3 = 2.625$

$x_3 > 0$

$[2.625, 2.5]$

$$x_4 = \frac{2.625 + 2.5}{2} = 2.5625$$

$f(2.5625) = 2 \times 2.5625 - 3\sin(2.5625) - 5$

$= -9.1274 \times 10^{-3}$

$f(2.5625) < 0.$

$$x_5 = \frac{2.5625 + 2.625}{2}$$ $[2.625, 2.5625]$

$x_5 = 2.5937$

$f(2.5937) = 2.5937 \times 2 - 3\sin(2.5937) - 5$

$f(2.5937) > 0 \quad = 7.88099$

$x_6 \Rightarrow [2.5937, 2.5625]$

$\Rightarrow x_6 = \dfrac{2.5937 + 2.5625}{2}$

$x_6 = 2.5781$

$f(2.5781) = 2.5781 \times 2 - 3\sin(2.5781) - 5$

$f(2.5781) = 0.00212 > 0$

$\Rightarrow x_7 \Rightarrow [\cancel{2.5}\ [2.5781, 2.5625]$

$x_7 = \dfrac{2.5781 + 2.5625}{2}$

$x_7 = 2.5703$

$\cancel{x_7}\ f(2.5703) = 6.0645 \times 10^{-3} > 0.$

$\Rightarrow \cancel{x_6 \& x_7} = \cancel{2.57}$

$x_8 \Rightarrow [2.5703, 2.5625]$

$x_8 = \dfrac{2.5703 + 2.5625}{2} = 2.5664$

$x_8 < 0 = -1.53146027 \times 10^{-3}$

Round off to 2 decimal place

$x_7 = 2.5703$
$x_8 = 2.5664$

$x_7 = 2.57$ $\quad x_8 = 2.57$

$x_9 = [3, 2.5703]$

$x_9 = \frac{3+2.5703}{2} = 2.7851$

$\Rightarrow \frac{x_7+x_8}{2} = \frac{2.57+2.57}{2}$

Q $f(x) = 3x + \sin(x) - e^x = 0$

$x = 0$

$3 \times 0 + \sin(0) - e^0$

$x = 0 \Rightarrow -1$

$x = 2$

$3 \times 2 + \sin(2) - e^2 = -1.354$

$x = 3$

$\boxed{x = 0}$

$3 \times 0 + \sin(0) - e^0 = -1$

$\boxed{x = 1}$

$3 \times 1 + \sin(1) - e^1 = 0.299$

$\therefore \quad [0, 1]$

$x_1 = \frac{0+1}{2} = 0.5$

$f(0.5) = 3 \times 0.5 + \sin(0.5) - e^{0.5} = -0.1399$

$f(0.5) < 0$

$[1, 0.5]$

$$x_2 = \frac{0.5+1}{2} = \frac{1.5}{2} = 0.75$$

$$f(0.75) = 3\times 0.75 + \sin(0.75) - e^{0.75} = 0.146$$

$$f(0.75) > 0$$

$[0.75, 0.5]$

$$x_3 = \frac{0.75+0.5}{2}$$

$$x_3 = 0.625$$

$$f(0.625) = 3\times 0.625 + \sin(0.625) - e^{0.625}$$

$$f(0.625) > 0 = 0.017$$

$\cancel{[0.625, 0.5]}$ $[0.5, 0.625] \Rightarrow x_4$

$$x_4 = \left[\frac{0.5+0.625}{2}\right]$$

$$x_4 = 0.5625$$

$$f(0.5625) = 3\times 0.5625 + \sin(0.5625) - e^{0.5625}$$

$f(0.5625) < 0 = -0.05773$

$\Rightarrow [0.5625, 1]$

$x_5 = \frac{0.5625+1}{2}$

$x_5 = 0.78125$

$f(0.78125) = 3 \times 0.78125 + \sin(0.78125) - e^{0.78125}$

$f(0.78125) > 0 = 0.173184$

$[0.5625, 0.78125]$

$x_6 = \frac{0.5625 + 0.78125}{2}$

$x_6 = 0.67189$

$f(0.6719) = 3 \times 0.6719 + \sin(0.6719) - e^{0.6719}$

$f(0.6719) > 0 = 0.0694$

$[0.5625, 0.6719]$

$$x_7 = \frac{0.5625 + 0.6719}{2}$$

$$x_7 = 0.6172$$

$$f(0.6172) = 3 \times 0.6172 + \sin(0.6172) - e^{0.6172}$$
$$= 8.6416 \times 10^{-3}$$

$f(0.6172) > 0$.

$[0.5625, 0.6172]$

$$x_8 = \frac{0.5625 + 0.6172}{2}$$

$$x_8 = 0.58985$$

$f(.5898) < 0 = -1.04844$

~~x_9~~

$[0.5895, 0.6172]$

$$x_9 = \frac{0.5895 + 0.6172}{2}$$

$$x_9 = 0.60335$$

$x_9 < 0 = -7.65 \times 10^{-3}$

$[\cdot 60335, 0\cdot 6172]$

$$x_{10} = \frac{0\cdot 60335 + 0\cdot 6172}{2}$$

$x_{10} = \cdot 6103$

$f(\cdot 6103) > 0 = 5\cdot 5793 \times 10^{-4}$

$[\cdot 6034, \cdot 6103]$

$$x_{11} = \frac{\cdot 6034 + 0\cdot 6103}{2}$$

$x_{11} = 0\cdot 60685.$

$$\boxed{\begin{aligned} x_{10} &= \cdot 6103 \\ x_{11} &= \cdot 6159 \end{aligned}}$$

Q $x^4 - x - 10 = 0$

$x = 0$

$0 - 0 - 10 = -10$

$x = 0.5$

$.5^4 - .5 - 10 = -10.43$

$x = 1$

$1 - 1 - 10 = -10$

$x = 1.5$

$1.5^4 - 1.5 - 10 = -6.437$

$x = 2$

$2^4 - 2 - 10 = 4$

$[1.5, 2]$

$x_1 = \frac{1.5+2}{2}$

$x_1 = 1.75.$

$f(1.75) = 1.75^4 - 1.75 - 10 = -2.3710$

$f(1.75) < 0$

$[1.75, 2]$

$$x_2 = \frac{1.75+2}{2} = 1.875$$

$$f(1.875) = 1.875^4 - 1.875 - 10 =$$

$f(1.875) > 0 = 0.4846.$

$[0.4846, 1.75]$

$$x_3 = \frac{0.4846+1.75}{2}$$

$x_3 = 1.1173.$

$$f(1.1173) = 1.1173^4 - 1.1173 - 10$$
$$= -9.5588$$

$f(1.1173) < 0$

$[1.1173, 2]$

$$x_4 = \frac{1.1173+2}{2} = 1.5586$$

$$f(1.5586) = 1.5586^4 - 1.5586 - 10$$

$f(1.5586) < 0 = -5.6574$

$[1.5886, 1.875]$

$$x_5 = \frac{1.5886 + 1.875}{2}$$

$$x_5 = 1.7318$$

$$f(1.7318) = 1.7318^4 - 1.7318 - 10$$
$$= -2.7370$$

$$f(1.7318) < 0$$

$[1.7318, 1.875]$

$$x_6 = \frac{1.7318 + 1.875}{2}$$

$$x_6 = 1.8034$$

$$f(1.8034) = 1.8034^4 - 1.8034 - 10$$

$$f(1.8034) < 0 \quad = -1.226$$

$[1.8034, 1.875]$

$$x_7 = \frac{1.8034 + 1.875}{2} = 1.8392$$

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$[1.8392,\ 1.875]$

$$x_8 = \frac{1.875 + 1.8392}{2}$$

$$x_8 = 1.8571$$

$f\ \cancel{x_8 > 0}$ $f(1.8571) > 0 = 0.0372$

$[1.8392,\ 1.8571]$

$$x_9 = \frac{1.8392 + 1.8571}{2}$$

$$x_9 = 1.8481$$

$f(1.8481) < 0 = -0.1826$

x_9 & $x_8 = 1.8481,\ 1.8571.$

$$\text{Approximate} = \frac{1.8\text{[illegible]}57 + 1.857}{2}$$

Approximate $= 1.86$

Q $x - \exp(-x) = 0$

$x - \exp^{(-x)}$

$\boxed{x = 0}$

$-e^{(0)} = -1$

$\boxed{x = 1}$

$1 - e^{-1} = 1$

$[0, 1]$

$x_1 = \frac{0+1}{2} = 0.5$

$f(0.5) = 0.5 - e^{-0.5}$

$f(0.5) < 0 = -0.1065$

$[0.5, 1]$

$x_2 = \frac{0.5+1}{2}$

$x_2 = \frac{1.5}{2} = 0.75$

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$[0.5, 0.75]$

$$x_3 = \frac{0.5 + 0.75}{2}$$

$$x_3 = 0.625$$

$f(0.625) = 0.089$

$f(0.625) > 0$

$\Rightarrow [0.5, 0.625]$

$$x_4 = \frac{0.5 + 0.625}{2}$$

$$x_4 = 0.563$$

$f(0.563) < 0 = -6.498 \times 10^{-3}$

$[0.563, 0.625]$

$$x_5 = \frac{0.563 + 0.625}{2}$$

$$x_5 = 0.594$$

$f(0.594) > 0 = 0.0418$

$[0.563, 0.594]$

$$x_6 = \frac{0.563 + 0.594}{2}$$

$x_6 = 0.57$[illegible]

$x_5, x_6 = 0.549, 0.584594$

x_6 Approximately $= 0.5464$

$f(0.5729) > 0 = 9.0122 \times 10^{-3}$

$[0.563, 0.5729]$

$$x_7 = \frac{0.563 + 0.573}{2}$$

$x_7 = 0.568$

$f(0.568) > 0 = 1.342 \times 10^{-3}$

~~0.568~~ $[0.563, 0.568]$

$$x_8 = \frac{0.563 + 0.568}{2}$$

$x_8 = 0.566$

$x_7 = 0.568$

$x_8 = 0.566$

Q $x - \sin x - 1/2$

$\boxed{x = 0}$

$0 - \sin(0) - 1/2 = -1/2$

$\boxed{x = 1}$

0.482

$[0, 1]$

$x_1 = \frac{0+1}{2} = 0.5$

$f(0.5) = 0.5 - \sin(0.5) - 0.5$

$f(0.5) < 0 = -8.7265 \times 10^{-3}$

$[0.5, 1]$

$x_2 = \frac{1.5}{2} = 0.75$

$f(0.75) = 0.75 - 0.5 - \sin(0.75)$

$f(0.75) > 0 = 0.2369$

~~0.75, 1~~

$[0.5, 0.75]$

$x_3 = \frac{0.5+0.75}{2} = 0.625$

$f(0.625) = 0.625 - \sin(0.625) - 0.5$

$f(0.625) > 0 = 0.114$

$[0.5, 0.625]$

$$x_4 = \frac{0.5 + 0.625}{2}$$

$x_4 = 0.5625.$

$f(0.5625) > 0 = 0.0526$

$[0.5, 0.5625]$

$$x_5 = \frac{0.5 + 0.5625}{2}$$

$x_5 = 0.5313$

$f(0.5313) > 0 = 0.022$

$[0.5, 0.5313]$

$$x_6 = \frac{0.5 + 0.5313}{2}$$

$x_6 = 0.5157)$

$f(0.5157) > 0 = 6.6699 \times 10^{-3}$ "

$[0.5, 0.5157]$

$$x_7 = \frac{0.5 + 0.5157}{2}$$

$x_7 = 0.5083.$

$x_6 = 0.5157$

$x_7 = 0.5173$

$\boxed{\text{Approx} = 1.4254}$

Q $\exp(-x) - 3\log x = 0$

$e^{-x} - 3\log x = 0$

$\boxed{x = 0}$

$e^0 - 3\log(0) \Rightarrow$ Not possible

$\cancel{1 - 3 = -2}$

$x = 0.5$

$e^{-0.5} - 3\log(0.5) = 1.5096$

$x = 1.5$

$e^{-1.5} - 3\log(1.5) = -0.3051$

$[0.5, 1.5]$

$$x_1 = \frac{0.5+1.5}{2}$$

$$\boxed{x_1 = 2}$$

$f(x_1) < 0 = -0.7697$

$[2, 0.5]$

$$x_2 = \frac{2.5}{2} = 1.25$$

$f(x_2) < 0 = -4.225$

$[1.25, 0.5]$

$$x_3 = \frac{0.5+1.25}{2}$$

$x_3 = 0.875$

$f(0.875) > 0 = 0.59083$

$[0.875, 0.5]$

$$x_4 = \frac{0.875+0.5}{2}$$

$x_4 = 0.6875$

~~0.6875~~ $e^{0.6875} = 3\log(0.6875)$

$= 0.99$//

$f(0.6875) > 0 = 0.99.$

$[1.25, 0.6875]$

$x_5 = \frac{1.25 + 0.6875}{2}$

$x_5 = 0.968$

$x_5 > 0 = 0.4222$

$x_6 \Rightarrow [\cancel{0}1.25, 0.968]$

$x_6 = \frac{1.25 + 0.968}{2}$

$x_6 = 1.109$

$f(1.109) > 0 = 0.1950$

$[1.109, \overset{1.25}{\cancel{0.6875}}]$

$x_7 = \frac{1.109 + 1.25}{2}$

$x_7 = 1.1795$

$f(1.1795) > 0 = 0.09233$

$x_6, x_7 = 1.109, 1.1795$

$$\frac{1.109 + 1.1795}{2}$$

$\boxed{1.144}$

Q $x^3 - x - 11$

$\boxed{x = 1}$ $\boxed{x = 0}$

$1 - 1 - 11 = -11$ $0 - 0 - 11 = -11$

$\boxed{x = 2}$

$2^3 - 2 - 11 = 8 - 2 - 11$

$= 6 - 11 = -5$

$\boxed{x = 2.5}$

$2.5^3 - 2.5 - 11 = 2.125$

$[2, 2.5]$

$x_1 = \frac{2 + 2.5}{2} = 2.25$

$f(2.25) = 2.25^3 - 2.25 - 11 < 0$

$= -1.8593$

$[2.25, 2.5]$

$x_2 = \frac{2.25 + 2.5}{2}$

$x_2 = 2.375$.

$f(2.375) = 2.375^3 - 2.375 - 11$

$f(2.375) > 0 = 0.0214$

$[2.375, 2.25]$.

$x_3 = \frac{2.375 + 2.25}{2}$

$x_3 = 2.3125$

$f(2.3125) < 0 = -0.9460$

$[2.3125, 2.375]$

$x_4 = \frac{2.3125 + 2.375}{2}$

$x_4 = 2.3437$

$f(2.3437) < 0 = -0.4691$

$[2.3437, 2.375]$

$$x_5 = \frac{2.3437 + 2.375}{2}$$

$x_5 = 2.3593$

$f(2.3593) < 0$ $\cancel{2.3593^3}$ -

$= -0.2259$

$[2.3593, 2.375]$

$$x_6 = \frac{2.3593 + 2.375}{2}$$

$x_6 = 2.3671$

Approx $= 2.3683, 2.3671$

Regular Falsi Method

$x^3 - x - 11 = 0$

$[2, 3]$

2.5

$2^3 - 2 - 11 = 8 - 2 - 11 = -5$

$2.5^3 - 2.5 - 11 = 2.5^3 - 2.5 - 11 = 2.125$

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a).$$

$$y - f(2) = \frac{f(2.5) - f(2)}{2.5 - 2}(x - 2)$$

$\boxed{y = 0}$

$$y - f(2) = \frac{f(2.5) - f(2)}{0.5}(x - 2)$$

$[2, 3]$

$2^3 - 2 - 11 = 8 - 2 - 11 = -5$ 27 11

$$y - f(2) = \frac{f(3) - f(2)}{3-2}(x-2)$$

$$y - f(2) = f(3) - f(2)\,(x-2)$$

$\boxed{y=0}$

$$x_1 = a - \frac{f(a)(b-a)}{f(b) - f(a)}$$

$$x_1 = 2 - \frac{f(2)(1)}{f(3) - f(2)}$$

$$x_1 = 2 - \frac{(-5)}{13+5}$$

$$x_1 = \frac{36+5}{18} = \frac{41}{18}$$

$$f(x_1) = \left(\frac{41}{18}\right)^3 - \frac{41}{18} - 11 = -1.460048$$

$f(x_1) < 0$

$$[x, b] \quad \left[\frac{41}{18}, 13\right]$$

$$x_2 = x_1 - \frac{f(x_1)(b - x_1)}{f(b) - f(x_1)}$$

$$x_2 = \frac{41}{18} - \frac{(-1.460048)\left[\cancel{\frac{47}{7}}\ 13 - \frac{41}{18}\right]}{\cancel{47}\ 13 + 1.460048}$$

$$x_2 = \frac{2.27777 - (-1.460048)(0.72222)}{13 + 1.460048}$$

$$x_2 = \frac{2.27777 + 1.05447}{13 + 1.460048}$$

$$x_2 = \frac{3.3324}{14.460048}$$

Q $x^3 - x - 11$

$$x_1 = a - \frac{f(a)(b-a)}{f(b)-f(a)}$$

$\boxed{x=1}$

$1^3 - 1 - 11 = -11$

$\boxed{x=2}$

$2^3 - 2 - 11 = 8 - 2 - 11$

$= 6 - 11 = -5$

$\boxed{x=3}$

$3^3 - 3 - 11 = 27 - 3 - 11$

$= 24 - 11$

$= 13$

$[2, 3]$

$$x_1 = 2 - \frac{f(a)(b-a)}{f(b)-f(a)}$$

$$x_1 = 2 - \frac{(-5)(1)}{13+5}$$

$$= 2 - \frac{(-5)}{18}$$

$$f(x_1) = \left(\frac{41}{18}\right)^3 - \frac{41}{18} - 11$$

$$f(x_1) = 11.8177 - 2.2777 - 11$$

$$f(x_1) < 0 = -1.46$$

$$[x_1, b]$$

$$\left[\frac{41}{18}, 3\right]$$

$$x_2 = x_1 - \frac{f(x_1)\,(b - x_1)}{f(b) - f(a)}$$

$$x_2 = \frac{41}{18} - \frac{(-1.46)\left(3 - \frac{41}{18}\right)}{13 - (-1.46)}$$

$$= \frac{41}{18} - \frac{(-1.46)\left(\frac{54-41}{18}\right)}{13 + 1.46}$$

$$= 2.2777 + \frac{(1.46)(0.722)}{13 + 1.46}$$

$$= 2.2777 + 0.07291$$

$$x_2 = 2.35061$$

$\cancel{f(x_2)} =$

$$[x_2, b] = [2.35061, 3]$$

$$x_3 = x_2 - \frac{f(x_2)(b - x_2)}{f(b) - f(x_2)}$$

$\cancel{x_3 = x}$

$\cancel{x_3 = 2.35061 - (-0.317626)[2.35061 - 2}$

$$x_3 = 2.35061 - \frac{(-0.317626)[3 - 2.35061]}{13 + 0.317626}$$

$$x_3 = 2.35061 - \frac{(-0.317626)[0.64939]}{13.317626}$$

$$x_3 = 2.35061 - \frac{(-0.317626)(0.64939)}{13.317626}$$

$$x_3 = 2.35061 + 0.015487$$

$$x_3 = 2.36609$$

$$f(x_3) = (2.36609)^3 - (2.36609) - 11$$
$$= -0.1198$$

$f(x_3) < 0$

$[2.36609, 3]$

$$x_4 = 2.36609 - \frac{f(x_3)(b - x_3)}{f(b) - f(x_3)}$$

$$x_4 = 2.36009 - \frac{(-0.1198)(3 - 2.36609)}{13 + 0.1198}$$

$$x_4 = 2.36009 - \frac{(-0.1198)(0.66331)}{13.1198}$$

$$x_4 = 2.36009 - \frac{(-0.07946)}{13.1198}$$

$$x_4 = 2.36009 + 6.\cancel{05}61656 \times 10^{-4}$$

$$x_4 = 2.36055$$

$$f(x_4) = 2.36055^3 - 2.36055 - 11$$

$$f(x_4) = -0.20707 < 0$$

$$[2.36055, 3]$$

$\cancel{x_5}$

$$x_5 = x_4 - \frac{f(x_4)(b - x_4)}{f(b) - f(x_4)}$$

$$x_5 = 2.36055 - \frac{(-0.20707)(3 - 2.36055)}{13 + \cancel{2}\,0.20707}$$

$$x_5 = 2.37057$$

$$f(x_5) = (2.37057)^3 - 2.37057 - 11$$

$$f(x_5) = -0.04890 < 0$$

Q $x^3 - 3x + 1 = 0 \quad [1, 2]$

$\Rightarrow$ $x = 1$

$1^3 - 3 + 1 = 1 - 3 + 1 = 2 - 3 = -1$

$x = 2$

$2^3 - 3 \times 2 + 1 = 8 - 6 + 1 = 2 + 1 = 3$

$$x_1 = \cancel{f}(a) - \frac{f(a)\,(b-a)}{f(b) - f(a)}$$

$$x_1 = 1 - \frac{(-1)\quad(2-1)}{3+1}$$

$$x_1 = 1 - \frac{(-1)(1)}{4}$$

$$x_1 = 1 + \frac{1}{4} = \frac{4+1}{4} = \frac{5}{4} = 1.25$$

$$f(x_1) = (1.25)^3 - 3 \times 1.25 + 1$$
$$= \frac{-51}{64} < 0 = -0.7968$$

$[1.25, 2]$

$$x_2 = x_1 - \frac{f(x_1)\,(b - x_1)}{f(b) - f(x_1)}$$

$$x_2 = 1.25 - \frac{(-0.7968)(2 - 1.25)}{3 + 0.7968}$$

$$x_2 = 1.25 - \frac{(-0.5976)}{3.7968}$$

$x_2 = 1.25 + 0.15739$

$x_2 = 1.40739$

$f(x_2) < 0 = -0.43448$

$[1.40739, 2]$

$$x_3 = 1.40739 - \frac{f(x_2)(b - x_2)}{f(b) - f(x_2)}$$

$$x_3 = 1.40739 - \frac{(-0.43448)(2 - 1.40739)}{3 + 0.43448}$$

$$x_3 = 1.40739 - \frac{(-0.43448)(0.59261)}{3.43448}$$

$$x_3 = 1.40739 + \frac{0.257477}{3.43448}$$

$x_3 = 1.40739 + 0.07496$

$x_3 = 1.48235$ //

$f(x_3) < 0 = -0.02824$

$[1.48235, 2]$

$$x_4 = 1.48235 - \frac{f(x_3)(b - x_3)}{f(b) - f(x_3)}$$

$$x_4 = \frac{1.48235}{1} - \left[\frac{(-0.02824)(3-1.48235)}{3+0.02824}\right]$$

$$x_4 = 1.48235 - \left[\frac{(-0.02824)(2.48235)}{3+0.02824}\right]$$

$$x_4 = 1.48235 + [0.02314]$$

$$x_4 = 1.50549$$

$$f(1.50549) < 0 = -0.10427$$

$$[1.50549, 2]$$

$$x_5 = x_4 - \frac{f(x_4)(b - x_4)}{f(b) - f(x_4)}$$

$$x_5 = 1.50549 + \frac{.10427(2-1.50549)}{3+0.10427}$$

$$x_5 = 1.50549 + \frac{.10427 \times 0.49451}{3.10427}$$

$$x_5 = 1.50549 + 0.01661$$

$$x_5 = \cancel{1.5}5\ 1.5221$$

$$f(1.5221) = (1.5221)^3 - 3 \times 1.5221 + 1$$

$$f(1.5221) < 0 = -0.03991$$

$$x_6 = x_5 - \frac{f(x_5)(b-x_5)}{f(b)-f(x_5)}$$

$$x_6 = 1.5221 - \frac{(-0.03491)(2-1.5221)}{3+0.03991}$$

$$x_6 = 1.5221 + \frac{0.01907}{3.03991}$$

$$x_6 = 1.528130$$

$$f(1.5283) = 1.5283^3 - 3\times1.5223+1$$
$$f(1.5283) < 0 = -0.01524$$

$\cancel{x_7 = x_6 - (-0.01524) f(2-}$

$[1.5283, 2]$

$$x_7 = 1.5283 - \frac{(-0.01524)[2-1.5283]}{3+0.01524}$$

$$x_7 = 1.5283 + \frac{(0.01524)[0.4717]}{3.01524}$$

$$x_7 = 1.53067$$

$$f(1.53068) < 0 = -5.6885\times10^{-3}$$

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Q $x^3 - 2x - 5 = 0 \quad (2,3)$

$2^3 - 2\times2 - 5 = 8-4-5$
$= 8-9$
$= -1$

$3^3 - 2\times3 - 5 = 27-6-5$
$= 27-11 = 16$

$[2,3]$

$$x_1 = a - \frac{f(a)(b-a)}{f(b)-f(a)}$$

$$x_1 = 2 - \frac{(-1)(3-2)}{16+1}$$

$$x_1 = 2 - \frac{(-1)(1)}{17}$$

$$x_1 = 2 + \frac{1}{17}$$

$$x_1 = \frac{35}{17} = 2.05882$$

$f(2.05882) < 0 = -0.39083$

$$x_2 = x_1 - \frac{f(x_1)(b-x_1)}{f(b)-f(x_1)}$$

$$x_2 = 2.05882 - \frac{(-0.39083)[3-2.05882]}{16+0.39083}$$

$$x_2 = 2.05882 + \frac{0.36784}{16.39083}$$

$$x_2 = 2.05882 + 0.02244$$

$$x_2 = 2.08126$$

$$f(2.08126) < 0 = -0.147244$$

$$[2.08126, 3]$$

$$x_3 = x_2 - \frac{f(x_2)(b - x_2)}{f(b) - f(x_2)}$$

$$x_3 = 2.08126 + \frac{0.147244(3 - 2.08126)}{16 + 0.147244}$$

$$x_3 = 2.08126 + \text{Ans}$$

$$x_3 = 2.08963$$

$$f(2.08963) = 2.08963^3 - 2 \times 2.08963 - 5 < 0$$

$$= -0.05477$$

$$x_3 = 2.08963$$

$$x_2 = 2.08126$$

$$[2.08963, 3]$$

$$x_4 = x_3 - \frac{f(x_3)(b - x_3)}{f(b) - f(x_3)}$$

$$x_4 = 2.08963 - \frac{(-0.05477)[3 - 2.08963]}{16 + 0.05477}$$

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~~$x_4 = 2.08963$~~

$x_4 = 2.09273$

$x_4 = 2.09273$
$x_3 = 2.08963$
$= 2.09863$

The Root is 2.09

Fixed point method

(1.) $x^3 - 3x + 1 = 0$
$[1, 2]$

$x = 1 \Rightarrow 1^3 - 3 + 1 = 1 + 1 - 3$
$= -1$

$x = 2 \Rightarrow 2^3 - 3 \times 2 + 1 = 8 - 6 + 1$
$= 2 + 1$
$= 3$

$x_0 = (3x-1)^{1/3} \quad [1, 2]$

$\phi_1'(x) = \frac{1}{3}(3x-1)^{-2/3} \times 3$

$\phi_1'(x) = \left| \frac{1}{\cancel{3}(3x-1)^{2/3}} \right| \times \cancel{3}$

$$\phi_1'(x)\Big|_{x=1} = \left|\frac{1}{(3\times1-1)^{2/3}}\right|$$

$$\phi_1'(x) < 1$$

$$x_1 = \phi(x_0)$$
$$x_1 = (3\times1-1)^{1/3}$$
$$x_1 = 2^{1/3} = 1.2599$$

$$x_2 = (3\times1.2599-1)^{1/3}$$
$$x_2 = 1.4060$$

$$x_3 = (3\times1.4060-1)^{1/3}$$
$$= (4.218-1)^{1/3}$$
$$= (3.218)^{1/3}$$
$$x_3 = 1.4763$$

$$x_4 = (3\times1.4763-1)^{1/3}$$

$$x_4 = 1.5079$$

$$x_5 = (3\times1.5079-1)^{1/3}$$
$$x_5 = 1.5217$$

$x_6 = (3 \times 1.5217 - 1)^{1/3}$

$x_6 = 1.5276$

$x_7 = (3 \times 1.5276 - 1)^{1/3}$

$x_7 = 1.5301$

$x_8 = (3 \times 1.5301 - 1)^{1/3}$

$x_8 = 1.5312$

Approx. $= 1.53$

Q $\cos x - x e^x = 0$

$\boxed{x=1}$ $\boxed{x=0}$

$\cos 1 - 1e^1$ $\cos 0 - 1e^0$

$\boxed{x=2}$ $1 - 1 = 0$

$\cos(2) - 2e^2 = -13.778$

$x_0 = \frac{\cos x}{e^x}$

$\phi_1'(x) = \frac{e^x \cos x - e^x \sin x}{e^x}$

$\phi_1'(x) = e^2 \cos(2) - e^{(2)} \sin 2$

$\phi_1'(x)_{x=2} \quad \dfrac{7.3845 - 0.2578}{7.3890}$

$\phi_1'(x)_{x=2} < 1$

$x_1 = \cancel{co}\, \phi(x_0)$

$x_1 = \dfrac{\cos x}{e^x}$

$x_1 = \dfrac{\cos(2)}{e^2}$

$x_1 = 0.13525$

$x_2 = \dfrac{\cos(x_1)}{e^{x_1}} = \dfrac{\cos(0.13525)}{e^{0.13525}}$

$x_2 = 0.87349$

$x_3 = \dfrac{\cos(x_2)}{e^{x_2}} = \dfrac{\cos(0.87349)}{e^{0.87349}}$

$x_3 = 0.41744$

$$x_4 = \frac{\cos(0.41744)}{e^{0.41744}}$$

$$x_4 = 0.65871$$

$$x_5 = \frac{\cos(0.65871)}{e^{0.65871}}$$

$$x_5 = 0.51748$$

$$x_6 = \frac{\cos(0.51748)}{e^{0.51748}}$$

$$x_6 = 0.59599$$

$$x_7 = \frac{\cos(0.59599)}{e^{0.59599}}$$

$$x_7 = 0.55098$$

$$x_8 = \frac{\cos(0.55098)}{e^{0.55098}}$$

$$x_8 = 0.57635$$

$$x_9 = \frac{\cos(0.57635)}{e^{0.57635}}$$

$$x_9 = 0.56191$$

$$x_{10} = \frac{\cos(0.58191)}{e^{0.58191}}$$

$$x_{10} = 0.57009$$

$$x_{11} = \frac{\cos(0.57009)}{e^{0.57009}}$$

$$x_{11} = .56544$$

$$x_{12} = \frac{\cos(0.56544)}{e^{0.56544}}$$

$$x_{12} = .56803$$

Ex
Q1

$x^3 - 3x + 1 = 0$

method of iteration.

$f(1) = 1 - 3 + 1 = 2 - 3 = -1$

$f(2) = 8 - 6 + 1 = 3.$

$[1, 2]$

$x = (3x-1)^{1/3}$

$\phi(x) = (3x-1)^{1/3}.$

$\phi'(x) = (3x-1)^{1/3 - 1}$

$\phi_1'(x) = \dfrac{1}{(3x-1)^{2/3}}$

$|\phi'(x)| < 1$ for $x_0 = 2$ btw $[1, 2]$

$x_1 = \phi(x_0) = (3 \times 2 - 1)^{1/3}$
$= (6-1)^{1/3}$
$= (5)^{1/3} = 1.7099$
$= 1.7189$

$x_2 = \phi(x_1) = (3 \times 1.7189 - 1)^{1/3}$
$= 1.6078$
$= 1.6168$
$= 1.6177$

$$x_3 = \phi(x_2) = (3 \times 1.6177 - 1)^{1/3}$$

$$= 1.5677.$$

$$x_4 = \phi(x_3) = (3 \times 1.5677 - 1)^{1/3}$$

$$= 1.5471$$

$$x_5 = \phi(x_4) = (3 \times 1.5471 - 1)^{1/3}$$
$$= 1.5384$$

$$x_6 = \phi(x_5) = (3 \times 1.5384 - 1)^{1/3}$$
$$= 1.5347$$

$$x_7 = \phi(x_6) = (3 \times 1.5347 - 1)^{1/3}$$
$$= 1.5332$$

$$x_8 = \phi(x_7) = (3 \times 1.5332 - 1)^{1/3}$$
$$= 1.5325$$

$$x_9 = \phi(x_8) = (3 \times 1.5325 - 1)^{1/3}$$

$$= 1.5322$$

$x_8 = x_9$ upto 3 decimal place

1.532.

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Q $3x - \log x = 6$

$f(x) = 3x - \log x - 6$

$\boxed{x=0}$

$\cancel{f(x)}\ f(0) = 0 - 1 - 6 = -7$

$\boxed{x=1}$

$\cancel{f(x)}\ f(1) = -3$

$\boxed{x=2}$

$f(2) = -0.3010$

$\boxed{x=3}$

$f(3) = 2.5228.$

$f(x) = 0\ [2, 3]$

$\left[\log_{10} x = \log_e x \times \log_{10} e\right.$

$3x = \log x + 6$

$x = (\log x + 6)\frac{1}{3}$

$\phi'(x) = \frac{1}{3}(\log_{10} x + 6)$

$\phi'(x) = \frac{1}{3}(\log_e x \times \log_{10} e)$

$\phi'(x) = \frac{1}{3}\left(\frac{\log e}{x}\right)$

$$\phi'(x) = \frac{1}{3} \frac{\log_{10} e}{x}$$

$$\phi'(x) = \frac{1}{3} \log e = \frac{0.4343}{x}$$

$$\phi'(x) = \frac{1}{3} \times \left[\frac{0.4343}{x}\right]$$

~~$\phi'(x) =$~~

$$\phi'(2) = 0.0723$$

$$\phi'(3) = 0.0482$$

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$\boxed{x_0 = 2}$

$\Rightarrow$ $x_1 = \phi(x_0) = \frac{1}{3}(6 + \log x)$

$x_1 = \frac{1}{3}(6 + \log(2))$

$x_1 = \frac{1}{3}(6 + \log 2)$

$x_1 = 2.1003$

$x_2 = \phi(x_1)$

$x_2 = \frac{1}{3}(6 + \log(2.1003))$

$x_2 = \frac{1}{3}(6 + 0.3222)$

$x_2 = 2.1074$

$x_3 = \phi(x_2)$

$x_3 = \cancel{\phi}\frac{1}{3}(6 + \log(2.1074))$

$x_3 = 2.1079$

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$x_4 = \phi(x_3) = \frac{1}{3}\left(6+\log(2.1079)\right)$

$x_4 = 2.1079 \quad = 2.1088$

$x_5 = \phi(x_4) = \frac{1}{3}\left(6+\log(2.1088)\right)$

$= 2.1080$

$x_4 = x_5$ upto 3 decimal place

2.108

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3. $3x - \sqrt{1+\sin x} = 0$

$f(x) = 3x - \sqrt{1+\sin x}$

$\boxed{x=0}$

$f(0) = -1$

$f(1) = 1.991 \approx 2$

$\therefore [0,1]$

$3x = \sqrt{1+\sin x}$

$x = \dfrac{\sqrt{1+\sin x}}{3}$

$x = \sqrt{\dfrac{1+\sin x}{3}}$

$\boxed{x=0} = 0.3333$

$\boxed{x=1} = 0.3362$

$\phi(x) = \dfrac{1}{3}(1+\sin x)^{1/2}$

$\phi'(x) = \dfrac{1}{6}(1+\sin x)^{1/2-1} \times \cos x$

$\phi'(x) = \dfrac{1}{6}\dfrac{\cos x}{\sqrt{1+\sin x}}$

$\boxed{x=0}$ $\phi'(0) = 0.1666$

$\boxed{x=1}$ $\phi'(1) = 0.1652$

$x_0 = 1$

$$x_1 = \phi_1(x_0)$$

$$x_1 = \sqrt{\frac{1+\sin(1)}{3}}$$

$$x_1 = \cancel{0.5823}\ 0.3362$$

$$x_2 = \phi(x_1)$$

$$x_2 = \cancel{\sqrt{\frac{1+\sin(0.5823)}{3}}}\ \sqrt{\frac{1+\sin(0.3362)}{3}}$$

$$x_2 = \cancel{0.5802}\ 0.3343$$

$x_3 = \frac{\sqrt{1+\sin(0.5802)}}{3}$

$x_3 = 0.5802$

$x_2 = x_3 = 0.580$

$x_3 = \frac{\sqrt{1+\sin(0.3343)}}{3}$

$x_3 = 0.3343$

(2) $x^3 + x^2 - 100 = 0$

$\Rightarrow \quad f(x) = x^3 + x^2 - 100$

$f(4) = 4^3 + 4^2 - 100 = -20$

$f(5) = 5^3 + 5^2 - 100 = 50.$

~~$x_0 = 4$~~ $\quad [4, 5]$

$\boxed{x_1 = \phi(x_0)}$

$\phi(x) = x^3 + x^2 - 100$

~~$\phi'(x) = 3x^2 + 2x$~~

$\phi(x) = x^2(x+1) = 100$

$= x = \frac{\sqrt{100}}{\sqrt{x+1}} = \frac{10}{\sqrt{x+1}}$

Newton Ramphson method

Q $x^3 - 3x + 1 = 0$ [1 2]

Special 1

$f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

$\Rightarrow$ $f(1) = 1^3 - 3 + 1 = \cancel{25} - 3 + 2 = -1$

$f(2) = 8 - 6 + 1 = 2 + 1 = 3$

$\Rightarrow$ Numerically 1 is lesser than 3.

$\Rightarrow$ so $\boxed{x_0 = 1}$

$f(x_0) = f(1) = -1$

$f'(x_0) = f'(1) = 3 - 3 = 0$

Not possible so we will take approximation here.

$\because x_0 = \frac{1+2}{2} = 1.5$

$\boxed{x_0 = 1.5}$

$f(x_0) = f(1.5) = 1.5^3 - 3 \times 1.5 + 1$

$= 3.375 - 4.5 + 1 = -0.125$

$f'(1.5) = 1.5^2 \times 3 - 3$

$= 3.75$

$\Rightarrow$ Now

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1.5 - \left(\frac{-0.125}{3.75}\right)$$

$$x_1 = 1.5 + 0.0333$$

$$x_1 = 1.5333$$

$$f(x_1) = (1.5333)^3 - 3\times1.5333 + 1) = 0.0049$$

$$f'(x_1) = 3\times(1.5333)^2 - 3 = 4.053$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1.5333 - \frac{0.0049}{4.053}$$

$$x_2 = \underline{\underline{1.5321}}$$

$$\because\ x_3 = (1.5321)^3 - 3\times1.5321 + 1$$

$$x_3 = 3.5963 - 4.5963$$

$$x_3 \qquad + 1$$

$$x_3 = -1 + 1 = 0$$

(2) $x^3 - 5x - 6$

$\Rightarrow$ $x = 0 \quad = -6$

$x = 1 \quad = 1 - 5 - 6$

$= -11 + 1 = -10$

$x = 2 \quad = 2^3 - 5 \times 2 - 6$

$= 8 - 10 - 6$

$= -16 + 8 = -8$

$x = 3 \quad = 3^3 - 5 \times 3 - 6$

$= 27 - 15 - 6$

$= 6.$

$[2, 3]$

$f(2) = 2^3 - 5 \times 2 - 6 \qquad f(3) = 3^3 - 5 \times 3 - 6$

$= -8 \qquad = 6$

$f'(2) = 3x^2 - 5 \qquad f'(3) = 3 \times 3^2 - 5$

$= 3 \times 2^2 - 5 \qquad = 27 - 5$

$= 12 - 5 = 7 \qquad = 22$

$\because \boxed{x_0 = 3}$

$f(x_0) = f(3) = 6.$

$\because f'(x_0) = 22$

$$x_1 = x_0 - \frac{f(x)}{f'(x_0)} = 3 - \frac{6}{22} = 2.7272$$

$$f(x_1) = (2.7272)^3 - 5 \times 2.7272 - 6$$

$$= 0.6478$$

$$f'(x_1) = 17.3128$$

$$\cancel{f(x_2)}\ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.7292 - \frac{0.6478}{17.3128}$$

$$= 2.7272 - 0.0374$$

$$\boxed{x_3 = 2.6898}$$

$$f(x_3) = (2.6898)^3 - 5 \times 2.6898 - 6$$

$$= 0.0117$$

$$f'(x_3) = 16.7050$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.6898 - \frac{0.0117}{16.7050}$$

$$\boxed{x_4 = 2.6890}$$

$$x_3 \approx x_4 = \underline{2.689}$$

(6) $xe^x = \cos x$

$xe^x - \cos x = f(x) \rightarrow f'(x) = e^x + xe^x + \sin x$

$x = 0 \quad = -1$

$x = 1 \quad = 1.7184$

$[0, 1]$

$\boxed{x_0 = 0}$

$f(x_0) = f(0) = -1$

$f'(x_0) = f'(0) = 1 + 0$

$$x_1 = x_0 - \frac{f(x_0)}{f(x_1)}$$

$$x_1 = 0 - \frac{(-1)}{(1)}$$

$\boxed{x_1 = 1}$

$$f(x_1) = e^1 - \cos(1) = 1.7184$$

$$f'(x_1) = e^1 + 1e^1 + \sin(1) = 5.4365$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1 - \frac{1.7184}{5.4365} = 1 - 0.3160 = 0.684$$

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$x_3 = 0.684$

$f(x_3) = 0.684\, e^{0.684} - \cos(0.684)$

$f'(x) = 0.684\, e^{0.684} + e^{0.684} + \sin(0.684)$

$f(x_3) = 0.3556$

$f'(x_3) = 2.6983$

$x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)}$

$x_4 = 0.684 - 0.1317$

$\boxed{x_4 = 0.553}$

$f(x_4) = 0.553\, e^{0.553} - \cos(0.553)$

$= -0.038$

$f'(x_4) = 2.7094$

$x_5 = 0.553 + 0.014$

$\boxed{x_5 = .567}$

$f(x_5) = 0.567\, e^{0.567} - \cos(0.567)$
$= -3.4693 \times 10^{-4}$
$f'(x_5) = 2.7724$

$$x_6 = .567 - \frac{-3.4693 \times 10^{-4}}{2.7724}$$

$$x_6 = .567 + (1.2513 \times 10^{-4})$$

$$\boxed{x_6 = 0.567}$$

Eg Q5 $\sqrt{12}$

$x = \sqrt{12}$
$x^2 = 12$
$f(x) = x^2 - 12$
$f'(x) = 2x$

$f(3) = 3^2 - 12 = 9 - 12$
$= -3$
$f(4) = 16 - 12 = 4$

$\Rightarrow f'(3) = 6$
$\boxed{x_0 = 3}$
$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 3.5$ $\qquad f(3.5) = 0.25$

$f'(3.5) = 7$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 3.5 - \frac{f(3.5)}{f'(3.5)}$$

$$= 3.5 - \frac{0.25}{7}$$

$$\boxed{x_2 = 3.464}$$

$f(x_2) = -7.04 \times 10^{-4}$ $\qquad f'(x_2) = 6.928$

$$x_3 = 3.464 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 3.464 - \frac{7.04 \times 10^{-4}}{6.928}$$

$$x_3 = 3.464 - 1.016 \times 10^{-4}$$

$$x_3 = 3.4638$$

$$\boxed{x_3 = 3.464}$$

(13) $\sqrt[3]{17}$

$\Rightarrow$ $x^3 - 17 = 0$

$\Rightarrow$ $f(x) = x^3 - 17$

$\Rightarrow$ $f'(x) = 3x^2$

$\Rightarrow$ $\boxed{x=0}$

$f(0) = -17$

$\boxed{x=1}$

$f(1) = 1 - 17$

$\boxed{x=2}$

$f(2) = 8 - 17 = -11$ $\quad f'(2) = 12$

$\boxed{x=3}$

$f(3) = 27 - 17 = \underline{\underline{20}}$

$[2, 3]$

$\boxed{x_0 = 2}$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \frac{(-9)}{12}$$

$$x_1 = 2 + 0.75$$

$$\boxed{x_1 = 2.75}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 2.75 - \frac{2.75^3 - 17}{3 \times 2.75^2}$$

$$x_2 = 2.75 - 0.1673$$

$$\boxed{x_2 = 2.5827}$$

$$x_3 = 2.5827 - 0.0113$$

$$\boxed{x_3 = 2.5714}$$

$$x_4 = 2.5714 - 3.0446 \times 10^{-4}$$

$$\boxed{x_4 = 2.5710}$$

$$\boxed{x_3 \,\&\, x_4}$$

Q $3x - \cos x - 1 = 0$

$f(x) = 3x - \cos x - 1$

$f'(x) = 3 + \sin x = (3)$

$f(0) = 3\times 0 - 1 - 1 = -2$

$f(1) = 3\times 1 - \cos 1 - 1$

$= 3 - \cancel{2}1 = 2$

$f(0) \Rightarrow f(x_0)$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 - \frac{(-2)}{3}$$

$$\boxed{x_1 = \frac{2}{3} = \underline{\underline{0.66666}}}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$f(x_1) \Rightarrow 3\times 0.66666 - \cos(0.66666) - 1$

$= -1.32321 \times 10^{-4}$

$f'(x_1) = 3.01163$

$x_2 = 0.66666 + \frac{1.32321\times 10^{-4}}{}$

$$\boxed{x_2 = 0.6666}$$

(14)

$\sqrt[3]{N}$

$x^3 - N = f(x)$

$f'(x) = 3x^2$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\boxed{x_{n+1} = x_n - \frac{(x^3 - N)}{3x_n^2}}$$

$\sqrt[5]{10}$

$x^5 - 10 = f(x)$

$f'(x) = 5x^4$

$x = 0 = 0 - 10 = -10$

$x = 1 = 1 - 10 = -9$ 4x4

$x = 2 = 2^5 - 10 = 32 - 10 = 22$

$[1, 2]$

$[x_0 = 1]$

$f(x_0) = f(1) = -9$

$f'(x_0) = f'(1) = 5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1 - \frac{(-9)}{5}$$

$$x_1 = 1 + 1.8 = 2.8$$

$$\boxed{x_1 = 2.8}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \qquad f(2.8) = 2.8^5 - 10 = 162.1036$$

$$f'(2.8) = 307.328$$

$$x_2 = 2.8 - \frac{162.103}{307.328}$$

$$x_2 = 2.8 - 0.5274$$

$$x_2 = 2.2726$$

$$\boxed{x_2 = 2.272}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \qquad f(x_2) = 50.5398$$

$$f'(x_2) = 133.230$$

$$x_3 = 2.272 - \frac{50.5398}{133.230}$$

$$x_3 = 2.272 - 0.3793$$

$$\boxed{x_3 = 1.892}$$

$$x_4 = 1.892 - \frac{14.2440}{64.0699}$$

$$x_4 = 1.892 - 0.2223$$

$$x_4 = 1.6697$$

$$\boxed{x_4 = 1.6787}$$

$$x_5 = 1.678 - \frac{3.3033}{39.6403}$$

$$x_5 = 1.678 - 0.0833$$

$$x_5 = 1.5947$$

$$\boxed{x_5 = 1.595}$$

$$x_6 = 1.595 - \frac{0.3229}{32.3603}$$

$$x_6 = 1.585$$

$$\boxed{x_6 = 1.594}$$

$x \sin x + \cos x$

1.3818

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(15) $x^3 - \sin x + 1 = 0$

=) $f(x) = x^3 - \sin x + 1$
$f(-x) = -x^3 + \sin x + 1$

$\frac{\pi \times 200}{180}$

~~$f(-4) = -63.75$~~

$f(0) = 1$
$f(1) = 0.84$
$f(2) = -2^3 + \sin(2) + 1$
$= -6$

$f'(-x) = -3x^2 + \cos x$
$f'(-1) = -2.459$

$[1, 2]$

$\boxed{x_0 = 1}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 1 + \frac{0.84}{2.468}$

$\boxed{x_1 = 1.340}$

$x_2 = 1.340 - \frac{0.432}{5.158}$

$\boxed{x_2 = 1.256}$

$x_3 = 1.256 - \frac{(-0.030)}{-4.422}$

$= 1.256 - 6.784 \times 10^{-3} = 1.258$

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(7) $x \sin x + \cos x = 0$

$f(x) = x \sin x + \cos x$

$f(3) = -0.5666$

$\cancel{2 \sin(2) + \cos(2)}$

$f(2) = 1.4024$

$\boxed{x_0 = 3}$ $f'(x) = x\cos x + \cancel{\sin x}$ $\sin x$

$f'(3) = -2.9699$

$x_1 = 3 - \frac{(-0.5666)}{(-2.9699)}$

$x_1 = 3 - 0.1907$

$\boxed{x_1 = 2.8093}$

LectureNotes.in

$x_2 = 2.8093 - \frac{(-0.0288)}{-2.6556}$

$x_2 = 2.8093 - 0.0108$

$x_2 = 2.7985$

$\boxed{x_2 = 2.8885}$

$x_3 = 2.8885 - \frac{(-0.2448)}{-2.7964}$

$x_3 = 2.8885 - 0.0875$

$\boxed{x_3 = 2.801}$

$$x_4 = 2.801 - \frac{(-6.8946 \times 10^{-3})}{-2.6401}$$

$$x_4 = 2.801 - 2.6114 \times 10^{-3}$$

$$x_4 = 2.798$$

$$x_5 = 2.798 - \frac{1.017 \times 10^{-3}}{-2.6344}$$

$$\boxed{x_5 = 2.798}$$

(8) $x\cos x = 0$

$\Rightarrow$ $\boxed{x = 2}$

$f(2) = -0.8322$

$f(3) = -2.969$

$f(4) = -2.614$

$f(5) = 1.418$

$\Rightarrow$ $\cancel{f(x) = \cos x\, x}$ =

$f'(x) = -x\sin x + \cos x$

$f'(5) = -5\sin(5) + \cos(5)$

$f'(5) = 5.078$

$\boxed{x_0 = 5}$

$$x_1 = 5 - \frac{1.418}{5.078} = 4.720$$

$\boxed{x_2 = 4.720}$

$$4.720 - \frac{0.035}{4.727}$$

$\boxed{x_3 = 4.712}$

$$x_4 = 4.712 - \frac{(-1.832 \times 10^{-3})}{4.7116}$$

$$x_4 = 4.712 + 3.8901 \times 10^{-4}$$

$\boxed{x_4 = 4.712}$

Regular False Method

(1.) $x^3 - 3x - 5 = 0$

$f(2) = 2^3 - 3\times2 - 5 = 8 - 6 - 5$
$= 2 - 5 = -3$

$f(3) = 3^3 - 3\times3 - 5 = 13$

$x_1 = \dfrac{a f(b) - b f(a)}{f(b) - f(a)}$ $[2, 3]$

$= \dfrac{2(13) - 3(-3)}{13 + 3}$

$= \dfrac{26 + 9}{16} = 2.1875$

$f(2.1875) = -\cancel{2.8949}\ -1.0949 < 0$
$= -2.\cancel{895}8 < 0$

$\Rightarrow$ $[\cancel{-2.8958}\ 2.1875, 3]$

$\Rightarrow$ ~~$x_2 = 2.1875(-2.8958) - 3$~~

$x_2 = \dfrac{2.1875(13) - 3\times(\cancel{-2.8958}\ -1.0949)}{13 + \cancel{2.8958}\ 1.0949}$

$x_2 = 2.25061$

$x_2 = 2.2506$

$f(x_2) = 2.2506^3 - 3 \times 2.2506 - 5.$

$= \cancel{-1.7884} < 0$
-0.21706

$[2.2506, 3]$

$$x_3 = \frac{2.2506(13) - 3 \times (\cancel{-1.7884}\ -.21706)}{13 + 0.21706}$$

$x_3 = 2.2186$

$f(x_3) = (2.2186)^3 - 3 \times 2.2186 - 5$
$= -0.7354 < 0$

$[2.2186, 3]$

$$x_4 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{2.2186(13) - 3 \times (-0.7354)}{13 + 0.7354}$$

$$= \frac{31.048}{13.7354}$$

$= 2.2604$

$$f(x_4) = 2.2604^3 - 3 \times 2.2604 - 5$$

$$f(2.2604) = -0.2318 < 0$$

$$[2.2604, 3]$$

$$x_5 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{2.2604(13) + 3 \times 0.2318}{13 + 0.2318}$$

$$= \frac{30.0806}{13.2318}$$

$$x_5 = 2.2733$$

$$f[2.2733] = -0.0717 < 0$$

$$[2.2733, 3]$$

$$x_6 = \frac{2.2733[13] + 0.0717 \times 3}{13 + 0.0717}$$

$$x_6 = 29.768$$

$$x_6 = 2.2772$$

$f(2.2772) = -0.0228.$

$[2.2772, 3]$

$$x_7 = \frac{2.2772(13) - 3\times(-0.0228)}{13.0228}$$

$$x_7 = \frac{29.672}{} = 2.2784$$

$f(2.2784) = -7.7828\times10^{-3}$

$[2.2784, 3]$

$$x_8 = \frac{2.2784\times13 + 3\times7.7828\times10^{-3}}{13+7.7828\times10^{-3}}$$

$$= \frac{29.7153}{13.0077} = 2.2852$$

$f(2.2852) = 0.0780$

$$= \frac{2.2784\times13 + 3\times0.0077}{13 + 0.0077}$$

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(3) $x^2 - \log x - 12 = 0$

$x = 0 = 0 - \log(0) - 12 = 0$

$= -1 - 12 = -13$

$x = 1 = 1 - \log(1) - 12$

$= 1 - 12 = -13.$

$x = 2 = 2^2 - \log(2) - 12$

$= 4 - \log(2)$

$x = 3 = -3.477$

$x_0 = 4 = 3.397.$

$[3, 4]$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$= 3 \times 3.397 + 3.477(4)$

$$x_1 = \frac{24.099 \; \cancel{+13.908}}{6.874}$$

$x_1 = 3.505819$

$f(x_1) = -0.254022$

$x(x-4-1) - 4(x-4-1)$

$x^2 - 4x - x - 4x + 16 + 4$

$[3.505819, \cancel{3.397}\ 4]$

$$x_2 = \frac{af(b) - b(f(a))}{f(b) - f(a)}$$

$$= \frac{3.505819 \times 3.397 + 4 \times 0.254022}{3.397 + 0.254022}$$

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Secant method

Newton Ramphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$\Downarrow$

This is not applicable if the derivative of f^n doesn't exist.

$$f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{\frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}}$$

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1})(f(x_n))}{f(x_n) - f(x_{n-1})}$$

☆ **Determination of Rate of Convergence of Secant method.**

⇒ Let α be the exact Root.

$$x_{n+1} - \varepsilon_{n+1} = \alpha. \quad \text{—} (1)$$

[ε_{n+1} is the error as x_{n+1} is ε_{n+1} away from α]

(Sum btw α & x_{n+1} is ε_{n+1})

Similarly

$$x_n - \varepsilon_n = \alpha. \quad \text{—} (2)$$

→ Secant method

$$\boxed{x_{n+1} = x_n - \frac{(x_n - x_{n-1})\, f(x_n)}{f(x_n) - f(x_{n-1})}}$$

$$\underbrace{\alpha + \varepsilon_{n+1}} = \alpha + \varepsilon_n - \frac{(\cancel{\alpha} + \varepsilon_n - \cancel{\alpha} - \varepsilon_{n-1})\, f(\alpha + \varepsilon_n)}{f(\alpha + \varepsilon_n) - f(\alpha + \varepsilon_{n-1})}$$

$$\varepsilon_{n+1} = \varepsilon_n - \frac{f(\alpha + \varepsilon_n)(\varepsilon_n - \varepsilon_{n-1})}{f(\alpha + \varepsilon_n) - f(\alpha + \varepsilon_{n-1})} \quad \text{—} (1)$$

$$f(\alpha+\varepsilon_n) = \overset{\nearrow 0}{f(\alpha)} + \frac{\varepsilon_n f'(\alpha)}{1!} + \frac{\varepsilon_n^2}{2!} f''(\alpha)$$

(since α is the exact so neglecting high terms)

$$f(\alpha+\varepsilon_n) = \varepsilon_n f'(\alpha) + \frac{\varepsilon_n^2}{2!} f''(\alpha)$$

$$f(\alpha+\varepsilon_n) = \left[1 + \frac{\varepsilon_n}{2!} \frac{f''(\alpha)}{f'(\alpha)}\right] \varepsilon_n f'(\alpha)$$

$$f(\alpha+\varepsilon_n) = \left[1 + \frac{M}{2!} \varepsilon_n\right] \varepsilon_n f'(\alpha)$$

$$\varepsilon_{n+1} = \varepsilon_n - \frac{\varepsilon_n f'(\alpha)(1+M\varepsilon_n)(\varepsilon_n - \varepsilon_{n-1})}{\varepsilon_n f'(\alpha)(1+M\varepsilon_n) - \varepsilon_{n-1} f'(\alpha)(1+M\varepsilon_{n-1})}$$

By the definition of Rate of convergence.

$$|e_{n+1}| = K|e_n|^P \text{ --- } (2).$$

$$e_n = K_1(e_{n-1})^P \text{ --- } (3)$$

$$e_{n+1} = M e_n \cdot e_{n-1} \text{ --- } (1)$$

from (1) & (2).

$$K|e_n|^P = M|e_n \cdot e_{n-1}|$$

dividing both sides by e_n

$$\frac{|e_n|^P}{e_n} = \frac{M}{K}|e_{n-1}|$$

$$(e_n)^{P-1} = \left(\frac{M}{K}\right)|e_{n-1}|$$

$$e_n = \left(\frac{M}{K}\right)^{\frac{1}{P-1}}|e_{n-1}|^{\frac{1}{P-1}}$$

from (3)

$$e_n = K_1(e_{n-1})^P$$

$$(e_{n-1})^P \times K_1 = \left(\frac{M}{K}\right)^{\frac{1}{P-1}}|e_{n-1}|^{\frac{1}{P-1}}$$

Since base are same variable power will be equal.

$$p = \frac{1}{p-1}$$

$(p = 1.1618)$ Rate of convergence by secant method.

Finite difference

Forward diff operator

$$\boxed{\Delta^n y_i = \Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i}$$

We define

$$\Delta f(x) = f(x+h) - f(x).$$

$$\boxed{\Delta y_0 = f(x_1) - f(x_0) = y_1 - y_0}$$

Δ is called forward diff. operator

$\Delta y_0, \Delta y_1 \; --- \; \Delta y_{n-1}$
called first diff ~~operator~~.
↳ forward

Second diff
↳ forward.

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$$

$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$$

Backward difference operator

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\cancel{\Delta^5 y_0}$
x_0	y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	
x_1	y_1	Δy_1	$\Delta^2 y_1$	$\Delta^3 y_1$		
x_2	y_2	Δy_2	$\Delta^2 y_2$			
x_3	y_3	Δy_3				
x_4	y_4					
x_5	y_5					

$$\boxed{\Delta^n y_n = \Delta^{n-1} y_n - \Delta^{n-1} y_n}$$

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Backward diff - Operator

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
x_0	y_0	∇y_1	$\nabla^2 y_1$	$\nabla^3 y_1$	$\nabla^4 y_1$	$\nabla^5 y_1$
x_1	y_1	∇y_2	$\nabla^2 y_2$	$\nabla^3 y_2$	$\nabla^4 y_2$	
x_2	y_2	∇y_3	$\nabla^2 y_3$	$\nabla^3 y_3$		
x_3	y_3	∇y_4	$\nabla^2 y_4$			
x_4	y_4	∇y_5				
x_5	y_5					

$$\boxed{\nabla f(x) = \nabla f(x) - \nabla f(x-h)}$$

$$\nabla y_1 = y_1 - y_0$$

$$\nabla y_2 = y_2 - y_1$$

$$\boxed{\nabla^{n} y_n = \nabla^{n-1} y_n - \nabla^{n-1} y_{n-1}}$$

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★ Relation between Nebla and delta

$\Delta f(x) = f(x+h) - f(x)$ —— (1)
$\nabla f(x) = f(x) - f(x-h)$ —— (2)

Replacing (2) with $(x+h)$

$\nabla f(x+h) = f(x+h) - f(x) = \Delta f(x)$ //

$\boxed{\nabla f(x+h) = \Delta f(x)}$ —— (3)

Now

$\nabla^2 f(x+2h) = \nabla f(x+2h) - \nabla f(x+h)$

$\nabla f(x+2h) - \nabla f(x+h)$

$\nabla f[(x+2h) - f(x+h)]$

$\nabla f(x+2h) - \nabla f(x+h)$

from (3)

$\Delta f(x+h) - \Delta f(x)$

$\Delta[f(x+h) - \Delta f(x)]$

$\underline{\underline{\Delta^2 f(x)}}$

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Ex-1 $f(x) = \dfrac{1}{(x+1)(x+2)(x+3)} = x^{-3}$

$= \boxed{h=1}$

$\dfrac{1}{[(x-1)+2][(x-1)+3][(x-1)+4]}$

$= (x-1)^{-3}$

$= -3(x-1)^{-4}$

$= +3\times4(x-1)^{-5}$

$= -3\times4\times5(x-1)^{-6}$

Ex 4 $\Delta \log f(x) = \log\left(1+\dfrac{\Delta f(x)}{f(x)}\right)$

$\Delta \log f(x) = \log\left[f(x+h) - \cancel{\log} f(x)\right]$

$\Delta \log f(x) = \log\left[\dfrac{f(x+h)}{f(x)}\right]$

$\Delta \log f(x) = \log\left[\dfrac{f(x)+f(x+h)-f(x)}{f(x)}\right]$

$\Delta \log f(x) = \log\left[1+\dfrac{\Delta f(x)}{f(x)}\right]$

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(5) $\Delta\left[\tan^{-1}\left(\frac{n-1}{n}\right)\right] = \tan^{-1}\left(\frac{1}{2n^2}\right)$

without losing generality taking
$\boxed{h=1}$

$\because$ ~~$\Delta \tan$~~ $\Delta f(x) = f(x+h) - f(x)$

$\tan^{-1}\left(\frac{x+1-1}{x+1}\right) - \tan\left(\frac{x-1}{x}\right)$

$\tan^{-1}\left(\frac{x}{x+1}\right) - \tan\left(\frac{x-1}{x}\right)$

$$\tan^{-1}\left(\frac{\frac{x}{x+1} - \frac{x-1}{x}}{1 + \left(\frac{x}{x+1} \times \frac{x-1}{x}\right)}\right)$$

$$\tan^{-1}\left(\frac{\dfrac{x^2-(x^2-1)}{x(x+1)}}{\dfrac{x(x+1)+(x-1)(x)}{x(x+1)}}\right)$$

$$\tan^{-1}\left(\frac{\dfrac{1}{x(x+1)}}{\dfrac{x^2+\cancel{x}+x^2-\cancel{x}}{\cancel{x(x+1)}}}\right)$$

$$\tan^{-1}\left(\frac{1}{2x^2}\right)$$

(b) $f(x) = \dfrac{x}{x^2+7x+12}$

$\Delta f(x) = ?$ taking diff of interval unity.

We know that

$$\Delta f(x) = f(x+h) - f(x)$$

$\Rightarrow \because \; x = x+1$ (diff in interval)

$$\Delta f(x) = f(x+1) - f(x)$$

$\boxed{\text{Now}}$

$$\frac{x}{(x+3)(x+4)} = \frac{A}{(x+3)} + \frac{B}{(x+4)}$$

$$A(x+4) + B(x+3) = 0$$

$$Ax + Bx = x$$
$$4A + 3B = 0$$

$$A + B = 1 \qquad \boxed{A = 1-B}$$
$$4A + 3B = 0$$
$$4(1-B) + 3B = 0$$
$$4 - 4B + 3B = 0$$
$$4 - B = 0$$
$$\boxed{B = 4}$$
$$\boxed{A = -3}$$

$$f(x) = \frac{-3}{x+3} + \frac{4}{x+4}$$

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$$\frac{-3}{x+4} + \cancel{\frac{4}{x+5}} \cancel{\frac{+3}{x+3}} \cancel{\frac{-4}{x+4}}$$

$$\Delta f(x) = \frac{-3}{x+4} \frac{-4}{x+4} + \frac{4}{x+5} + \frac{3}{x+3}$$

$$\Delta f(x) = \frac{-7}{x+4} + \frac{4}{x+5} + \frac{3}{x+3}$$

(7.) $f(x) = ab^{cx}$

$$\Delta f(x) = ab^{c(x+h)} - ab^{c(x)}$$

$$= ab^{cx} b^{ch} - ab^{cx}$$

$$= ab^{cx}(b^{ch} - 1)$$

$$\Delta f(x) = (ab)^{cx}(b^{ch} - 1)$$

$$\Delta^2 f(x) = \Delta f(x) \Delta$$

$$\Rightarrow \cancel{(ab)^{cx}}(b^{ch} - 1)\,\Delta(ab^{cx})$$

$$= (b^{ch} - 1)\left[ab^{c(x+h)} - ab^{cx}\right]$$

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$(b^{ch}-1)\left[ab^{cx}b^{ch}-ab^{cx}\right]$

$(b^{ch}-1)\ ab^{cx}\left[b^{ch}-1\right]$

$(b^{ch}-1)^2\ ab^{cx}$

(8) $(\Delta-\nabla)x^2$

$=$

$\Delta x^2-\nabla x^2$

$\left[(x+h)^2-(x)^2\right]-\left[(x)^2-(x-h)^2\right]$

$\cancel{x^2}+h^2+2hx-\cancel{x^2}-\cancel{x^2}+\cancel{x^2}+h^2-2hx$

$(2h^2)$

(9.) $\Delta^n \sin x$ taking $\boxed{h=1}$

$\Rightarrow \Delta'(\sin x) = \sin(x+1)-\sin x.$

$2\cos\left(\frac{x+1+x}{2}\right)\cdot\sin\left(\frac{x+1-x}{2}\right)$

$2\cos\left(x+\frac{1}{2}\right)\ \sin\left(\frac{1}{2}\right)$

$2\sin\left(\frac{\pi}{2}+x+\frac{1}{2}\right)\sin\left(\frac{1}{2}\right)$

$$2\sin\left(\frac{1}{2}\right) \sin\left[\frac{\pi}{2}+x+\frac{1}{2}\right]$$

$$\Delta^2 \sin x = \Delta f(x) \Delta$$

$$\Delta^2_{\sin x} = (\Delta \sin(x))\Delta.$$

$$\Delta^2_{\sin x} = 2\sin\left(\frac{1}{2}\right)\Delta\left[\sin\left(\frac{\pi}{2}+x+\frac{1}{2}\right)\right]$$

$$2\sin\left(\frac{1}{2}\right)\left[\sin\left(\frac{\pi}{2}+x+1+\frac{1}{2}\right) - \sin\left(\frac{\pi}{2}+x+\frac{1}{2}\right)\right]$$

$$2\sin\left(\frac{1}{2}\right) \quad 2\cos\left(\frac{\frac{\pi}{2}+x+1+\frac{1}{2}+\frac{\pi}{2}+x+\frac{1}{2}}{2}\right)$$

$$\sin\left(\frac{\cancel{\frac{\pi}{2}}+\cancel{x}+1+\frac{1}{2}-\cancel{\frac{\pi}{2}}-\cancel{x}-\frac{1}{2}}{2}\right)$$

$\frac{3}{2}-\frac{1}{4}$

$$2\sin\left(\frac{1}{2}\right)\frac{2}{2}\cos\left[\frac{\pi}{2}+x+1\right]\sin\left(\frac{1}{2}\right)$$

$$\left[2\sin\left(\frac{1}{2}\right)\right]^2 \sin\left[\frac{\pi}{2}+\left(\frac{\pi}{2}+x+1\right)\right]$$

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(10) $\Delta^{10}\left[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)\right]$

$\Delta^{10} y = \Delta^{10}(24x^{10})$ [interval = 2]

~~$y = 24\Delta^{10}(x^{10})$~~

degree of polynomial = 10

coeff of degree of polynomial

$= -1 \times -2 \times -3 \times -4$

$= 24$

$\Delta^{10} y = \Delta^{10}\, 24(x^{10})$

$= 24 \times 2^{10} \times 10!$

[Formula for theorem of finite diff.]

$\Rightarrow \Delta^r f(x) = \text{const} \quad r = n$

$= 0 \quad r > n.$

$$\boxed{\Delta^n x^n = n!\, \underbrace{h^n}_{\text{degree}}} \rightarrow [\text{interval}]$$

(17.)

$$y = \frac{1}{(3x+1)(3x+4)(3x+7)}$$

$$y = \frac{1}{27\left(x+\frac{1}{3}\right)\left(x+\frac{4}{3}\right)\left(x+\frac{7}{3}\right)}$$

$$y = \frac{1}{27\left(x-\frac{2}{3}+1\right)\left(x-\frac{2}{3}+2\right)\left(x-\frac{2}{3}+3\right)}$$

$$y = \frac{1}{27\left(x-\frac{2}{3}\right)^{3}}$$

$$y = \frac{1}{27}\left(x-2/3\right)^{-3}$$

$$\Delta y = \frac{-3}{27}\left(x-\frac{2}{3}\right)^{(-4)}$$

$$\Delta^2 y = \frac{-3\times-4}{27}\left(x-\frac{2}{3}\right)^{-5}$$

$$\Delta^2 y = \frac{12}{27}\left(x-\frac{2}{3}\right)^{-5}$$

$$\Delta^2 y = \frac{12}{27} \frac{1}{\left(x-\frac{2}{3}+1\right)\left(x-\frac{2}{3}+2\right)\left(x-\frac{2}{3}+3\right)\left(x-\frac{2}{3}+4\right)\left(x-\frac{2}{3}+5\right)}$$

$$\Delta^2 y = \frac{12}{27} \frac{3^5}{(3x+1)(3x+2)(3x+7)(3x+10)(3x+13)}$$

$$\Delta^2 y = \frac{12}{27} \frac{\times (3)^5}{(3x+1)(3x+4)(3x+7)(3x+10)(3x+13)} //$$

Q $\dfrac{\Delta^2 (5x+12)}{x^2+5x+6}$

$\Delta = \cancel{5(x+1)+12}$

$\dfrac{5x+12}{x^2+5x+6}$

Other difference Operator

Here $\Delta \Rightarrow E$

Avg $\Rightarrow \mu$

$\because \; E f(x) = f(x+h)$ ~~$= f(x)$~~

$\because \; E^{-1} f(x) = f(x-h)$

$\rightarrow$ Q.

Ans $\quad E = 1 + \Delta$

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta f(x) = \cancel{E - f(x)} \; E f(x) - f(x)$$

$$\Delta f(x) = (E-1) f(x)$$

$$(\Delta + 1) = E //$$

(2) $\nabla = 1 - E^{1}$

$\cancel{E^{-1}} =$

$\nabla f(x) = f(x) - f(x-h)$

$\nabla f(x) = f(x) - E^{1} f(x)$

$\nabla f(x) = f(x)\,(1 - E^{-1})$

$\nabla = (1 - E^{-1})$

(3.) $\delta = E^{1/2} - E^{-1/2}$

$\delta f(x) = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)$

$= E^{1/2} f(x) - E^{-1/2} f(x)$

$\delta f(x) = \left(E^{1/2} - E^{-1/2}\right) f(x)$

$\delta = \left(E^{1/2} - E^{-1/2}\right)$

(4) $$\mu = \frac{E^{1/2} + E^{-1/2}}{2}.$$

$$\mu f(x) = \frac{f(x+h/2) + f\left(x-\frac{h}{2}\right)}{2}$$

$$\mu f(x) = \frac{E^{1/2} f(x) + E^{-1/2} f(x)}{2}$$

$$\mu \cancel{f(x)} = \left(\frac{E^{1/2} + E^{-1/2}}{2}\right)\cancel{f(x)}$$

(5) $$\delta = E^{1/2} \nabla.$$

$$\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$$

$$\delta f(x) = E^{1/2} f(x) - E^{-1/2} f(x)$$

$$\delta f(x) = \left(E^{1/2} - E^{-1/2}\right) f(x)$$

$$\delta \cancel{f(x)} = E^{1/2}\left(1 - E^{-1}\right)\cancel{f(x)}$$

$$\delta = E^{1/2} \nabla$$

$E^{1/2}$

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(6) $E = e^{hd}$.

Using Taylors series $y = f(x)$.

$$f(x+h) = f(x) + h f'(x) + \frac{h^2 f''(x)}{2!} - - -$$

$$E f(x) = f(x) + hD f(x) + \frac{h^2 D^2}{2!} f(x)$$

$$E f(x) = \left[1 + hD + \frac{h^2 D^2}{2!}\right] f(x)$$

$$E = \left[1 + hD + \frac{h^2}{2!} D^2\right]$$

$$\boxed{E = e^{hD}}$$

(7) $D = \frac{1}{h}\left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - - -\right]$

$$E = e^{hD}$$

$$\log E = hD$$

$[E = 1 + \Delta]$

$$\log(1+\Delta) = hD$$

$$\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} = Dh$$

$$D = \frac{1}{h}\left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3}\right] //$$

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Proof

(1) $E\nabla = \nabla E = \Delta$

$E\nabla = (E(1-E^{-1})$

$E\nabla = E - 1 = \Delta$

$\nabla E = (1-E^{-1})E$

$\boxed{\Delta = E - 1}$

(2) $(E^{1/2}+E^{-1/2})(1+\Delta)^{1/2} = (2+\Delta)$

$\cancel{(E^{1/2}+E^{-1/2})(1+E-1)^{1/2}}$

$\cancel{E-1=\Delta}$

$\boxed{\Delta = E^{-1}}$

$(E=\Delta+1)$

$(E^{1/2}+E^{-1/2})(E)^{1/2}$

$(E+1) = (\Delta+1)+1$

$= \Delta+2$

$= \boxed{2+\Delta}$

③ $\nabla\Delta = \Delta - \nabla = \delta^2$

Imp

$\boxed{\Delta = E - 1}$

$\boxed{\nabla = 1 - E^{-1}}$

$(E - 1)(1 - E^{-1})$

$E(1 - E^{-1}) - 1(1 - E^{-1})$

$E - 1 - 1 + E^{-1}$

$E + E^{-1} - 2$

$\boxed{\delta = E^{1/2} - E^{-1/2}}$

(nebla)

$1 - E^{-1}$

$(E^{1/2} - E^{-1/2})^2$

$\underline{\delta^2}$

(E)

$(1 + \text{delta})$

$\boxed{\Delta = E - 1 \quad \nabla = 1 - E^{-1}}$

$(E - 1) - (1 - E^{-1})$

$E - 2 + E^{-1}$

$E + E^{-1} - 2$

$(E^{1/2} - E^{-1/2})^2 = \delta^2$ //

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(4) $E^{1/2} = \mu + \frac{1}{2}\delta.$

$\mu = \left(\frac{E^{1/2} + E^{-1/2}}{2}\right)$

$\delta = \left(E^{1/2} - E^{-1/2}\right)$

$E^{1/2} = \left(\frac{E^{1/2} + \cancel{E^{-1/2}}}{2}\right) + \left(\frac{E^{1/2} - E^{-1/2}}{2}\right)$

$E^{1/2} = E^{1/2}$

(5) $\mu\delta = \frac{\Delta}{2} + \frac{\Delta E^{-1}}{2}$

$E = 1 + \Delta$

$(\Delta = E - 1)$

$\frac{E-1}{2} + \frac{(E-1)E^{-1}}{2}$

$\frac{(E-1)}{2} + \frac{(1-E^{-1})}{2}$

$\left(E - E^{-1}\right)$

$$\left(\frac{E^{1/2}+E^{-1/2}}{2}\right) \quad \left(E^{1/2}-E^{-1/2}\right)$$

$$\Downarrow \qquad\qquad \downarrow$$

$$\mu \qquad\qquad \underline{\underline{\delta}}.$$

(6) $1-e^{-hD} = \nabla$

$E = e^{hD}$

$\frac{1}{h}\log E = D.$

$\log E = Dh$

$E = e^{Dh}$

$E^{-1} = e^{-Dh}$ [Removing log]

(7) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

$\Delta = E-1 \quad \nabla = 1-E^{-1}$

$\cancel{\nabla = E^{-1}-1}$

$E = 1+\Delta$

$\cancel{\nabla = E^{-1}-1}$

$\frac{E-1}{1-E^{-1}} - \frac{1-E^{-1}}{E-1}$

$\nabla = f(x)-f(x-h)$

$= f(x) - E^{-1}(f(x))$

$\nabla = f(x)(1-E^{-1})$

$$\frac{(E-1)^2-(1-E^{-1})^2}{(E-1)(1-E^{-1})}$$

$$\frac{E^2+1-2E-\left(1^2+(E^{-1})^2-2E^{-1}\right)}{(1-E^{-1})(E-1)}$$

$$\frac{E^2+\cancel{1}-2E-\cancel{1}-(E^{-1})^2+2E^{-1}}{(1-E^{-1})(E-1)}$$

$$\frac{E^2-E^{-2}-2E+2E^{-1}}{(1-E^{-1})(E-1)}$$

$$E^2-1-2E-E^{-2}+2E^{-1}+1$$

$$\frac{(E-1)^2}{(1-E^{-1})(E-1)}-\frac{(1-E^{-1})^2}{(1-E^{-1})(E-1)}$$

$$\frac{(E-1)}{(1-E^{-1})}-\frac{(1-E^{-1})}{(E-1)}$$

$$\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}$$ //

(13) $y_4 = y_3 + \Delta y_2 + \Delta^2 y_1 + \Delta^3 y_1$

$= y_3 + (E-1)y_2 + (E-1)^2 y_1 + (E-1)^3 y_1$

$= y_3 + (E-1)y_2 + (E^2+1-2E)y_1 + [E^3 - 1 - 3E(E+1)]y_1$

$= y_3 + (E-1)y_2 + (E^2-2E+1)y_1 + (E^3-1-3E^2+3E)y_1$

$= y_3 + (E-1)y_2 (E^2 - 2E + \cancel{1} + E^3 - \cancel{1} - 3E^2 + 3E)y_1$

$= y_3 + (E-1)y_2 (-2E^2 + E^3 + E)y_1$

$= \cancel{y_3} + (\cancel{y_3} - y_2) \cancel{-2}y_3 - y_4 + y_1$

$(E-1)y_2$

$(y_3 - y_2)$

wumisha (1) bdlta hai

$= \cancel{y_3} + \cancel{y_3} - \cancel{y_2} + \cancel{y_3} - \cancel{2y_2} + \cancel{y_1}$

$+ y^4 - \cancel{y_1} - \cancel{3y_3} + \cancel{3y_2}$

$= y^4$

14. $\Delta^2 y_2 = \nabla^2 y_4$

$(E-1)^2 y_2 = (1-E^{-1})^2 y_4$ $\quad$ $\cancel{\Delta = f}$

$\Delta f(x) = f(x+1) - f(x)$

$(E^2+1-2E)\,y_2$ $\quad$ $\Delta f(x) = Ef(x) - f(x)$

$\boxed{(y_4 - 2y_3 + y_2)}$ $\quad$ $\Delta f(x) = f(x)(E-1)$

$= (E-1)$

$(1^2 + (E^{-1})^2 - 2E^{-1})\,y_4$

$\boxed{y_4 + y_2 - 2y_3}$

15. $u_0 = 2 \quad u_1 = 11 \quad u_2 = 80$
$u_3 = 200 \quad u_4 = 100 \quad u_5 = 8.$

x	u	Δu	$\Delta^2 u$	$\Delta^3 u$	$\Delta^4 u$	$\Delta^5 u$
0	2	9				
			60	−9		
1	11	69		~~−262~~	−262	761
2	80		51	−271		
3	200	180	−220	~~499~~	499	
4	100	−100		228	—	
5	8	−92	8			

$\nabla^5 u_5 = \Delta^5 u_0 = 761$

$\nabla^5 u_5 = (1 - E^{-1})^5 u_5.$

1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$u_5 - 5E^{-1} + 10E^{-2} - 10E^{-3} + 5E^{-4} - E^{-5}$

(14) Δe^x

$$\Delta e^x = e^{x+h} - e^x$$
$$= e^x(e^h - 1)$$

$$\Delta^2 e^x = \Delta\Delta(e^x)$$
$$= \Delta\left[e^x(e^h-1)\right]$$
$$= (e^h-1)\Delta e^x$$
$$= (e^h-1)\, e^x (e^h-1)$$
$$= (e^1-1)\, e^x (e^1-1)$$

$0 =$

$$\Delta e^x = e^{x+1} - e^x$$
$$= e^x(e-1)$$

(2) $\tan^{-1}(x)$

$$\tan^{-1}(x+h) - \tan^{-1}(x)$$
$$\tan^{-1}\left(\frac{x+h-x}{1+(x+h)(x)}\right)$$
$$\tan^{-1}\left(\frac{h}{\quad}\right.$$

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(3) $\Delta 3^x$

$3^{(x+1)} - 3^x$

$\Delta = \Delta 3^{(x+h)} - \Delta 3^x$

3

$= 3^x(3^h - 1) = 3(3^x - 1)$

(4) $\left(\frac{2^x}{x!}\right)$

$\frac{2^{(x+h)}}{(x+h)!} - \frac{2^x}{x!}$

$\frac{2^{(x+1)}}{(x+1)!} - \frac{2^x}{x!}$

$\frac{2^{(x+1)}x! - 2^x(x+1)!}{(x+1)!\,x!}$

$\frac{2^x \cdot 2\, x! - 2^x(x+1)!}{(x+1)!\,x!}$

$(x+1)\,(x!)^2$

$\frac{2^x \times 2\, x!}{(x+1)!\,x!} - \frac{2^x(x+1)!}{(x+1)!\,x!}$

$\frac{2^x(2 - x - 1)}{(x+1)!}$

$\frac{2^x(1-x)}{(x+1)!}$

$2^x\left(\frac{2}{(x+1)} - \frac{1}{x!}\right)$

$2^x\left(\frac{2^x}{(x+1)!} - \frac{(x+1)!}{(x+1)(x!)}\right)$

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$$\frac{2^n 2 n! - 2^n (n+1)!}{(n+1)!\, n!}$$

$$\Rightarrow \frac{2^n \cancel{n!}\left(2 - (n+1)\right)}{(n+1)!\, \cancel{n!}}$$

$$\Rightarrow \frac{2^n}{(n+1)!}\left(1 - n\right)$$

(15) $\Delta\left[\dfrac{1}{f(x)}\right] = \dfrac{-\Delta f(x)}{f(x)\,f(x+1)}$

$$\frac{1}{f(x+1)} - \frac{1}{f(x)}$$

$$\frac{f(x) - f(x+1)}{f(x)\,f(x+1)}$$

$\because \Delta f(x) \Rightarrow f(x+1) - f(x)$

but $\Delta f(x) = \dfrac{1}{f(x) - f(x+1)}$

$\therefore \quad \dfrac{-\Delta f(x)}{f(x)\,f(x+1)}$

(16)

(1) $\Delta x e^x$.

$(x+h)e^{(x+h)} - xe^x$ $\quad$ $xe^{(x+h)} + he^{(x+h)} - xe^x$

$\cancel{(x+h)e^x \cdot e^h - xe^x}$ $\quad$ $xe^x \cdot e^h + he^x e^h - xe^x$

$e^x(xe^h + he^h - x)$

$\cancel{e^x[(x+h)e^h - x]}$ $\quad$ $e^x((x+h)e^h - x)$

$xe^x \cdot e^h + he^x e^h - xe^x$.

$xe^x(e^h - 1) + he^x e^h$

$(x+h)e^x \cdot e^h -$

(2) $\Delta \sin x$

$\sin(x+h) - \sin x$.

$2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)$

$2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)$

$2\cos\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)$

(4) $\Delta^2(e^{ax+b})$

$$= e^{a(x+h)+b} - e^{ax+b}$$

$$= e^{ax+b}(e^{ah+b}-1)$$

$$= e^{ax+b}(e^{ah+b}-1)$$

$$= \cancel{\Delta(e^{ax+b}) = e^{ax+b}(e^{ah+b}-1)}$$

$$\Delta(e^{ax+b}) = e^{(ax+ah)+b} - e^{ax+b}$$

$$\Delta(e^{ax+b}) = e^{ax+b} \times e^{ah+b} - e^{ax+b}$$

$$\Delta(e^{ax}+b) = e^{(ax+b)}(e^{ah+b}-1)$$

$$\Delta^2(e^{ax+b}) = \Delta(\Delta e^{ax+b})$$

$$= \Delta\left[e^{(ax+b)}(e^{ah+b}-1)\right]$$

$$= [\Delta e^{ax+b}](e^{ah+\cancel{b}}-1)$$

$$= e^{ax+b}(e^{ah+\cancel{b}}-1)^2$$

17

1. $\dfrac{x+1}{(x-1)(x-2)}$

$$\frac{A}{(x-1)} + \frac{B}{(x-2)} = x+1$$

$$A(x-2) + B(x-1) = x+1$$

$$Ax + Bx = x$$
$$-2A - B = 1.$$

$$(A+B) = 1$$
$$-2A - B = 1$$

$$\boxed{A = 1 - B}$$
$$-2(1-B) - B = 1$$
$$-2 + 2B - B = 1$$
$$\boxed{B = 3}$$

$$\boxed{A = 1 - 3 = -2}$$

$$\frac{-2}{(x-1)} + \frac{3}{(x-2)} = f(x)$$

$$\Delta f(x) = \frac{-2}{x} + \frac{3}{(x-1)} + \frac{2 \cancel{+} 3}{(x-1)(x-2)}$$

$$= \frac{-2(x-1)(x-2) + \cancel{3}5x(x-2) - 3x(x-1)}{x(x-1)(x-2)}$$

$$\frac{-2x(x-2)+2(x-2)+5x^2-10x-3x^2+3x}{(x)(x-1)(x-2)}$$

$$\frac{\cancel{-2x^2}+4x+2x-4\cancel{+5x^2}-10x\cancel{-3x^2}+3x}{x(x-1)(x-2)}$$

$$\frac{6x-10x+3x-4}{x(x-1)(x-2)}$$

$$\frac{-x-4}{\cancel{(x)}(x-1)(x-2)}$$

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(18) $\Delta^2\left(\frac{5x+12}{x^2+5x+6}\right) = \frac{2(5x+16)}{(x+2)(x+3)(x+4)(x+5)}$

?

(19) ?

$$\frac{2(5x+16)}{(x+2)(x+3)(x+4)(x+5)}$$

$$\frac{A}{(x+2)} + \frac{B}{(x+3)} + \frac{C}{(x+4)} + \frac{D}{(x+5)}$$

$$A(x+3)(x+4)(x+5) + B(x+2)(x+4)(x+5) + C(x+2)(x+3)(x+5) + D(x+2)(x+3)(x+4)$$

$\cancel{A}x($

$y|x=$ tan...

Δ (A, B)

$x = \tan x$ $\tan x - x = 0$ 0.5

$\phi(x) = \tan$.

$\phi'(x) = \sec^2 x$.

$\phi'(x) < 1$

$0.5 \le 1$

$x = 1 > 0$

$= 0$ ≥ 0

.35

$x = 1$ < 0

0.5

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(20) $\Delta^n (e^{2x+3})$

$\Delta^n \left(e^{2(x+h)+3} - e^{2x+3}\right)$

$\Delta^n \left(e^{(2x+2h)+3} - e^{2x+3} \right.$

$\Delta^n (e^{2x+3}) = e^{2x+3} (e^{2h}-1)^n$

(21) $\Delta^n \left(\frac{1}{x}\right)$

$\Delta^n \left(\frac{1}{x+h} - \frac{1}{x}\right)$

$\frac{\cancel{x} - \cancel{x} - h}{(x+h)x}$

$\Delta^1 \left(\frac{1}{x}\right) = \frac{-1}{(x+1)(x)}$

$\Delta^2 \left(\frac{1}{x}\right) = \Delta\left(\Delta\left(\frac{1}{x}\right)\right)$

$= \Delta\left(\frac{-1}{(x+1)x}\right)$

$= \Delta\left(\frac{-1}{(x+2)(x+1)} + \frac{1}{(x+1)(x)}\right)$

$= \cancel{\Delta}\left(\frac{-x + x+2}{(x)(x+1)(x+2)}\right)$

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21. $\Delta^n a^x$

$\Delta a^x = a^{x+h} - a^x$

$\Delta(a^x) = a^x (a^h - 1)$

$\Delta^2 (a^x) = \Delta(a^x)$

$= \cancel{\Delta(a^x)(a^h-1)^n}$

a^{x+h}

$= \cancel{\Delta(a^x)(a^h-1)}$

$= \cancel{a^x \Delta(a^h-1)}$

$= \cancel{a^x [a^{(h+1)} - 1 - a^h + 1]}$

$= a^x \; \cancel{a^h} (\; a^h$

$= \Delta \, \Delta(a^x)$

$= \Delta [a^x (a^h - 1)]$

$= \Delta [a^{x+h} (a^h - 1) - a^x [a^h - 1]]$

$= a^x a^h (a^h - 1) - a^x (a^h - 1)]$

$= a^x [a^h (a^h - 1)^2]$

(2) $\Delta^n (e^{ax+b})$.

$\Delta = e^{a(x+h)+b}$

$\underset{(e^{ax+b})}{\Delta} = e^{(ax+b)} e^{(ah+b)} - e^{(ax+b)}$

$= e^{(ax+b)} (e^{ah} - 1)$ cony

$\underset{(e^{ax+b})}{\Delta^2} = \Delta \, \Delta(e^{ax+b})$.

$= \Delta \left[e^{a(x+h)+b} \cdot (e^{ah} - 1) - e^{ax+b} (e^{ah} - 1) \right]$

$= e^{ax+b} e^{ah} (e^{ah} - 1) - e^{ax+b} (e^{ah} - 1)$

$= e^{ax+b} \left[e^{ah} (e^{ah} - 1) - (e^{ah} - 1) \right]$

$= e^{ax+b} (e^{ah} - 1)(e^{ah} - 1)$

$= e^{ax+b} (e^{ah} - 1)^n$

(22.)

(1) $\Delta^3\left[(1-x)(1-2x)(1-3x)\right]$

$-3\times2-1\ 3!$

$-6\times3!$

(2.) $\Delta^3\left[(1-ax)(1-bx)(1-cx)\right]$

$-\Delta^3\cdot abc\,x^3$

$-abc\ x^3$

$-abc\ 3! = -6abc$

(3) $\Delta^4\left[(1-2x)(1-3x)(1-5x)(1-6x)\right]$

$\Delta^4\ 180\ x^4$

$180\ 4!$

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2) $\Delta^{10}\left[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)\right]$

$\Delta^{10} - 4 \times -3 \times -2 \times -1 \cdot (x^{10})$

$\cancel{24 \times 10!}$

$24 \times 2^{10} \times 10!$ //

(2B)

$\cancel{y_1 = 18 \quad y_2 = 20, \quad y_3 = 24 \quad y_0 = 9}.$

y

(3) $y = \dfrac{1}{x(x+3)(x+6)}$

$= \dfrac{1}{(x-1+1)(x-1+4)(x-1+7)}$

$y = (x-1)^{-3}$

$y = -3(x-1)^{-4}$

$$\Delta^2 y = 12\,(x-1)^{-5}$$

$$= \frac{12}{(x-1)^5}$$

$$= \frac{12}{(x-1)^5}$$

$$= \frac{12}{(x-1+1)(x-1+4)(x-1+7)(x-1+10)(x-1+13)}$$

$$\Delta^2 y = \frac{12 \times 3^5}{x(x+3)(x+6)(x+9)(x+12)}$$ //

$\left(\frac{E^{1/2}+E^{-1/2}}{2}\right) \times E^{1/2}-E^{-1/2}$

$\frac{E+\cancel{E^0}-\cancel{E^0}-E^{-1}}{2}$ $\quad \frac{E-E^{-1}}{2}$

$\frac{E-\cancel{1}}{2}+\frac{\cancel{1}-E^{-1}}{2}$

$\frac{\Delta}{2}+$

$1-e^{-hd}=\nabla$

$D=\frac{1}{h}\log E$

$Dh=\log E$

$E=e^{hD}$

$\frac{1}{E}=\frac{1}{e^{hD}}$

$e^{-hD}=E^{-1}$

$\nabla= E^{-1}$

$E^{-1}=1-\nabla$

$e^{-hD}=1-\nabla$

$\boxed{\nabla=1-e^{-hD}}$

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6.2

(10) $\frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{\delta^2}{4}} = \Delta$.

$\frac{1}{2}\delta^2 + \frac{\delta}{2}\sqrt{4+\delta^2}$

$\frac{1}{2}\delta\left[\delta + \sqrt{4+\delta^2}\right]$

$\delta = E^{1/2} - E^{-1/2}$

$\frac{1}{2}\delta\left[E^{1/2} - E^{-1/2}\ \sqrt{4+(E^{1/2}-E^{-1/2})^2}\right]$

$\frac{1}{2}\delta\left[E^{1/2} - E^{-1/2}\ \sqrt{4+(E^{1/2})^2+(E^{-1/2})^2 - 2}\right.$

$\frac{1}{2}\delta\left[E^{1/2} - E^{-1/2}\ \sqrt{4-2+(E^{1/2})^2+(E^{-1/2})^2}\right.$

$\frac{1}{2}\delta\left[E^{1/2} - E^{-1/2}\ \sqrt{(E^{1/2}+E^{-1/2})^2}\right.$

$\frac{1}{2}\delta\left[E^{1/2} - \cancel{E^{-1/2}} + E^{1/2} + \cancel{E^{-1/2}}\right]$

$\frac{1}{\cancel{2}}\ \cancel{2}\,\delta\ E^{1/2}$

$\left[E^{1/2} - E^{-1/2}\right] E^{1/2}$

$\boxed{E - 1 = \Delta}$

(11) ?

(12) $h=1 \quad (\Delta+\nabla)^2 f(x) : x^2+x$

$\cancel{(\Delta^2+\nabla^2+2\Delta\nabla)(x^2+x)}$

$\cancel{[(E-1)^2+(1-E)^{-2}]}(x^2+x)$

$\cancel{E^2+1-2E+1-E^{-2}-2E^{-1}}$

$(\Delta+\nabla)^2 f(x)$

$(E-\cancel{1}+\cancel{1}-E^{-1})^2(x^2+x)$

$(E-E^{-1})^2(x^2+x)$

$(E^2+E^{-2}-2)(x^2+x)$

$E^n f(x) = f(x+nh)$

$E^2(x^2+x) = (x+2)^2+(x+2)$

Similarly

$E^{-2}(x^2+x) = (x-2)^2+(x-2)$

$\cancel{f(x) = x^2+x}$

$-2(x^2+x)$

$\cancel{x^2}+4+\cancel{2x}+\cancel{x}+2+\cancel{x^2}+4-\cancel{2x}$
$+\cancel{x}-2 \quad -\cancel{2x^2}-\cancel{2x}$

$\underline{\underline{8}}$

13. $y_4 = y_3 + \Delta y_2 + \Delta^2 y_1 + \Delta^3 y_1$

$\Delta = E - 1$

$y_4 = y_3 + (E-1)y_2 + (E-1)^2 y_1 + (E-1)^3 y_1$

$= y_3 + Ey_2 - y_2 + (E^2 + 1 - 2E) y_1$

$= y_3 + y_3 - y_2 + y_3 + y_1 - 2y_2$

$a^3 - b^3 - 3ab(a-b)$

$+ [E^3 - 1^3 - 3E(E-1)] y_1$

$(E^3 - 1 - 3E^2 + 3E) y_1$

(14) $\Delta^2 y_2 = \nabla^2 y_4$

$(E-1)^2 y_2$

(15) $(1-E^{-1})^5 u_5 = \nabla^5 u_5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$(1 - 5E^{-1} + 10E^{-2} - 10E^{-3} + 5E^{-4} - E^{-5})u_5$

$u_5 - 5u_4 + 10u_3 - 10u_2 + 5u_1 - u_0$

$8 - 500 + 2000 - 800 + 55 - 2$

76

(12.) $y_0 + y_6 = 2.37$

$y_1 + y_5 = 2.92$

$y_2 + y_4 = 3.01$

$\Delta^6 y_0$

$(E-1)^6 y_0$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

$(E^6 - 6E^5 + 15E^4 - 20E^3 + 15E^2 - 6E^1 + 1) y_0$

$y_6 - 6y_5 + 15y_4 - 20y_3 + 15y_2 - 6y_1 + y_0$

$(y_0 + y_6) - 6(y_5 + y_1) + 15(y_2 + y_4)$

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$2.37 - 6\times 2.92 + 15\times 3.01$

$y_3 = 1.5$

(ii) $y_0 + y_8 = 1.924$

$y_1 + y_7 = 1.959$

$y_2 + y_6 = 1.982$

$y_3 + y_5 = 1.996$

$\Delta^8 (E-1)^8 y_0$

$u_8 - 8u_7 + 28u_6 - 56u_5 + 70u_4$
$-56u_3 + 28u_2 - 8u_1 + u_0 = 0.$

$(y_0 + y_8) - 8(y_1 + y_7) + 28(y_2 + y_6)$
$-56(y_5 + y_3) + 70y_4 = 0$

$1.924 - 8\times 1.959 + 28\times 1.982$
$-56(1.996) = -70y_4$

$y_4 = 1.0004$ //

8

x	1	2	3	4	5
y	2	5	7	-	32

$(E-1)^4 u_0$

$(E^4 - 4E^3 + 6E^2 - 4E + 1) u_0 = 0$

$u_4 - 4u_3 + 6u_2 - 4u_1 + u_0$

$32 - 4u_3 + 6\times7 - 4\times5 + 1 = 0$

$4u_3 = 32 + 42 - 20 + 1$

$u = 13.75$

$\boxed{u = 14}$

$1 - 6u_5 + 15u_4 - 20u_3 + 15u_2$
$-6u_1 + u_0 = 0$

$$20u_3 = 1 - 6 \times 390 + 15 \times 350$$
$$\cancel{-20\times} + 15 \times 260$$
$$- 6 \times 220 + 200$$

$$u_3 = 284$$

3

$(E-1)^6$

$E^6 - 6E^5 + 15E^4 - 20E^3 + 15E^2$
$-6E + 1] u_0$

$y_6 - 6y_5 + 15y_4 - 20y_3 + 15y_2$
$-6y_1 + y_0$

$430 - 6 \times 390 + 15 \times 350 - 20y_3 + 15 \times 260$
$-6 \times 220 + 200 = 0$

$$\boxed{20y_3 = 306}$$

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Interpolation

(1) Forward Interpolation

⇒ $\phi(x)$ be interpolating polynomial.

$y = f(x) \quad x_0 < x < x_n$.

$$\phi(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2)$$

$$\phi(x) = f(x) = y$$

at $\boxed{x = x_0}$

$$\phi(x_0) = a_0 = F(x_0) = y_0.$$

$$\boxed{a_0 = y_0}$$

at $\boxed{x = x_1}$

$$\phi(x_1) = a_0 + a_1(x_1 - x_0)$$

$$y_1 = a_0 + a_1 h$$

$$y_1 = y_0 + a_1 h$$

$$\boxed{\frac{y_1 - y_0}{h} = a_1}$$

$$\boxed{a_1 = \frac{\Delta y_0}{h}}$$

$$\phi(x_1) = a_0 + a_1(x_1 - x_0) + a_2(x - x_0)(x - x_1)$$

$$\phi(x_1) = y_0 + \frac{\Delta y_0}{h}(x_1 - x_0)$$

$\boxed{\text{at } x = x_2}$

$$\phi(x_2) = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$$

[illegible]

$$\frac{(y_2-y_1)-(y_1-y_0)}{2h^2}$$

$$\frac{\Delta y_1 - \Delta y_0}{2h^2} = \frac{\Delta^2 y_0}{2h^2}.$$

$$\phi(x) = y_0 + \frac{\Delta y_0}{h}(x-x_0) + \frac{\Delta^2 y_0}{2h^2}(x-x_0)(x-x_1) - - - .$$

$$\Rightarrow \quad y(p) = y_0 + p\Delta y_0 + \frac{p(p-1)\Delta^2 y_0}{2!} + \frac{p(p-1)(p-2)\Delta^3 y_0}{3!}$$

$$x = x_0 + ph$$

$$\boxed{p = \frac{x-x_0}{h}}$$

(1.) $y(\cancel{15})$

x	8	10	12	14	16
$f(x)$	1000	1900	3250	5400	8950

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y$	$\Delta^4 y$
8	1000				
10	1900	900			
12	3250	1350	450	350	
14	5400	2150	800		250
16	8950	3550	1400	600	

$\boxed{f(9)}$

$x = 9$ $\boxed{h = 2}$

$x_0 = 8$

$$p = \frac{x - x_0}{h} = \frac{9-8}{2} = \frac{1}{2}$$

$\boxed{p = 0.5}$

$$y(0.5) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$y_0 = 1000$

$\Delta y_0 = 900$

$\Delta^2 y_0 = 450$

$\Delta^3 y_0 = 350$

$\Delta^4 y_0 = 250.$

$$y(0.5) = 1000 + 0.5 \times 900 - 0.5 \times 0.5 \times 450 + 0.5 \times -0.5 - 1.5 \times 350 + 0.5 \times -0.5 \times -1.5 - 2.5 \times 250$$

$$= 1000 + 450 - \frac{112.5}{2!} + \frac{131.25}{3!} - \frac{234.37}{4!}$$

$= \cancel{1234.38}$

$1000 + 450 - 56.25 + 21.875 - 9.765$

$\cancel{1405.86}$

1406