

## Engineering

Application of science is engineering.

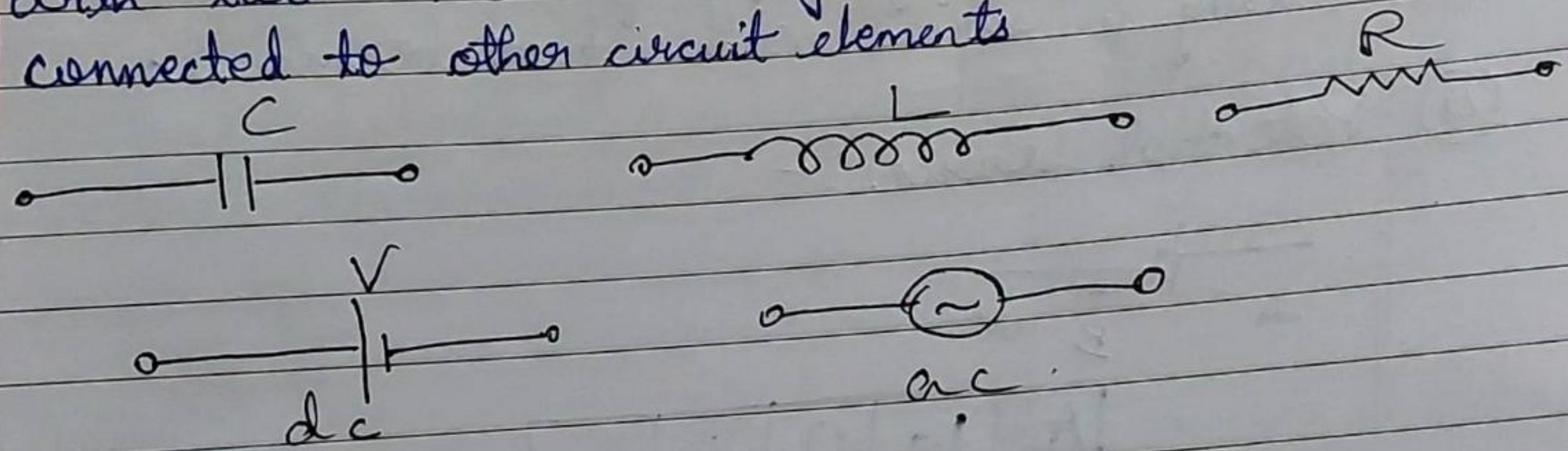
Electrical Engineering →

The branch of Engineering in which we deal with power generation, power transmission and power utilisation

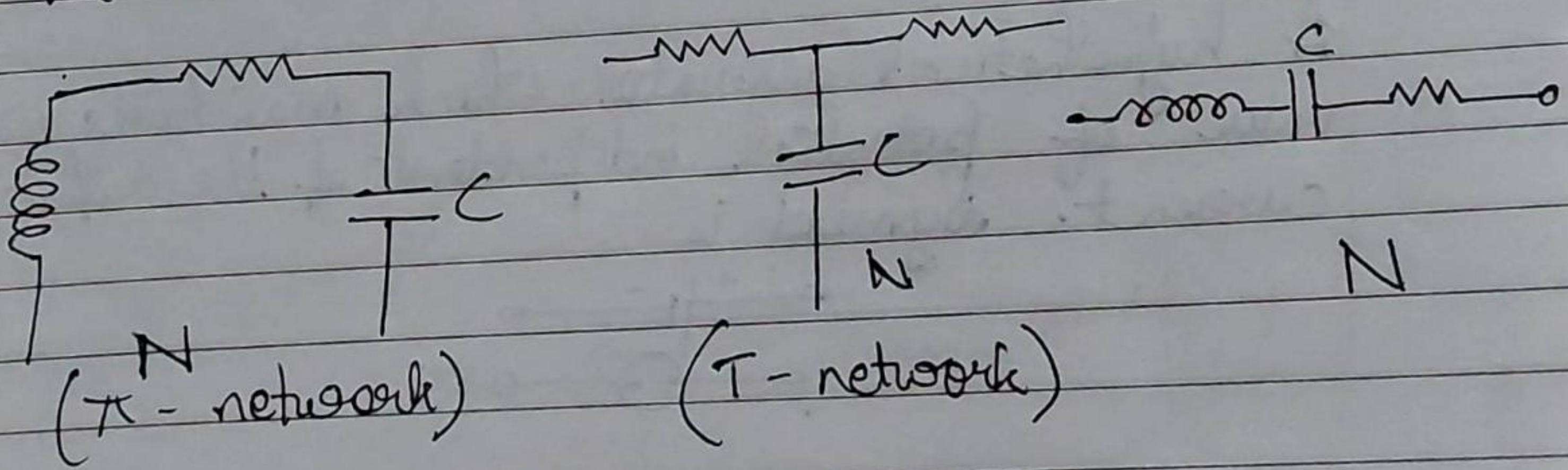
Terms used in Electrical Engineering -

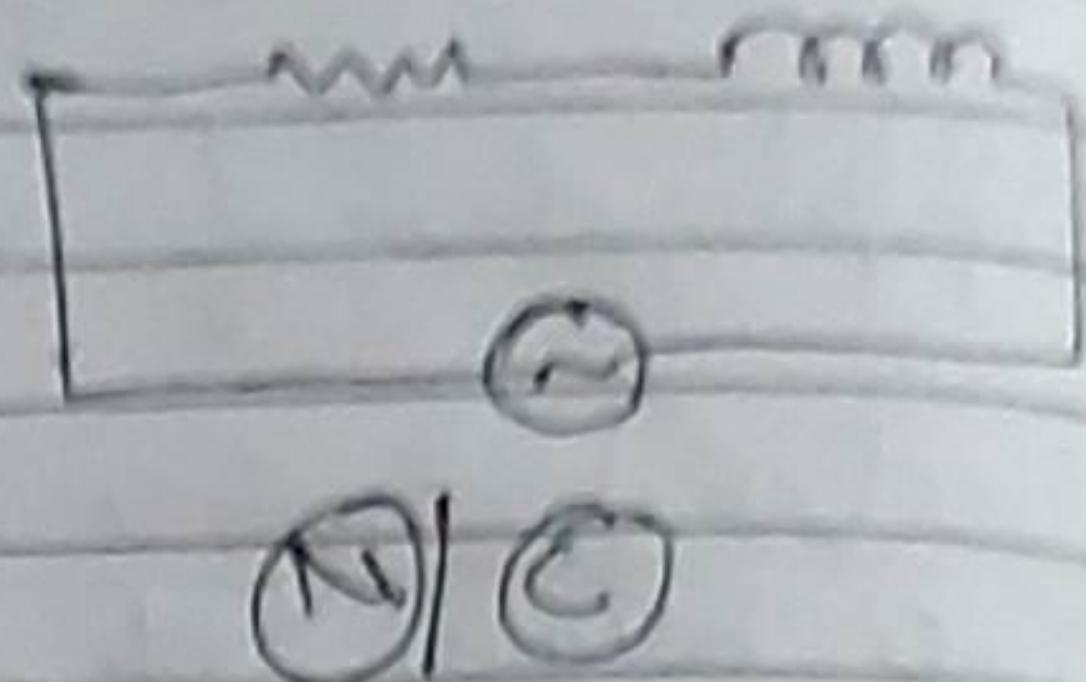
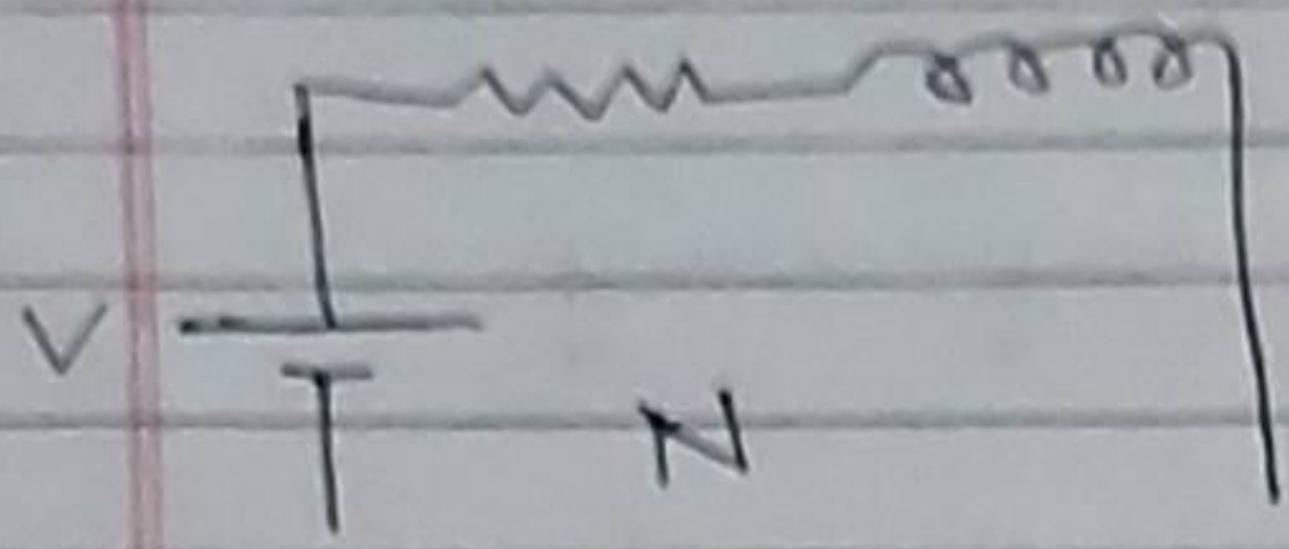
① Circuit Elements:-

Any individual circuit element (inductor, resistor, capacitor, and generator etc, wire etc) with two terminal by which it can be connected to other circuit elements



② Network and Circuit:-

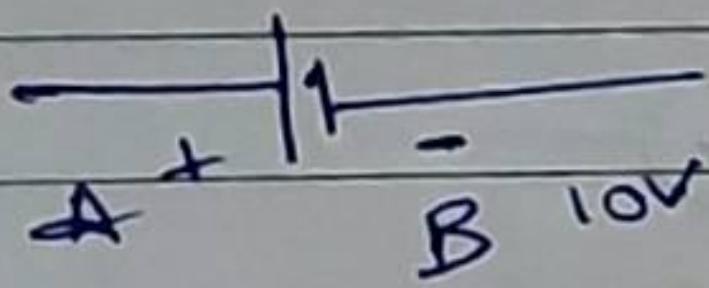




- \* Any inter-connection of circuit element is called network.
- \* Closed, energised network is called circuit
- \* All circuits are network  
Network  $\supset$  circuit

③ Branch:- A group of circuit elements usually in series with two terminal.

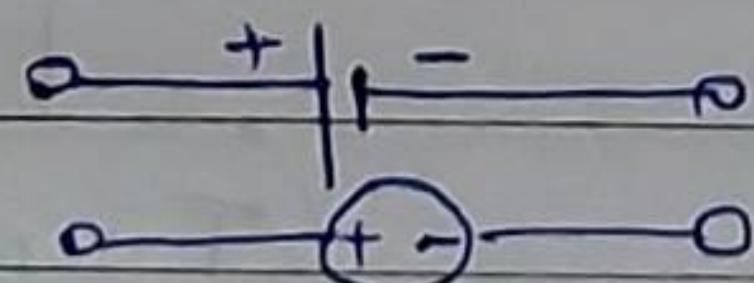
④ Potential Source :-



A	10	0	-10	20
B	0	-10	-20	10

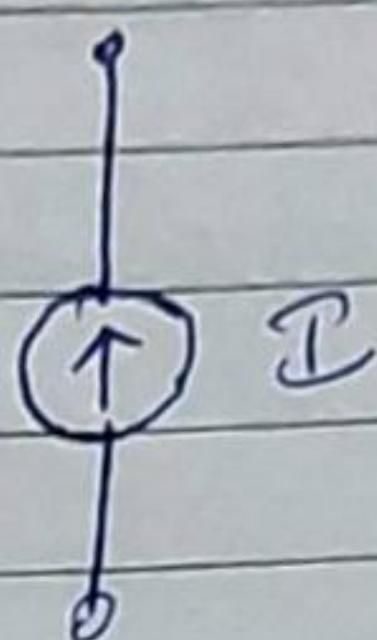
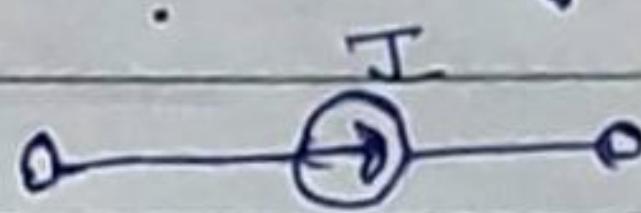
All are correct.

A hypothetical generator which maintains its value of potential independent of the output current. Symbol is



## 5 Current Source:-

A hypothetical generator which maintains its output current independent of the voltage across its terminal. It is represented by

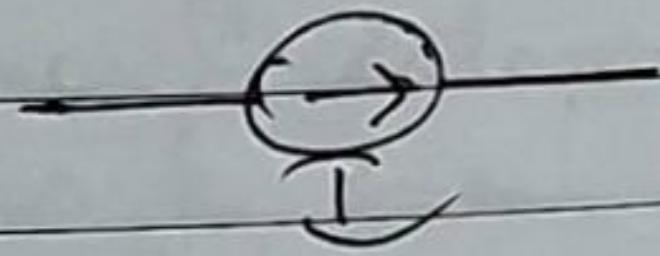
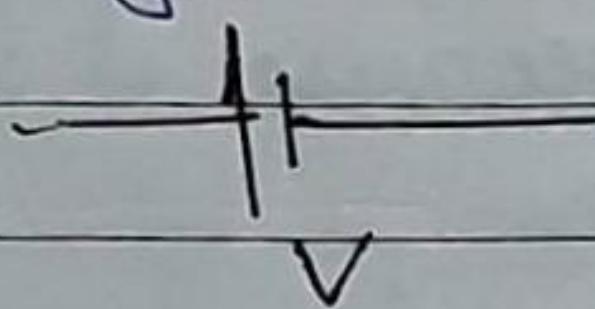


## Active and Passive Elements:-

\* An active source is one which supplies energy for long time to the circuit. Hence, it has the ability to electrically control the flow of charge.

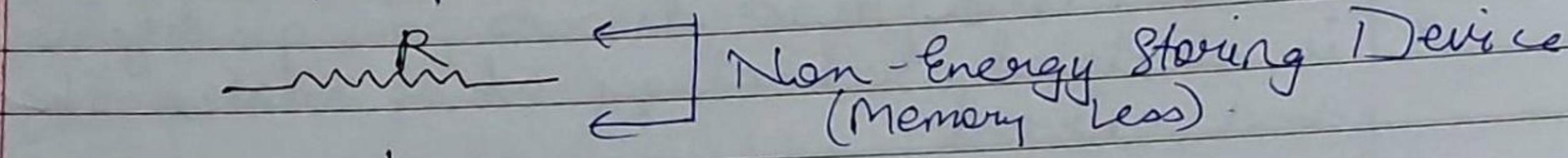
It has internal amplification property.

Eg → Voltage Source and current source



\* A passive element is one which receives energy or supply for less time and then either dissipate it to heat or stores it in an electric or magnetic field.

Eg → Resistance, Capacitor, Inductor



(Stores energy  
in magnetic  
field)

(Stores energy  
in electric  
field)

Energy Storing Device  
(With Memory)

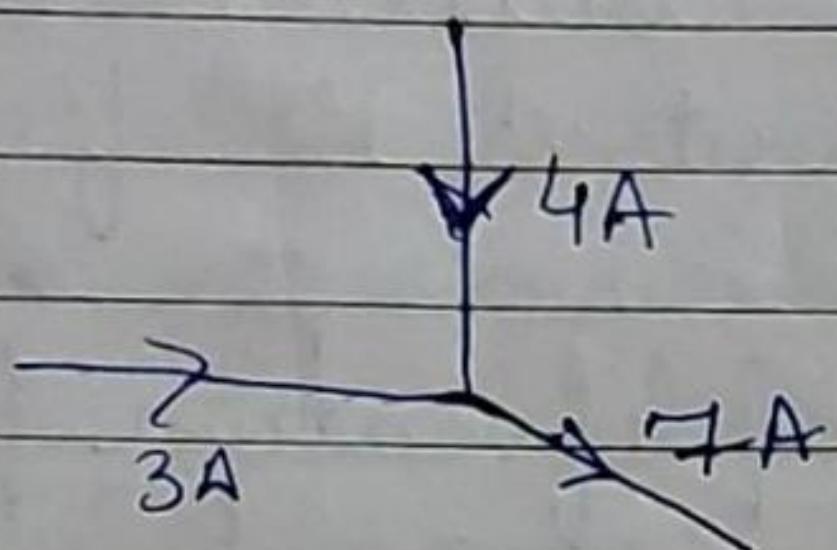
## Active and Passive Networks:-

A network containing circuit element with active source is called active network.

A network containing circuit element without active source is called passive network.

- \* It is custom that the direction of positive charge is the direction of the current.

$$\begin{array}{c} +Q \quad \oplus \rightarrow \quad I \rightarrow \\ -Q \quad \ominus \rightarrow \quad \leftarrow I \end{array}$$



- \* Current is a scalar quantity, because it does not obey triangle law of addition.

## Phasor and Vector:

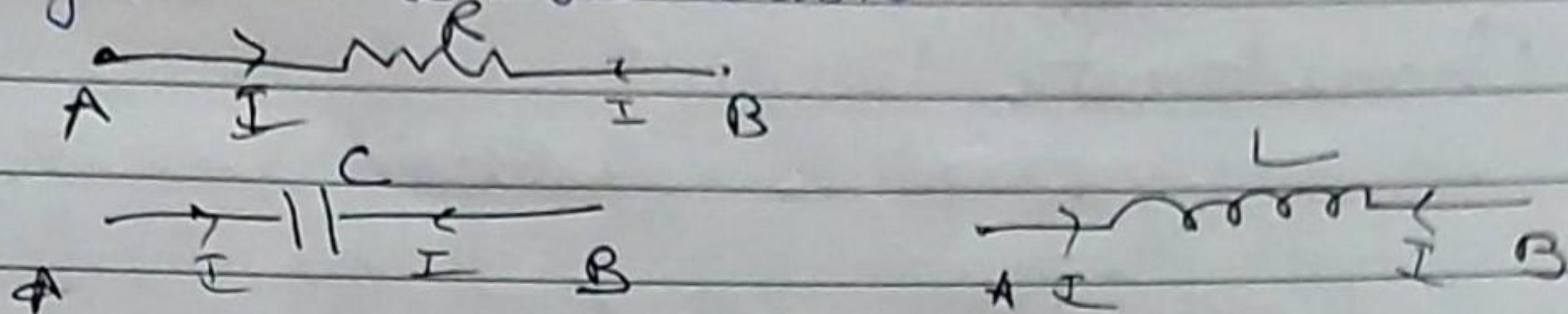
Vector is a generalised multi-dimensional quantity having both magnitude and direction.

Phasor is a two dimensional quantity used in electrical technology which relates voltage and current.

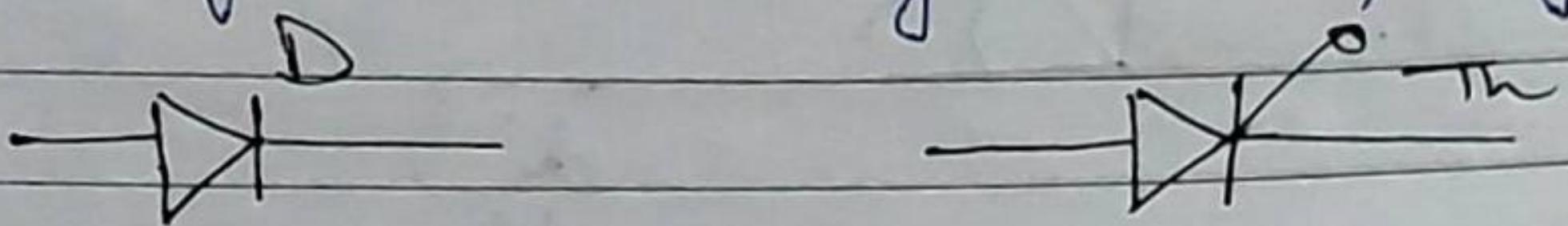
## Unilateral and Bilateral elements:-

- Current in a R
- Voltage across a R

In a bilateral element the same relationships between current and voltage exist for current flowing in either direction.



A unilateral element has different current and voltage relationship for two possible direction of current. Eg. Diode, Thyristor



Inductor :- Inductance can be characterised as that property of the circuit element by which energy is capable of being stored in a magnetic field.

Inductance felt in a circuit only when there is a changing current. Circuit element may have inductance by the virtue of its geometrical and magnetic properties.

$$V_L = L \frac{di}{dt}$$

$i = \text{constant}$  (DC)

$$\frac{di}{dt} = 0$$

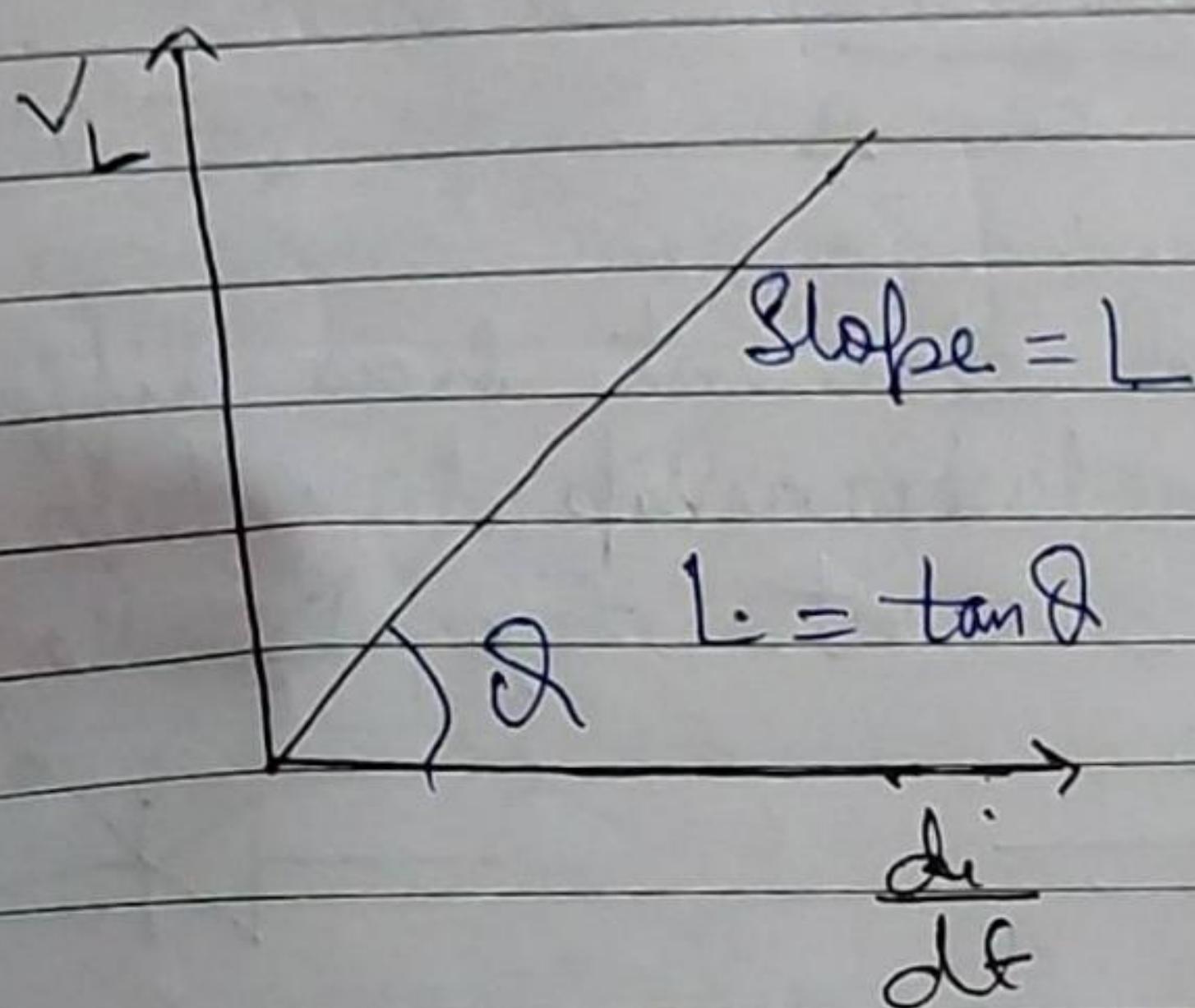
$$\frac{dI}{dt}$$

$$V_L = 0$$

Inductor behave like a wire if  $V_L = 0$

Unit  $\rightarrow$  Volt-sec or H

A  
Characteristic



A current in an inductor ~~can~~ cannot change in zero time (suddenly)

$$I_L(0^-) = I_L(0^+)$$

Finite change in current in zero time require an infinite voltage to appear across the inductor which is physically not possible.  $i-t$  graph is continuous.

Energy stored  $\rightarrow$

$$t=0, i=0; t=t, i=i$$

$$W = \int_{t=0}^{t=t} V_L i dt$$

$$\begin{aligned}
 &= \int L \left( \frac{di}{dt} \right) i dt \\
 &= \int L \cdot i di \\
 \boxed{W = \frac{1}{2} L i^2 I}
 \end{aligned}$$

Capacitor :- Capacitance can be characterised as that property of the circuit element in which energy is capable of being stored in an electric field. Its influence appears in a electric circuit only when there is changing potential difference across the capacitor.

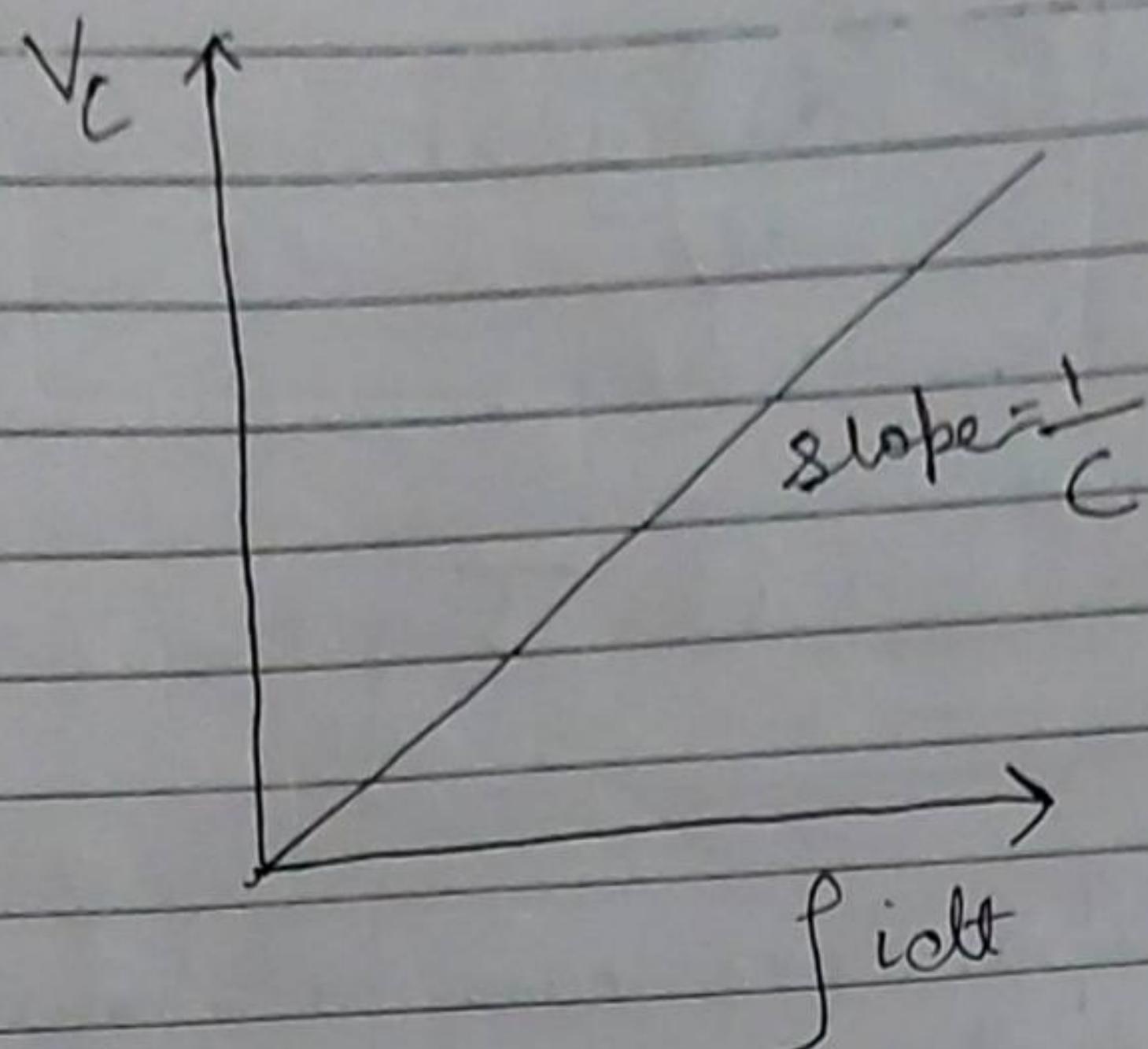
$$\begin{aligned}
 q &= CV_C \\
 \frac{dq}{dt} &= C \frac{dV_C}{dt} \\
 \boxed{i = C \frac{dV_C}{dt}}
 \end{aligned}$$

i = current

~~$$\begin{aligned}
 V_C &\equiv i = 0 \\
 V_C &= \text{constant} \\
 i &= 0 \text{ (open circuit)}
 \end{aligned}$$~~

Unit :- A - see or F  
V

Characteristic



Voltage across a capacitor cannot change in zero time:

$$V_c(0^-) = V_c(0^+)$$

Finite change in voltage in zero time require infinite current which is physically not possible.

Energy stored  $\rightarrow$   $t=0, V_c=0$ ;  $t=t, V_c=V_c$

$$W = \int_0^t V_c i dt$$

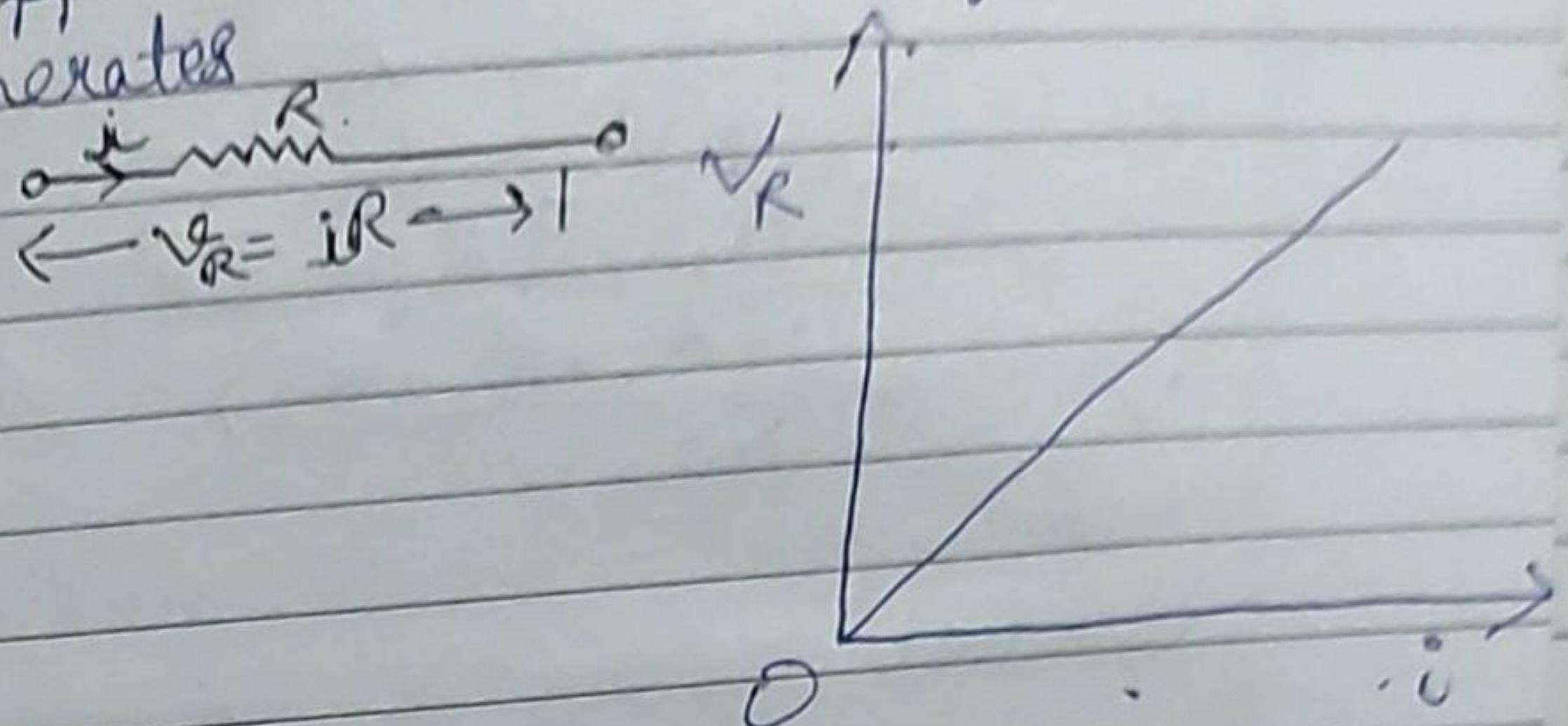
$$= \int_{V_c}^{V_c} V_c C \frac{dV_c}{dt} dt$$

$$= \int_0^C C V_c dV_c$$

$$\boxed{\int W = \frac{1}{2} C V_c^2}$$

Resistor:- Every circuit element have some degree of resistance.

Resistor offers opposition to the flow of current and hence heat generates



Energy loss :-

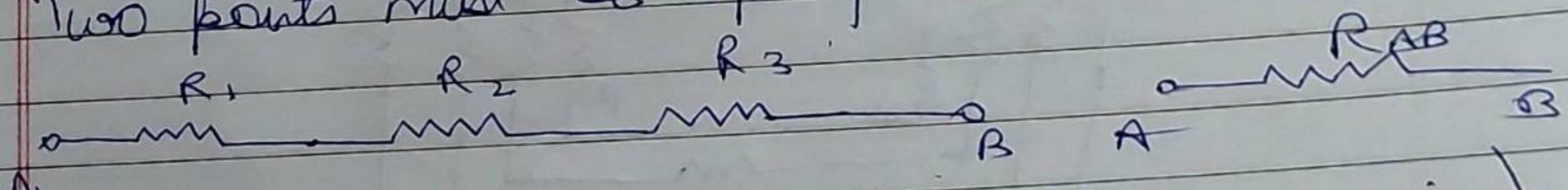
$$dw = \int_0^t V_R i dt$$

$$w = V_R i \int_0^t dt \quad (\text{If } i = \text{const})$$

$$P dw = \left( \frac{dw}{dq} \right) \left( \frac{dq}{dt} \right) (dt)$$

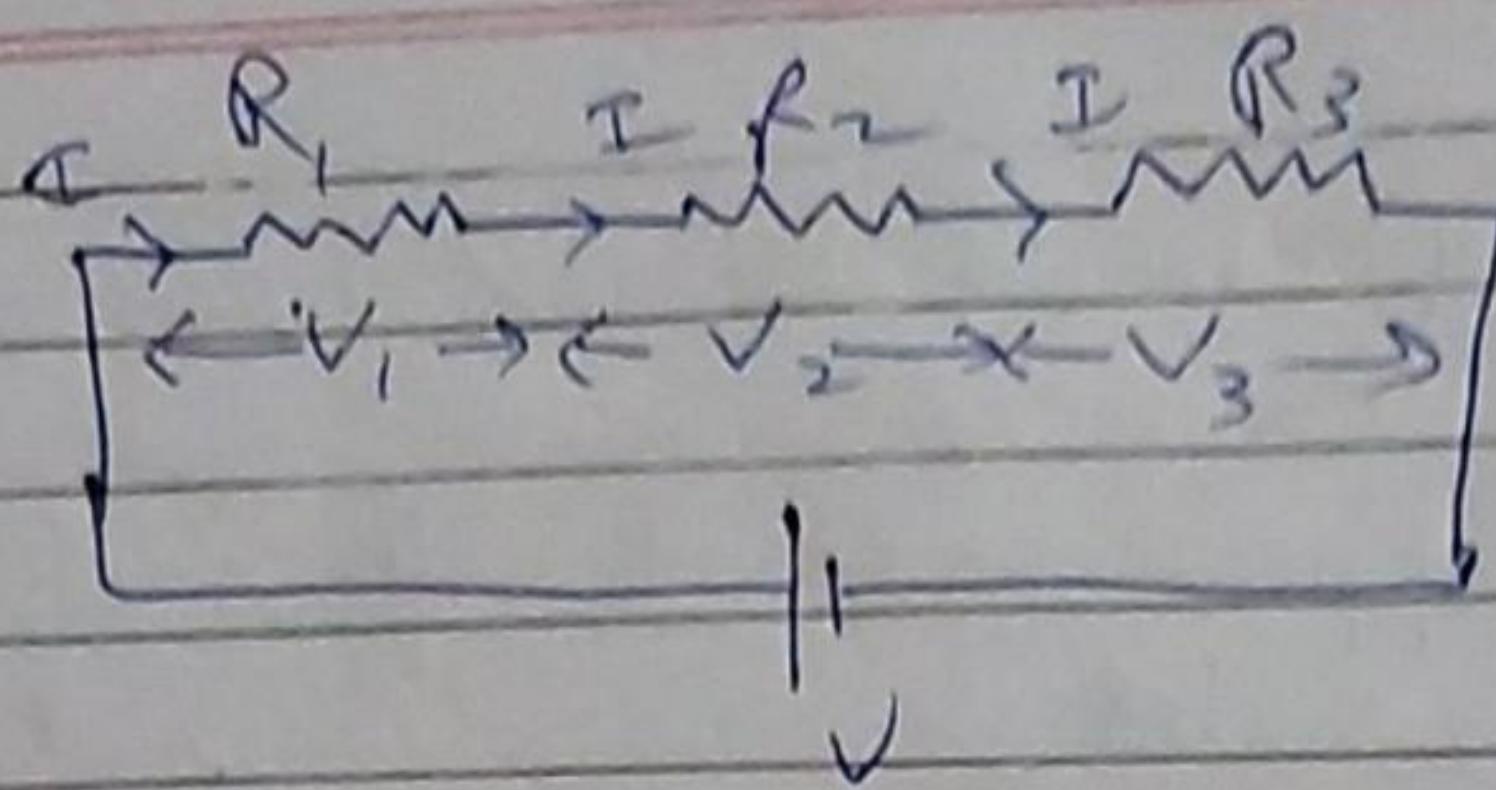
Series and parallel connection  $\rightarrow$

Two points must be specified



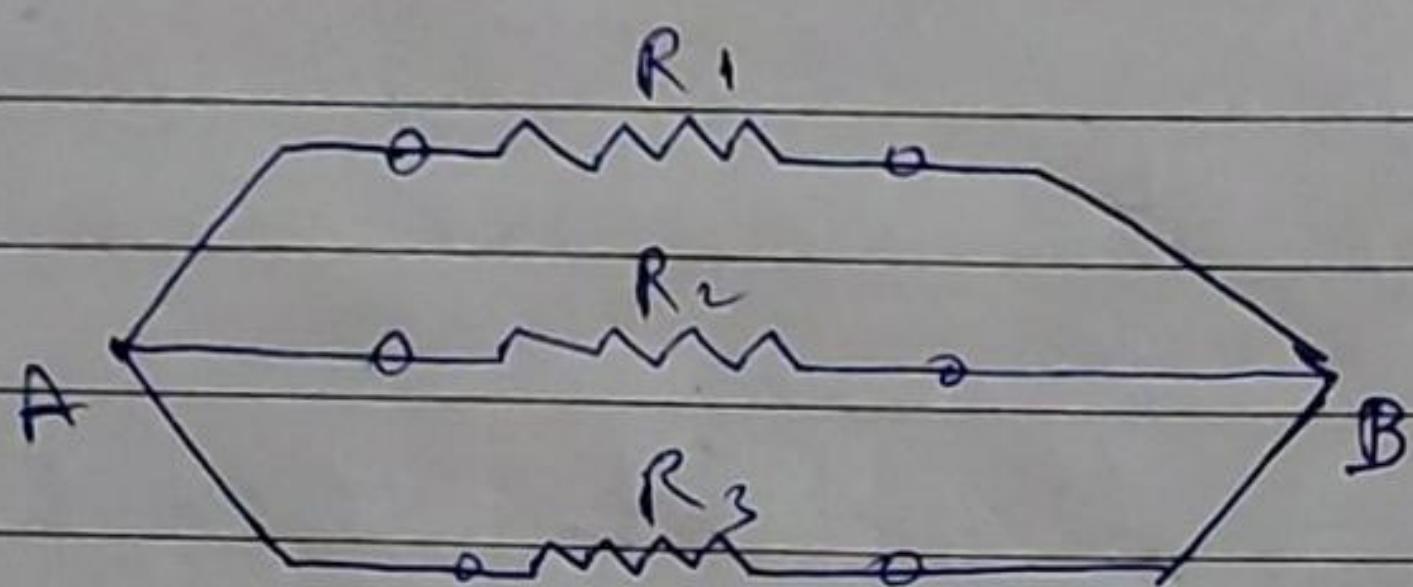
$$R_{AB} = R_1 + R_2 + R_3 \quad (\text{Circuit in series})$$

equal  $\neq$  equivalent



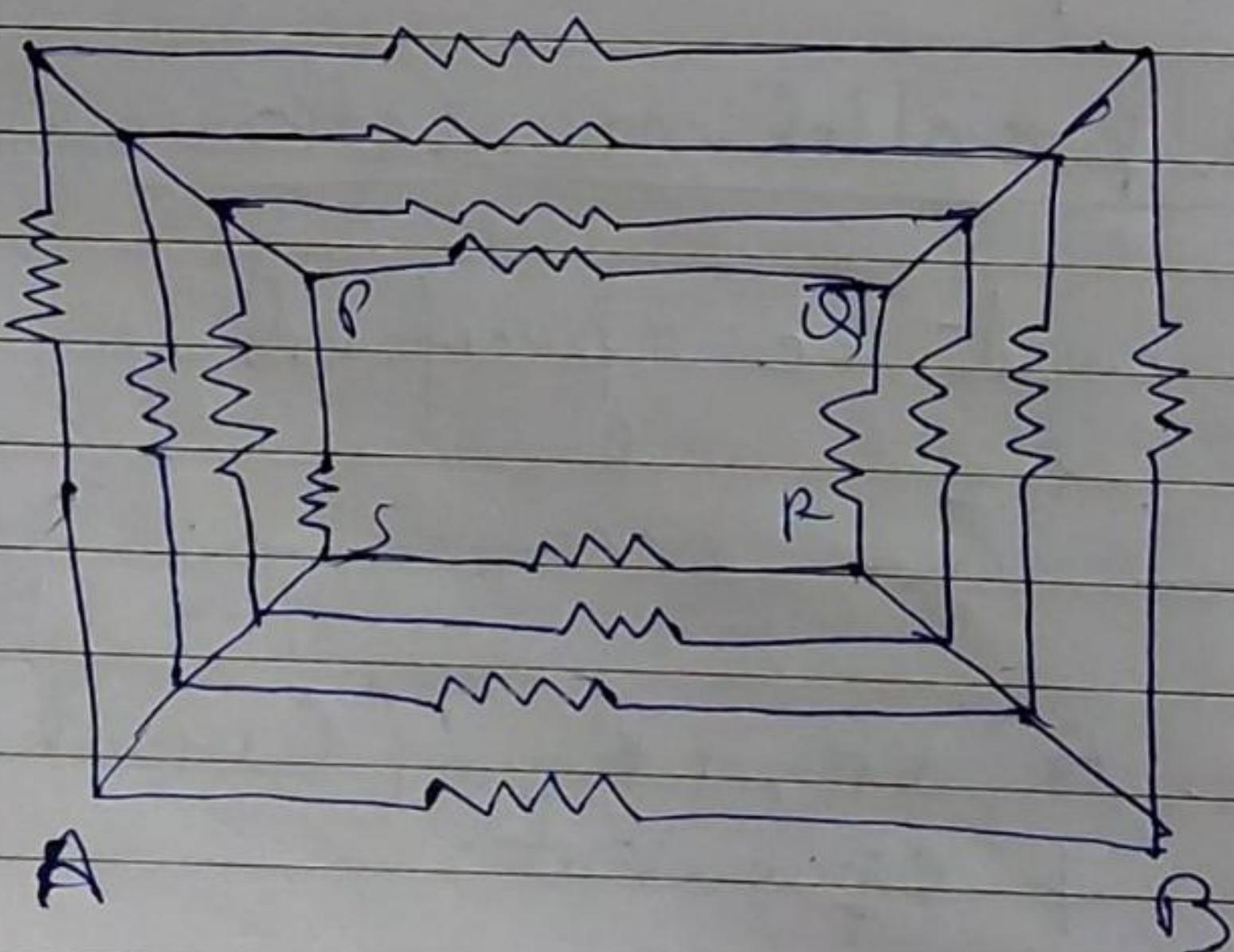
$$\begin{aligned}
 V &= V_1 + V_2 + V_3 \\
 &= IR_1 + IR_2 + IR_3 \\
 &= I(R_1 + R_2 + R_3) \\
 V &= I(R_{\text{eq}}).
 \end{aligned}$$

Parallel:-

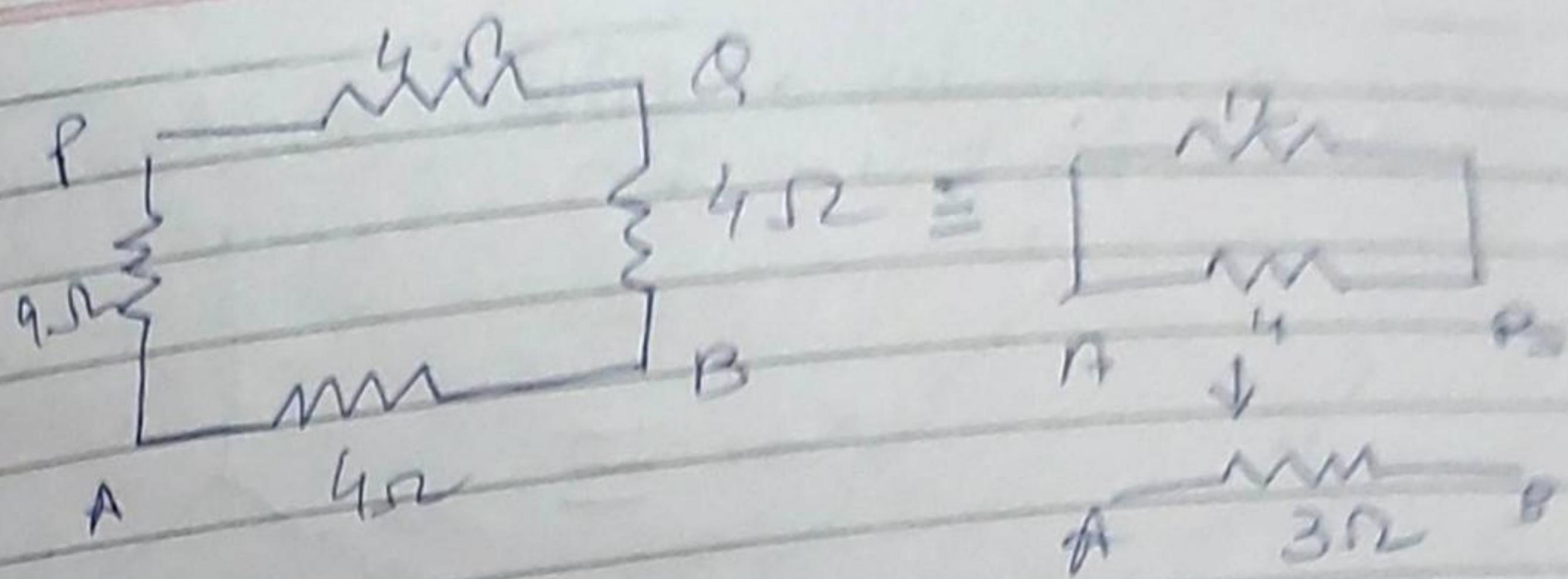


$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Q Calculate the equivalent resistance

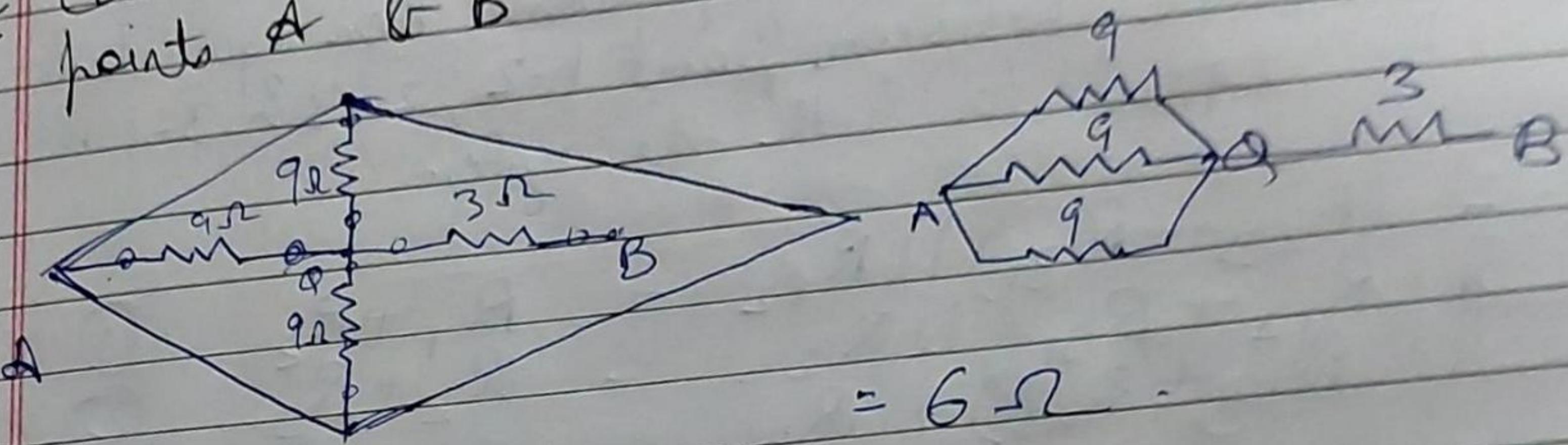


$$R = 16 \Omega$$

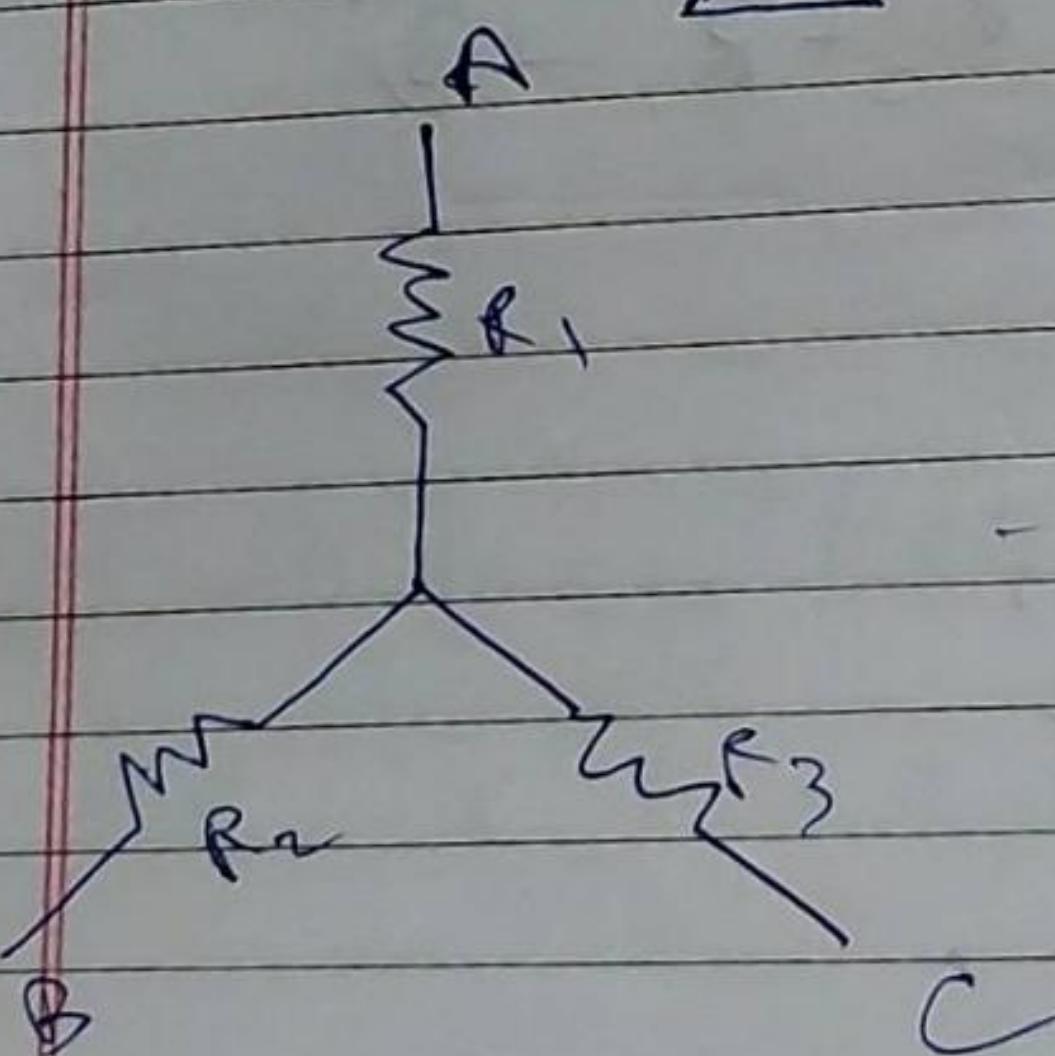
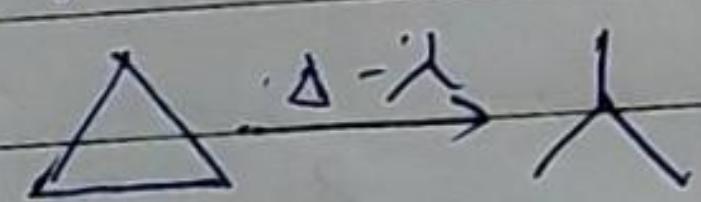
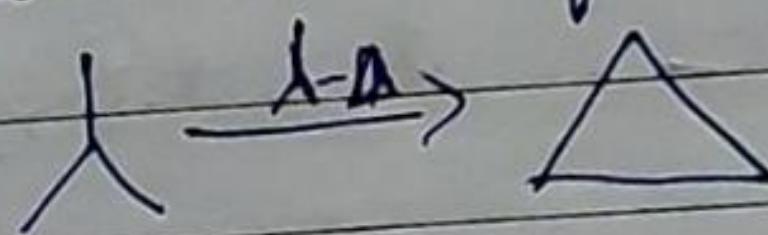


$$R_{AB} = 3\Omega$$

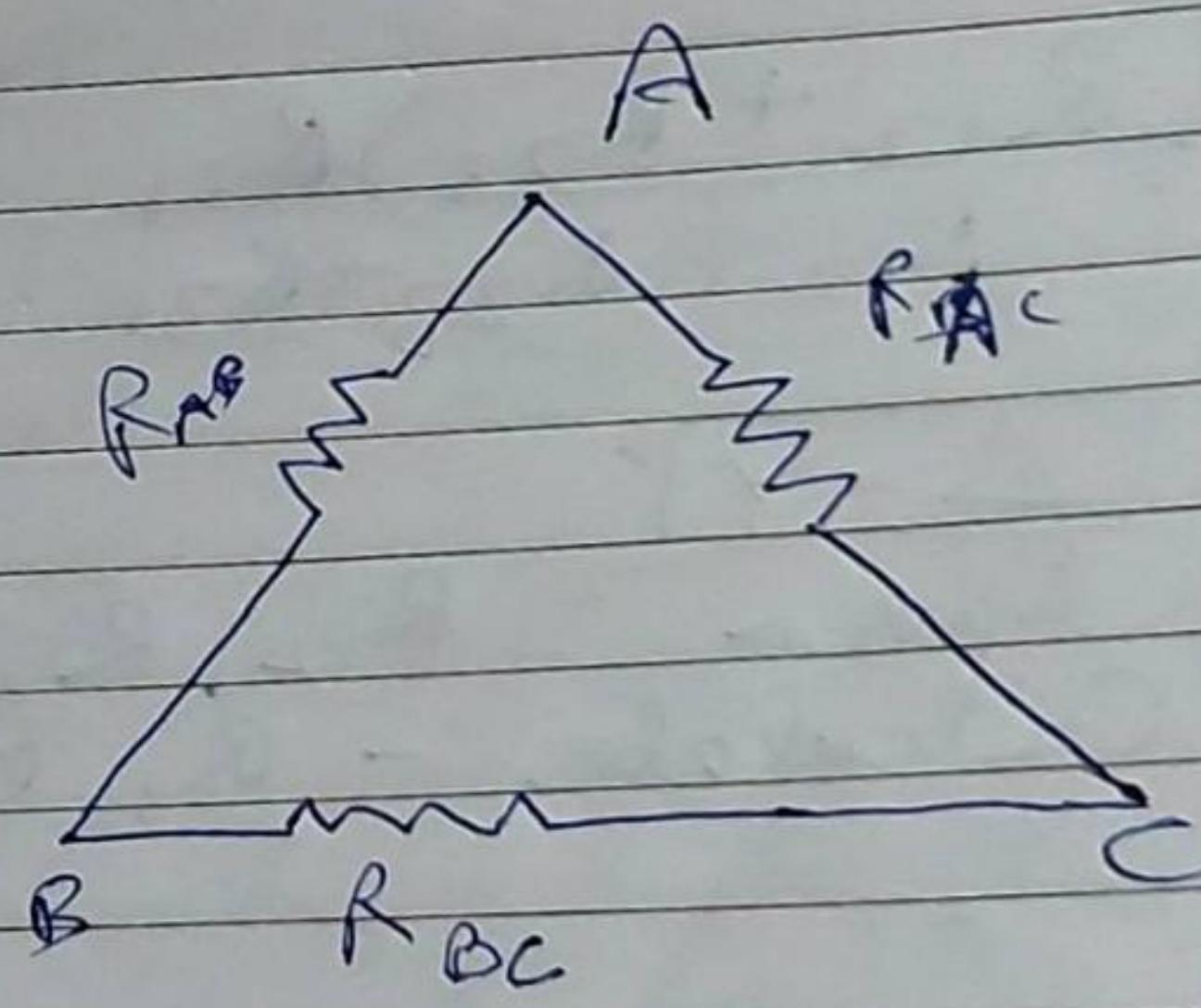
& calculate the equivalent resistance between points A & B



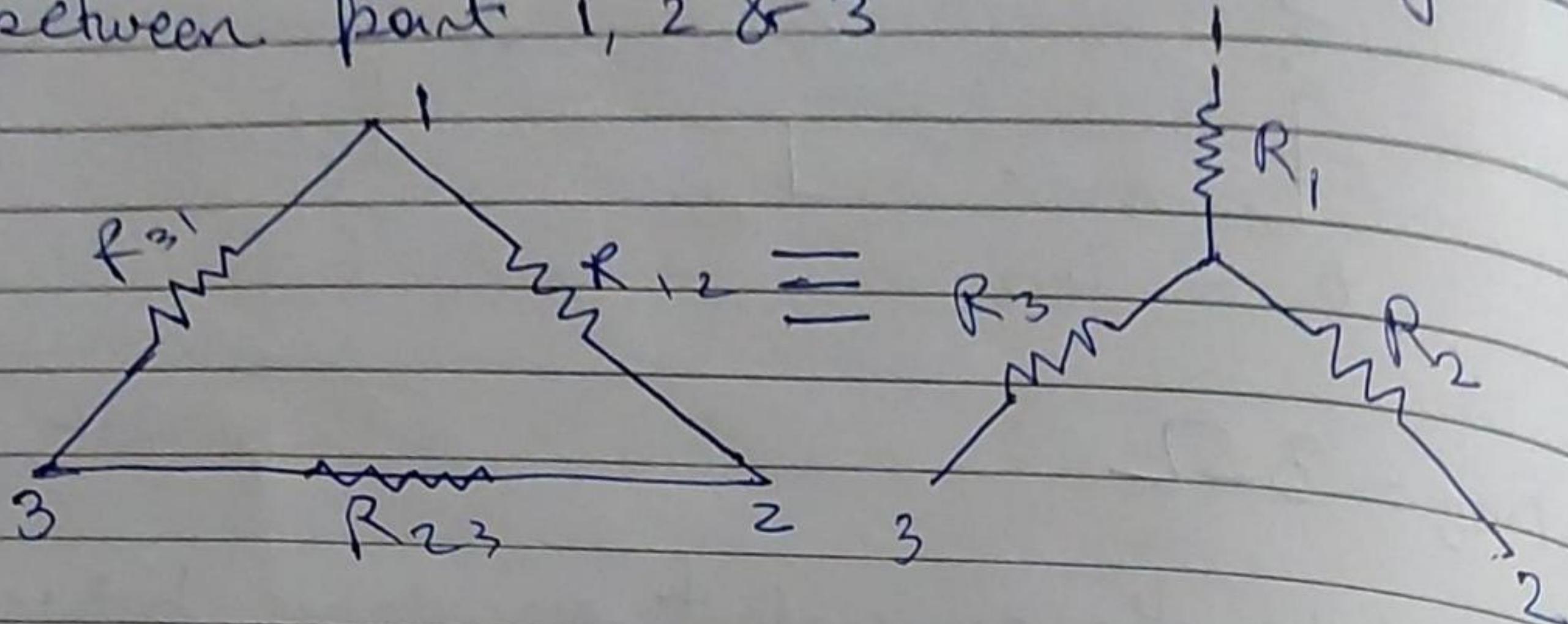
Star-delta transformation → The replacement of delta by equivalent star system is known as Delta-star transformation and vice-versa.



----->



Let us consider a delta-connected system between point 1, 2 & 3



If two systems are equivalent then equivalent resistance between point 1-2, 2-3 & 3-1 must be equal for both the system

$$\boxed{1-2} \quad (R_{23} + R_{31}) \parallel R_{12} = R_1 + R_2 \quad (1)$$

$$\boxed{2-3} \quad (R_{12} + R_{31}) \parallel R_{23} = R_2 + R_3 \quad (2)$$

$$\boxed{3-1} \quad (R_{12} + R_{23}) \parallel R_{31} = R_3 + R_1 \quad (3)$$

$$\frac{(R_{23} + R_{31}) R_{12}}{R_{12} + R_{23} + R_{31}} = R_1 + R_2 - (1')$$

$$\frac{(R_{12} + R_{31}) R_{23}}{R_{12} + R_{23} + R_{31}} = R_2 + R_3 - (2')$$

$$\frac{(R_{12} + R_{23}) R_{31}}{R_{12} + R_{23} + R_{31}} = R_3 + R_1 - (3')$$

$\Delta - Y'$

Given :-  $R_{12}$ ,  $R_{23}$ ,  $R_{31}$

Calculate :-  $R_1$ ,  $R_2$ ,  $R_3$

$\Delta - Y$

Given -  $R_1, R_2, R_3$   
Calculate -  $R_{12}, R_{23}, R_{31}$

Adding  $\textcircled{1}' + \textcircled{2}' + \textcircled{3}'$

$$2(R_1 + R_2 + R_3) = (R_{23} + R_{31})R_{12} + (R_{12} + R_{31})R_{23} + (R_{12} + R_{23})R_{31}$$

$$R_1 + R_2 + R_3 = \frac{(R_{23} + R_{31})R_{12} + (R_{12} + R_{31})R_{23} + (R_{12} + R_{23})R_{31}}{2(R_{12} + R_{23} + R_{31})} \quad \textcircled{4}$$

For  $R_1$ , subtract Eq  $\textcircled{2}'$  from  $\textcircled{4}$

$$R_1 = \frac{(R_{23} + R_{31})R_{12} + (R_{12} + R_{31})R_{23} + (R_{12} + R_{23})R_{31} - 2(R_{12} + R_{31})R_{23}}{2(R_{12} + R_{23} + R_{31})}$$

$$= \frac{(R_{23} + R_{31})R_{12} + (R_{12} + R_{23})R_{31} - (R_{12} + R_{31})R_{23}}{2(R_{12} + R_{23} + R_{31})} \quad \textcircled{5}$$

$$R_1 = \frac{R_{12}R_{31} + R_{12}R_{31}}{2(R_{12} + R_{23} + R_{31})} \quad \textcircled{6}$$

likewise  $R_2 = \frac{R_{23}R_{12}}{R_{12} + R_{23} + R_{31}}$  &  $R_3 = \frac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$   $\textcircled{7}$

Rearranging Eq  $\textcircled{4}$  we get

~~$$R_1 + R_2 + R_3 = \frac{R_{12}R_{23} + R_{12}R_{31} + R_{23}R_{31}}{(R_{12} + R_{23} + R_{31})}$$~~

Multiplying  $\textcircled{5} \& \textcircled{6}$ ,  $\textcircled{6} \& \textcircled{7}$ ,  $\textcircled{7} \& \textcircled{5}$  and adding

$$\text{all we get } R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}^2R_{31}R_{23} + R_{23}^2R_{12}R_{31} + R_{31}^2R_{12}R_{23}}{(R_{12} + R_{23} + R_{31})^2}$$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12} R_{23} R_{31}}{(R_{12} + R_{23} + R_{31})^2} (R_{12} + R_{23} + R_{31})$$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12} R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} - \textcircled{8}$$

Dividing  $\textcircled{8}$  by  $\textcircled{7}$  we get

$$\frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{R_{12} R_{23} R_{31}}{R_{12} R_{23}} - \frac{R_{12} R_{23}}{R_{23}}$$

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Likewise

$$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_1 = \frac{R_{12} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{23} \cdot R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{31} \cdot R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

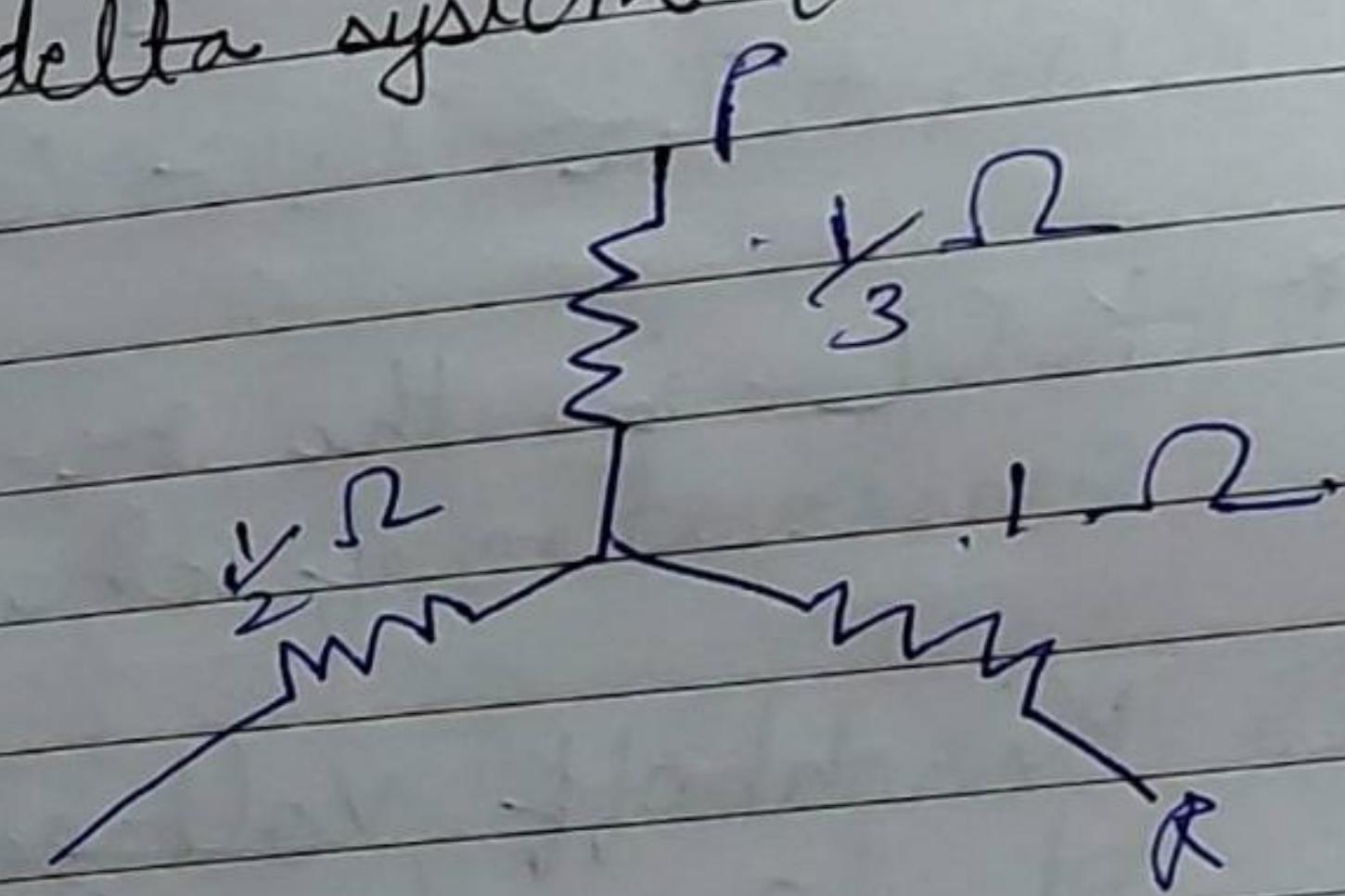
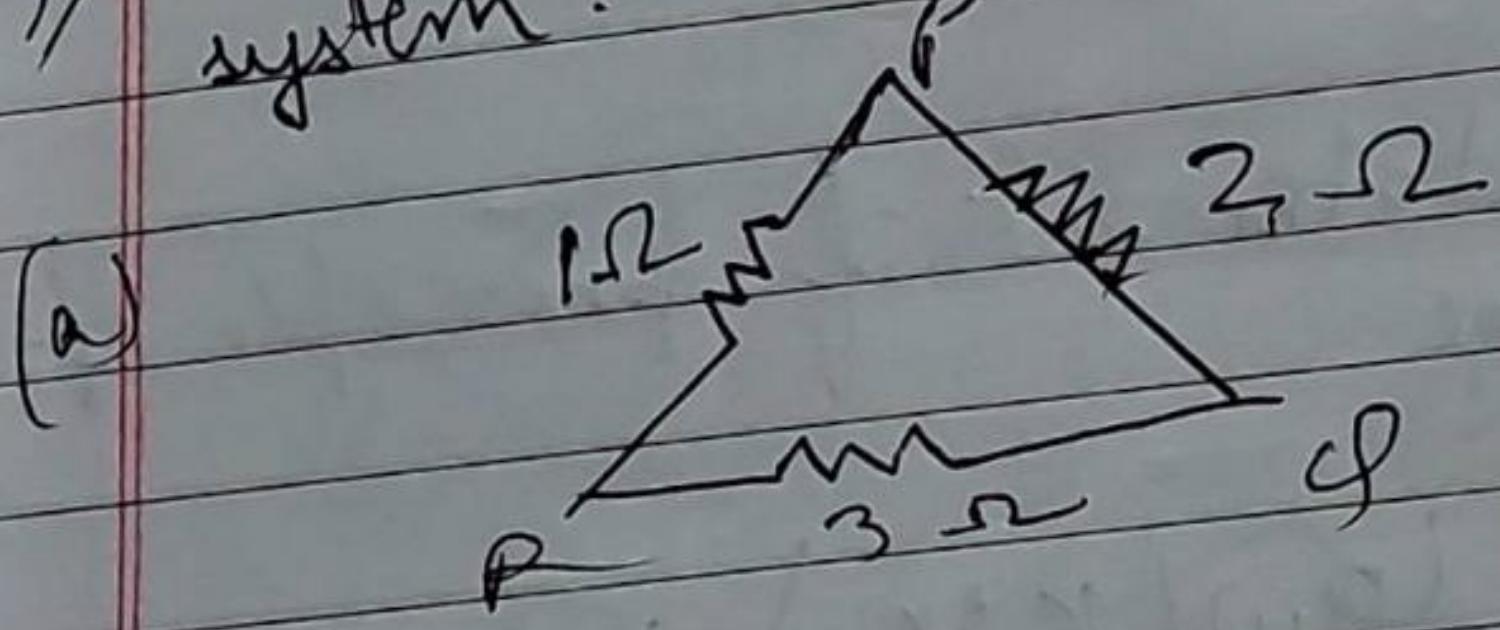
$$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

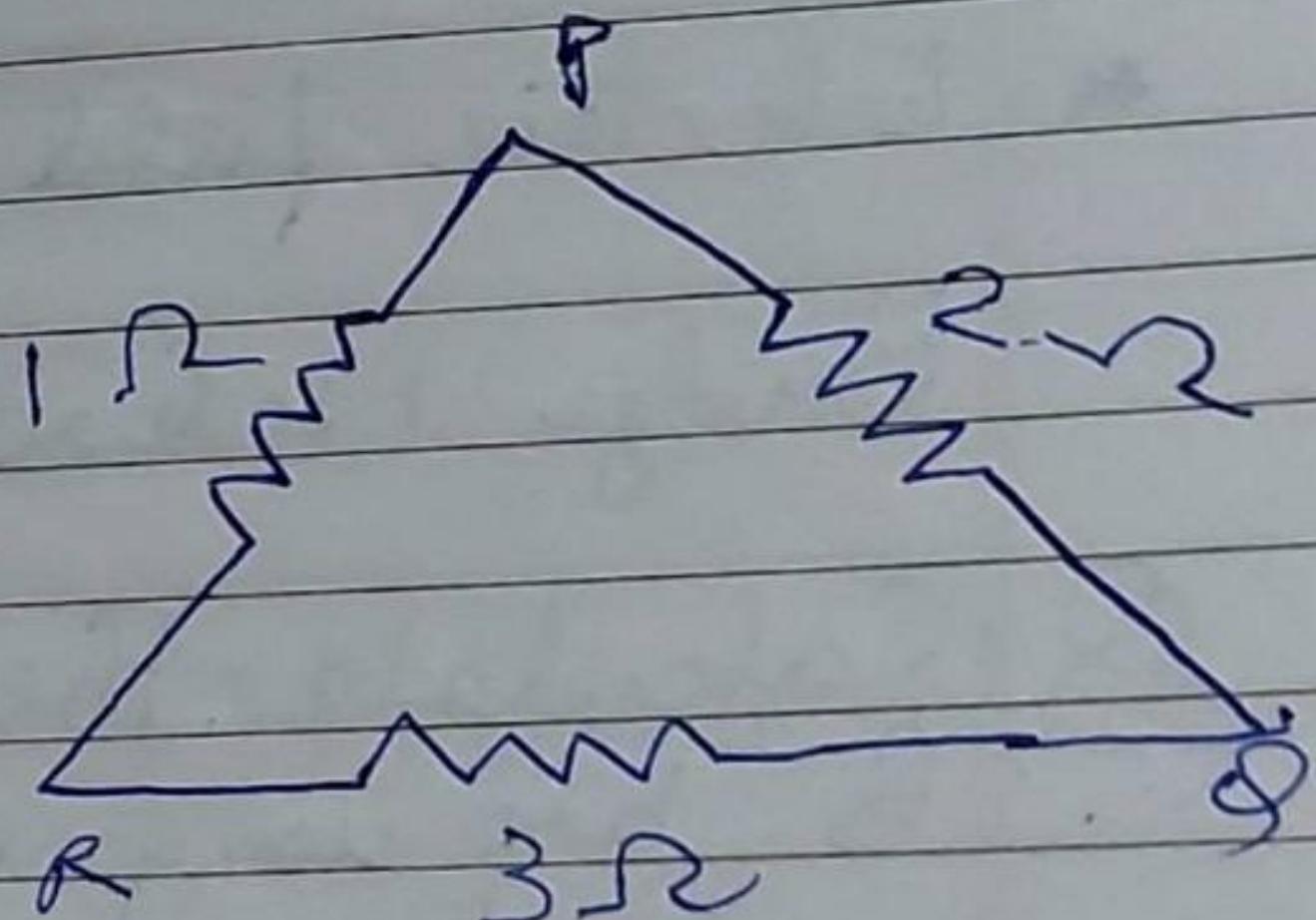
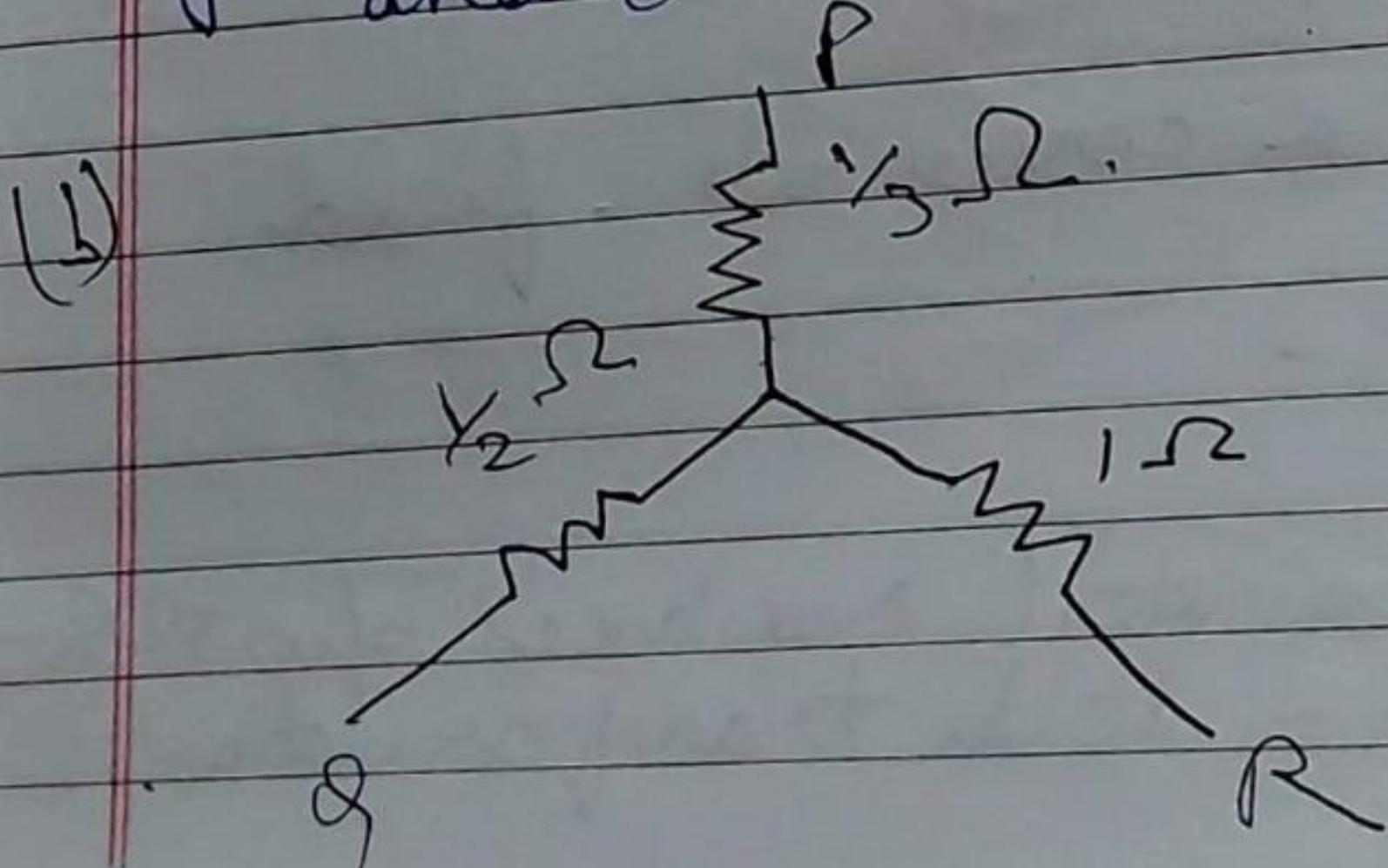
} Δ - Y

} Y - Δ

Q) Transform the given delta system into star system.



For point P, Multiply 2 resistances connected to P and divide it by ~~sum~~ summation of all

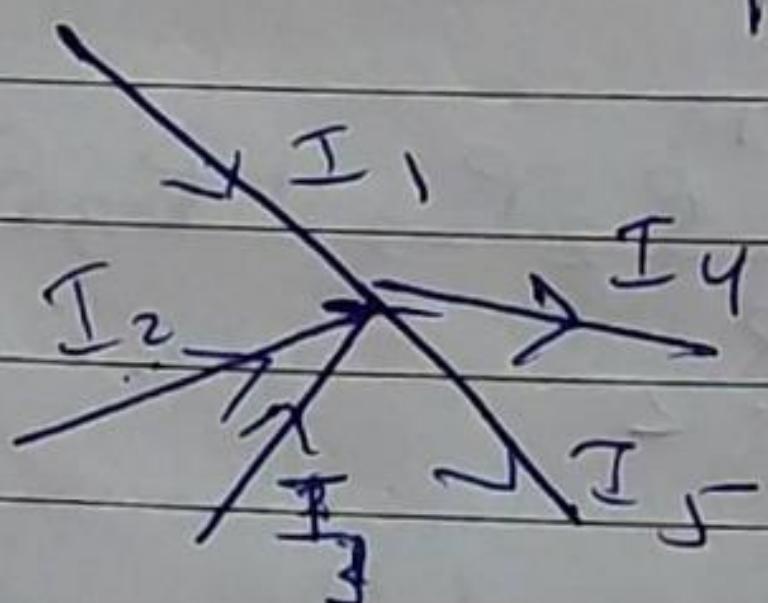


$$\left(\frac{1}{3}\right)(1) + \frac{1}{2}(1) + \frac{1}{3} \times \frac{1}{2} = \frac{2+3+6}{3} = 2\Omega$$

### Kirchhoff's Law

There are two laws :-

- ① Kirchhoff current law (KCL) → Sum of the current entering and leaving a junction point at any instant is equal to zero.



$$I_1 + I_2 + I_3 = I_4 + I_5$$

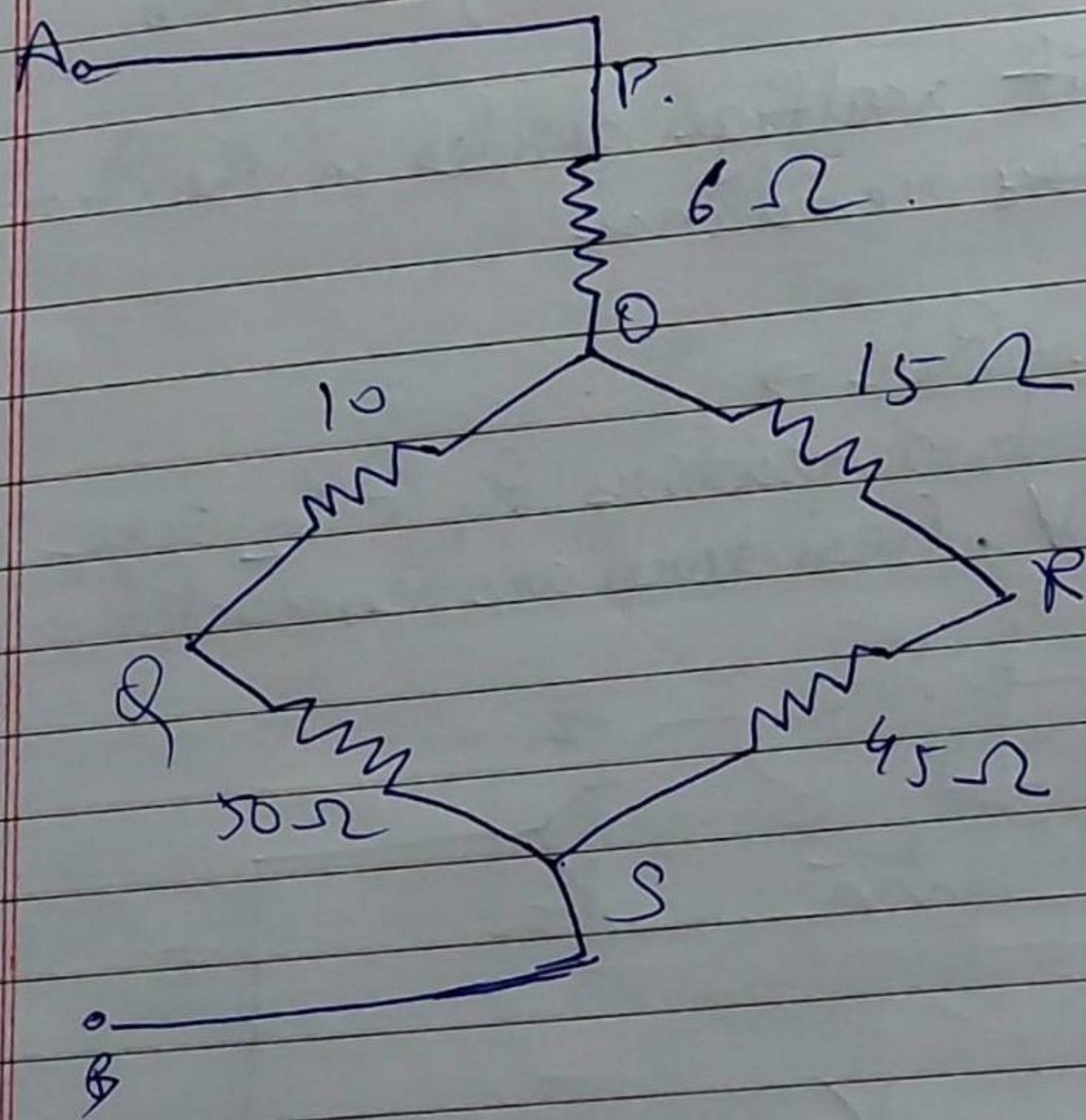
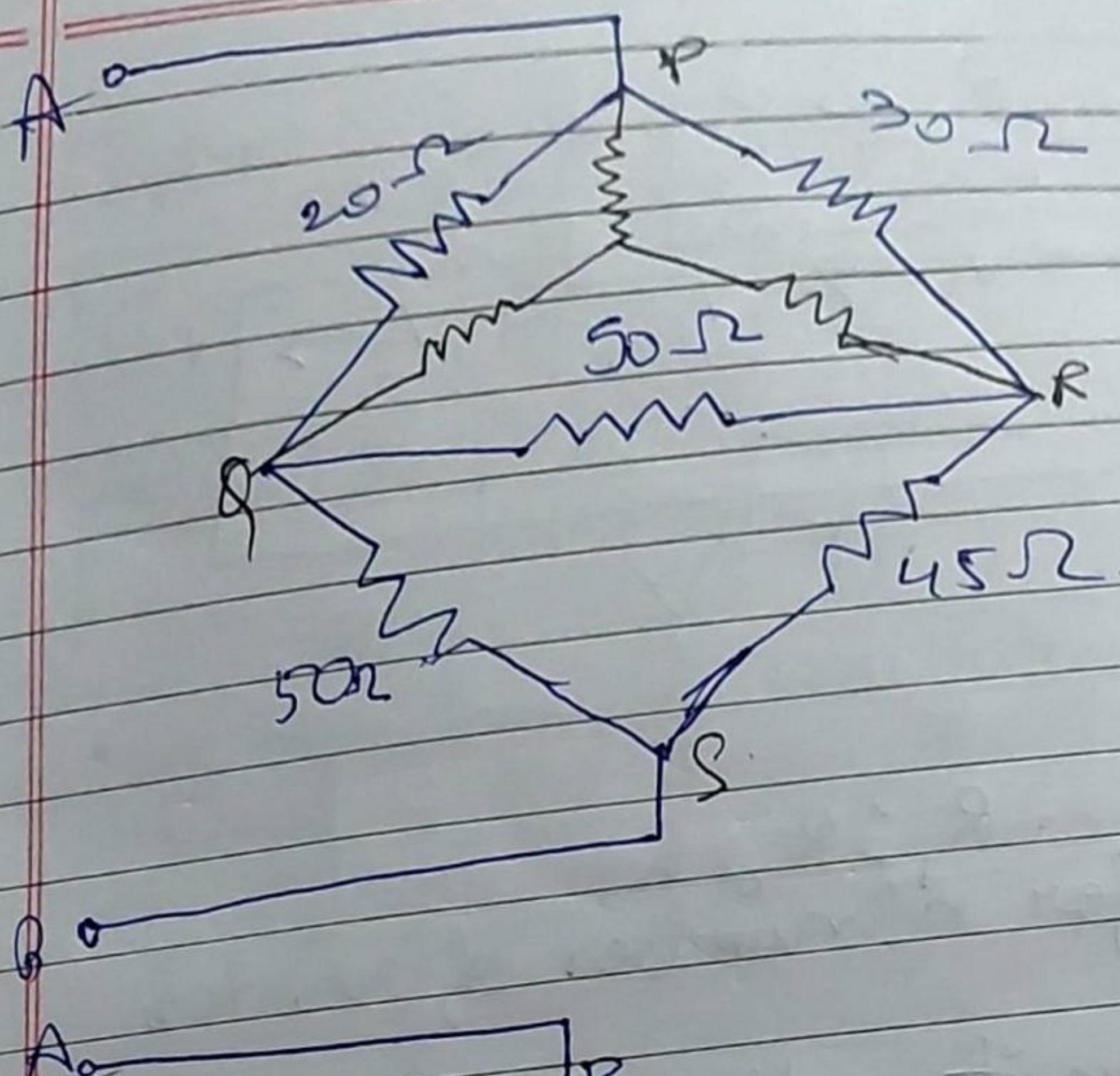
- \* KCL is the re-statement of principle of conservation of charge

- ② Kirchhoff Voltage Law (KVL) → Re-statement of law of conservation of energy.  
At any instant of time the sum of voltage in a closed circuit is zero.

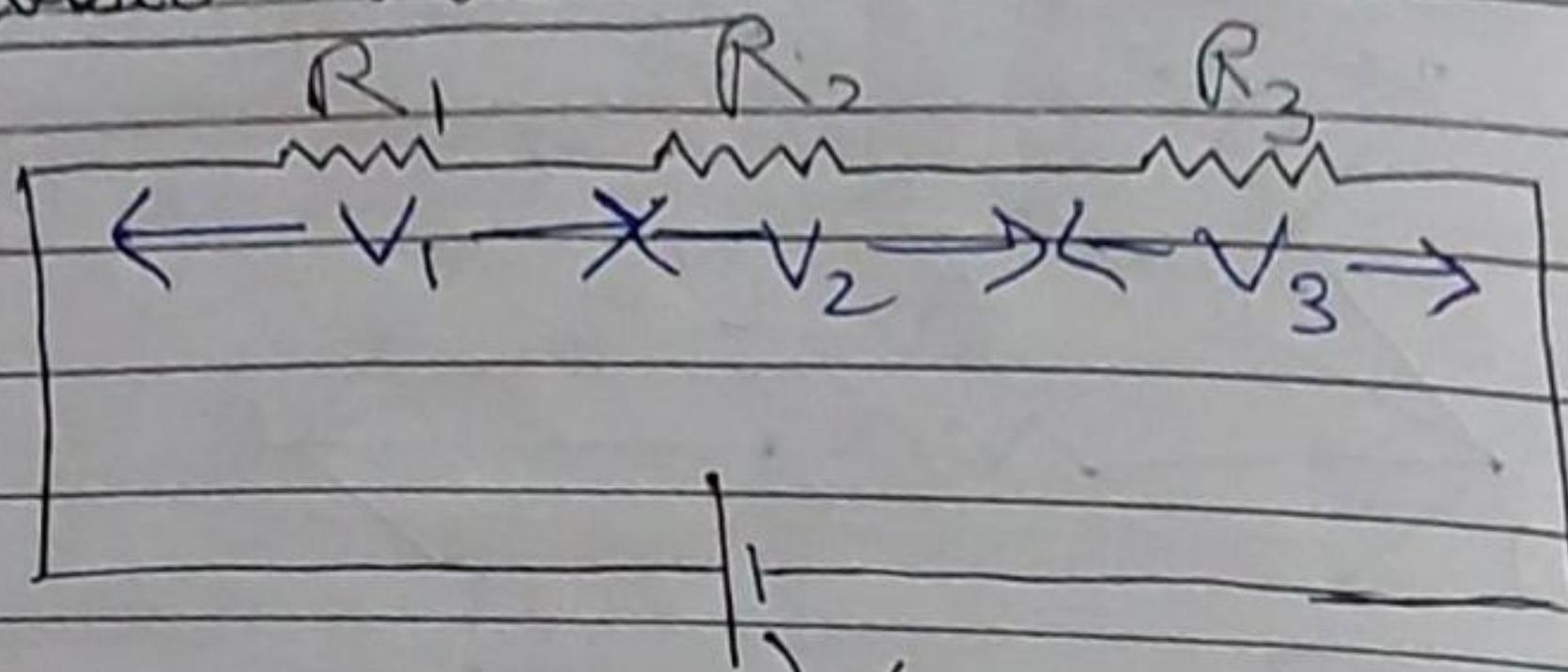
- \* Electric field is a conservative field.

### Bridge Network :-

- Q Calculate the equivalent resistance b/w A & B by using star - delta transformation



Voltage Division Rule →



$$V_1 = IR_1$$

$$V_2 = IR_2$$

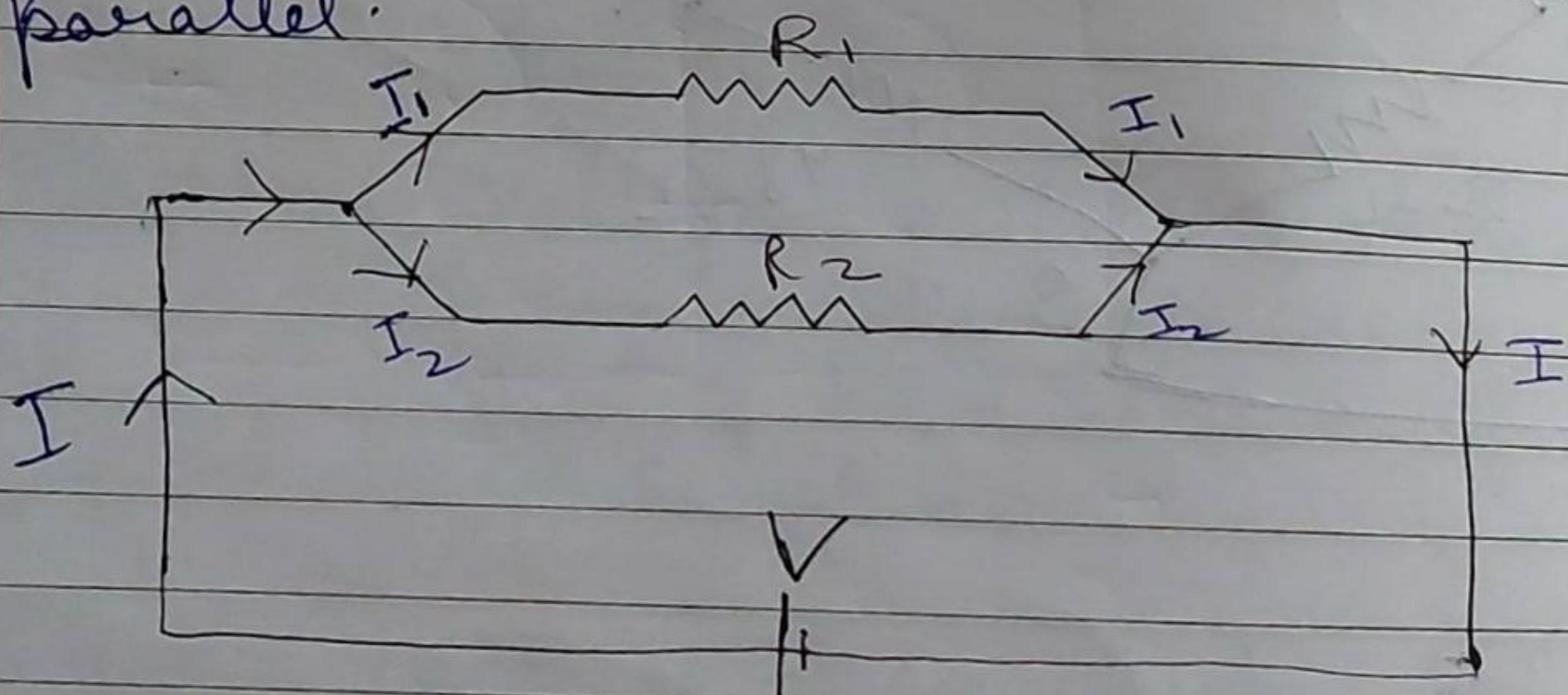
$$V_3 = IR_3$$

\*  $V_1 : V_2 : V_3 \therefore R_1 : R_2 : R_3$   
It is valid for  $n$  number of resistors

In a series circuit voltage divides in the ratio of their ~~resistances~~ resistance

Current Division Rule →

Let us consider two resistors  $R_1$  &  $R_2$  across a voltage source  $V$ . Resistors are connected in parallel.



$$V = I_1 R_1 \quad - \textcircled{1}$$

$$V = I_2 R_2 \quad - \textcircled{2}$$

$$V = I R_{\text{eq}} \quad - \textcircled{3}$$

$$V = I \left( \frac{R_1 R_2}{R_1 + R_2} \right) \quad - \textcircled{3}$$

$$I_1 R_1 = I_2 R_2 = I \left( \frac{R_1 R_2}{R_1 + R_2} \right)$$

$$I_1 = I \left[ \frac{R_2}{R_1 + R_2} \right]$$

For more than 2 resistors result can be obtained.

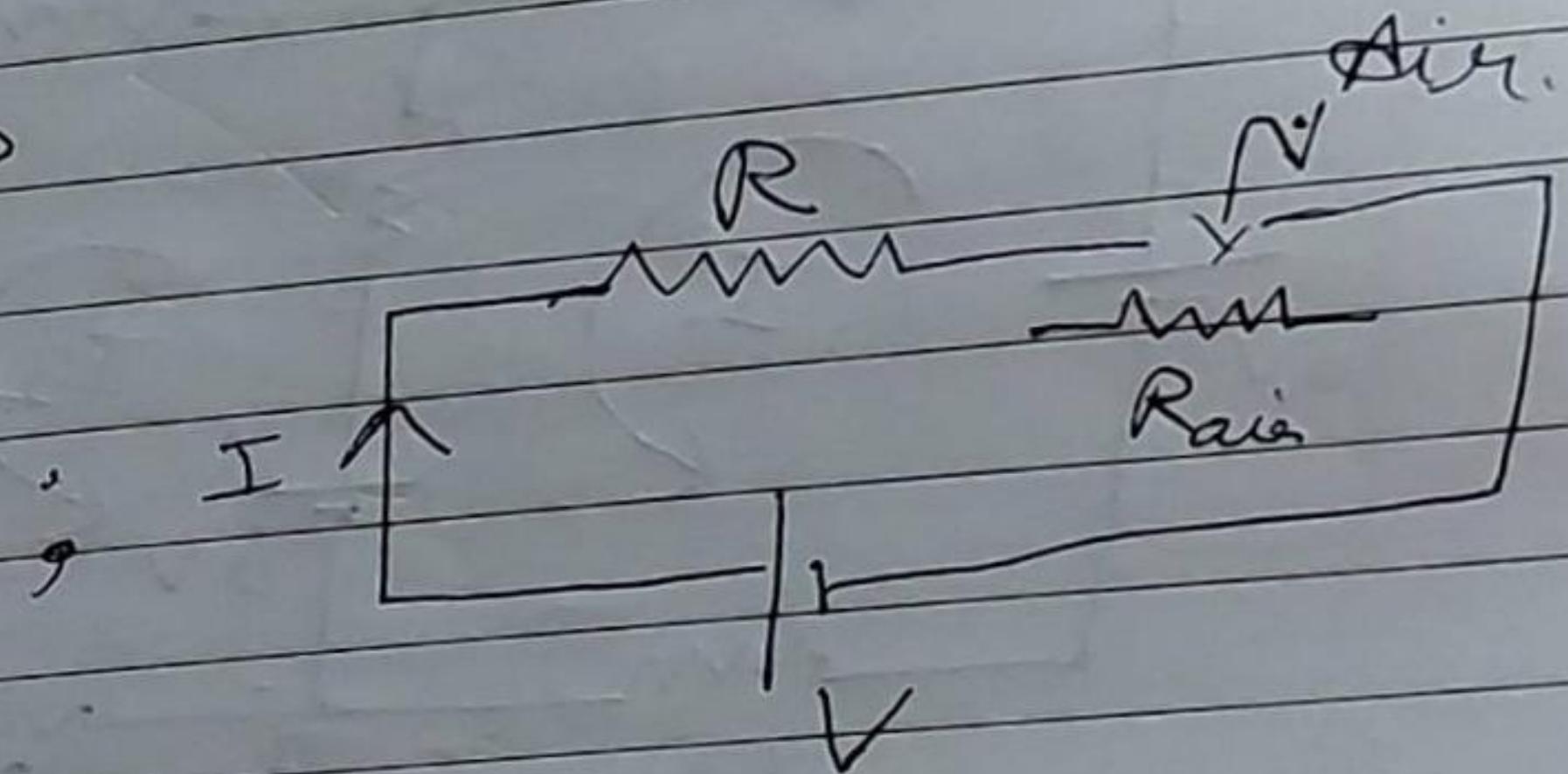
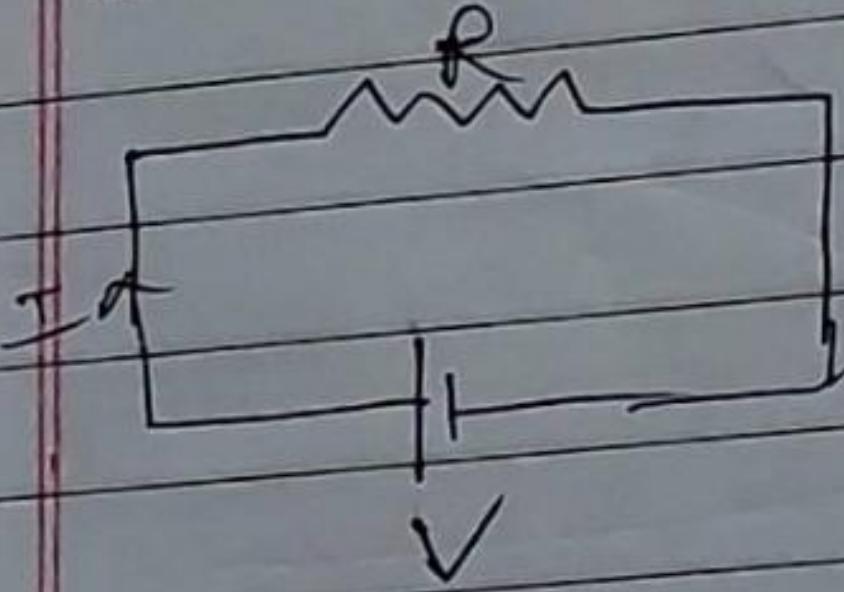
$$\text{If } R_2 = 0, \quad I_1 = 0 \\ (\text{short}) \quad I_2 = I$$

$$R_2 = \infty$$

$$I_1 = I \left( \frac{1}{\left( \frac{R_1}{R_2} + 1 \right)} \right) - I$$

$$I_2 = 0$$

Circuit and Air  $\rightarrow$



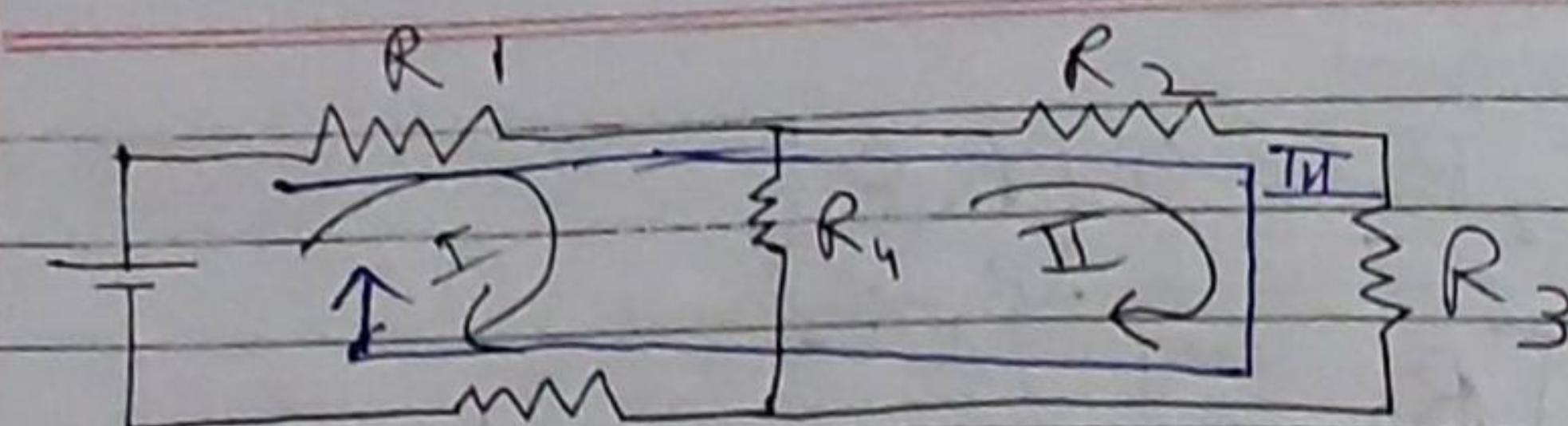
$$I = \frac{V}{R}$$

$$I = \frac{V}{R + R_{air}} \rightarrow 0$$

Mesh Analysis:-

mesh  $\rightarrow$  Mesh does not contain any closed path inside it.

Loop can contain closed path inside it.



(I) & (II) Mesh Loop  
(III)

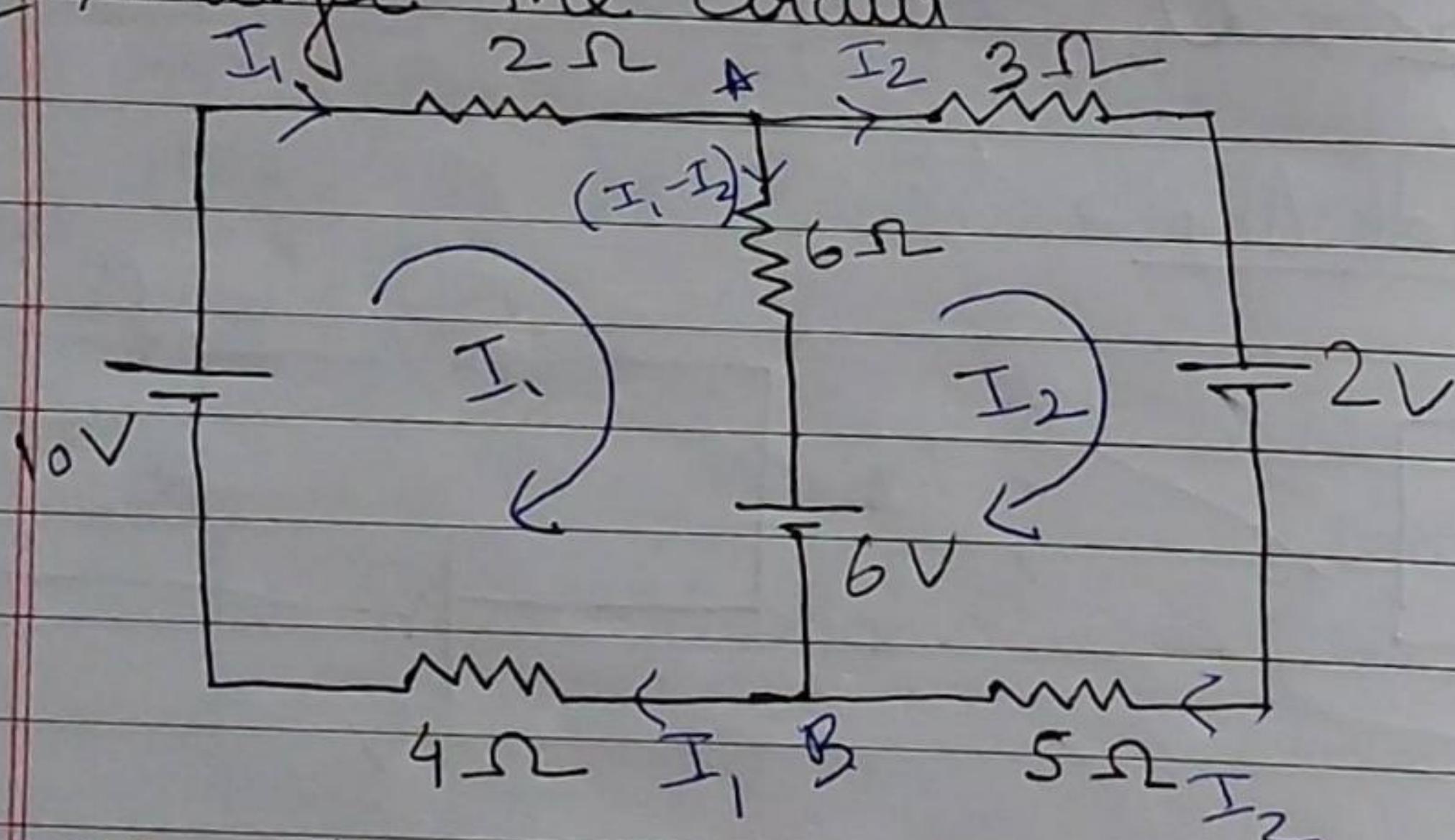
Given

Calculate

Circuit }  
I/O Analysis → V, I  
(o/p)

Mech analysis is valid only for planar circuit network

Q) Analyse the circuit



- 1) Identify the number of mesh.
- 2) Draw the mesh current
- 3) Draw the current in each and every branch
- 4) KVL in mesh I

$$2I_1 + 6(I_1 - I_2) + 6 + 4I_1 - 10 = 0$$

$$12I_1 - 6I_2 = 4$$

$$6I_1 - 3I_2 = 2$$

KVL in mesh II:

$$3I_2 + 2 + 5I_2 - 6 - 6(I_1 - I_2) = 0$$

$$14I_2 - 6I_1 = 4$$

$$-6I_2 + 6I_1 = 2$$

$$11I_2 = 6$$

$$I_2 = \frac{6}{11}$$

$$-\frac{18}{11} + 6I_1 = 2$$

$$6I_1 = 2 + \frac{18}{11}$$

$$I_1 = \frac{22 + 18}{11 \times 6}$$

$$I_1 = \frac{40}{66} = \frac{20}{33}$$

$$I_1 = \frac{20}{33} A, I_2 = \frac{6}{11} A$$

$2\Omega$   $\frac{20}{33} A, \frac{40}{33} V$

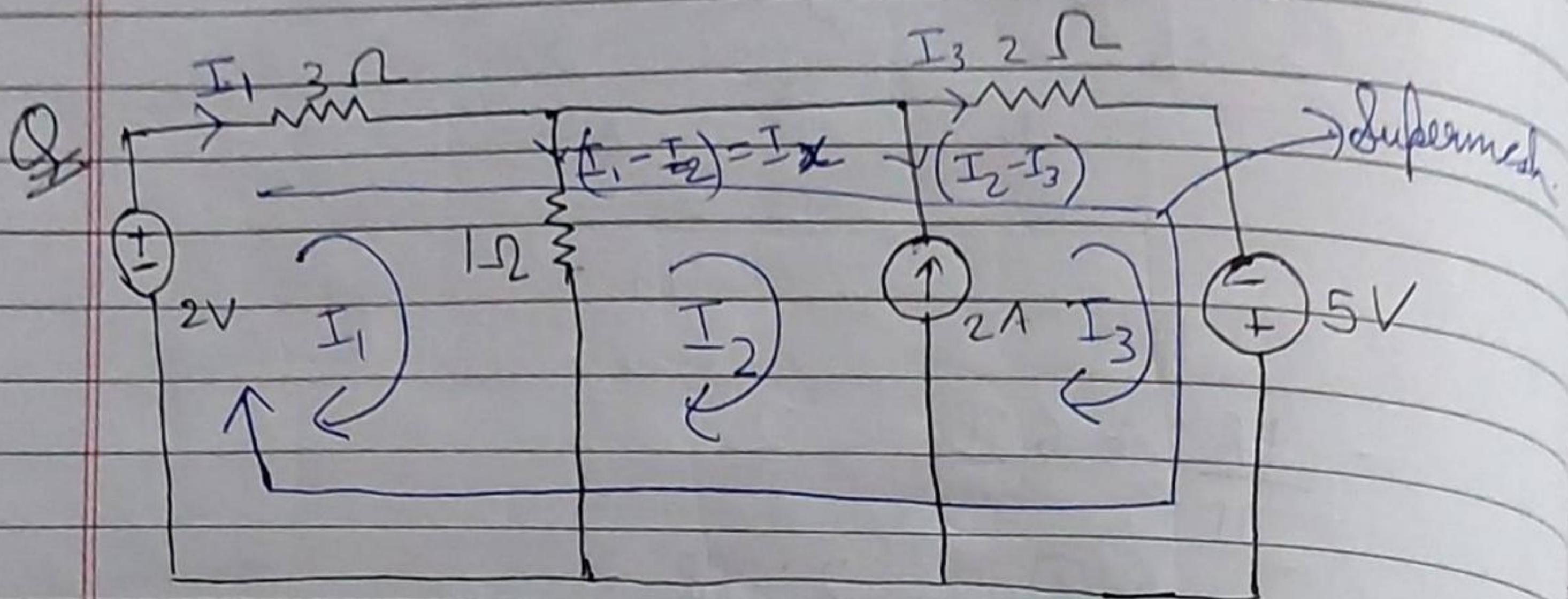
$3\Omega$   $\frac{6}{11} A, \frac{18}{11} V$

$6\Omega$   $\frac{2}{33} A, \frac{12}{33} V$   
(A to B)

$5\Omega$   $\frac{6}{11} A, \frac{30}{11} V$

$4\Omega$   $\frac{20}{33} A, \frac{80}{33} V$

## Super Mesh



$$\text{KVL}(1) \bullet 3I_1 + 1(I_1 - I_2) - 2 = 0 - \textcircled{1}$$

$$\begin{aligned} \text{KVL}(\text{Super-mesh}) \quad & 3I_1 + 2I_3 - 5 - 2 = 0 - \textcircled{2} \\ & 3I_1 + 2I_3 = 7 \end{aligned}$$

$$I_2 - I_3 = -2 - \textcircled{3}$$

$$\begin{aligned} \bullet 4I_1 - I_2 - 2 &= 0 \\ I_2 &= 4I_1 - 2 \end{aligned}$$

$$4I_1 - 2 - I_3 = -2$$

~~$$4I_1 - I_3 = -2$$~~

$$8I_1 - 2I_3 = 0$$

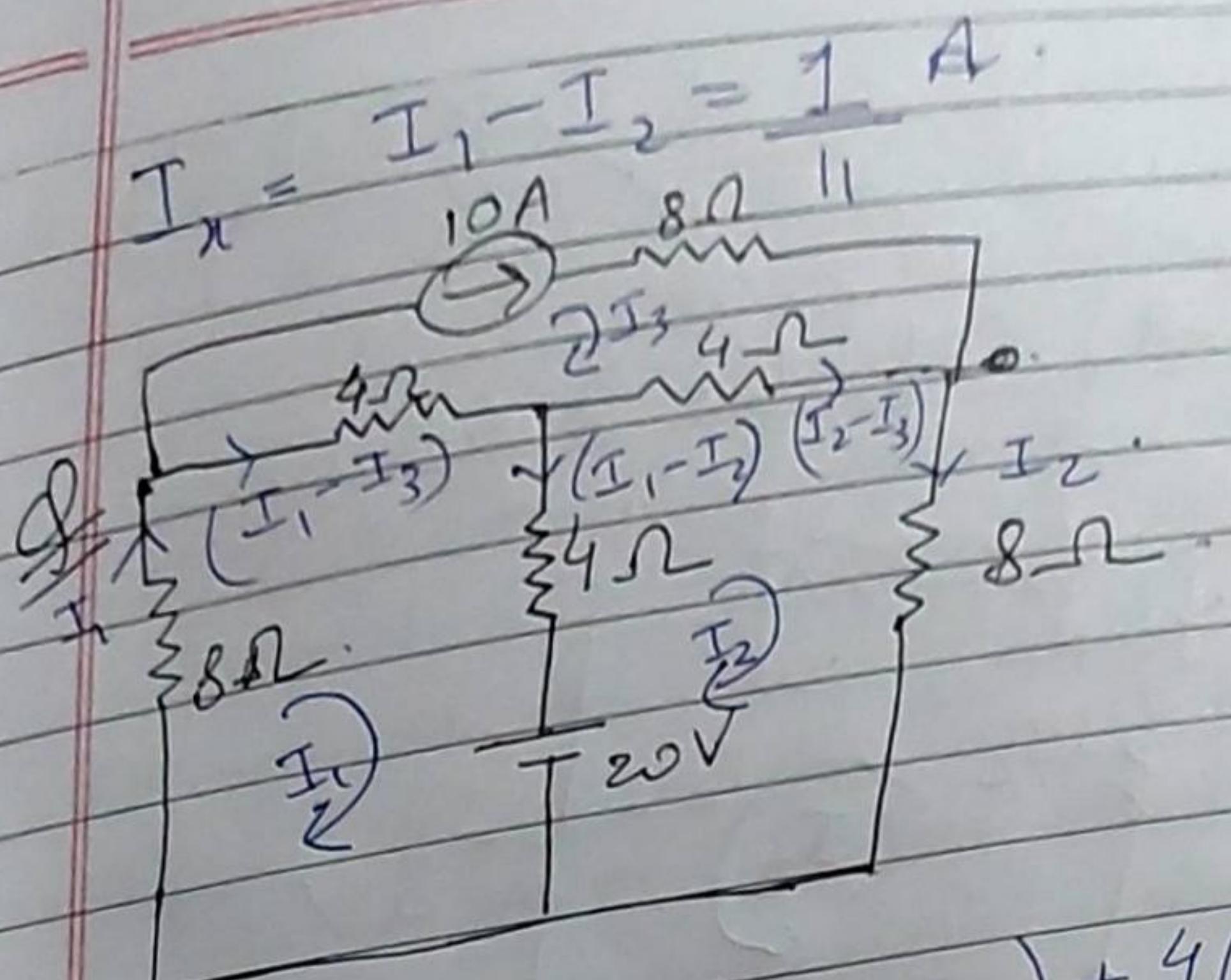
$$3I_1 + 2I_3 = 7$$

$$11I_1 = 7$$

$$I_1 = \frac{7}{11} \text{ A}$$

$$I_2 = \frac{6}{11} \text{ A}$$

$$I_3 = \frac{28}{11} \text{ A}$$



$$I_3 = 10 \text{ A}$$

KVL (Mesh I)  $\rightarrow 4(I_1 - I_2) + 4(I_1 - I_3) + 20 + 8I_1 = 0$

$$16I_1 - 4I_2 - 4I_3 = -20$$

$$4I_1 - I_2 = 5$$

KVL (Mesh II)  $\rightarrow 4(I_2 - I_3) + 8I_2 - 20 - 4(I_1 - I_2) = 0$

$$16I_2 - 4I_1 = 60$$

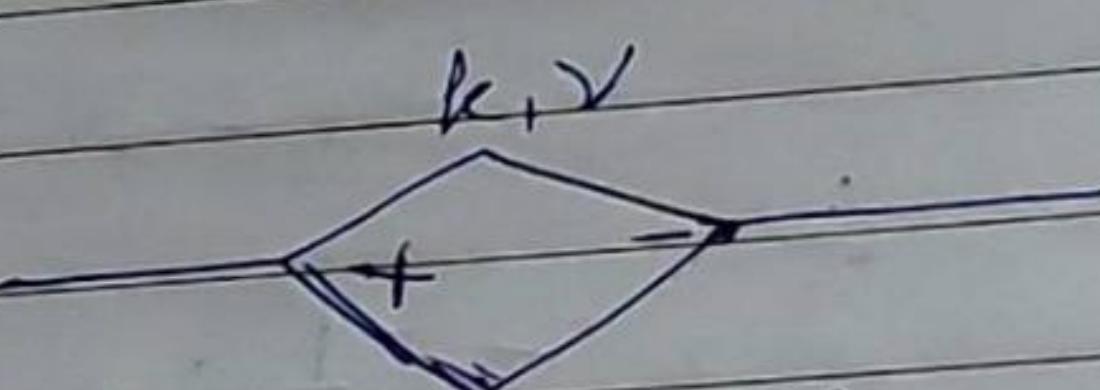
$$4I_2 - I_1 = 15$$

Solving both eq<sup>n</sup> get  $I_1$  &  $I_2$

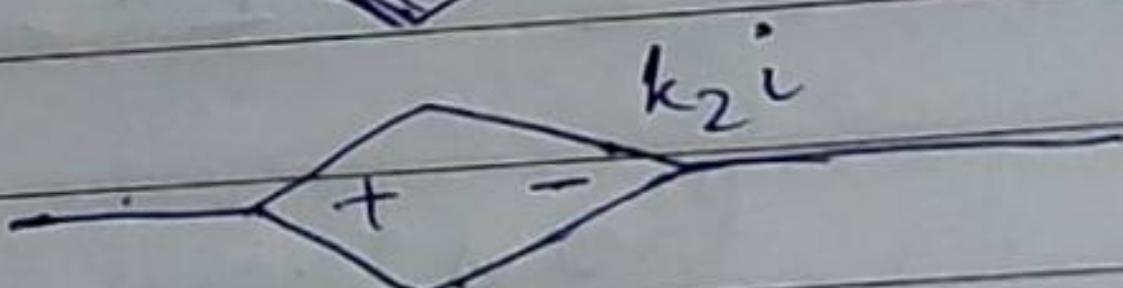
Dependent / Controlled Source  $\rightarrow$

These are basically operational amplifiers.  
These are of four types

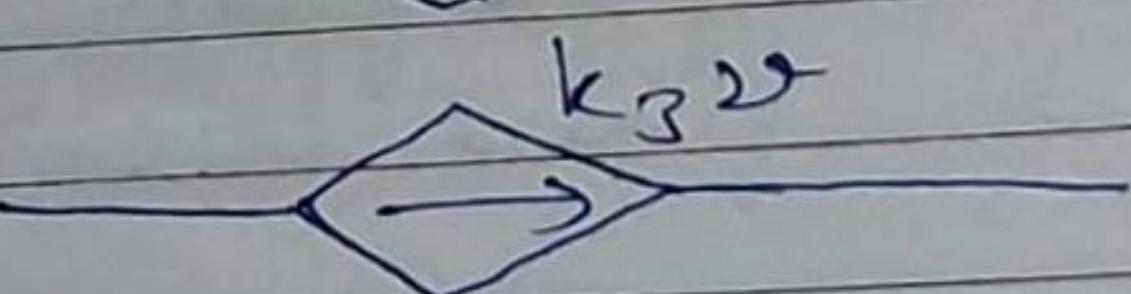
Voltage Controlled Voltage Source (VCVS)



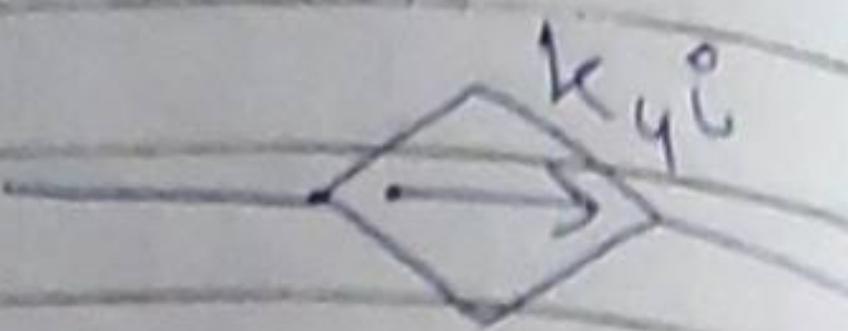
Current Controlled Voltage Source (CCVS)



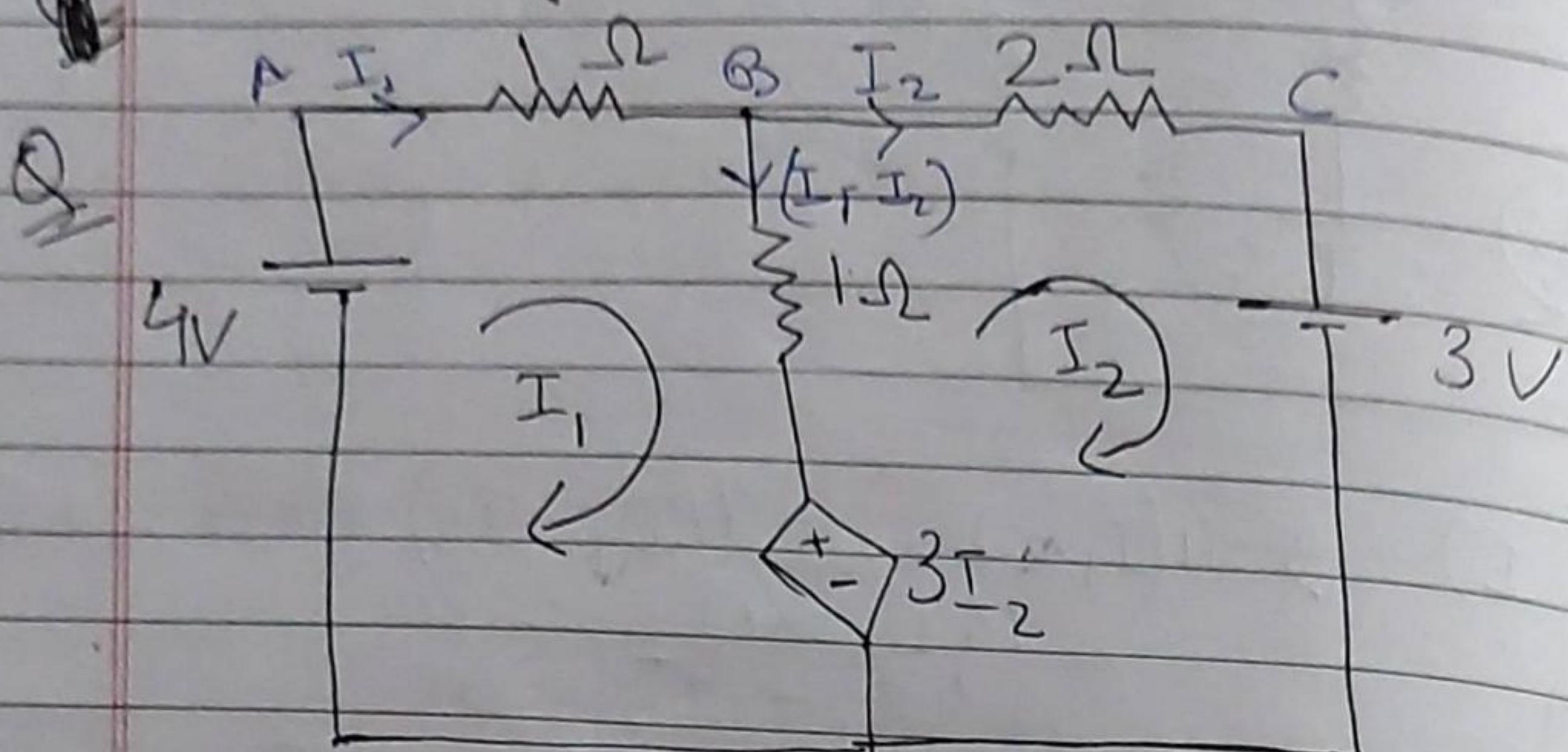
Voltage Controlled Current Source (VCCS)



## Current Controlled Current Source (CCCS)



Mesh Analysis with Dependent Source.



$$\text{I Mesh } I_1 + (I_1 - I_2) + 3I_2 - 4 = 0$$

$$\text{II Mesh } 2I_2 + 3 - 3I_2 - (I_1 - I_2) - 0$$

$$2I_1 + 2I_2 - 4 = 0$$

$$I_1 + I_2 = 2 \quad \textcircled{1}$$

$$2I_2 + 3 - 3I_2 - I_1 + I_2 = 0$$
~~3A~~

$$3 - I_1 = 0$$

$$I_1 = 3$$

$$I_2 = -1$$

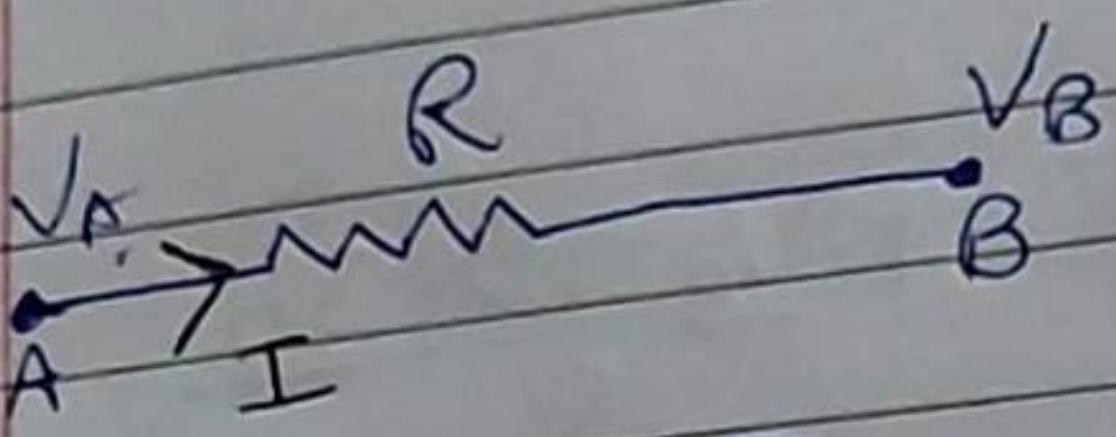
$\therefore I_2$  flows from C to B  
 $I_1$  flows from A to B

## Nodal Analysis $\rightarrow$

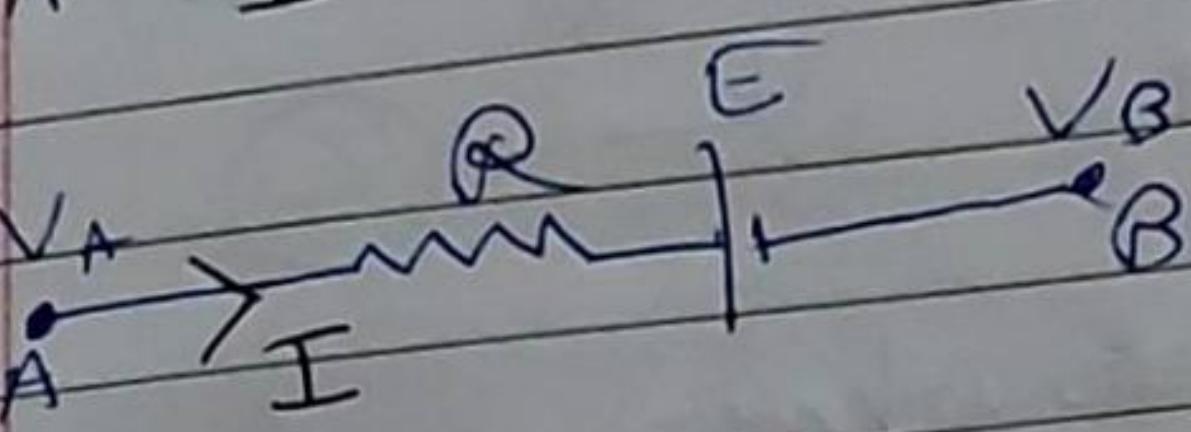
It is based on Kirchhoff current Law.

One of the node is taken as reference at zero on datum node and the potential difference between each of the other nodes and the reference node is expressed in terms of an unknown voltage and at every node KCL is applied.

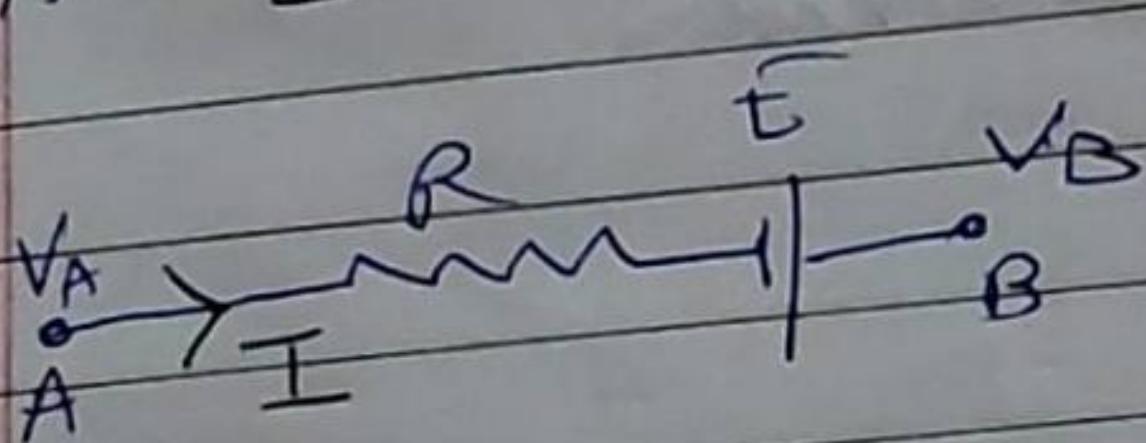
If there are  $n$  number of nodes then there will be  $(n-1)$  number of nodal equations in term of  $(n-1)$  number of unknown variable of nodal voltage.



$$I = \frac{V_A - V_B}{R}$$

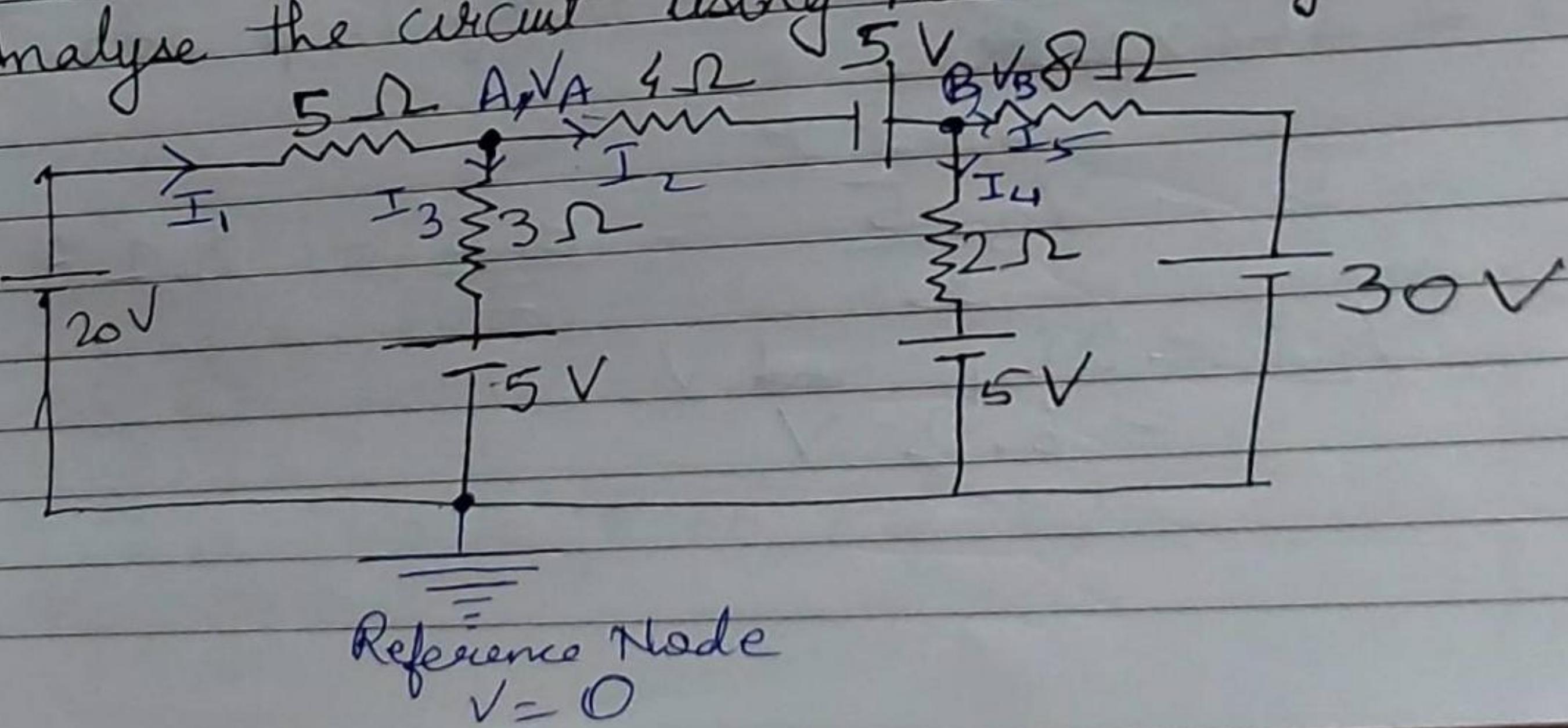


$$I = \frac{V_A - V_B - E}{R}$$



$$I = \frac{V_A - V_B + E}{R}$$

Q) Analyse the circuit using nodal analysis -



- 1) Locate reference node
- 2) Identify nodes and name it
- 3) Locate unknown voltage at node
- 4) Draw current in each and every branch
- 5) Apply KCL at node A

$$I_1 = I_2 + I_3$$

$$\frac{0 - V_A + 20}{5} = \frac{V_A - V_B + 5}{4} + \frac{V_A - 0 - 5}{3} - 0$$

- 6) Apply KCL at node B

$$I_2 = I_4 + I_5$$

$$\frac{V_A - V_B + 5}{4} = \frac{V_B - 0 - 5}{2} + \frac{V_B - 0 - 30}{8}$$

②

- (7) There are two eq<sup>n</sup> & two unknown

$$V_A = 9.2 \text{ V}$$

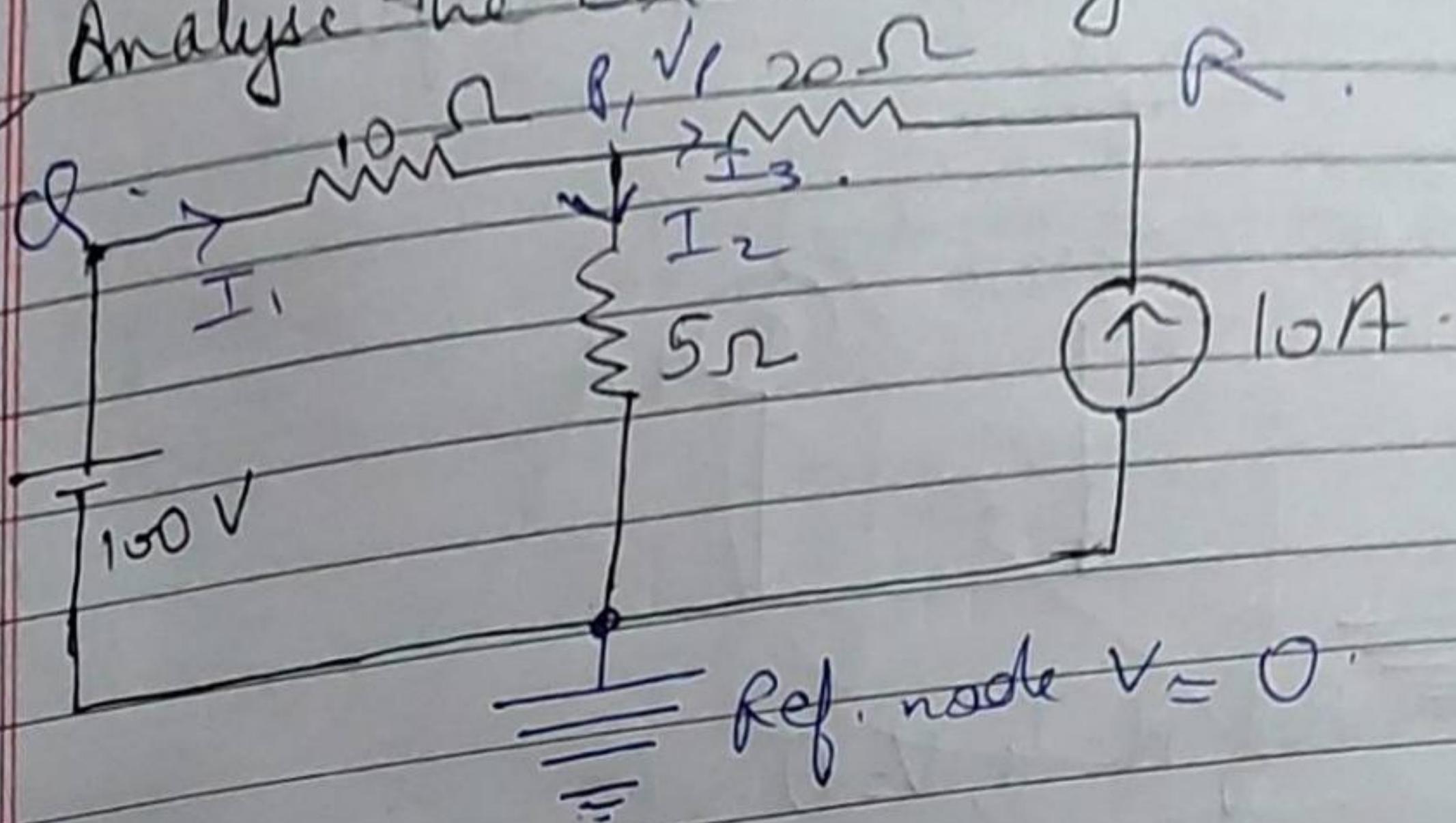
$$V_B = 11.2 \text{ V}$$

- \* Calculate the current & voltage drop across  $2 \Omega$  resistor

$$I_4 = \frac{V_B - 5}{2} = \frac{6.2}{2} = 3.1 \text{ A}$$

$$V_{2\Omega} = I_4 \times 2 \text{ V} \\ = 6.2 \text{ V}$$

Q) Analyse the circuit using nodal analysis.



(R)  $I_1 = I_2 + I_3$

$$\frac{0 - V_p + 100}{10} = \frac{(V_p - 0)}{5} + (-10)$$

$$\frac{100 - V_p}{10} = \frac{V_p}{5} - 10$$

$$\frac{100 - V_p}{2} = V_p - 50$$

$$100 - V_p = 2V_p - 100$$

$$3V_p = 200$$

$$V_p = \frac{200}{3} V$$

(10)  $I_1 = \frac{0 - V_p + 100}{10} = \frac{10}{3} A (\text{Q to P})$

$$V_{10\Omega} = \frac{100}{3} V$$

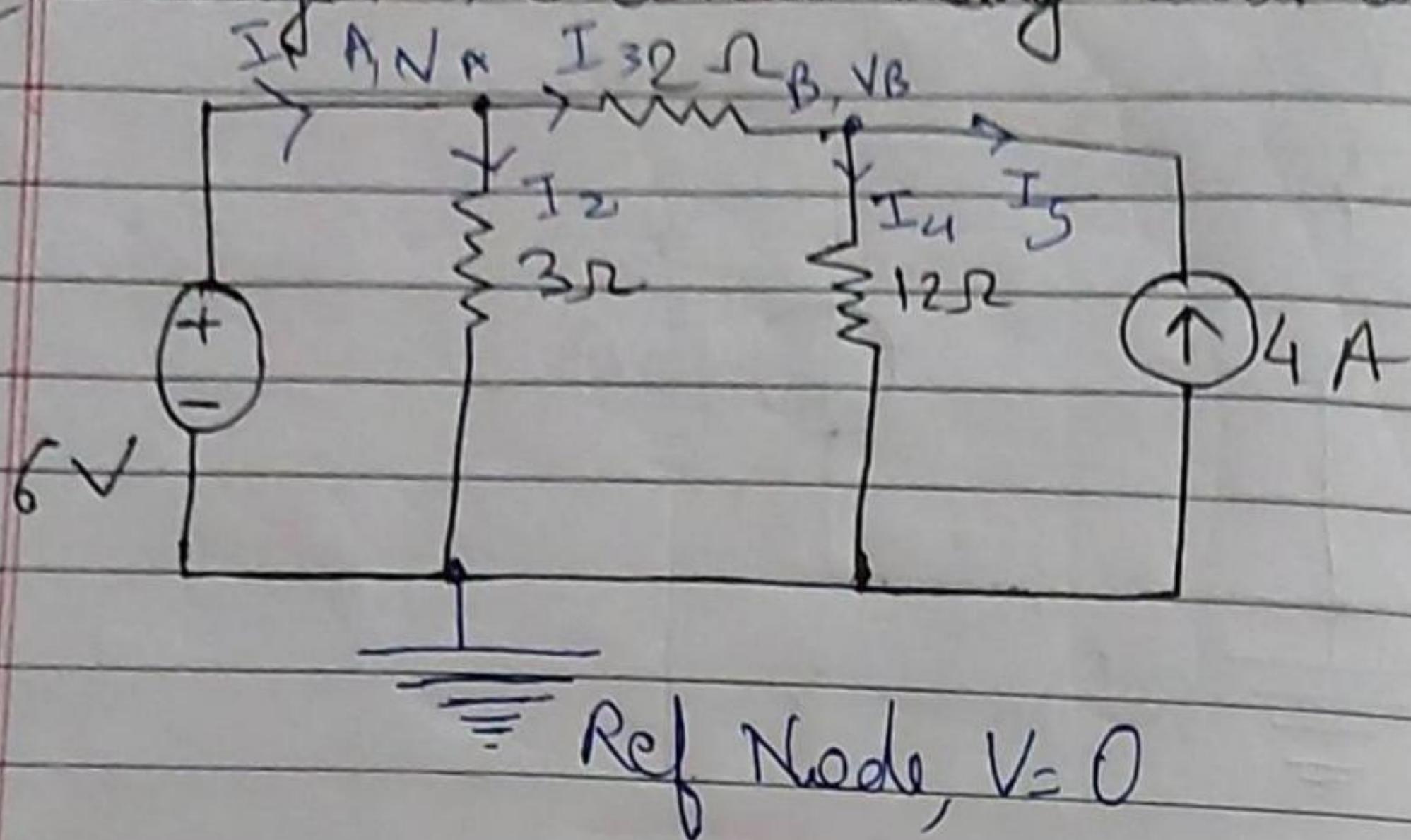
(20)  $I_3 = 10 A (R \text{ to P})$

$$V_{20\Omega} = 200 V$$

(5)  $I_2 = \frac{40}{3} A (P \text{ to Ref. V})$

$$V_{5\Omega} = \frac{200}{3} V$$

Q Analyse the circuit using nodal analysis.



$$V_A = 6V$$

(B)

$$I_3 = I_4 + I_5$$

$$\frac{V_A - V_B}{2} = \frac{V_B - 0 + (-4)}{12}$$

$$\frac{6 - V_B}{2} = \frac{V_B}{12} - 4$$

$$\frac{3 - V_B}{2} = \frac{V_B}{12} - 4$$

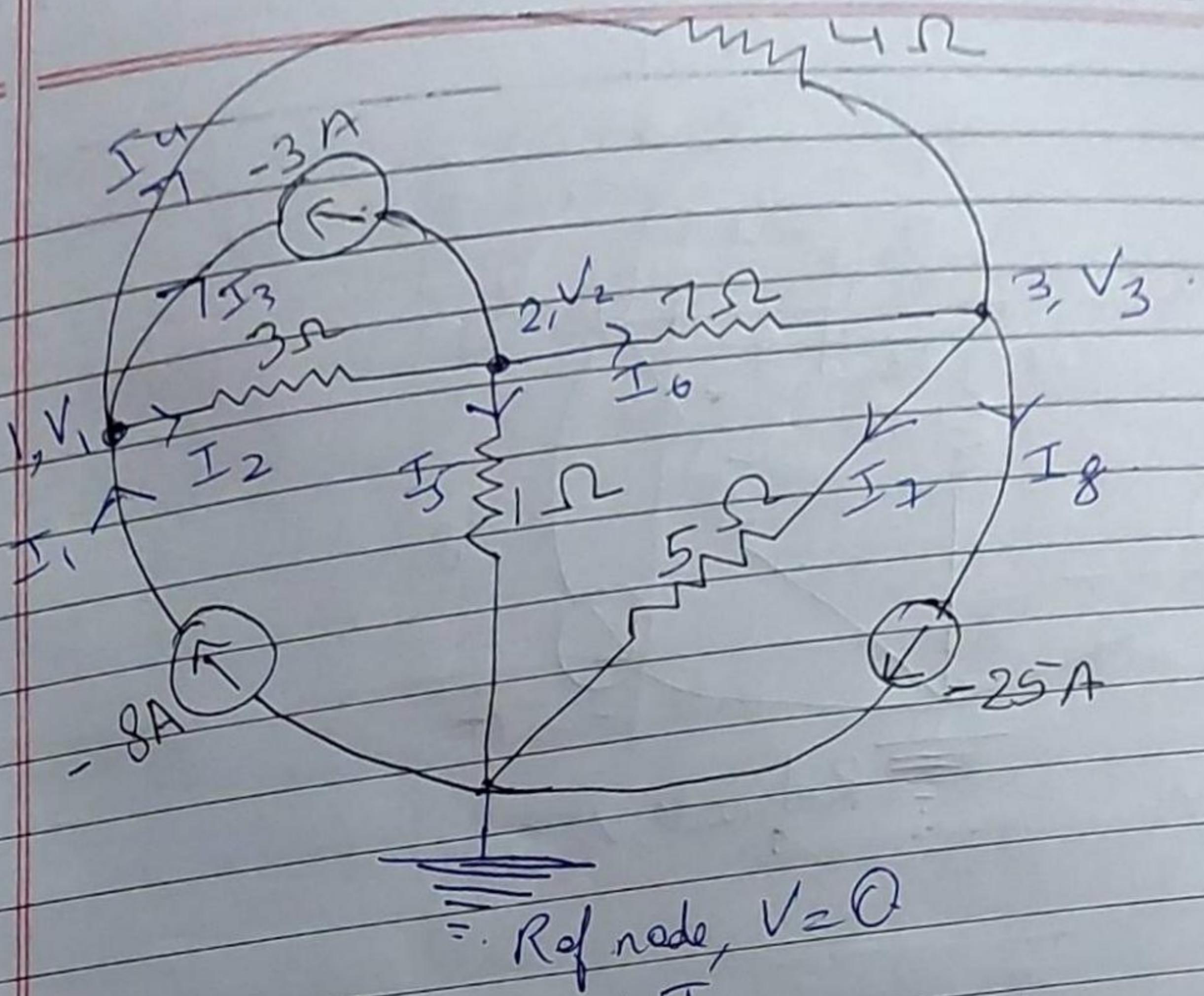
$$\frac{V_B}{12} + \frac{V_B}{2} = 7$$

$$\frac{6V_B + V_B}{12} = 7$$

$$7V_B = 12 \times 7$$

$$V_B = 12V$$

Q Analyse the circuit using nodal analysis



$\therefore R$  of node,  $V = 0$

$$\textcircled{1} \quad I_1 = I_2 + I_3 + I_4$$

$$(-8) = \frac{V_1 - V_2}{3} + 3 + \frac{V_1 - V_3}{4}$$

$$\textcircled{2} \quad I_2 + I_3 = I_5 + I_6$$

$$\frac{V_1 - V_2}{3} + 3 = \frac{V_2 - 0}{1} + \frac{V_2 - V_3}{7}$$

$$\textcircled{3} \quad I_4 + I_6 = I_7 + I_8$$

$$\frac{V_1 - V_3}{4} + \frac{V_2 - V_3}{7} = \frac{V_3 - 0}{5} + (-25)$$

Solving above three equations we get

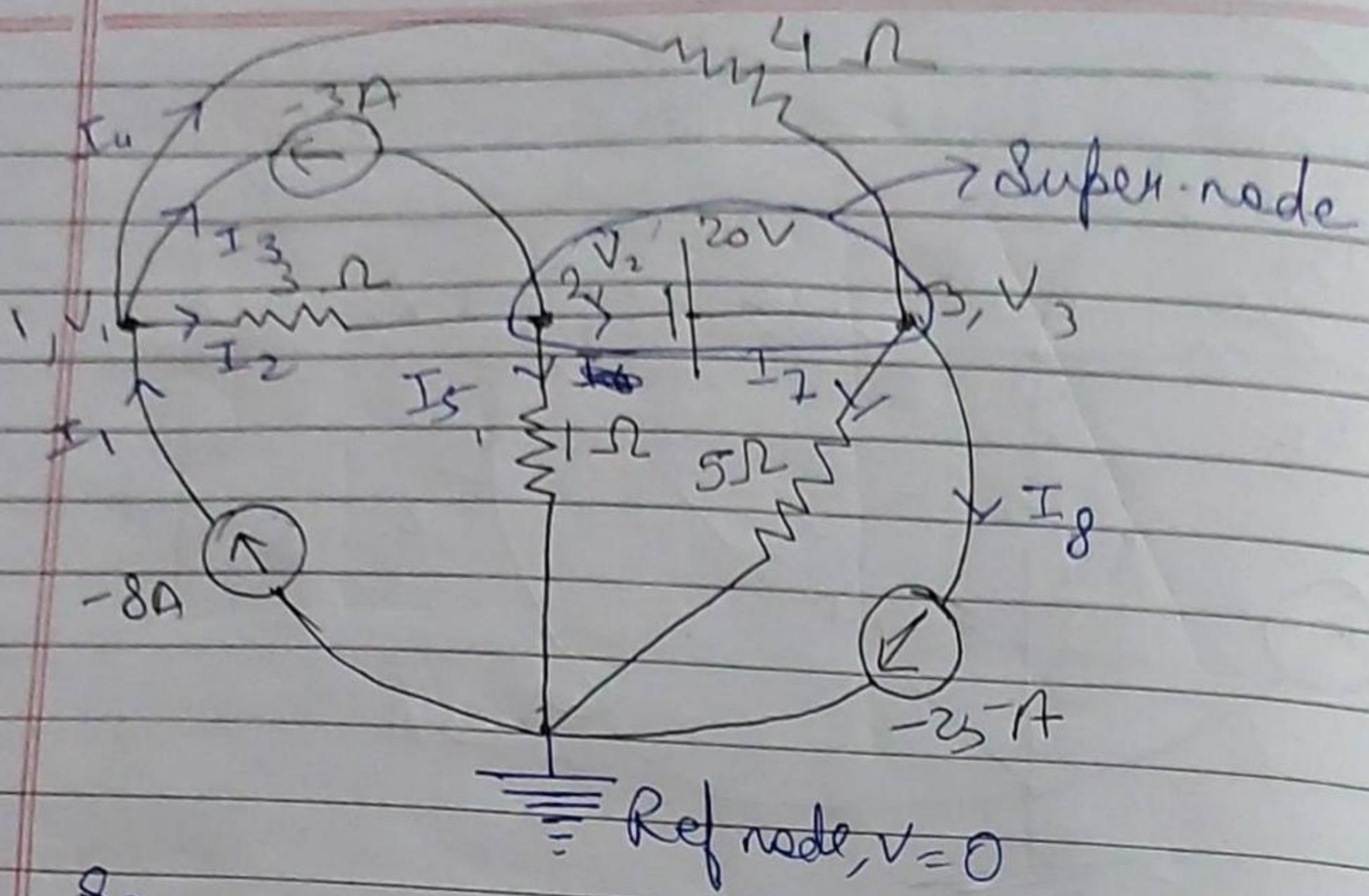
$$V_1, V_2, V_3 \text{ as}$$

$$V_1 = 5.412 V$$

$$V_2 = 7.736 V$$

$$V_3 = 46.32 V$$

Hence, circuit is ~~analysed~~ analysed.



- ① Same
- ② KCL at super node

$$\frac{I_2 + I_3 + I_4}{V_1 - V_2} + \frac{3}{(3)} + \frac{(V_1 - V_3)}{4} = \frac{-I_5 + I_7 + I_8}{V_2 - 0} + \frac{V_3 - 0}{1} + (-25)$$

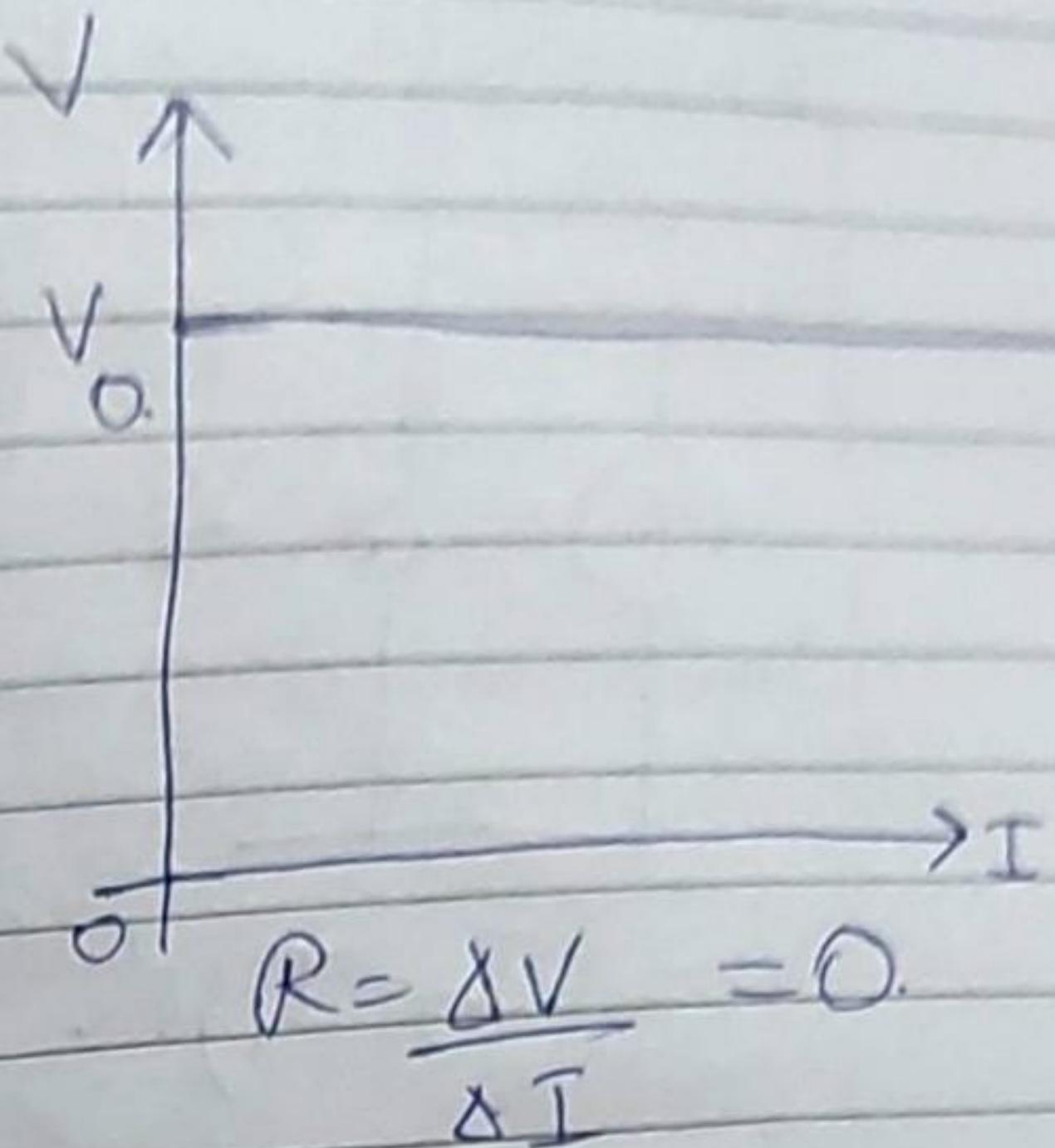
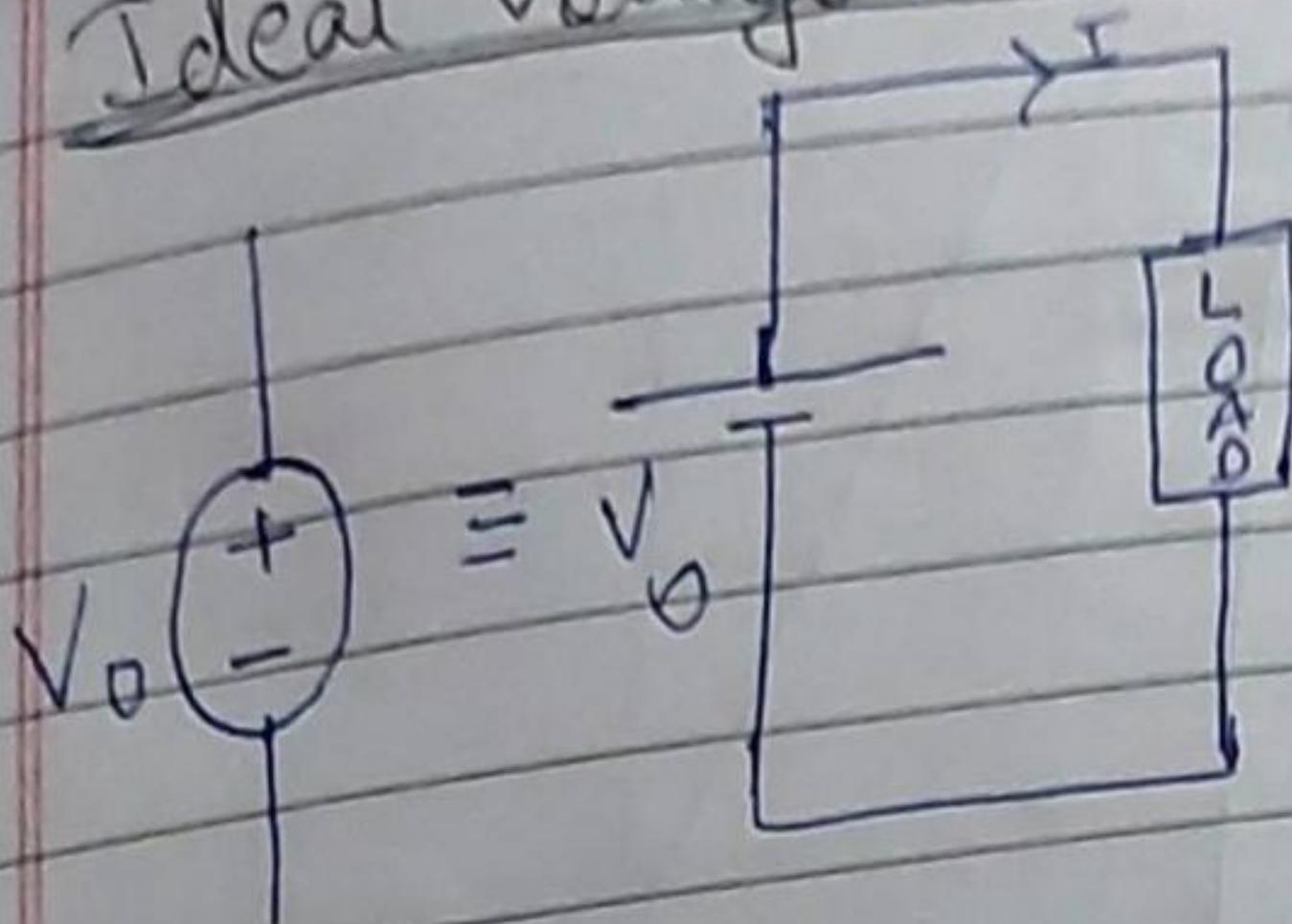
$$\textcircled{3} \quad V_3 - V_2 = 20.$$

$$V_1 = 1.07V$$

$$V_2 = 10.5V$$

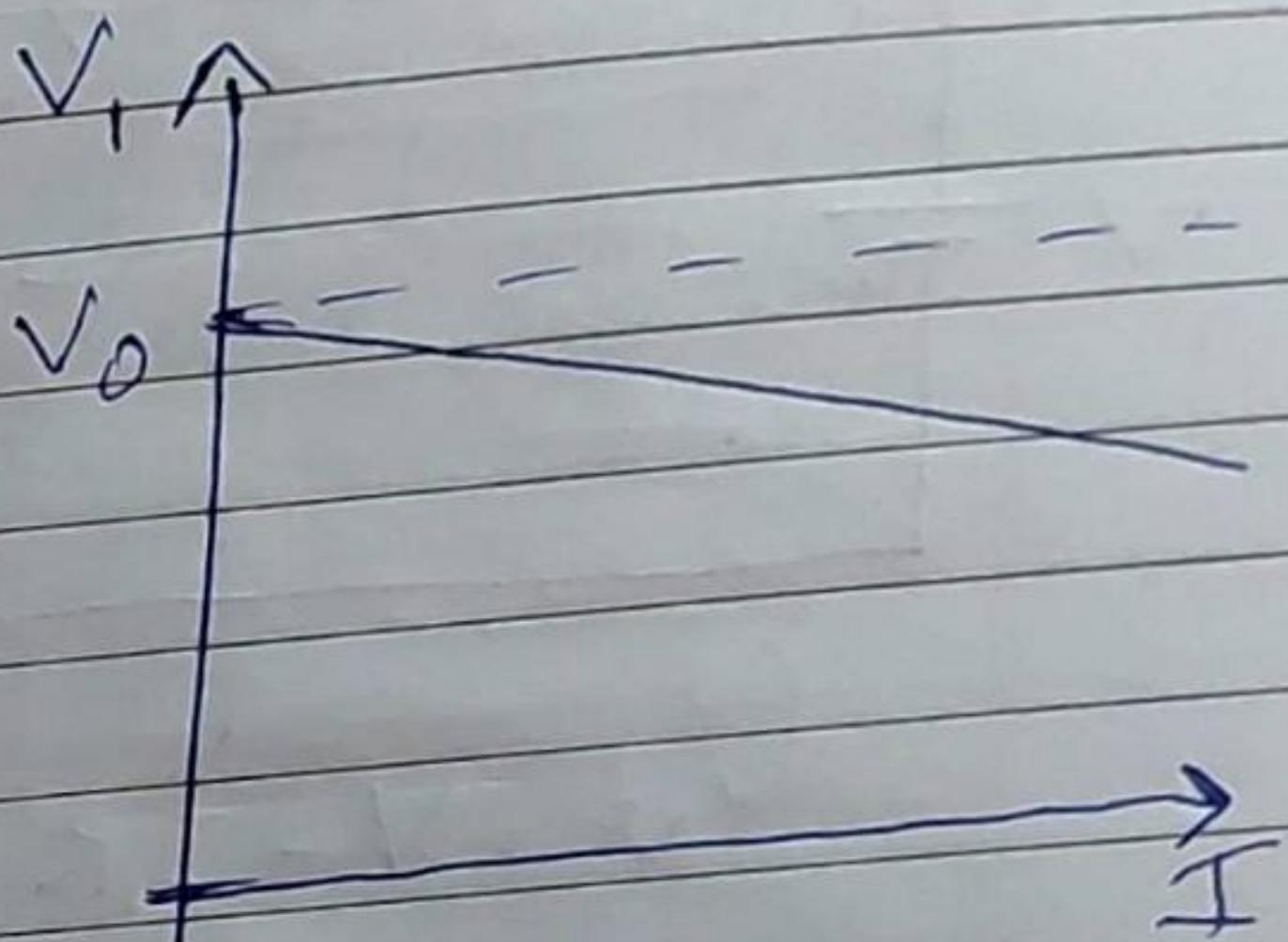
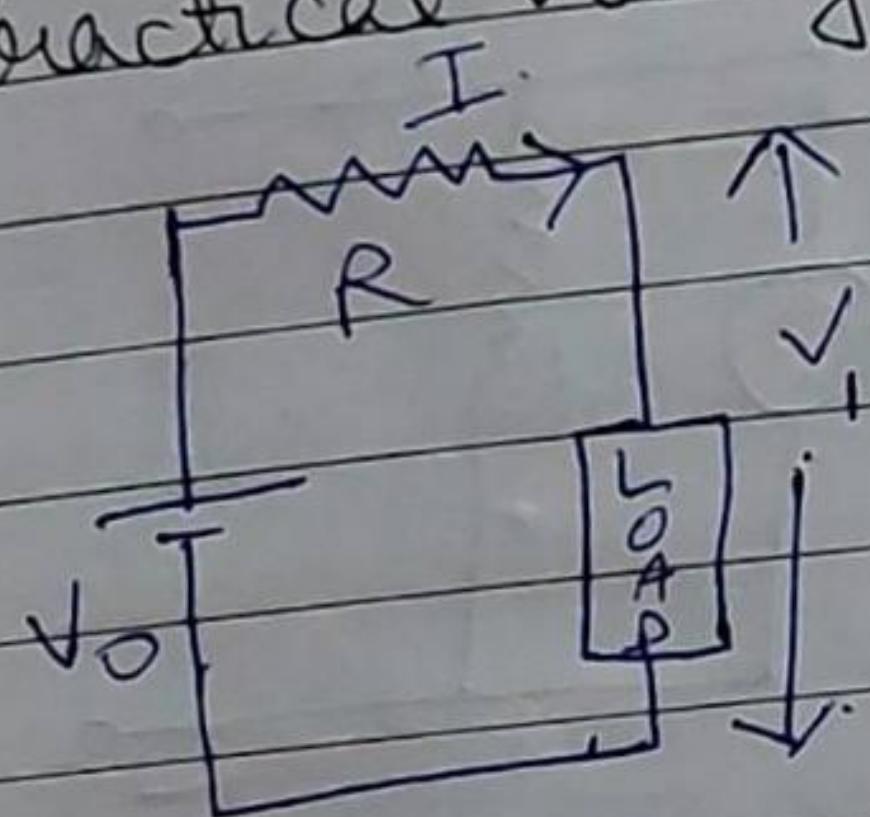
$$V_3 = 30.5V$$

Ideal Voltage Source →



Ideal voltage source has zero internal resistance.

Practical Voltage Source →

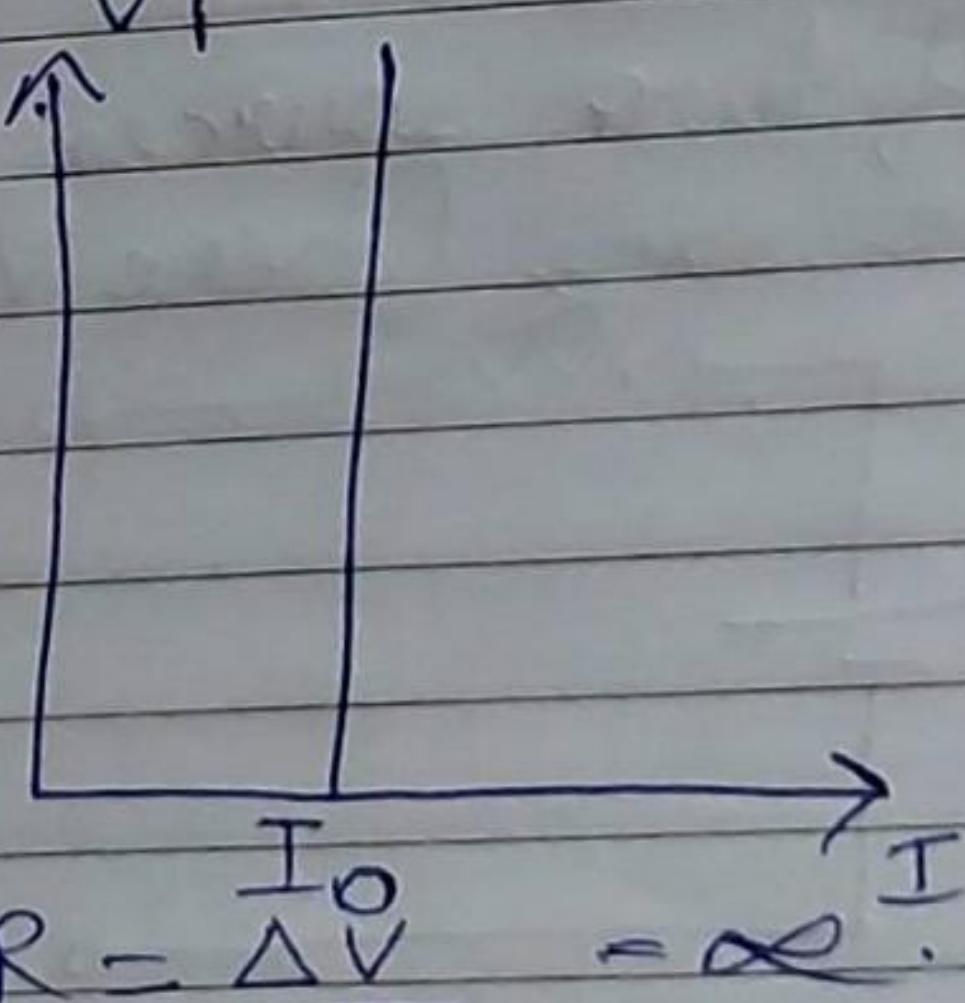
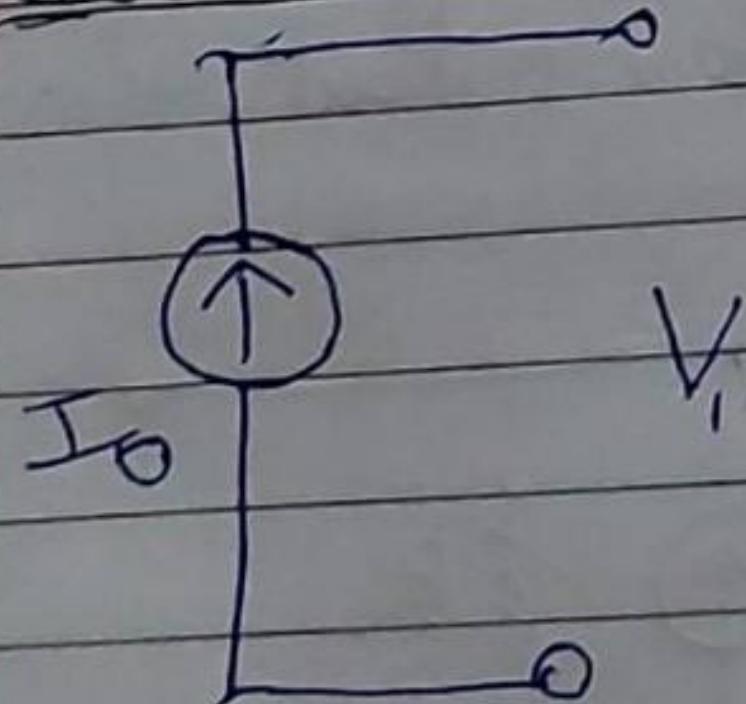


$$V_1 = V_o - IR$$

$$V_1 = -RI + V_o$$

$$Y = MX + C$$

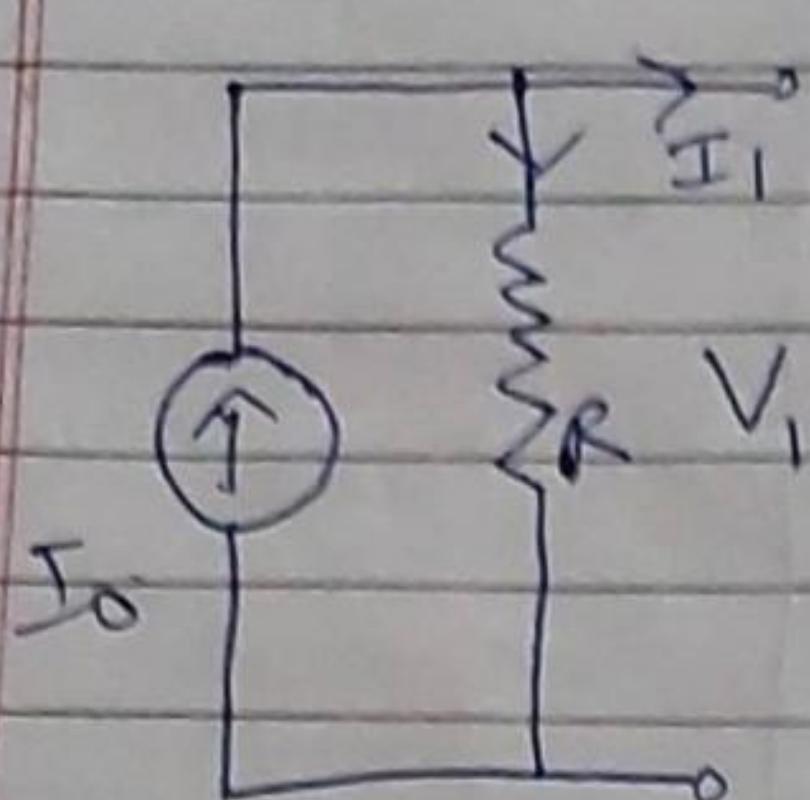
Ideal Current Source →



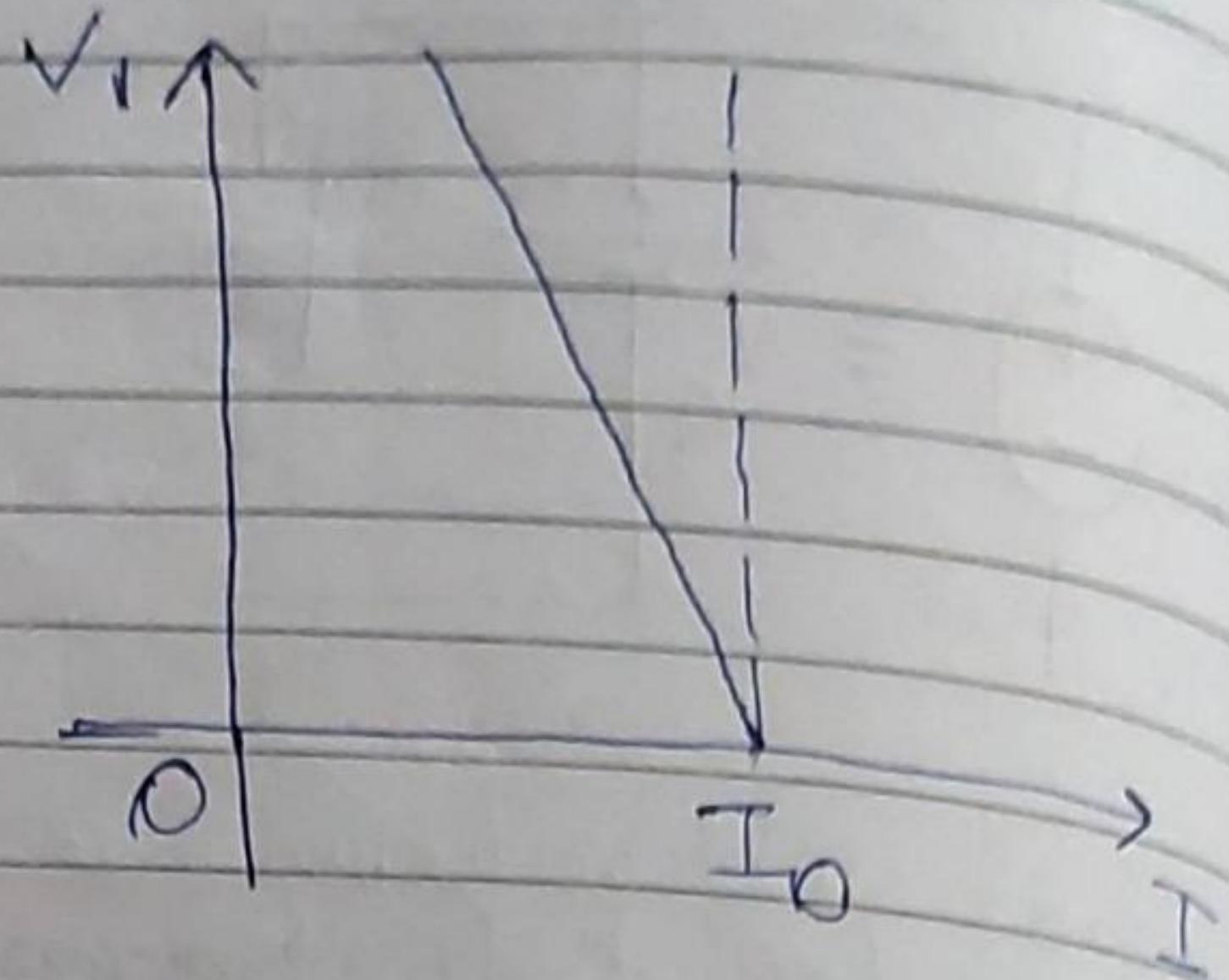
Ideal current source has infinite internal resistance.

$$R = \frac{\Delta V}{\Delta I} = \infty$$

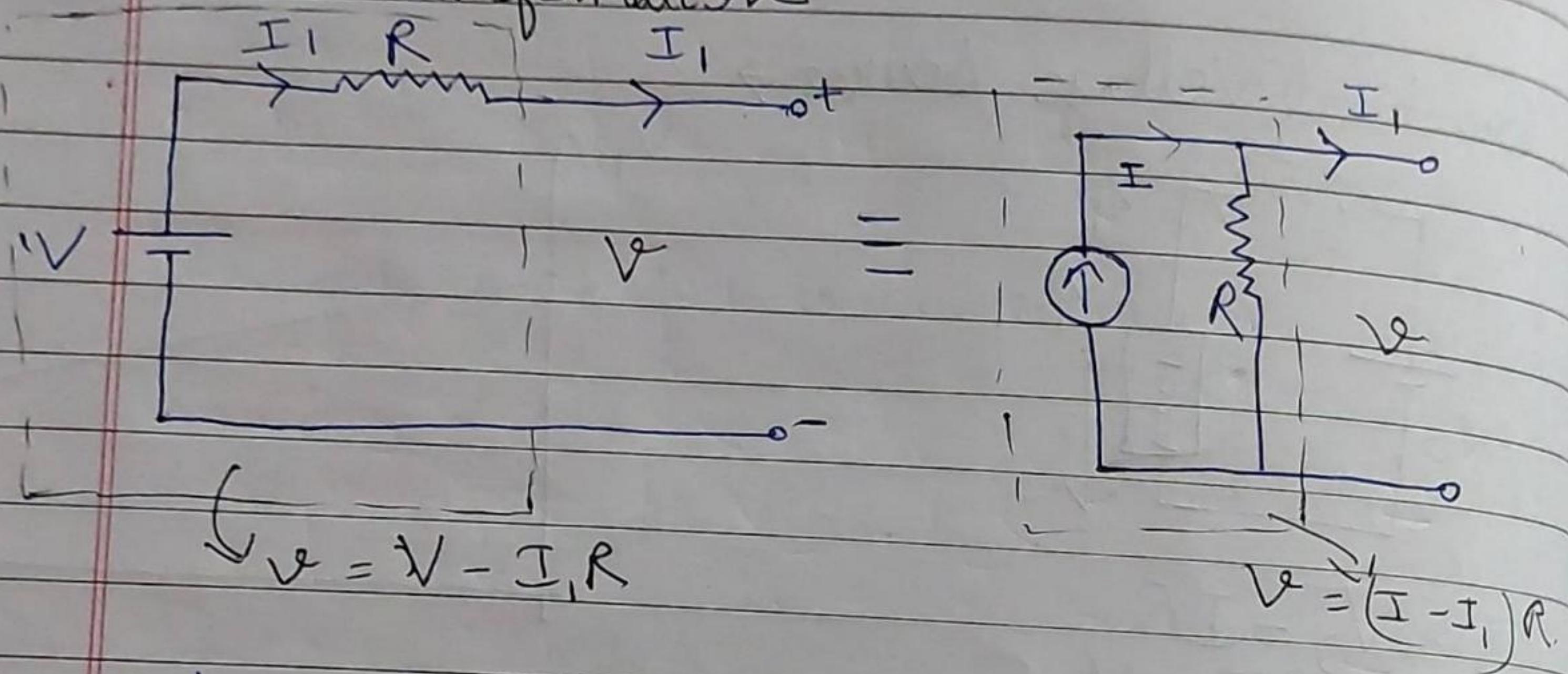
Practical current source  $\rightarrow$



$$V_1 = (I_0 - I_1)R$$



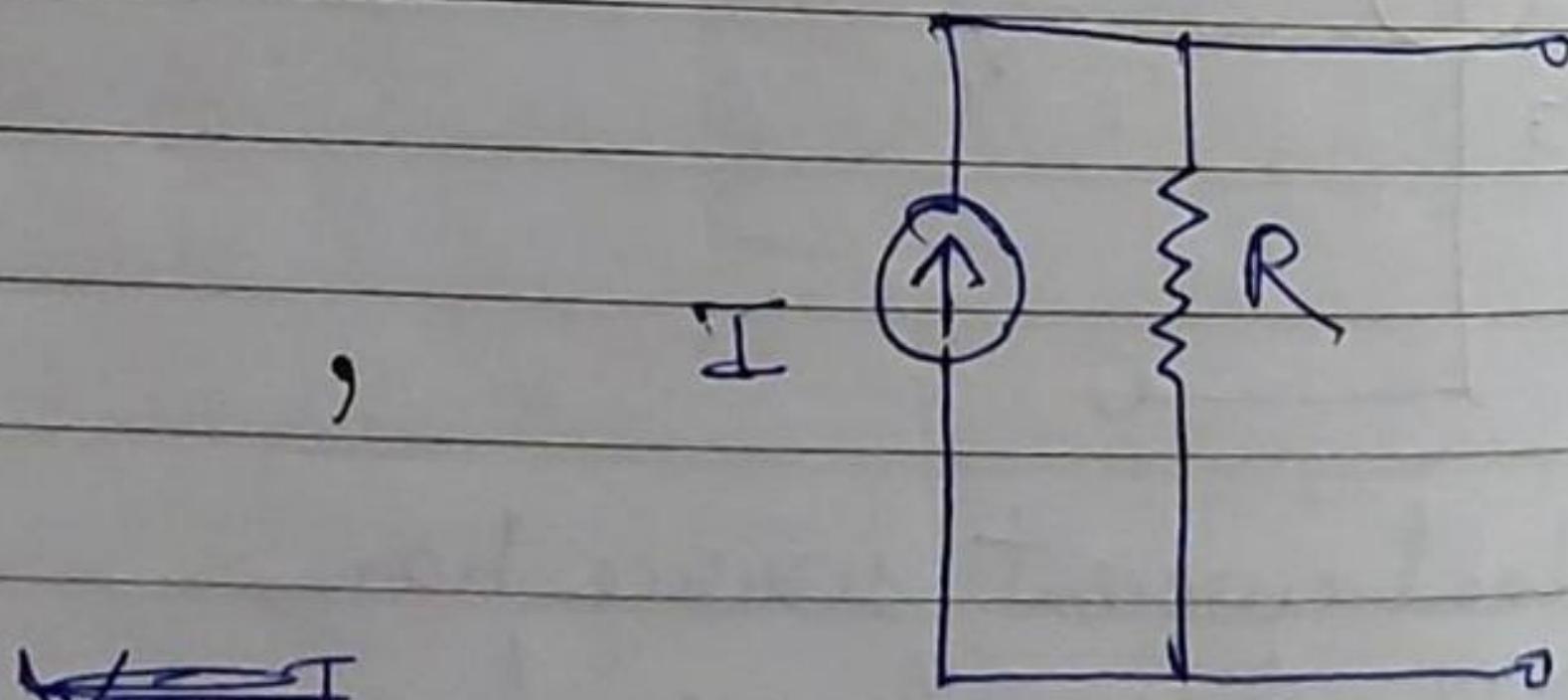
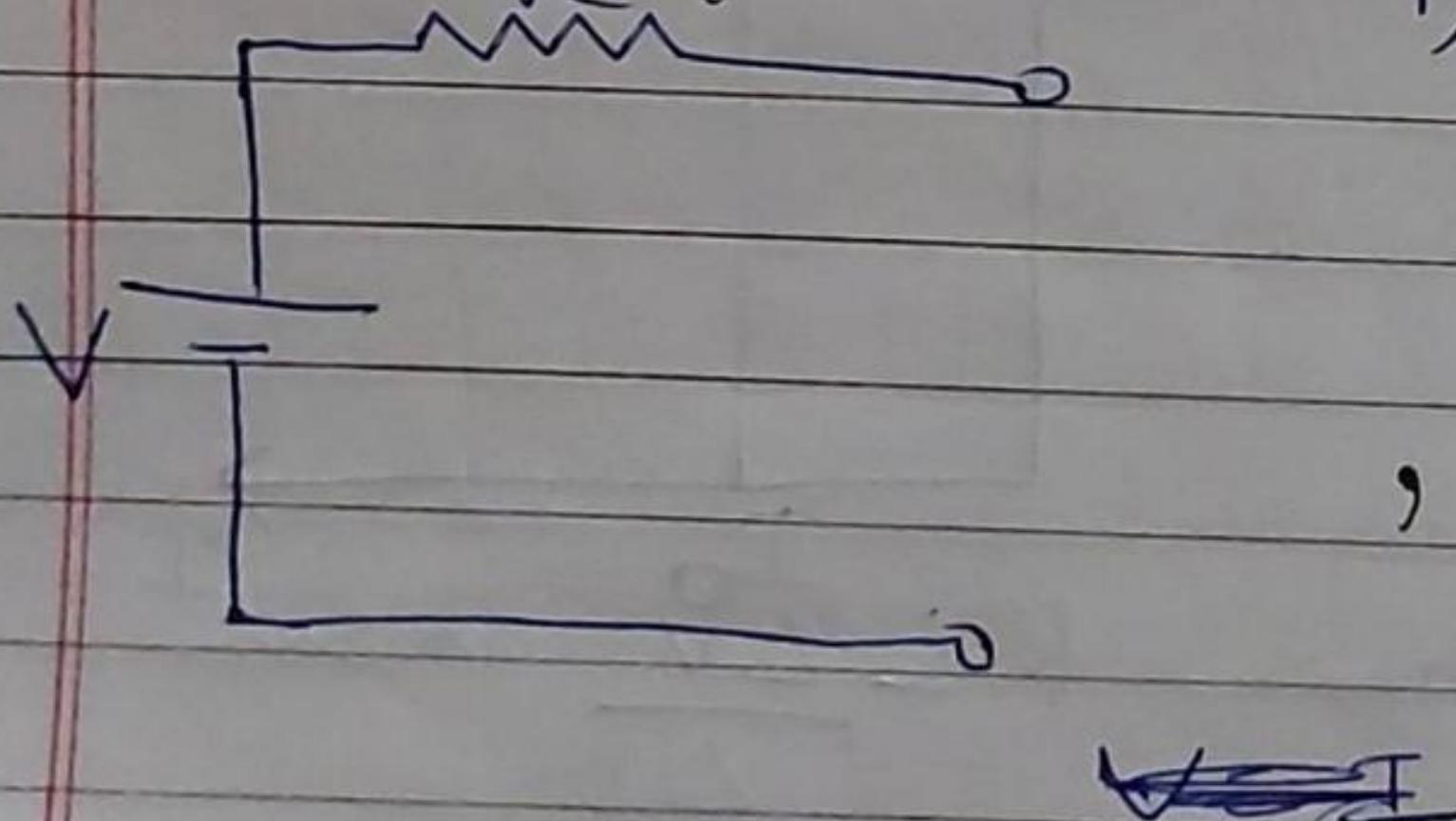
Source transformation



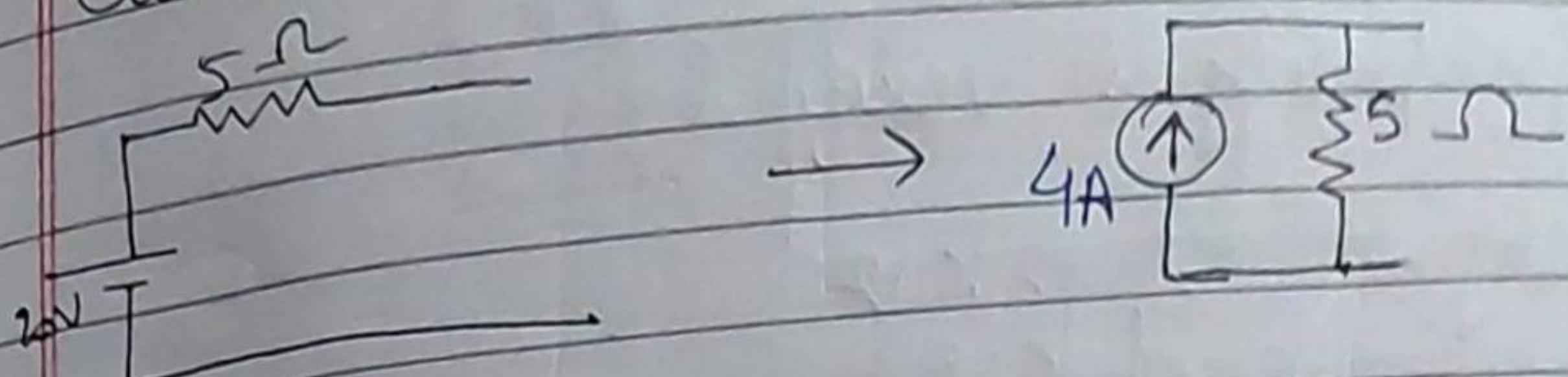
$$V - I_1 R = IR - I_1 R$$

$$V = IR$$

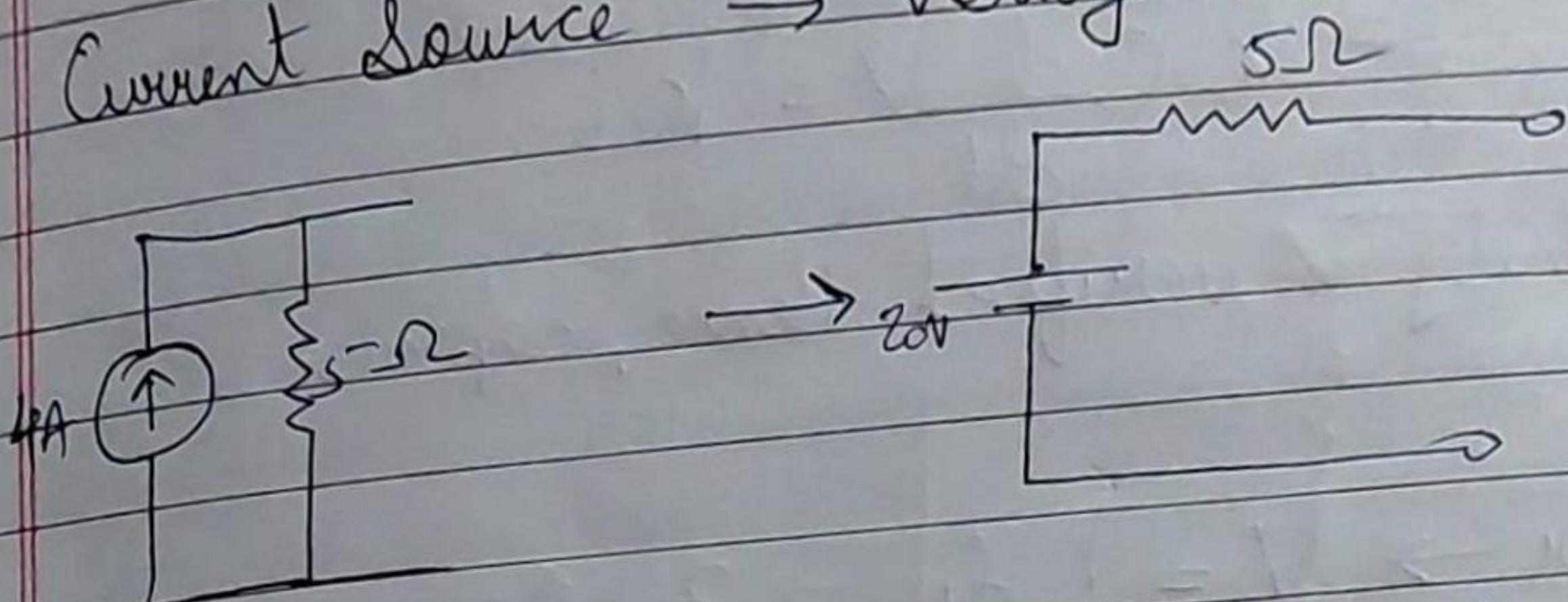
If two sources are equivalent then output should be same (here  $V$  &  $I_1$ )



Current Voltage Source to current source



Current Source  $\rightarrow$  Voltage Source



Linearity - A element is said to be linear if it satisfies homogeneity property and additive property.

$$(i) \text{ Additive} \rightarrow y = f(x)$$

$$x_1 \rightarrow [f] \rightarrow y_1$$

$$x_2 \rightarrow [f] \rightarrow y_2$$

$$y_1 = f(x_1), \quad y_2 = f(x_2)$$

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$= y_1 + y_2$$

$$(ii) \text{ Homogeneity} \rightarrow f(kx) = kf(x) = Ky$$

Capacitor: -  $i = C \frac{dv}{dt}$        $v_1 \rightarrow i_1$   
 $v_2 \rightarrow i_2$

$$i_1 = C \frac{dV_1}{dt}, \quad i_2 = C \frac{dV_2}{dt}$$

$$C \frac{d}{dt} (V_1 + V_2) = C \frac{d}{dt} V_1 + C \frac{d}{dt} V_2 = i_1 + i_2$$

Additive property is verified

Homogeneity -  $V \rightarrow kV$

$$i_V = C \frac{d}{dt} kV$$

$$= k C \frac{d}{dt} V = ki$$

Homogeneity is verified. Hence, capacitor is linear element.

Inductor  $\rightarrow V = L \frac{di}{dt}$   $\rightarrow$  By this. Similarly we can say inductor is also a linear element.

Resistor  $\rightarrow V = iR$ ,  $i_1 \rightarrow V_1$ ,  $i_2 \rightarrow V_2$

$$V_1 = i_1 R, V_2 = i_2 R$$

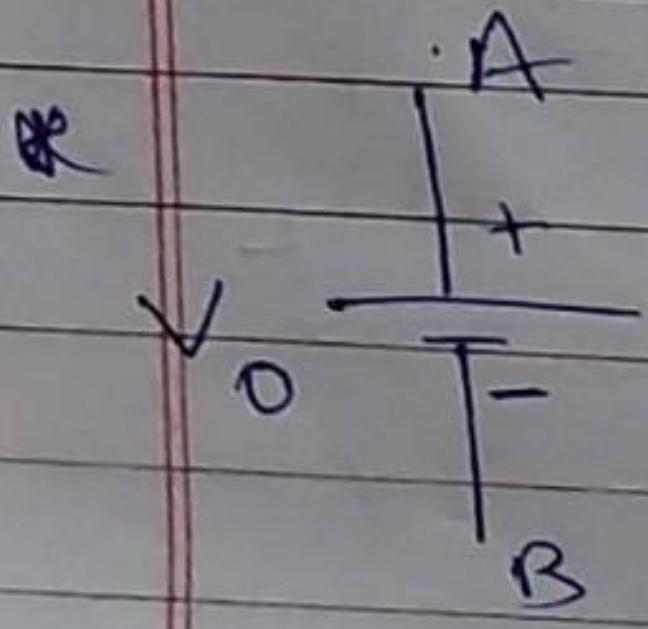
$$(i_1 + i_2)R = i_1 R + i_2 R = V_1 + V_2$$

Additive property is verified.

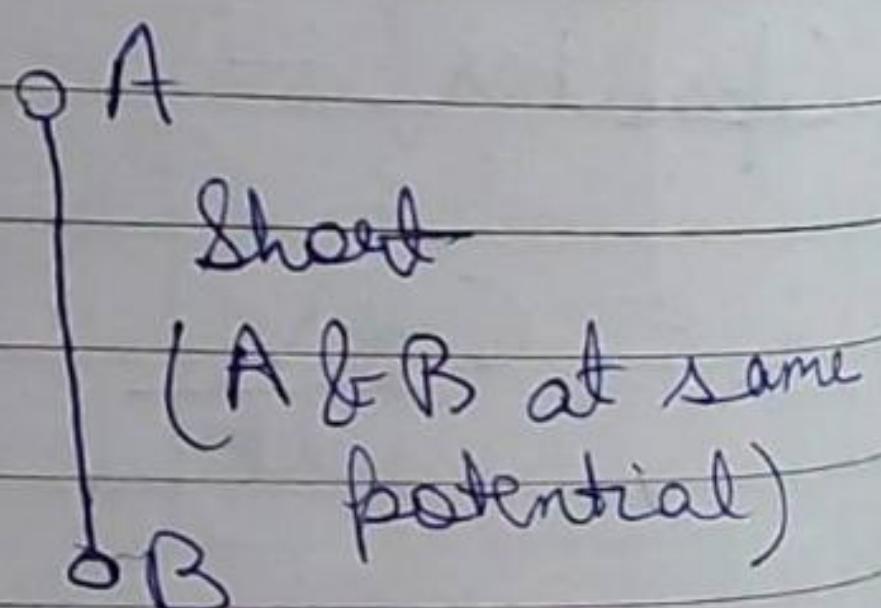
Homogeneous property -  $i = ki$   
 $V = k i R = k V$

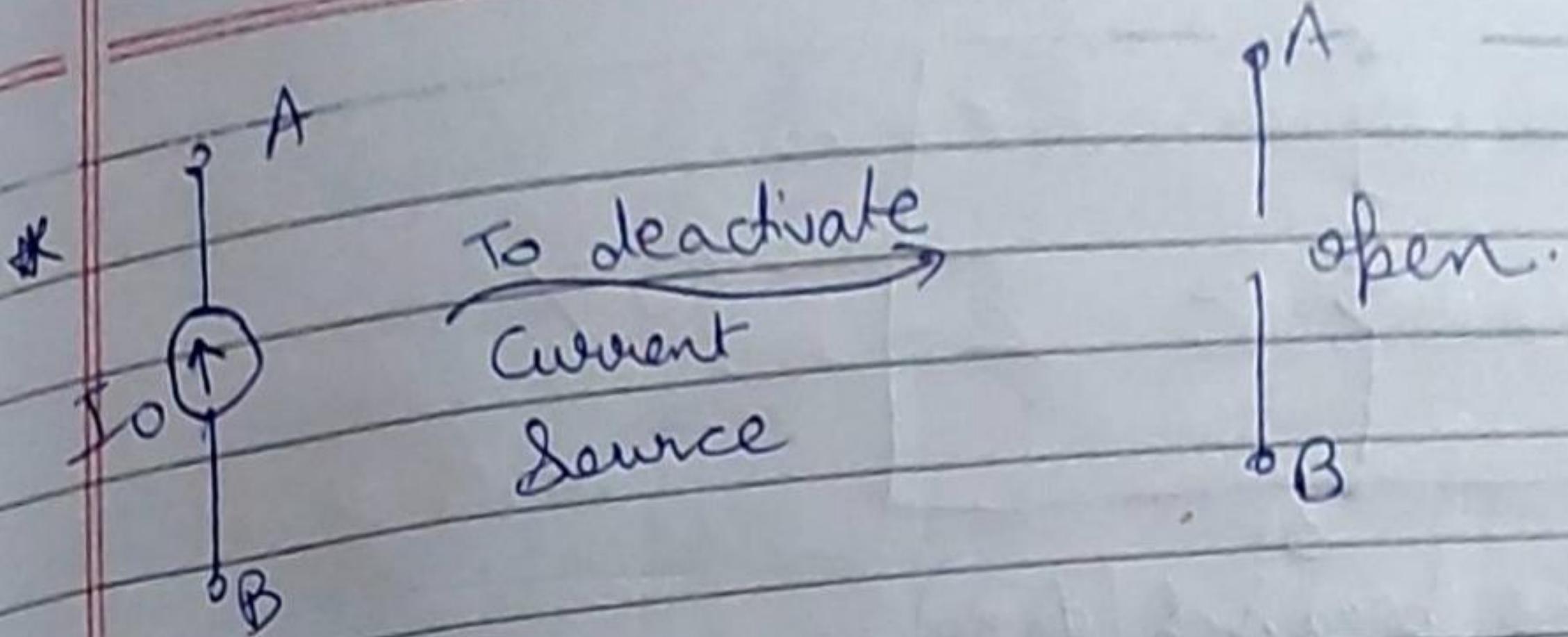
Hence, Resistor is a linear element.

'Deactivating Voltage Source':-



To deactivate  
Voltage  
source



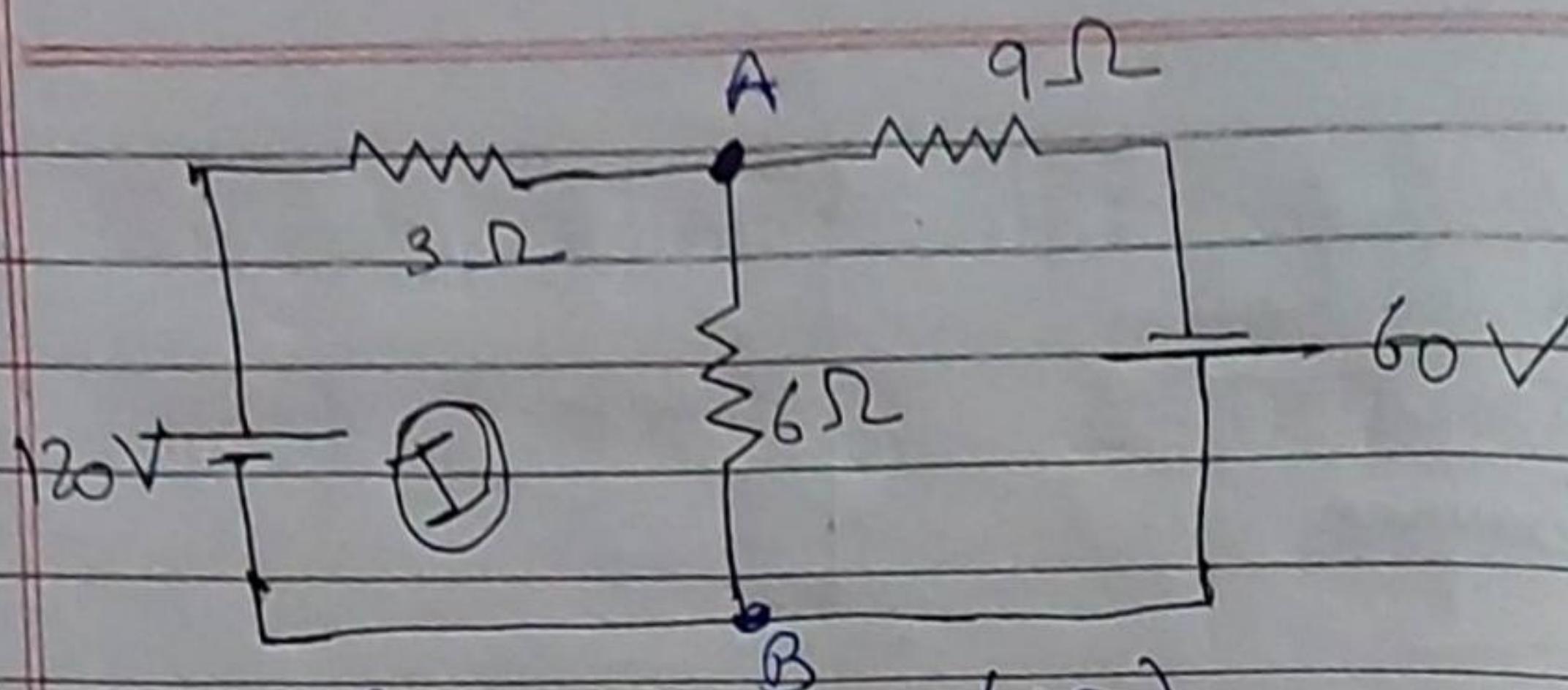


### Superposition Theorem -

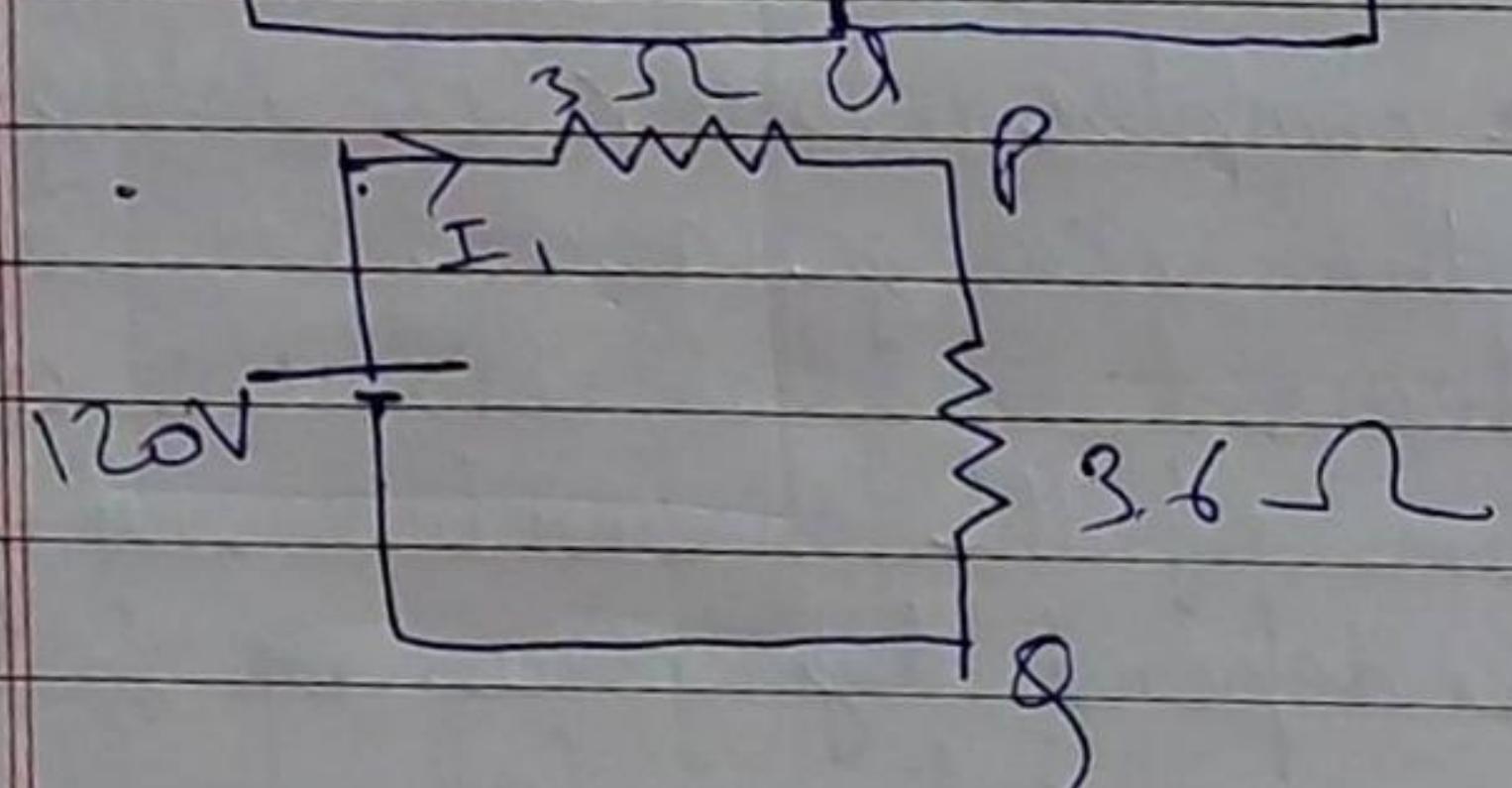
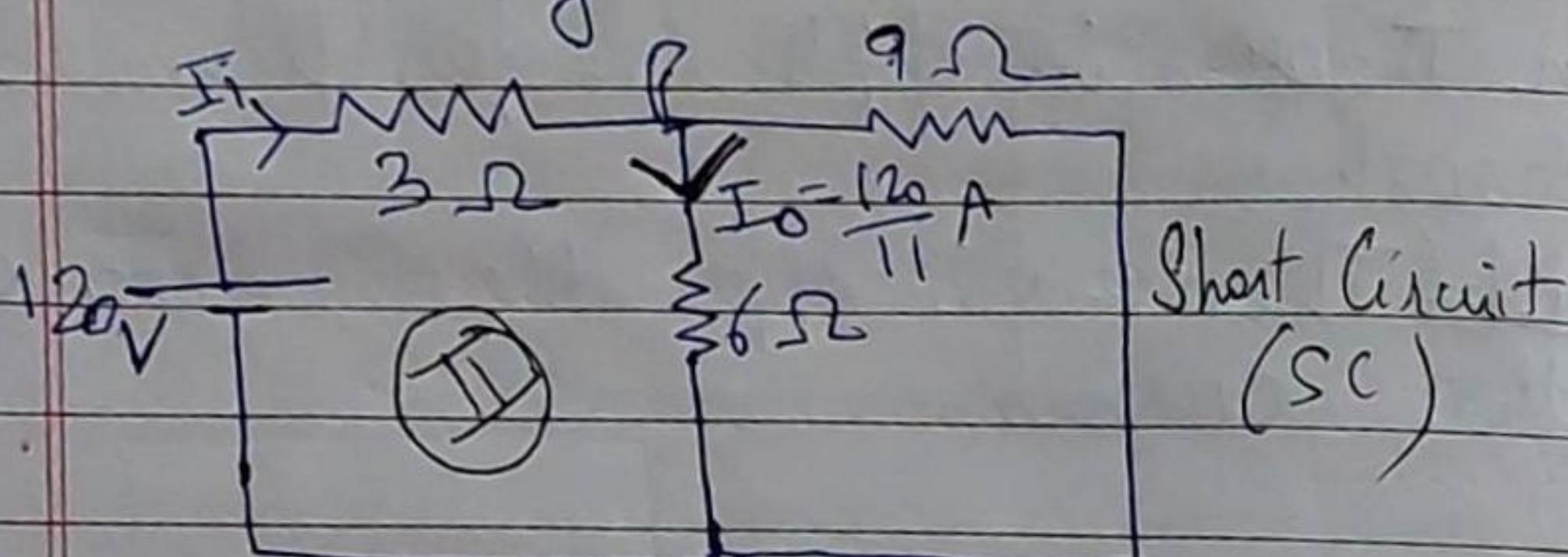
- (i) Superposition Theorem is applicable for linear and bilateral network.
- (ii) Superposition ~~Theorem~~ ~~Net~~ Theorem is applicable for two or more than two sources.

In any multi-source complex network consisting of linear bilateral elements, the voltage across or current through any given element of the network is equal to the algebraic sum of the individual voltages or currents, produced independently across or in that element by each source acting independently, when all the remaining sources are replaced by their respective internal resistances i.e. independent voltage source must be replaced by short circuit while independent current sources must be replaced by open circuit.

Q Calculate the current in  $6\Omega$  resistor by super-position theorem.



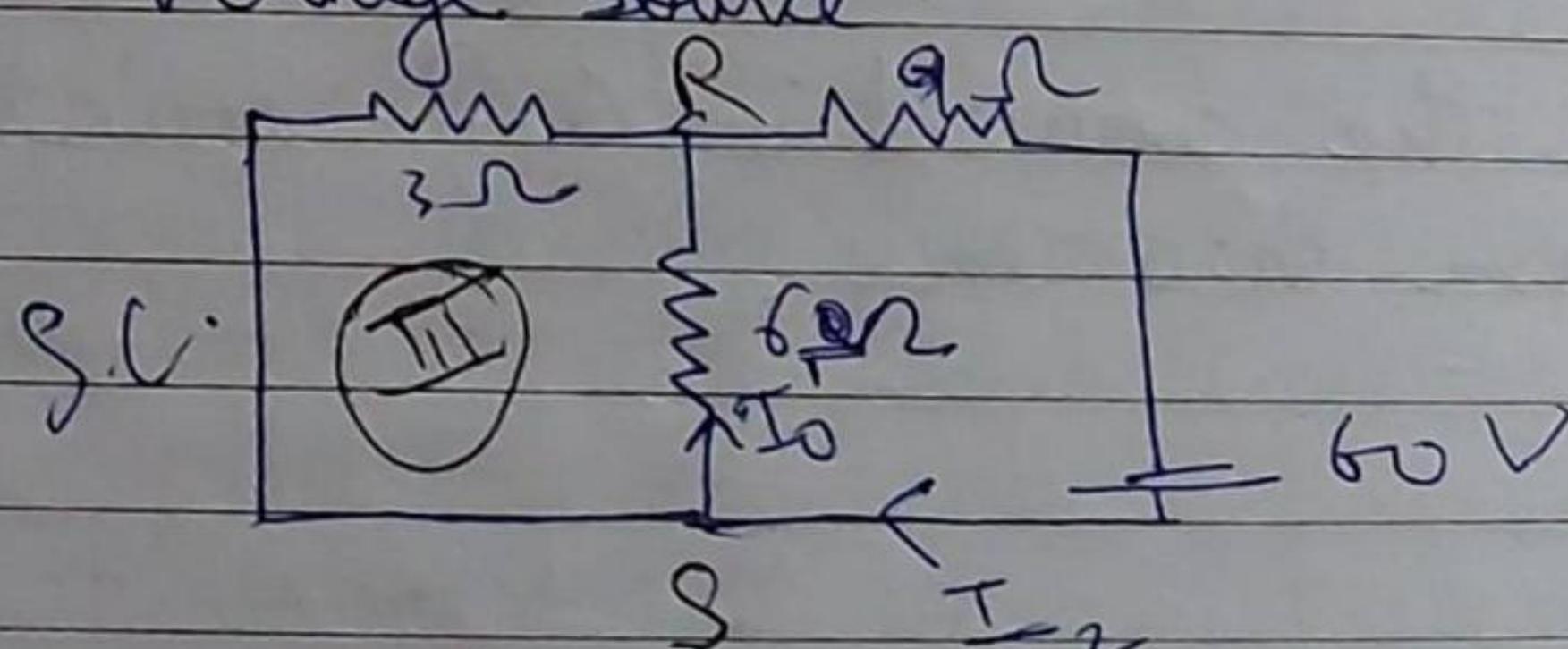
- (i) Identify the branch (AB)
- (ii) Consider only 120V voltage source. Hence, 60V voltage source must be deactivate.



$$I_1 = \frac{120}{3+3.6} = \frac{120}{6.6} = \frac{200}{11} A$$

$$I_0 = \frac{200}{11} \left[ \frac{3.6}{9+6} \right] A \quad (\text{By current div. rule})$$

- (iii) Consider only 60V voltage source & deactivate 120V voltage source



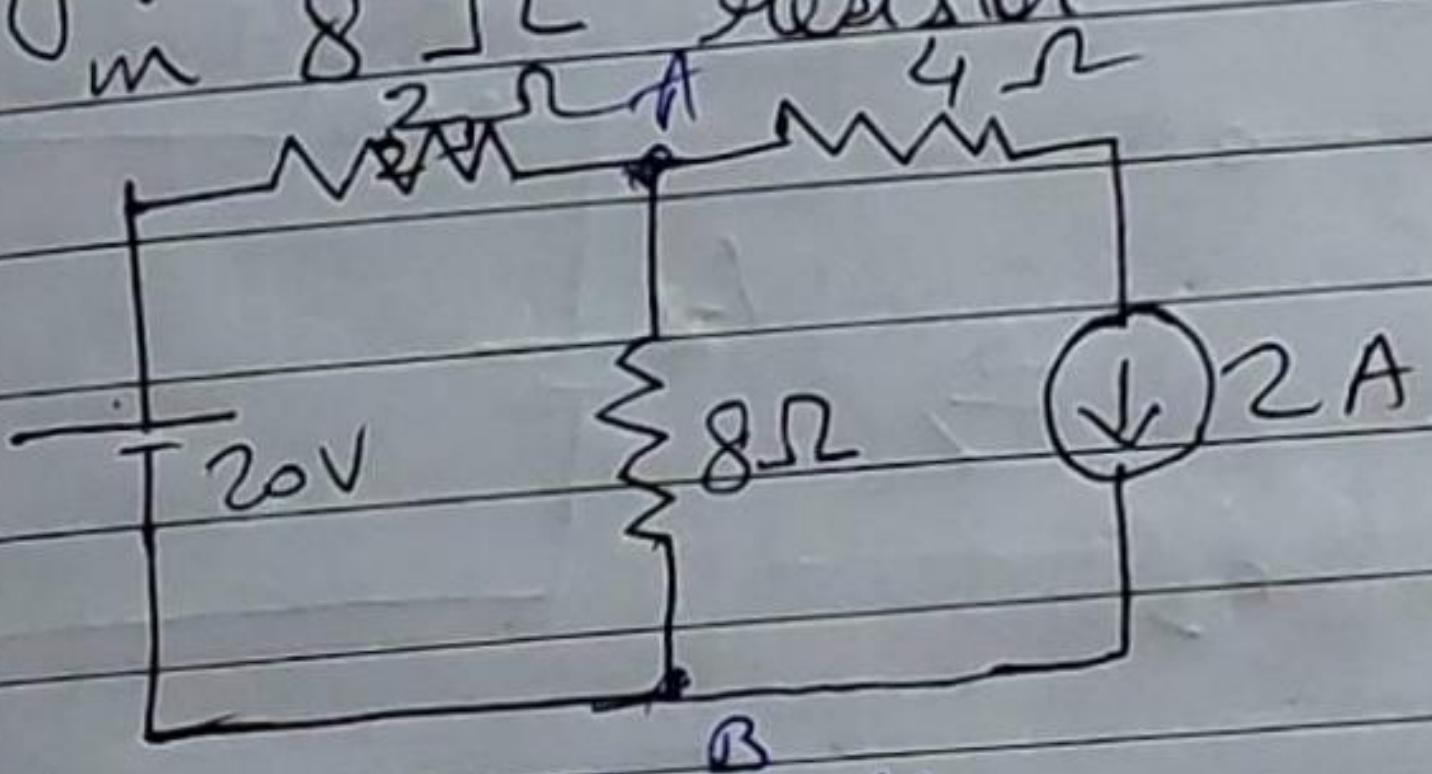
$$I_2 = \frac{60}{9} A$$

$$I_0' = \left( \frac{3}{3+6} \right) \frac{60}{11} = \frac{20}{11} \text{ A} \quad (\text{By current division Rule})$$

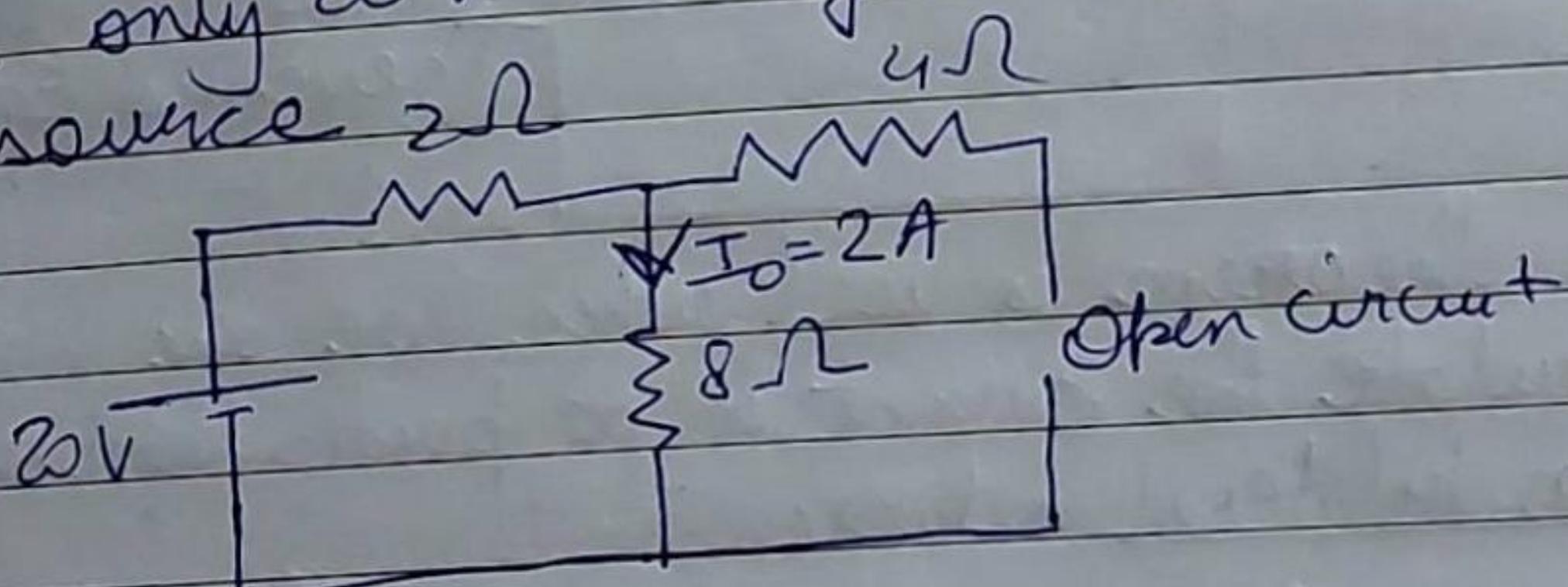
(ii) Using Superposition Theorem  
 Current in Branch AB =  $\left( \frac{120}{11} - \frac{20}{11} \right) \text{ A}$   
 $= \frac{100}{11} \text{ A}$  from A to B

$$(I = I_1 + I_2)$$

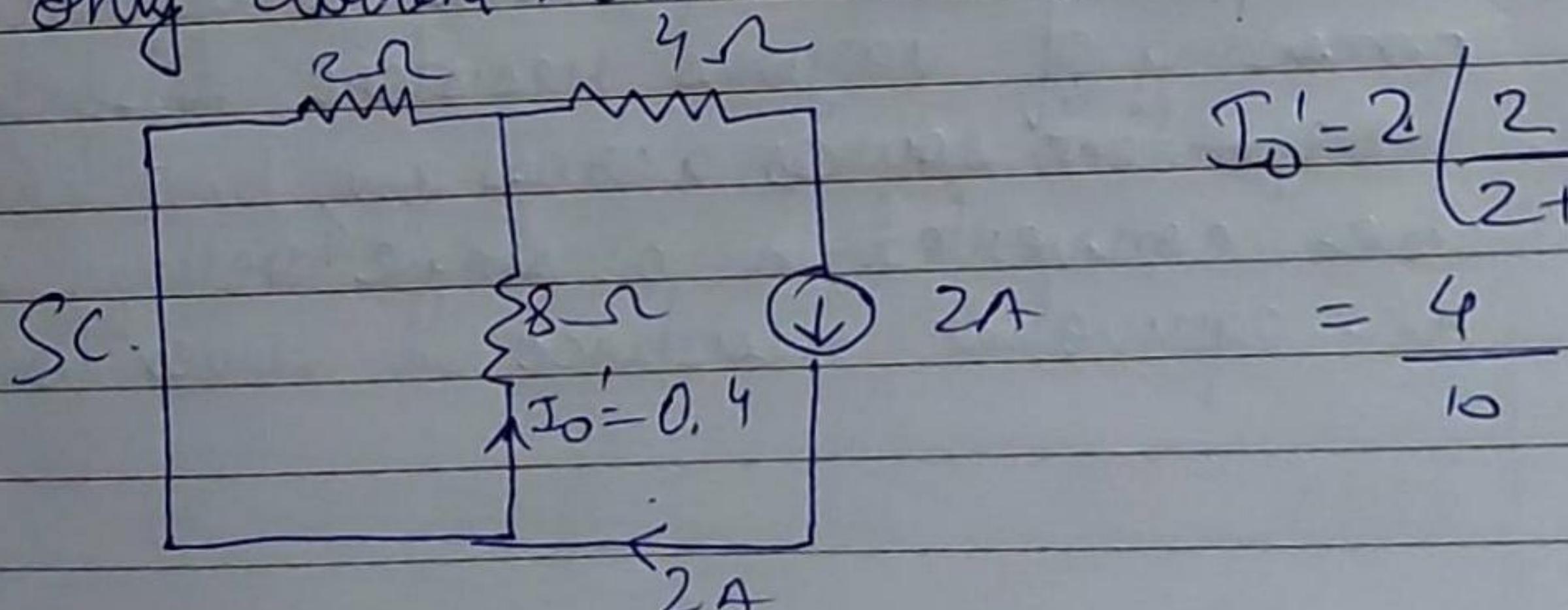
& By using super-position theorem calculate the current in  $8\Omega$  resistor.



Consider only  $20\text{V}$  voltage source and deactivate current source  $2\text{A}$



Consider only current source and deactivate voltage source



$$I_0' = 2 \left( \frac{2}{2+8} \right)$$

$$= \frac{4}{10}$$

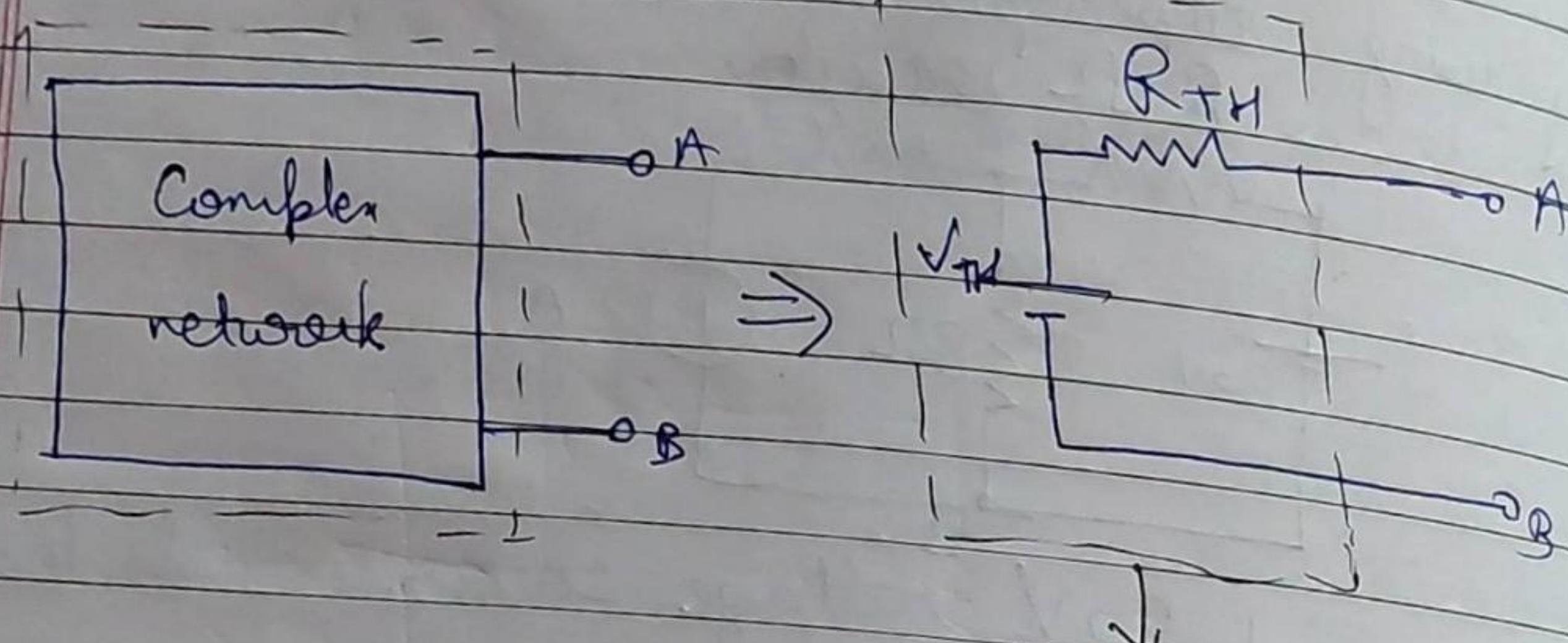
## Using Super-position Theorem

$$\text{Current in branch AB} = 2 - 0.4 = 1.6 \text{ A}$$

(from A  $\rightarrow$  B)

- \* The super-position theorem does not apply to the power because power is proportional to the square of the current which is not linear.
- \* Dependent source does not eliminated.

## Thevenin's Theorem $\rightarrow$ - - -



Thevenin's Equivalent Circuit.

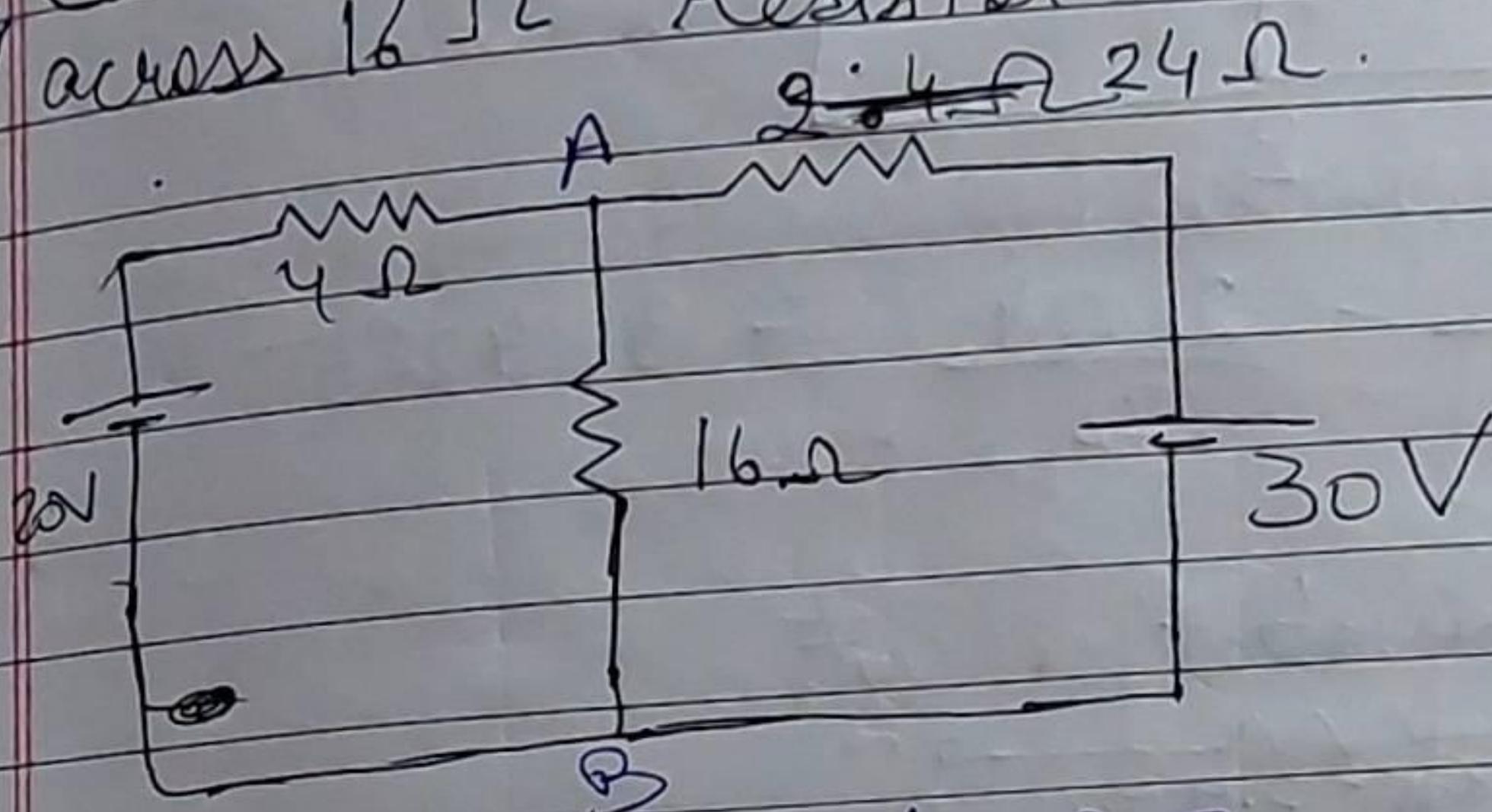
This theorem is used when we are interested to calculate the current in a particular branch of a linear bilateral network.

A complex network (active, bilateral and lossy) consisting of various resistors, dependent and independent sources across any two terminal can be converted in a single voltage source and an equivalent resistance in series with the source.

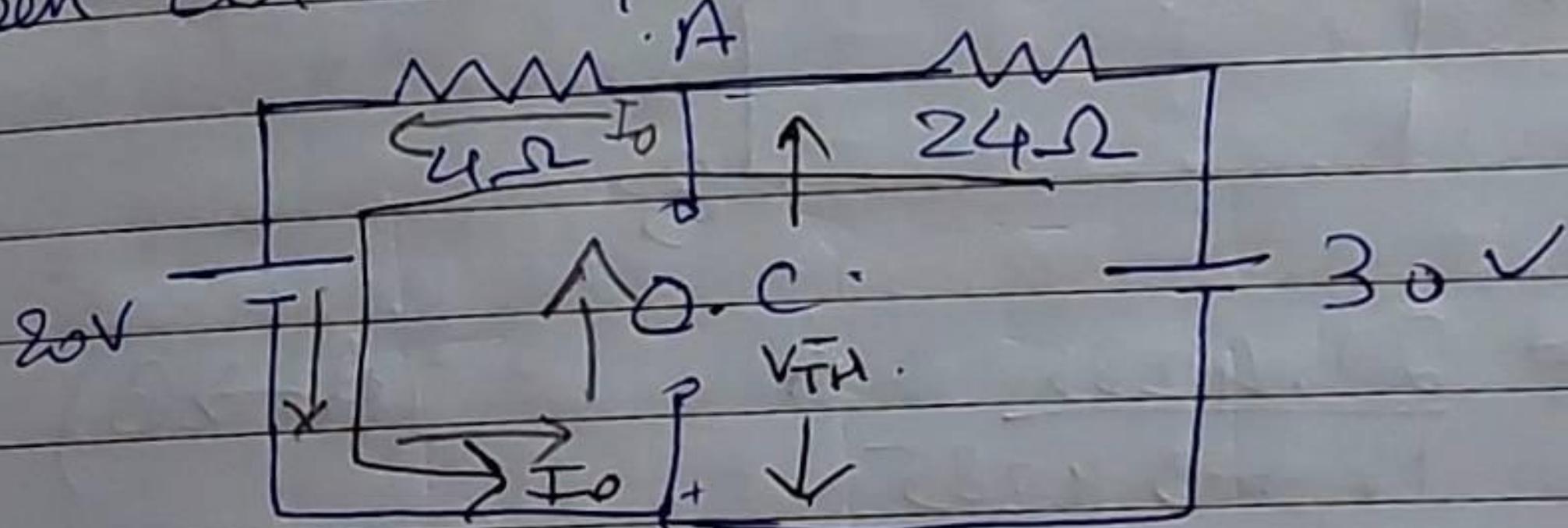
$V_{TH} \rightarrow$  It is the open circuit voltage across the two terminals.

$R_{TH} \rightarrow$  It is the equivalent resistance b/w terminals A & B when sources are replaced by their internal resistances i.e. voltage source should be short circuited & current source should be open circuited.

Q) Calculate the current using thevenin's equivalent across  $16\Omega$  resistor.



- Identify the branch AB.
- Open circuit b/w terminal A & B.



Applying KVL

$$24I_0 + 4I_0 + 20 - 30 = 0.$$

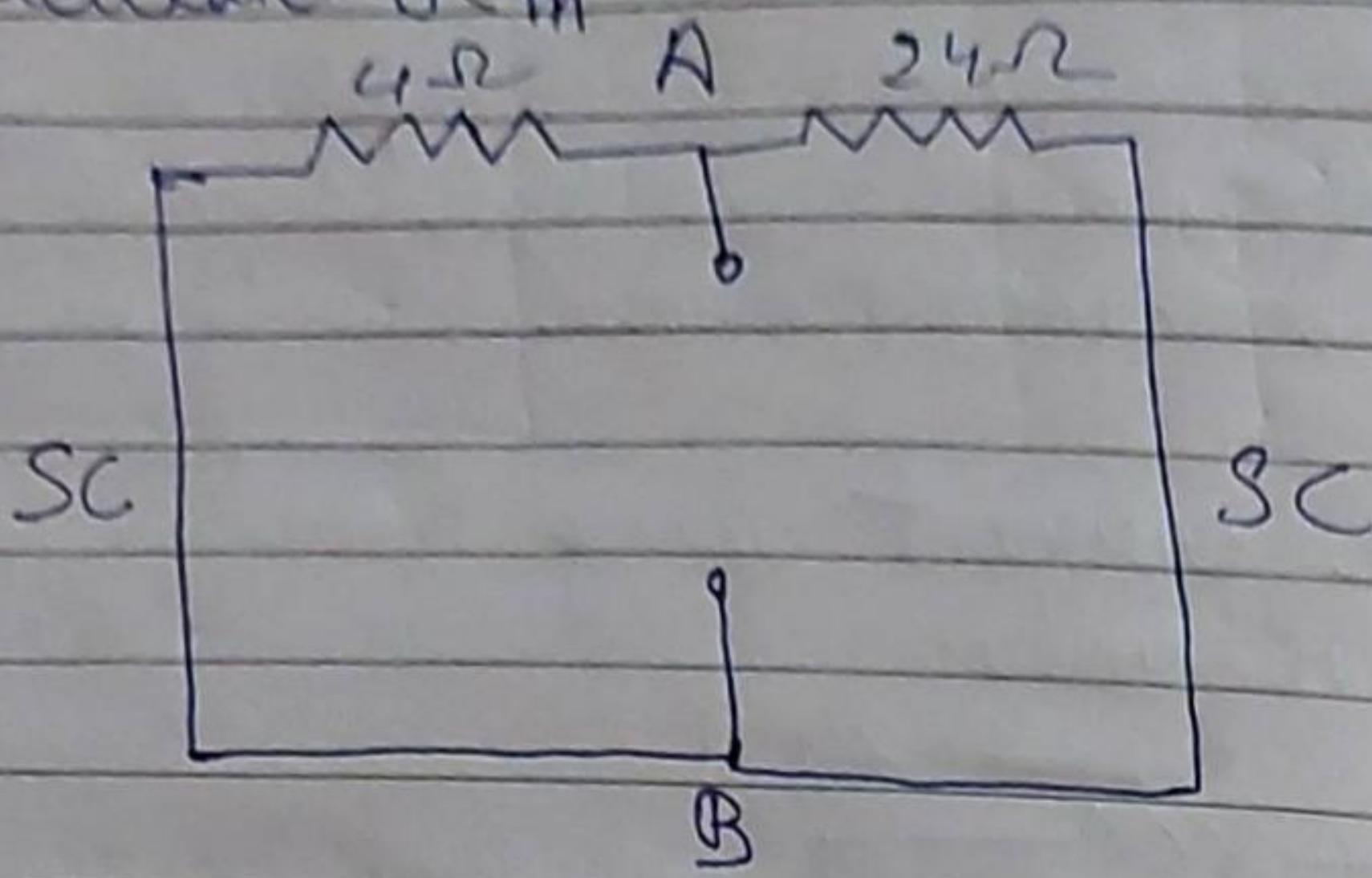
$$I_0 = \frac{10}{28} A = 0.36 A$$

$$4I_0 + 20 + V_{TH} = 0$$

$$V_{TH} = -(20 + 4 \times 0.36) = -21.4285 V$$

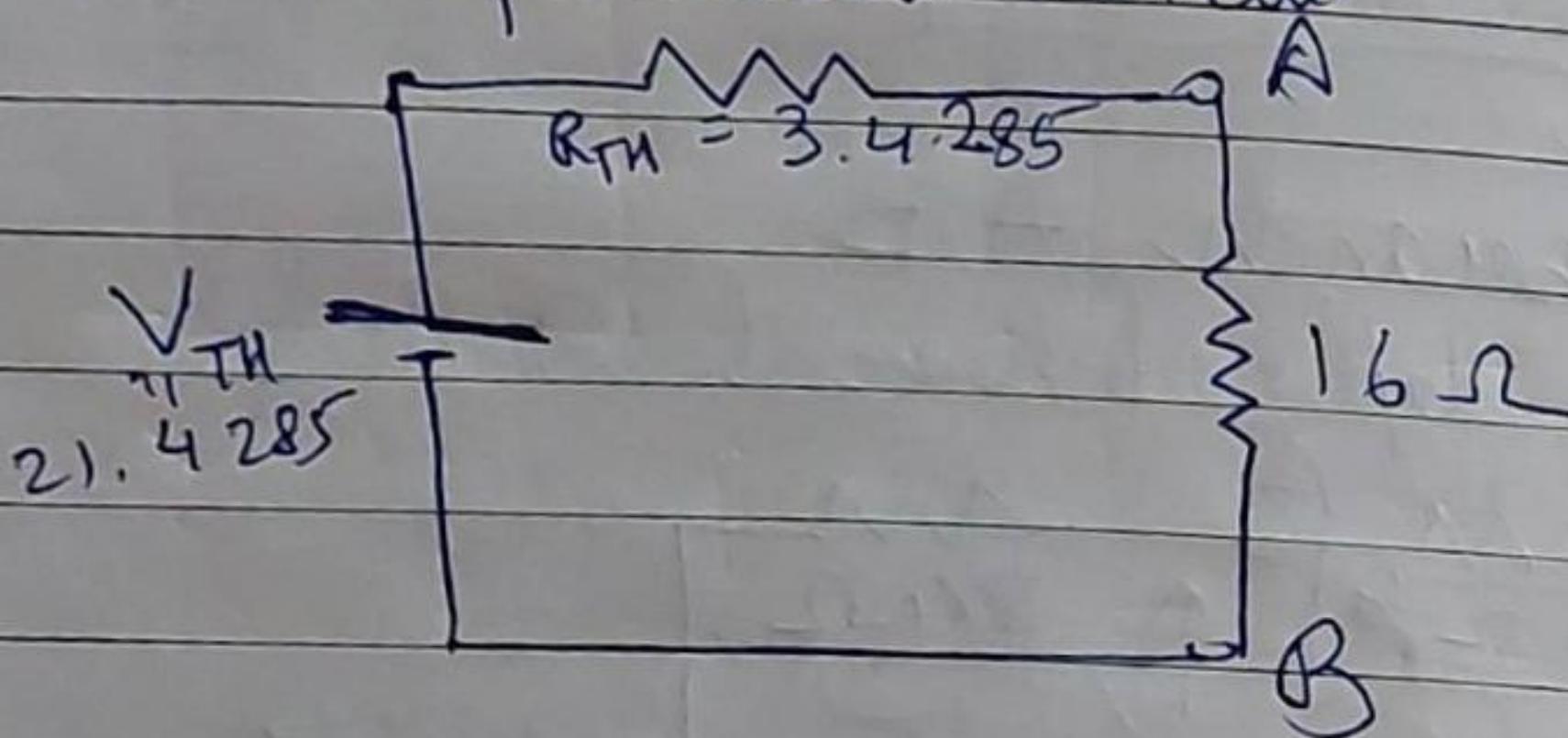
$$V_{TH} = 21.4285 \text{ V} \text{ (B at Lower Potential)}$$

(iii) Calculate  $R_{TH}$



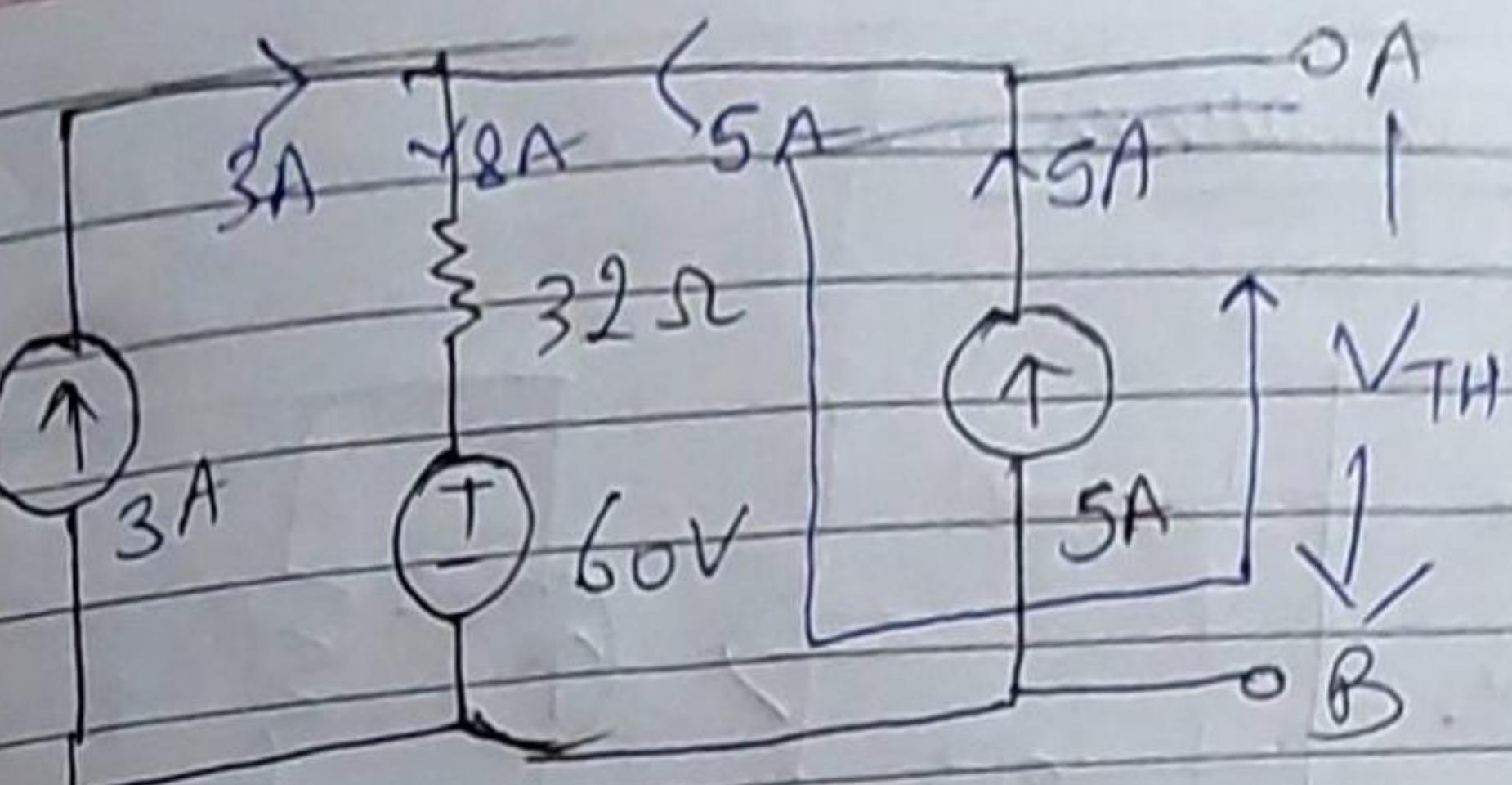
$$R_{TH} = \frac{4 \times 24}{24 + 4} = \frac{2 \times 24}{28} = 3.4285$$

(iv) Thevenin's equivalent circuit

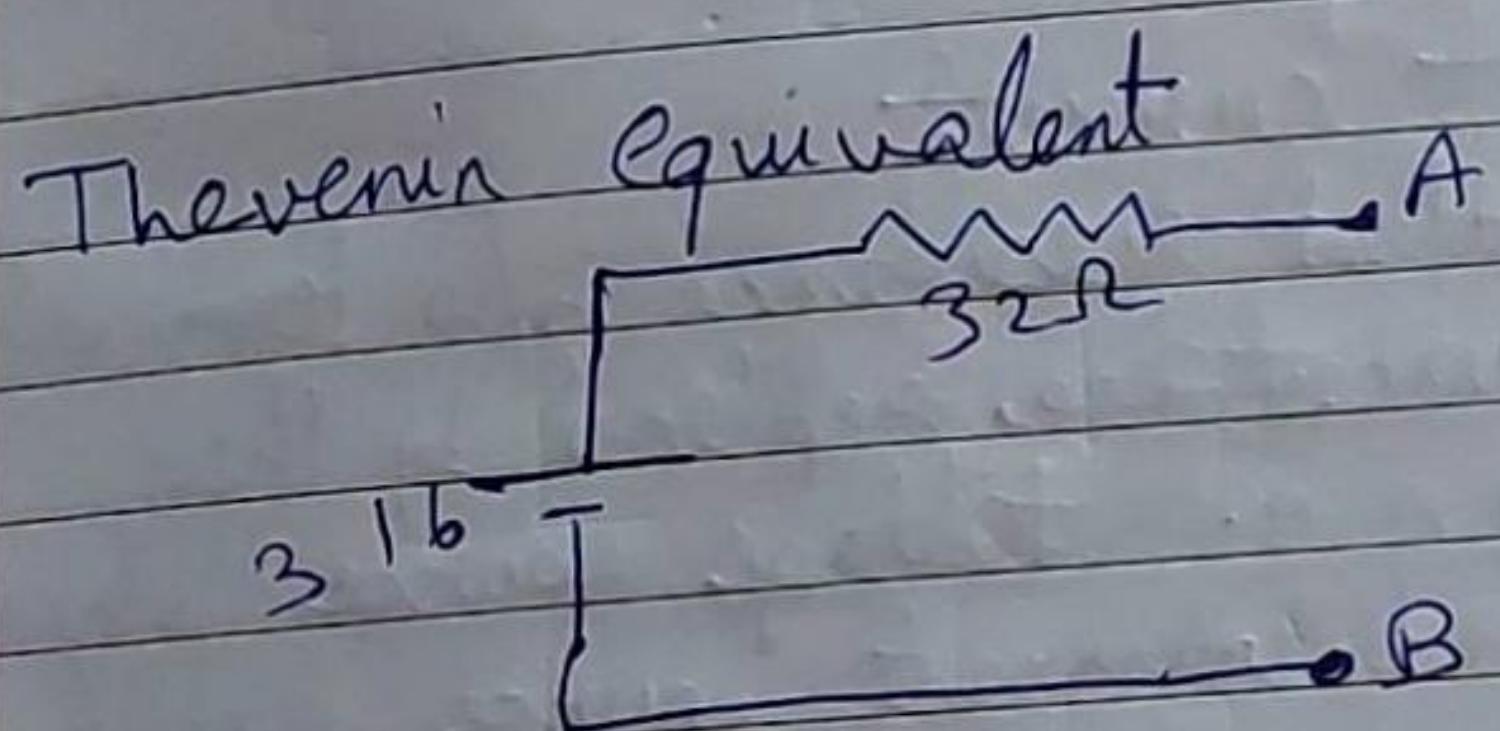
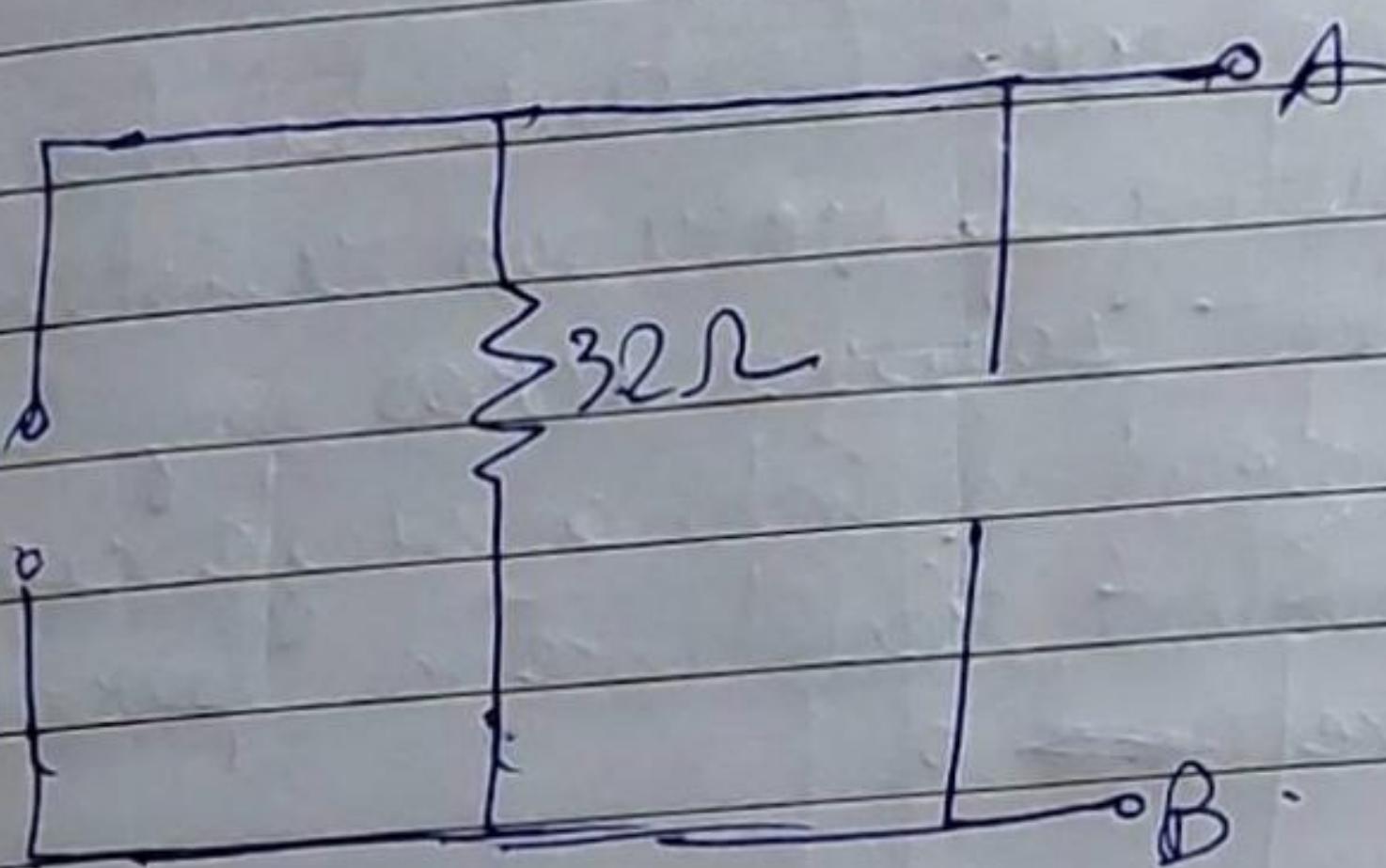


$$\begin{aligned} I_{16\Omega} &= \frac{21.4285}{16 + 3.4285} \text{ A} && (\text{A} \rightarrow \text{B}) \\ &= 1.1029 \text{ A} && (\text{A} \rightarrow \text{B}) \end{aligned}$$

~~Q~~ Obtain calculate the thevenin equivalent between terminal A & B.



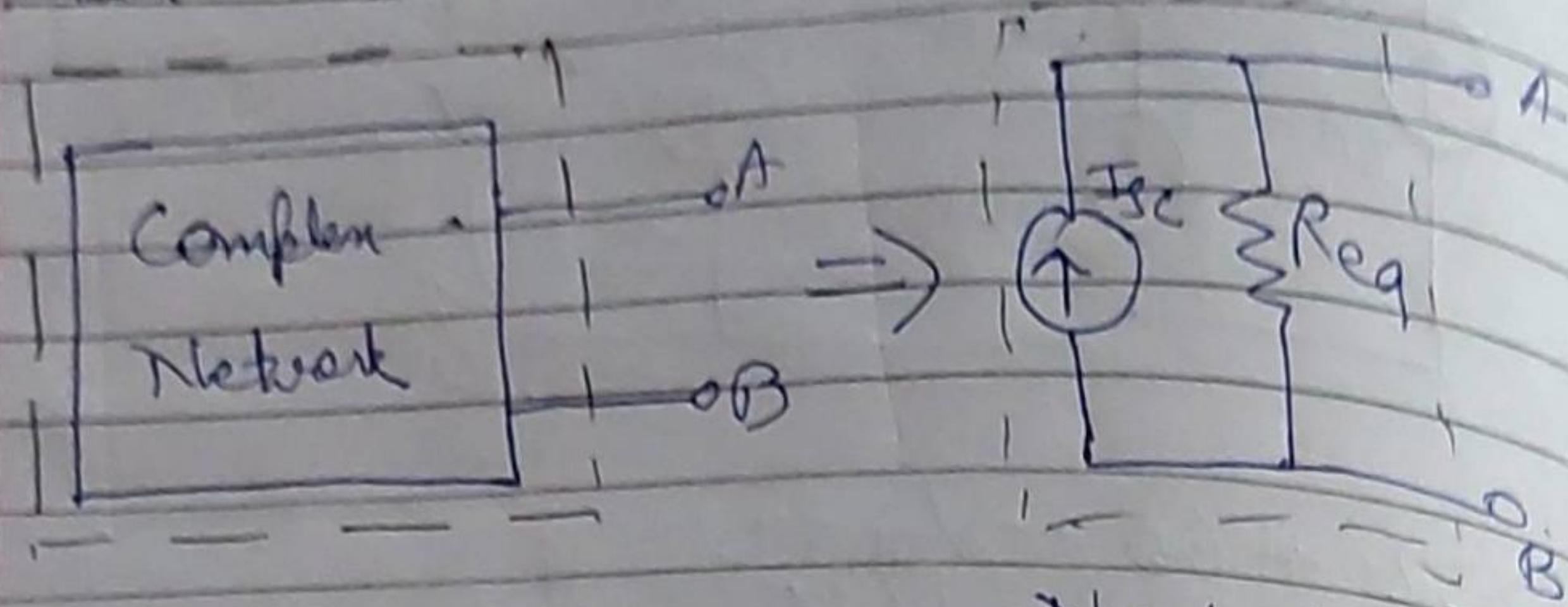
$$8 \times 32 + 60 - V_{TH} = 0 \\ V_{TH} = 316 \text{ V} \quad (\text{B at lower potential})$$



Limitation of Thevenin Theorem →

- ① Not applicable for the circuit consisting of non-linear elements.
- ② Not applicable to the unilateral networks.
- ③ There should not be magnetic coupling between the load and the circuit.
- ④ In the Load side there should not be controlled source.

Norton's Theorem →



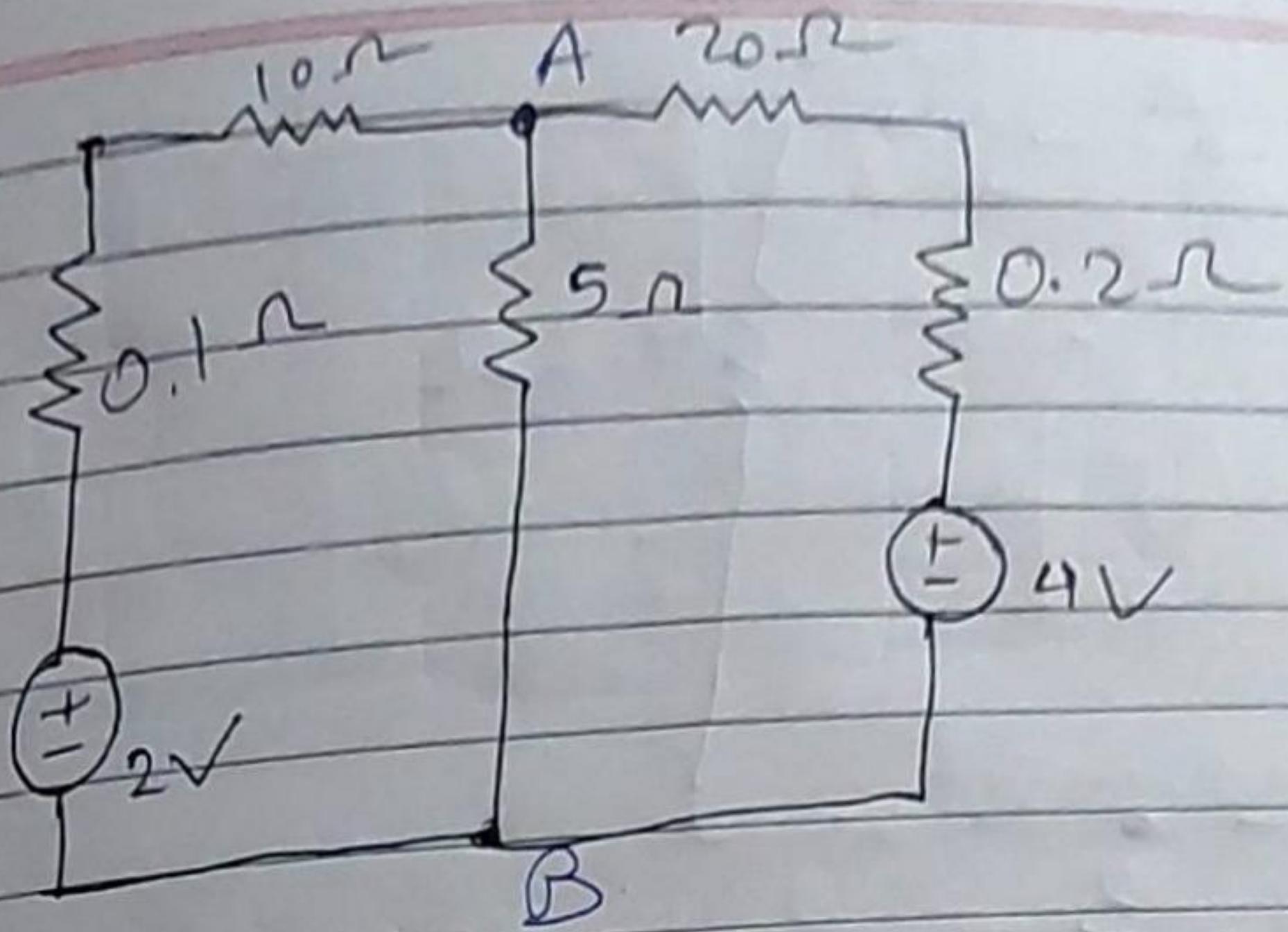
Norton's equivalent circuit

A complex network (active bilateral and linear) consisting of various resistors, dependent and independent sources across any two terminals can be converted into a single current source and an equivalent resistance in parallel with the current source.

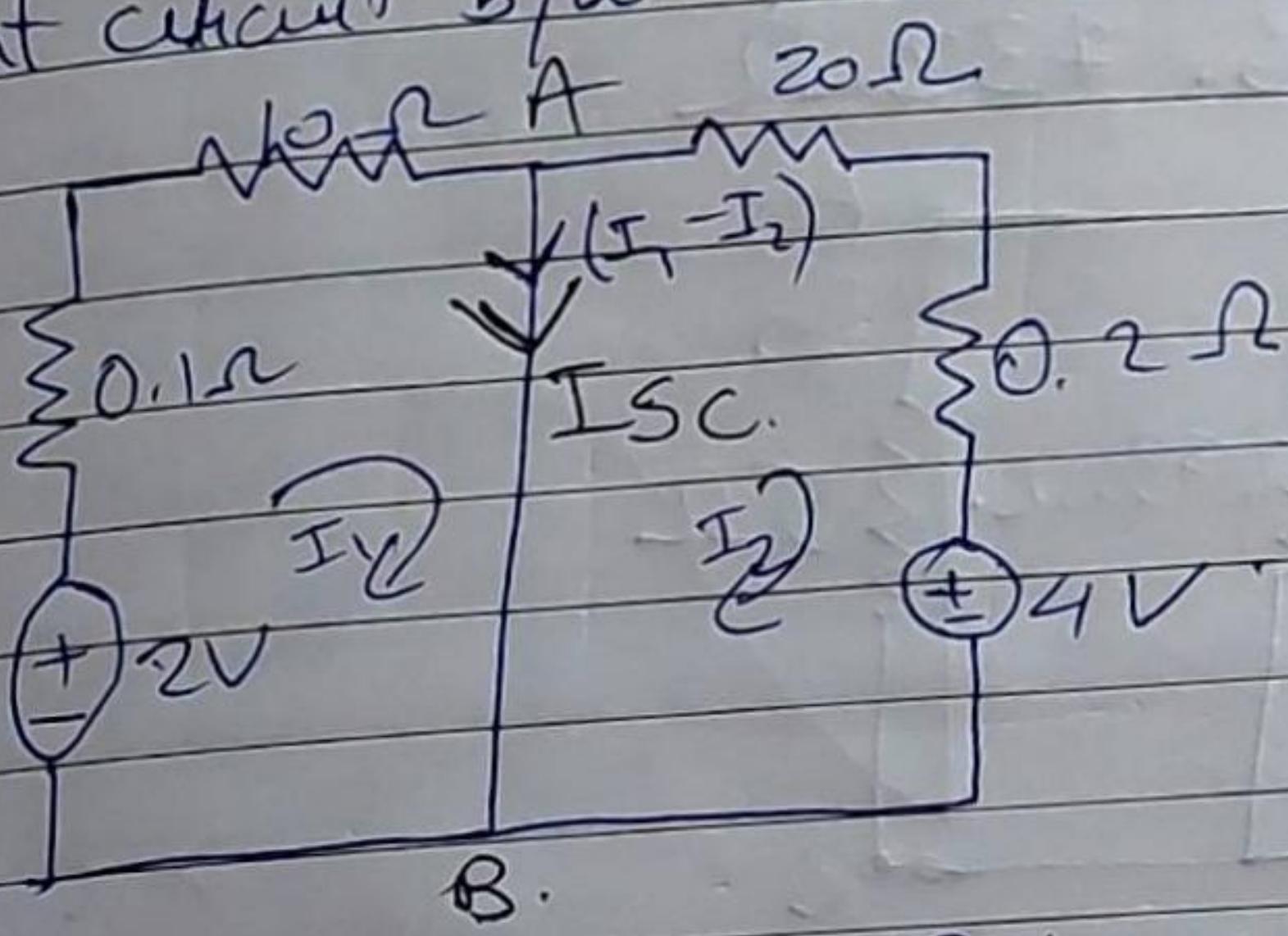
$I_{sc}$  → short circuit current

$R_{eq}$  → It is equivalent resistance b/w terminal A & B when sources are replaced by their internal resistances i.e. voltage source should be short circuit and current source should be open circuit.

Q. Calculate the current across  $5\Omega$  resistor by using Norton's theorem.



- (i) Mark terminal A & B  
 (ii) Short circuit b/w terminal A & B

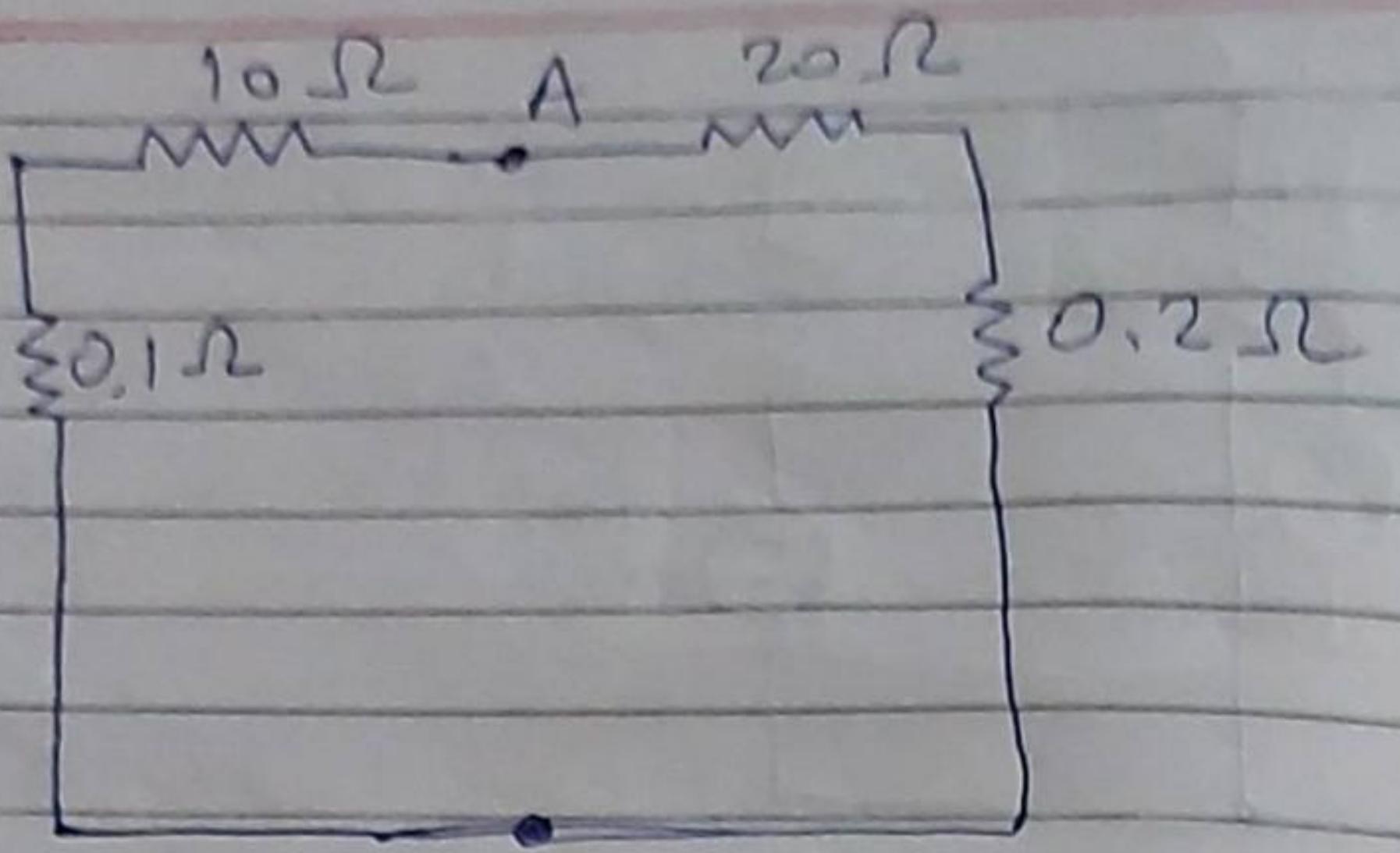


$$\text{KVL} \quad 10I_1 - 2 + 0.1I_1 = 0 \\ 10.1I_1 = 2 \\ I_1 = \frac{2}{10.1} = 0.198 \text{ A}$$

$$20I_2 + 0.2I_2 + 4 = 0$$

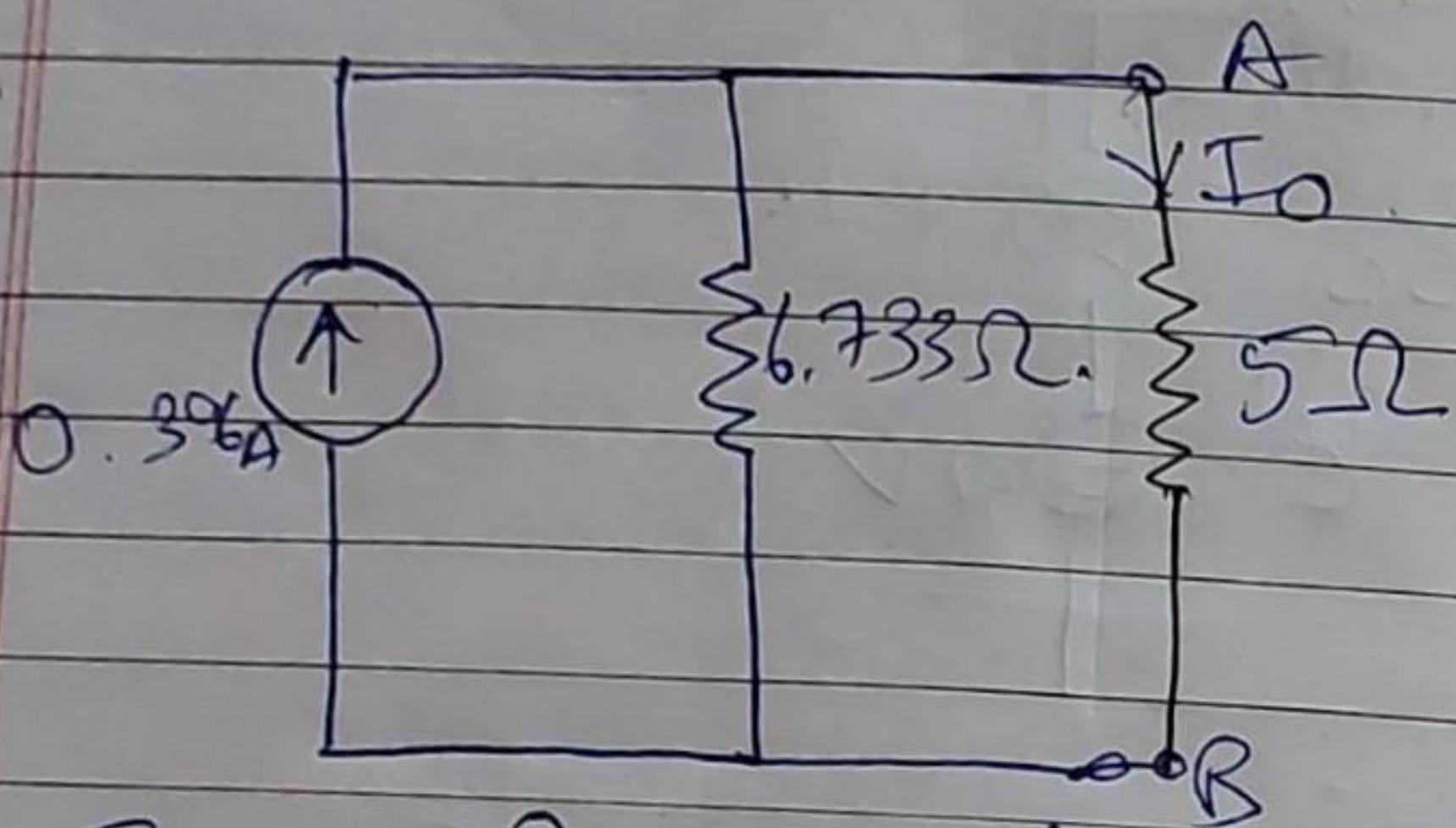
$$20.2I_2 = -4 \\ I_2 = \frac{-4}{20.2} = -0.198 \text{ A}$$

$$I_{SC} = 0.396 \text{ A. (A to B)}$$



$$R_{eq} = \frac{(0.1)(20.2)}{(20.2) + (0.1)} \Omega$$

$$= 6.733 \Omega$$

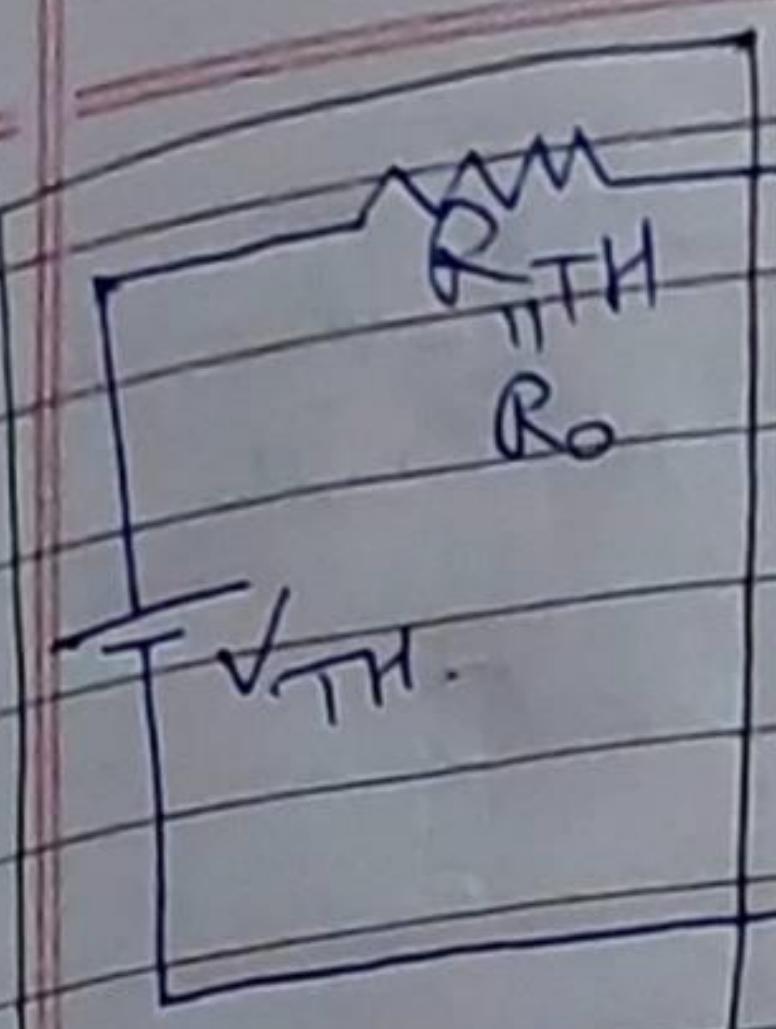


$$I_o = 0.396 \left( \frac{6.733}{6.733 + 5} \right)$$

$$= \frac{0.396 \times 6.733}{11.773}$$

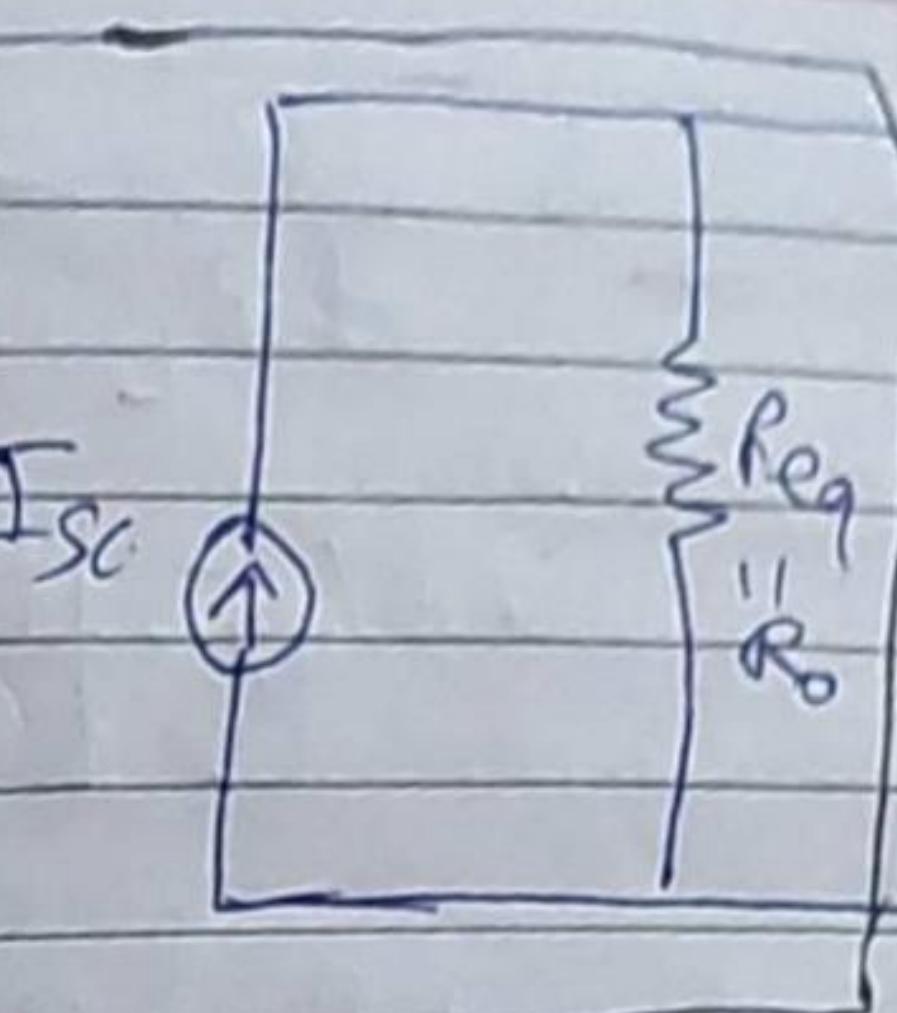
$$= 0.2272 \text{ A}$$

Thevenin - Norton Equivalence →



Thevenin's eq.  
circuit.

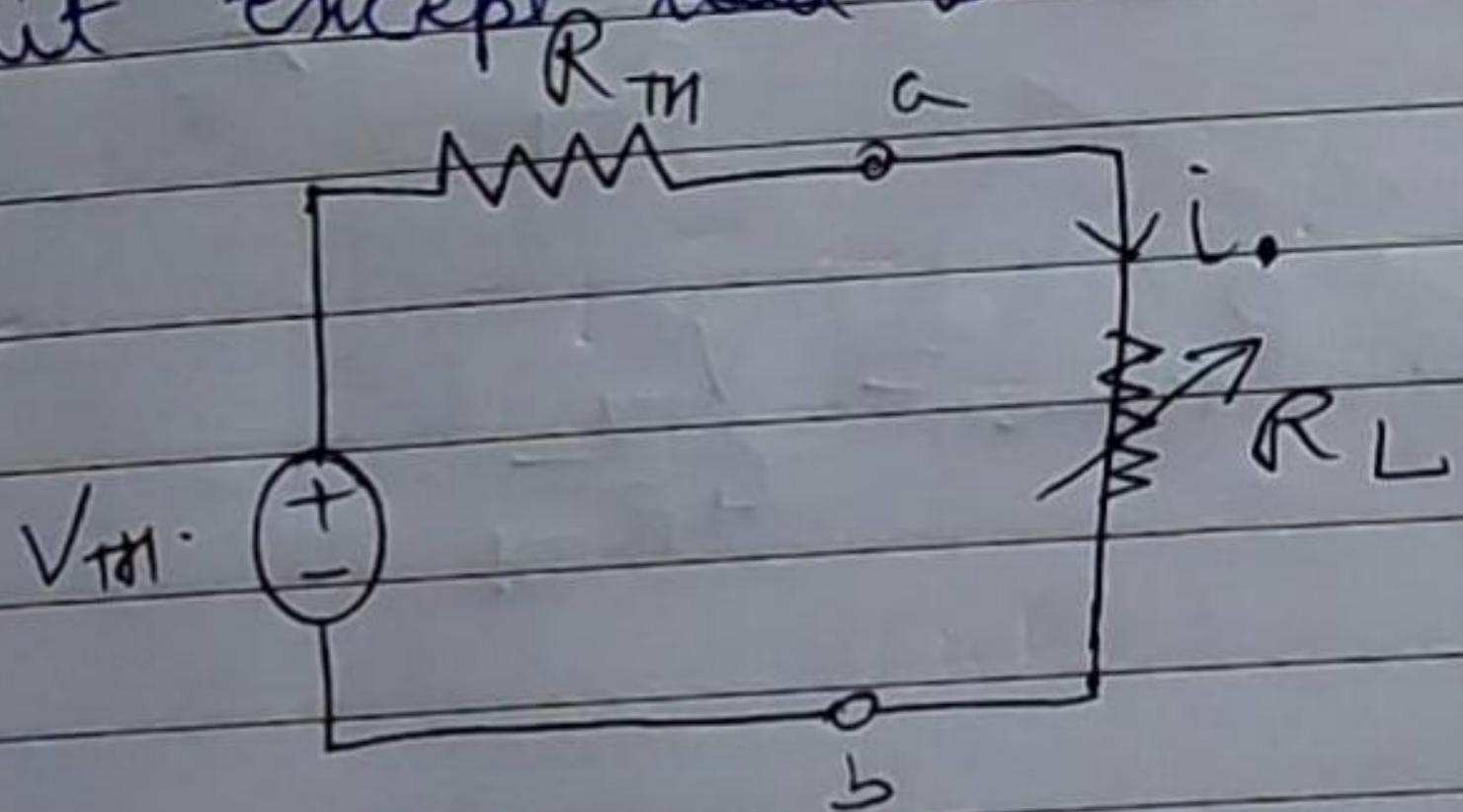
$$V_{TH} = R_{TH} I_{SC} = R_o I_{SC}$$



Norton's eq. circuit

### Maximum Power Transfer Theorem

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. The Thevenin equivalent of entire circuit except load is



i.e. Here,  $R_L$  is variable

The power delivered to the load is -

$$P = i^2 R_L = \left( \frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L$$

\* Maximum power theorem states that maximum power is transferred to the load when the load resistance equals the Thevenin's resistance as seen from the load ( $R_L = R_{TH}$ )

$$\text{Proof} \rightarrow \frac{dP}{dR_L} = V_{TH}^2 \left[ \frac{(R_{TH} + R_L)^2 - 2R_L(R_{TH} + R_L)}{(R_{TH} + R_L)^3} \right]$$

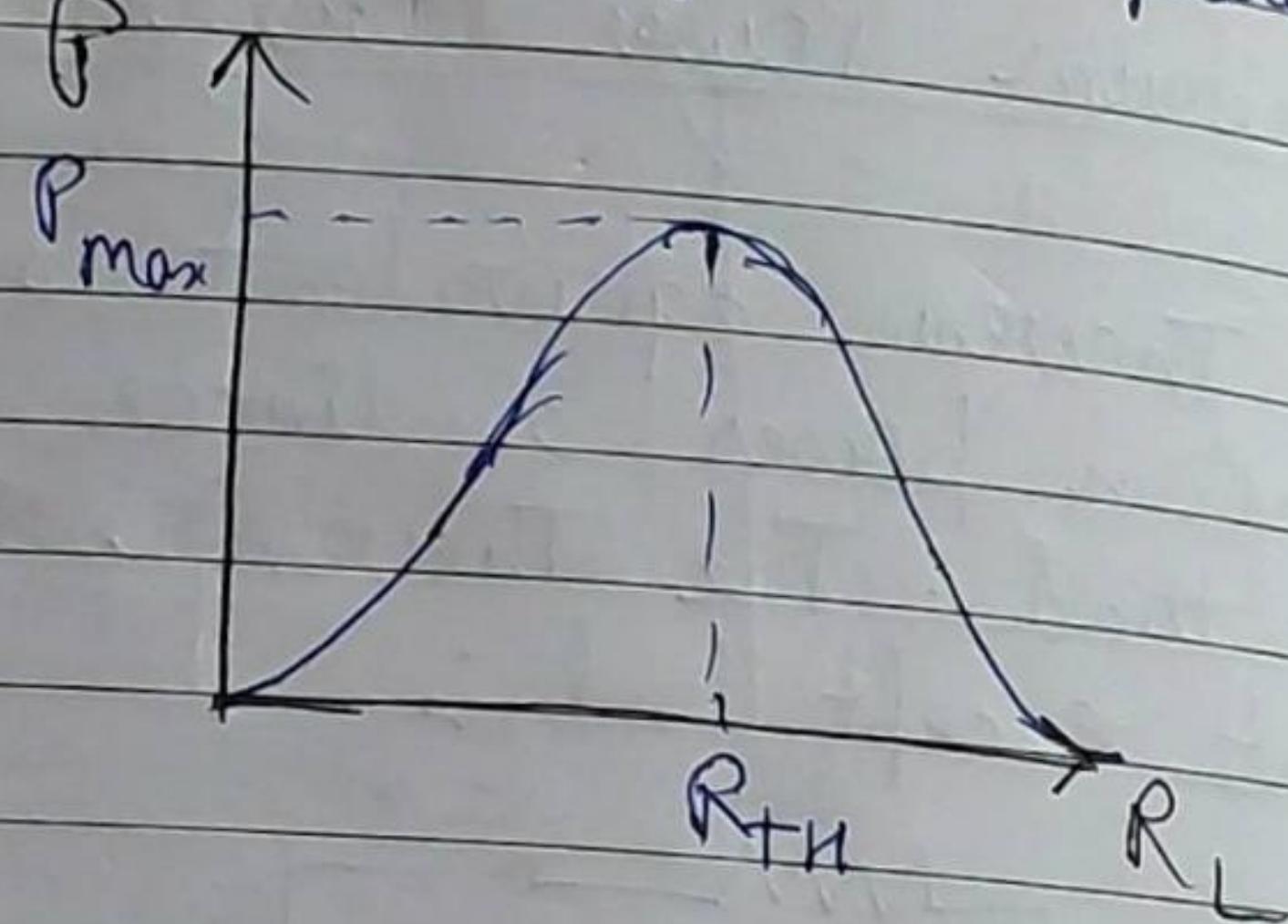
$$= V_{TH}^2 \left[ \frac{R_{TH} + R_L - 2R_L}{(R_{TH} + R_L)^3} \right] = 0$$

$$\Rightarrow R_{TH} + R_L - 2R_L = 0$$

$$\boxed{\begin{aligned} R_{TH} - R_L &= 0 \\ R_{TH} &= R_L \end{aligned}}$$

i.e. maximum power transfer takes place when

$$R_{TH} = R_L$$



$$\text{Maximum power } P_{max} = \frac{V_{TH}^2}{4R_{TH}}$$