Electromagnetics :-

maxwells electromagnetic eq. are based upon well known basic laws such as Gauss's law of electrostatics, Gauss's law of magnetostatics, Faraday law of electromagnetic induction and ampere circuital law. When the electric and magnetic fields are changing very rapidly in space and time then the varying electric field gives the magnetic field and vice-versa. We therefore consider electromagnetic fields by a set of equations known as maxwells equation of electromagnetics.

Note→ closed surfaces ~~alone~~ encloses volume and closed line encloses open surface.

Gauss's theorem :-

$$\oint_s \vec{A}\cdot d\vec{s} = \int_v (\nabla\cdot\vec{A})\cdot dv$$

Stokes theorem :-

$$\oint_l \vec{A}\cdot d\vec{l} = \int_s (\vec{\nabla}\times\vec{A})\, d\vec{s}.$$

Derivation of maxwell's Equations :-

① **maxwell's first eq :-** Gauss law of electrostatics states that the electric flux over a hypothetical closed surface is $\frac{1}{\varepsilon_0}$ times the total charge enclosed within the surface.

$$\oint_S \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}\int_V \rho\, dv$$

Now applying gauss divergence theorem:-

$$\oint_S \vec{D}\cdot d\vec{s} = \int_V \rho\, dv$$

since $\vec{D} = \varepsilon_0 \vec{E}$

$$\int_V (\nabla\cdot\vec{D})\, dv = \int_V \rho\, dv$$

$$\boxed{\nabla\cdot\vec{D} = \rho}$$

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

② maxwells second eq :- Gauss's law of magnetostatics states that net magnetic flux passing through any closed surface is 0 because magnetic monopoles does not exist over any closed surface. As a result there exists equal and opposite magnetic poles. Hence net magnetic flux becomes 0.

$$\oint_S \vec{B}\cdot d\vec{s} = 0$$

Applying gauss divergence theorem:-

$$\int_V (\nabla\cdot\vec{B})\, dv = 0$$

$$\nabla\cdot\vec{B} = 0$$

③ Maxwells third eq :-

$$\oint_l \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot d\vec{s} \rightarrow ①$$

According to faraday's law emf induced around a closed circuit is equal to the negative times rate of change of magnetic flux linked with the circuit.

$$Emf = -\frac{\partial \phi_m}{\partial t} \rightarrow ②$$

where $\phi_m = \oint_s \vec{B}\cdot d\vec{s}$

$$\therefore Emf = -\frac{\partial}{\partial t}\oint_s \vec{B}\cdot d\vec{s} \rightarrow ③$$

But we know that the emf in terms of electric field can be expressed as :-

$$Emf = \oint_l \vec{E}\cdot d\vec{l} \rightarrow ④$$

Now equating eq. ③ and ④ we get :-

$$\oint_s \vec{E}\cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

Now applying stokes theorem in LHS , we get :-

$$\int_s (\nabla \times E)\, ds = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

$\vec{\nabla} \cdot \vec{A}$ = divergence A.
$\vec{\nabla} \times \vec{A}$ = curl A.

④ maxwell's fourth eq:- Ampere circuital law states that the line integral of $\vec{H}$ around any closed path is equal to the total current within that path.

mathematically it can be written as:-

$$\oint_l \vec{H} \cdot d\vec{l} = \text{Total current}(I) =$$

conduction current + displacement current.

But the current can also be expressed in terms of current density as:-

∴ from eq ① and ② we get:-

$$\oint_l \vec{H} \cdot d\vec{l} = \int_s \vec{J} \cdot d\vec{s} \rightarrow ①$$

Applying stokes law:-

$$\oint_s (\nabla \times \vec{H}) \cdot d\vec{s} = \int_s \vec{J} \cdot d\vec{s} \rightarrow ②$$

$$\int_s (\text{curl } \vec{H} - \vec{J}) \cdot d\vec{s} = 0 \rightarrow ③$$

$$\text{curl } \vec{H} = \vec{J} \rightarrow ④$$

It can seen from eq ④ that It is valid only for static charge and insufficient for time varying field. To show this let us take the divergence of eq. ④ :-

$$\text{div}(\text{curl } \vec{H}) = \text{div}(\vec{J})$$

div of any curl.

$$\nabla \cdot (\text{curl } \vec{H}) = \nabla \cdot \vec{J}$$

$\boxed{\nabla \cdot \text{curl } \vec{H} = 0}$

(An Identity)

$$0 = \nabla \cdot \vec{J}$$

$$\text{div} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\text{div } \vec{J} = -\frac{\partial \rho}{\partial t} \rightarrow ⑤$$

$\therefore$ eq ⑤ may be valid if $\frac{\partial \rho}{\partial t} = 0$

$\therefore$ charge density should be static one.

Hence to include time varying field maxwell suggested that Ampere law should be modified that current density $\vec{J}$ should be replaced by $\vec{J} + \vec{J_d}$ whered $\vec{J_d}$ is current density for displacement current.

Now eq ④ becomes :-

$$\text{curl } \vec{H} = \vec{J} + \vec{J_d} = I \rightarrow ⑥$$

Now taking divergence we get :-

$$\text{div}(\text{curl } \vec{H}) = \text{div}(\vec{J} + \vec{J_d})$$

$$0 = \text{div}(\vec{J} + \vec{J_d})$$

$$\text{div } \vec{J} = -\text{div } \vec{J_d} \rightarrow ⑦$$

$$\text{div } \vec{J_d} = -\left(\frac{-\partial \rho}{\partial t}\right) \quad \text{From eq ⑤}$$

Now from maxwell's first eq :-

$$\nabla \cdot \vec{D} = \rho$$

$$\text{div } \vec{J_d} = \frac{\partial}{\partial t}(\nabla \cdot \vec{D})$$

$$\text{div } \vec{J_d} = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\text{div } \vec{J_d} = \text{div}\left(\frac{\partial \vec{D}}{\partial t}\right)$$

$$\vec{J_d} = \frac{\partial \vec{D}}{\partial t}$$

$$\text{curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

Above eq. is modified form of Ampere circuital law. The term which maxwell added to ampere's law is ~~time var~~ includes the time varying field is known as displacement current.

Displacement current → It is first introduced by maxwell. They assumed that as the current in the conductor produces the magnetic field but changing electric field in vacuum or dielectric also produces magnetic field. This means changing electric field is equivalent to a current which flows as long as the field is changing. This equivalent current is known as displacement current which also produces magnetic field.

Characteristics of displacement current →

(i) displacement current is a current which produces magnetic field (ii) Magnitude of displacement current is equal to the rate of change of electric displacement.

(iii) Displacement current solve the purpose to make total ~~cut~~ current continous across the discontinuity in a conduction current eg → A battery charging a capacitor produces a closed current loop in terms of total current.

Wave equations in free space :-

$$\nabla \cdot \vec{D} = \rho \rightarrow (a)$$
$$\nabla \cdot \vec{B} = 0 \rightarrow (b)$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow (c)$$
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \rightarrow (d)$$
$\rightarrow (1)$

conditions of free space are given by :-

$$B = \mu_0 \vec{H}, \quad \vec{D} = \varepsilon_0 \vec{E}, \quad \vec{J} = \sigma \vec{E}$$
$$\because \sigma = 0, \quad \vec{J} = 0$$
$$\rho = 0$$
$\rightarrow (2)$

$\therefore$ from (1) and (2)

$$\nabla \cdot \vec{E} = 0 \quad (a)$$
$$\nabla \cdot \vec{H} = 0 \quad (b)$$
$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad (c)$$
$$\nabla \times \vec{H} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (d)$$
(3)

taking curl in eq (3) (c).

$$\nabla \times (\nabla \times \vec{E}) = \nabla \times \left(-\mu_0 \frac{\partial \vec{H}}{\partial t}\right)$$

$$\nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t} (\nabla \times \vec{H})$$

$\nabla \cdot \vec{E} \Downarrow 0$ (3(a))

$\nabla \times \vec{H} \Downarrow \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ · (3)(d)

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \rightarrow (4)$$

Similarly :-

$$\nabla^2 \vec{H} - \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} = 0 \rightarrow (5)$$

$$\nabla^2 \varphi = \frac{1}{v^2} \frac{\partial^2 \varphi}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$$

$$\nabla^2 \varphi - \frac{1}{v^2} \frac{\partial^2 \varphi}{\partial t^2} = 0 \rightarrow (6)$$

since $\mu_0 = 4\pi \times 10^{-7}$ weber/A-m

$\varepsilon_0 = 8.85 \times 10^{-12}$ Farad/m

$$v = \sqrt{\frac{4\pi}{4\pi \mu_0 \varepsilon_0}} = \sqrt{\frac{4\pi \times 9 \times 10^9}{4\pi \times 10^{-7}}}$$

$$= 3 \times 10^8 \ ms^{-1} = c$$

Q. Show that electromagnetic wave travels with less than the speed of light in case of dielectric medium.

⇒

$$\left.\begin{aligned} \nabla \cdot \vec{D} &= \rho \rightarrow (a) \\ \nabla \cdot \vec{B} &= 0 \rightarrow (b) \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \rightarrow (c) \\ \nabla \times \vec{H} &= \vec{J} + \vec{J}_d \rightarrow (d) \end{aligned}\right] \leftarrow (1)$$

$\vec{B} = \mu \vec{H}$, $\vec{J} = \sigma \vec{E}$, $\vec{D} = \varepsilon \vec{H}$

$\rho = 0$, $\sigma = 0$, $\vec{J} = 0$

$$\nabla^2 \vec{E} - \mu \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$v = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_r \varepsilon_r \mu_0 \varepsilon_0}}$$

$$= \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

from ④, ⑤, ⑥ :-

$$\frac{1}{v^2} = \mu_0 \varepsilon_0 \Rightarrow v = \frac{1}{\sqrt{\mu_0 \cdot \varepsilon_0}}$$

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0 \rightarrow ⑦ \qquad \nabla^2 \psi - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = 0 \rightarrow ⑨$$

$$\nabla^2 \vec{H} - \frac{1}{c^2}\frac{\partial^2 \vec{H}}{\partial t^2} = 0 \rightarrow ⑧$$

soln. of eq. ⑦ and ⑧ may be of the form :-

$$\vec{E} = \vec{E_0}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} \rightarrow ⑩$$

$$\vec{H} = \vec{H_0}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} \rightarrow ⑪$$

$$\vec{k} = \hat{i} k_x + \hat{j} k_y + \hat{k} k_z$$

$$\vec{r} = \hat{i} x + \hat{j} y + \hat{k} z$$

$$\nabla \cdot \vec{E} = 0 \rightarrow (A)$$

$$\nabla \cdot \vec{H} = 0 \rightarrow (B)$$

Applying ⑩ and ⑪ to Ⓐ and Ⓑ we get :-

$$\nabla \cdot \vec{E} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left[\vec{E_0}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right]$$

$$\vec{E_0} = \hat{i} E_{0x} + \hat{j} E_{0y} + \hat{k} E_{0z}$$

$$= \cancel{\frac{\partial}{\partial x} E_{0x} e^{i(\vec{k}\cdot\vec{r}-\omega t)}} + \cancel{\frac{\partial}{\partial y} E_{0y} e^{i(\vec{k}\cdot\vec{r}-\omega t)}} + \frac{\partial}{\partial}$$

$$\nabla\cdot\vec{E} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left[(\hat{i}\vec{E}_{0x} + \hat{j}E_{0y} + \hat{k}E_{0z})e^{i(k_x x + k_y y + k_z z - \omega t)}\right]$$

$$= \frac{\partial}{\partial x}\left[E_{0x} e^{i(k_x x + k_y y + k_z z - \omega t)}\right] + \frac{\partial}{\partial y}\left[E_{0y} e^{i(k_x x + k_y y + k_z z - \omega t)}\right]$$

$$+ \frac{\partial}{\partial z}\left[E_{0z} e^{i(k_x x + k_y y + k_z z - \omega t)}\right].$$

$$= E_{0x} e^{i(k_x x + k_y y + k_z z - \omega t)} \cdot (ik_x) + E_{0y} e^{i(k_x x + k_y y + k_z z - \omega t)} \cdot i(k_y)$$

$$+ E_{0z} e^{i(k_x x + k_y y + k_z z - \omega t)} \cdot i(k_z)$$

$$= \left[E_{0x} \cdot ik_x + E_{0y} \cdot ik_y + E_{0z} \cdot ik_z\right] e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$= i(k_x E_{0x} + k_y \cdot E_{0y} + k_z E_{0z}) \cdot e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$= i\left[(\hat{i}k_x + \hat{j}k_y + \hat{k}k_z)\cdot(\hat{i}E_{0x} + \hat{j}E_{0y} + \hat{k}E_{0z})\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right].$$

$$= i(\vec{k}\cdot\vec{E_0}) \cdot e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\nabla\cdot\vec{E} = i(\vec{k}\cdot\vec{E}) \rightarrow (12)$$

since the application of free space condition to maxwell first eq gives :-

$$\nabla\cdot\vec{E} = 0$$

From eq (12) :-

$$i(\vec{K}\cdot\vec{E}) = 0$$

$i \neq 0$

$\vec{K}\cdot\vec{E} = 0 \Rightarrow \vec{K}$ and $\vec{E}$ are mutually $\perp$s to each other.

From eq (13) :-

$$\nabla\cdot\vec{H} = 0$$

$$\nabla\cdot\vec{H} = i[k\cdot\vec{H}]$$

$$i[\vec{K}\cdot\vec{H}] = 0$$

$i \neq 0 \Rightarrow \vec{K}\cdot\vec{H} = 0$

$\Rightarrow \vec{K}$ and $\vec{H}$ are mutually $\perp$s to each other.

Therefore electromagnetic waves are transverse in nature.

Application of free space conditions to maxwell's equation gives :-

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -\mu_0\frac{\partial\vec{H}}{\partial t} \rightarrow (13)$$

$$\nabla\times\vec{H} = \varepsilon_0\frac{\partial\vec{E}}{\partial t} \rightarrow (14)$$

$$\nabla\times\vec{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_0x & H_0y & H_0z \end{vmatrix}$$

Finally :-

$$\vec{K}\times\vec{E} = \mu_0\omega\vec{H} \rightarrow (15)$$

$$\vec{k}\times\vec{H} = \varepsilon_0\omega\vec{E} \rightarrow (16)$$

$$\nabla \times \vec{E} = i(\vec{k} \times \vec{E})$$

$$\nabla \times \vec{E} = \frac{-\partial B}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t} = -\mu_0 \frac{\partial}{\partial t} \left| H_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} \right.$$

$\therefore$

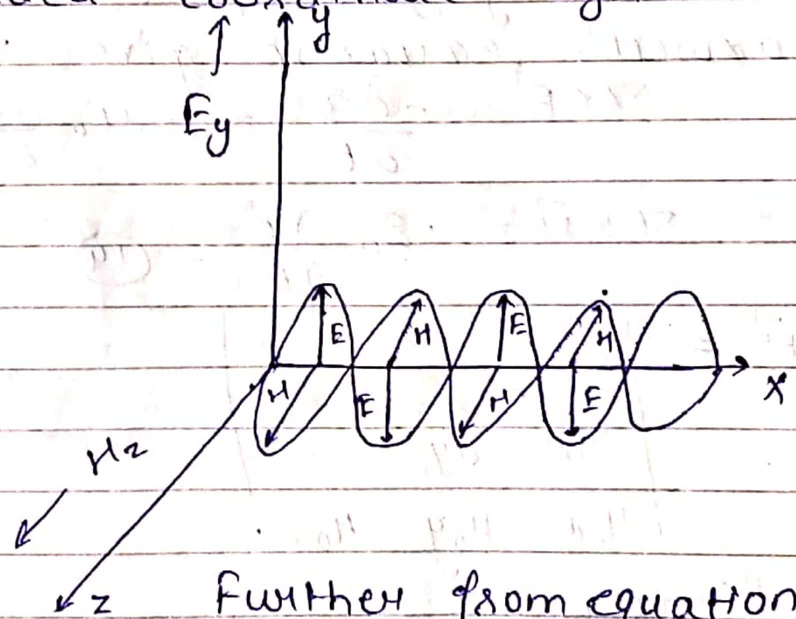
$$\vec{k} \times \vec{E} = \mu_0 \omega \vec{H}$$

$$\vec{k} \times \vec{H} = -\varepsilon_0 \omega \vec{E}$$

from eq (15) it is clear that $\vec{H}$ is $\perp$ to $\vec{k}$ and $\vec{E}$.

Also from eq (16) it is clear that $\vec{E}$ is $\perp$ to $\vec{k}$ and $\vec{H}$.

In this way we can see that in a plane electromagnetic wave, vectors $\vec{E}$, $\vec{H}$, $\vec{k}$ forms a set of orthogonal vectors. which also forms a right handed coordinate system. as shown in fig.



Further from equation

$$\vec{k} \times \vec{E} = \mu_0 \omega \vec{H}$$

$$\vec{H} = \frac{(\vec{k} \times \vec{E})}{\mu_0 \omega}$$

$$\vec{H} = \frac{k}{\mu_0 \omega}(\hat{n} \times \vec{E})$$

$$= \frac{1}{\mu_0 c}(\hat{n} \times \vec{E}) \qquad \because k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

$$|\vec{H}| = \frac{1}{\mu_0 c} E$$

$$H = \frac{1}{\mu_0 c} E$$

or $$\frac{E}{H} = Z_0 = \mu_0 c = \mu_0 \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$= \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}}$$

(wave impedance) $Z_0 = 376.6\ \Omega$.

Propagation of EM waves in conducting media :-

$$\left.\begin{aligned} \nabla \cdot \vec{D} &= \rho && \to (a) \\ \nabla \cdot \vec{B} &= 0 && \to (b) \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} && \to (c) \\ \nabla \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} && \to (d) \end{aligned}\right] \to ①$$

Conditions of conducting media :-

$$\left.\begin{aligned} &\sigma, \rho = 0 \text{ (No charge on surface)} \\ &\vec{J} = \sigma \vec{E}, \ \vec{B} = \mu \vec{H}, \ \vec{D} = \varepsilon_0 \vec{E} \end{aligned}\right\} \to ②$$

Applying in eq ① :-

$$\left.\begin{aligned} \nabla \cdot \vec{E} &= 0 && \to (a) \\ \nabla \cdot \vec{H} &= 0 && \to (b) \end{aligned}\right\} \to ③$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial H}{\partial t} \rightarrow (c)$$
$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon_0 \frac{\partial E}{\partial t} \rightarrow (d)$$
$\Bigg\} \rightarrow (3)$

taking the curl of eq. (3) (c) :-

$$\nabla \times (\nabla \times \vec{E}) = \nabla \times \left(-\mu_0 \frac{\partial H}{\partial t}\right)$$

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t} (\nabla \times \vec{H})$$

$$0 - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t}\left(\sigma \vec{E} + \varepsilon_0 \frac{\partial E}{\partial t}\right)$$

$$-\nabla^2 \vec{E} = -\mu_0 \sigma \frac{\partial \vec{E}}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial^2 t}$$

$$\nabla^2 \vec{E} = \mu_0 \sigma \frac{\partial \vec{E}}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

Similarly :-

$$\nabla^2 \vec{H} = +\mu_0 \sigma \frac{\partial \vec{H}}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2}$$

and now for the case of insulators :-
$\sigma = 0$

Now,

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$
$$\nabla^2 \vec{H} = \mu \varepsilon \frac{\partial^2 H}{\partial t^2}$$

most imp

Solution of Electromagnetic wave conducting media case :-

wave equation for electric field is given by:

$$\nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu \sigma \frac{\partial \vec{E}}{\partial t} = 0 \rightarrow (1)$$

Let us suppose that the wave is travelling along x-direction. The above eq. reduces to

$$\frac{\partial^2 \vec{E}}{\partial x^2} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu \sigma \frac{\partial E}{\partial t} = 0 \rightarrow (A)$$

The solution of eq. (A) is given by :-

$$E = E_0 e^{i(k_x x - \omega t)}$$

$$= E_0 e^{i(k_x x - \omega t)} \longrightarrow (2)$$

Substituting the values of $\frac{\partial^2 \vec{E}}{\partial x^2}$; $\frac{\partial^2 \vec{E}}{\partial t^2}$

and $\frac{\partial \vec{E}}{\partial t}$ by operating eq (2) :-

$$\frac{\partial \vec{E}}{\partial x} = E_0 e^{i(k_x x - \omega t)} \cdot i(k_x)$$

$$\frac{\partial^2 \vec{E}}{\partial x^2} = E_0 i^2 k_x^2 e^{i(k_x x - \omega t)}$$

$$= - E_0 k_x^2 e^{i(k_x x - \omega t)} = -k_x^2 E$$

$$\frac{\partial \vec{E}}{\partial t} = E_0 e^{i(k_x x - \omega t)} \cdot i(-\omega) = -i\omega E$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = - E_0 \omega^2 e^{i(k_x x - \omega t)} = - \omega^2 E$$

from eq (A) :-

$$\cancel{-k_x^2 E + \mu \epsilon \omega^2 E}$$

$$-k_x^2 \vec{E} + \mu \epsilon \omega^2 \vec{E} + \mu \sigma i \omega \vec{E} = 0$$

since $\vec{E} \neq 0$.

$$-K_x^2 + i\omega\mu\sigma + \omega^2\mu\varepsilon = 0 \rightarrow (3)$$

It is clear from eq (3) that K_x must be a complex quantity can be written as :-

$$K_x = \alpha + i\beta \rightarrow \text{phase constant.}$$

α — Attenuation constant. β — phase constant.

Now substituting the value of K_x in eq (3) we get :-

$$-(\alpha + i\beta)^2 + i\omega\mu\sigma + \omega^2\mu\varepsilon = 0$$

$$-(\alpha^2 - \beta^2 + 2i\alpha\beta) + i\omega\mu\sigma + \omega^2\mu\varepsilon = 0$$

$$\beta^2 - \alpha^2 - 2i\alpha\beta + i\omega\mu\sigma + \omega^2\mu\varepsilon = 0$$

or

$$(\beta^2 - \alpha^2 + \omega^2\mu\varepsilon) + i(\omega\mu\sigma - 2\alpha\beta) = 0$$

It is an identity :-

$\therefore$

$$\beta^2 - \alpha^2 + \omega^2\mu\varepsilon = 0$$

$$\alpha^2 - \beta^2 = \omega^2\mu\varepsilon = 0 \rightarrow (4)$$

$$\omega\mu\sigma - 2\alpha\beta = 0$$

$$2\alpha\beta = \omega\mu\sigma$$

$$\beta = \frac{\omega\mu\sigma}{2\alpha} \rightarrow (5)$$

put (5) in eq (4) :-

$$\alpha^2 - \frac{\omega^2\mu^2\sigma^2}{4\alpha^2} = \omega^2\mu\varepsilon$$

$$4\alpha^4 - \omega^2\mu^2\sigma^2 = 4\alpha^2\omega^2\mu\varepsilon$$

$$4\alpha^4 - 4\alpha^2\omega^2\mu\varepsilon = \omega^2\mu^2\sigma^2$$

$$4\alpha^4 - 4\alpha^2\omega^2\mu\varepsilon - \omega^2\mu^2\sigma^2 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4\omega^2\mu\varepsilon \pm \sqrt{16\omega^4\mu^2\varepsilon^2 + 16\omega^2\mu^2\sigma^2}}{8}$$

$$= \frac{\omega^2\mu\varepsilon \pm \sqrt{\omega^4\mu^2\varepsilon^2 + \omega^2\mu^2\sigma^2}}{2}$$

$$\alpha^2 = \frac{\omega^2\mu\varepsilon \pm \mu\omega\sqrt{\omega^2\varepsilon^2 + \sigma^2}}{2}$$

$$= \omega^2\mu\varepsilon\left[\frac{1}{2} \pm \frac{1}{2}\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}}\right]$$

In above eq. considering + sign only because the -ve sign makes the whole quantity imaginary.

$$\alpha = \omega\sqrt{\mu\varepsilon}\left[\frac{1}{2} + \frac{1}{2}\left(1 + \frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2}\right]^{1/2}$$

$$\beta = \frac{\omega\mu\sigma}{2\alpha}$$

Now substituting the values of α and β in eq (B) :-

finally substituting the value of k_x in :-

$$E = E_0 e^{i(k_x x - \omega t)}$$

we get :-

$$E = \underbrace{E_0 \exp(-\beta x)}_{\text{Amplitude}} \cdot \underbrace{\exp[i(\alpha x - \omega t)]}_{\text{phase}}.$$

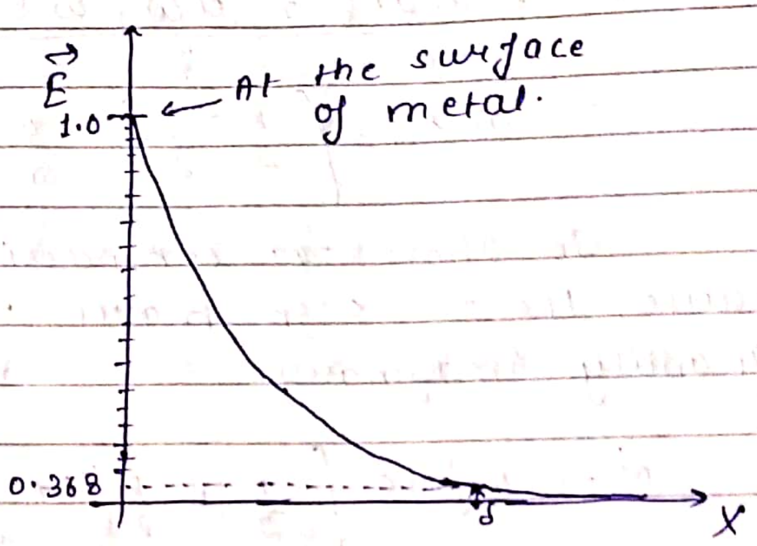
Above eq. represents that the wave gets attenuated exponentially for good conductor.

$$\frac{\sigma}{\omega\varepsilon} \gg 1$$

$$\Rightarrow \alpha \approx \beta \approx \left(\frac{\omega\mu\sigma}{2}\right)^{1/2}$$

Skin depth or Depth of penetration:-

The skin depth is defined as the depth at which electric field strength is reduced to $\frac{1}{e}$ times to its initial value as shown in the fig.



The amplitude of strength of electric field of an EM wave is decreases by the factor $e^{-\alpha x}$ where α is the attenuation constant. The distance for which electric field or amplitude of EM wave decreases by a factor e^{-1} or $\frac{1}{e}$ to its initial value will satisfy the eq.

$$\alpha x = 1 .$$

If δ is the depth of penetration then $x = \delta$

$$\alpha \delta = 1$$

$$\delta = \frac{1}{\alpha} .$$

$$\alpha \approx \beta \approx \left(\frac{\omega \mu \sigma}{2} \right)^{1/2}$$

$$\delta = \frac{1}{\left(\frac{\omega\mu\sigma}{2}\right)^{1/2}} = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$$

$$= \left(\frac{2}{2\pi\nu\mu\sigma}\right)^{1/2} = \left(\frac{1}{\pi\nu\mu\sigma}\right)^{1/2}$$

$$\delta = \sqrt{\frac{1}{\pi\nu\mu\sigma}}$$

(A) $\mu = \frac{V_f \rightarrow \text{in free space}}{V_m \rightarrow \text{in dielectric}}$

$$V_f = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$V_m = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

$$\mu = \frac{\frac{1}{\sqrt{\mu_0\varepsilon_0}}}{\frac{c}{\sqrt{\mu_r\varepsilon_r}}} = \sqrt{\mu_r\varepsilon_r}$$

$$\boxed{\mu = \sqrt{\mu_r \varepsilon_r}}$$

Poynting Theorem :-

Poynting vector → The important characteristic of EM wave is that they transfer energy through space when they from source to receiving point. This energy can be expressed in terms of electric and magnetic field strength.

"The amount of energy passing through unit surface area to the direction of the propagation of the wave is known as poynting vector represented by $\vec{P}$. Mathematically this can be represented as:-

$$\vec{P} = \vec{E} \times \vec{H}.$$

It's unit is $J/m^2 s$ or $Watt/m^2$.

Poynting Theorem:-

from ~~max~~ maxwell's ③ and ④ eqn:-

$$(\nabla \times \vec{E}) = -\frac{\partial \vec{B}}{\partial t} \rightarrow (A)$$

$$(\nabla \times \vec{H}) = \vec{J} + \frac{\partial \vec{D}}{\partial t} \rightarrow (B)$$

taking (.) product of $\vec{H}$ with eq (A) and $\vec{E}$ with eq (B) :-

$$\vec{H} \cdot (\nabla \times \vec{E}) = H \cdot \left(-\frac{\partial \vec{B}}{\partial t}\right) = H \cdot \left(-\mu \frac{\partial \vec{H}}{\partial t}\right)$$

$$= -\mu H \cdot \frac{\partial \vec{H}}{\partial t} = -\frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2\right] \rightarrow (C)$$

Similarly,

$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{E} \cdot \vec{J} + E \cdot \frac{\partial \vec{D}}{\partial t}$$

$$= \vec{E} \cdot \vec{J} + \vec{E} \cdot \left(\varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$= \vec{E} \cdot \vec{J} + \frac{\varepsilon}{2} \frac{\partial}{\partial t}(\vec{E} \cdot \vec{E})$$

$$= \vec{E} \cdot \vec{J} + \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2\right) \rightarrow (D)$$

Now substracting eq (D) from eq (C) we get:-

$$\vec{H}\cdot(\nabla\times\vec{E})-\vec{E}\cdot(\nabla\times\vec{H}) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right)-\vec{E}\cdot\vec{J}$$
$$-\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2\right)$$
$$= -\left[\vec{E}\cdot\vec{J}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2+\frac{1}{2}\varepsilon E^2\right)\right].$$

$$\nabla\cdot(\vec{E}\times\vec{H}) = -\left[E\cdot\vec{J}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2+\frac{1}{2}\varepsilon E^2\right)\right] \rightarrow (E)$$

$$\nabla\cdot(\vec{E}\times\vec{H}) = -\vec{E}\cdot\vec{J}-\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2+\frac{1}{2}\varepsilon E^2\right) \rightarrow (F)$$

Now consider the surface ~~as~~ bounds the volume V and integrating the above relation over the volume we get:-

$$\int_V \nabla\cdot(\vec{E}\times\vec{H})\,dV = -\frac{\partial}{\partial t}\int_V\left(\frac{\mu H^2}{2}+\frac{\varepsilon E^2}{2}\right)dV-\int_V(\vec{E}\cdot\vec{J})\,dV$$

Now applying gauss divergence in LHS. we get:-

$$\oint_S (\vec{E}\times\vec{H})\cdot d\vec{s} = -\frac{\partial}{\partial t}\int_V\left(\frac{\mu H^2}{2}+\frac{\varepsilon E^2}{2}\right)dV-\int_V(\vec{E}\cdot\vec{J})\,dV$$

(1) The term $\frac{\mu H^2}{2}+\frac{\varepsilon E^2}{2}$ represents the total Energy stored due to electric and magnetic fields in volume V. Therefore the term $-\frac{\partial}{\partial t}\int_V\left(\frac{\mu H^2}{2}+\frac{\varepsilon E^2}{2}\right)dV$ represents the rate of decrease of stored energy due to electric and magnetic field.

(II) IInd term is generalised form of Joule's law.

(III) IIIrd term $\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{s}$ represents the rate of flow of energy enclosing the volume V therefore $\vec{E} \times \vec{H}$ gives the rate of flow of energy through unit area enclosing the volume V. which is ~~dep~~ denoted by $\vec{P}$ is known as poynting given by:-

$$\boxed{\vec{P} = \vec{E} \times \vec{H}}$$

Q. If the magnitude of $\vec{H}$ in a plane wave is 1 amp/m, find the magnitude of $\vec{E}$ for a plane wave in free space.

⇒ $Z_0 = \frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.6$ ohms.

Q. If the earth receives 2 cal min^{-1} cm^{-2} solar energy. What are the amplitudes of electric and magnetic fields of radiation.

⇒

$$\vec{P} = \vec{E} \times \vec{H} \Rightarrow |\vec{P}| = |\vec{E} \times \vec{H}|$$

$$= EH \sin 90 = EH$$

$$|\vec{P}| = P = \frac{2 \times 4.2}{60 \times (10^{-2})^2} = \frac{2 \times 4.2 \times 10^4}{60}$$

$$= 1400.$$

$$EH = 1400 \rightarrow (1)$$

$$Z_0 = \frac{E}{H} = 376.6$$

$$376.6\,H^2 = 1400$$

$$H^2 = \frac{1400}{376.6}$$

$$H = 1.92 \text{ A/m}$$

$$E = 726.06 \text{ V/m}$$

Q. Assuming that all energy from a 1000 watt lamp is radiated uniformly, calculate the avg. values of intensity of electric and magnetic fields of radiations at a distance of 2m from ~~length~~ lamp.

⇒ given :-

$$P_{Av.} = \frac{P}{4\pi r^2} = \frac{1000}{4\pi \times (2)^2} = \frac{1000}{16\pi} \text{ W}$$

$$EH = \frac{1000}{16\pi}, \quad \frac{E}{H} = 376.6 \text{ ohms.}$$

$$376.6\,H^2 = \frac{1000}{16\pi}$$

$$H^2 = \frac{1000}{16\pi \times 376.6}$$

$$H = 0.23 \text{ A/m} \qquad E = 86.6 \text{ V/m}$$

(A) In conducting medium ; $\boxed{\mu_r = 1}$