UNIT-3 [ELECTROMAGNETIC THEORY]

MAXWELL'S EQUATIONS (DIFFERENTIAL & INTEGRAL FORM)

① $\vec{\nabla}\cdot\vec{D} = \rho$ [Gauss Law of Electrostatics]

② $\vec{\nabla}\cdot\vec{B} = 0$ [Gauss Law of magnetostatics]

③ $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$ [Faraday's Law of electromagnetic induction]

④ $\vec{\nabla}\times\vec{H} = \vec{J} + \dfrac{\partial D}{\partial t}$ [Modified Ampere's Circuital Law]

$\nabla\cdot$ → Divergence $\nabla\times$ → Curl

∇ → Gradient

★ D → Displacement Density

$\vec{D} \propto \vec{E}$ → Electric Field

$$\boxed{\vec{D} = \epsilon_0\vec{E}}$$

★ ρ → Volume charge density
★ B → Magnetic field or Magnetic flux density
★ E → Electric field
★ H → Magnetic Field Intensity
★ J → current density $\Rightarrow \dfrac{I}{A}$

④

★ $\dfrac{\partial D}{\partial t}$ → Displacement current

① $\oint_S \vec{E}\cdot\vec{ds} \Rightarrow \dfrac{q}{\epsilon_0}$

② (closed surface)

Net field $\vec{B} = 0$

$$\boxed{\mathrm{Div}\,\vec{B} \Rightarrow 0}$$

③ Emf $\Rightarrow -\dfrac{d\phi_B}{dt}$

$$\boxed{\int_l \vec{E}\cdot\vec{dl} \Rightarrow -\dfrac{d}{dt}\oint_S \vec{B}\cdot\vec{ds}}$$

② $\boxed{\oint_S \vec{B}\cdot\vec{ds} = 0}$

W.k.t from Maxwell II diff. eqn :-

$\vec{\nabla}\cdot\vec{B} = 0$

Apply volume Integral :-

$\int_V (\vec{\nabla}\cdot\vec{B})\, dV = 0$

By Gauss divergence Theorem

$\boxed{\oint_S \vec{B}\cdot ds = 0}$ Hence Prooved

③ $\boxed{\oint_l \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}\int \vec{B}\cdot\vec{ds}}$

W.k.t from Maxwell III D.E :-

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

Taking surface Integral

$\oint_S (\vec{\nabla}\times\vec{E})\cdot\vec{ds} = -\frac{\partial}{\partial t}\oint_S \vec{B}\cdot\vec{ds}$

Apply Stokes Theorem [line Integral $\rightleftharpoons$ Surface Integral]

$\boxed{\oint_l \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}\oint_S \vec{B}\cdot\vec{ds}}$ Hence Prooved

INTEGRAL FORM :-

① $\oint_S \vec{D}\cdot d\vec{s} = \int_V \rho\, dv$

② $\oint_S \vec{B}\cdot d\vec{s} = 0$

③ $\oint_l \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\oint_S \vec{B}\cdot d\vec{s}$

④ $\int_l \vec{H}\cdot d\vec{s} = \int_S J\cdot ds + \int_S \left[\frac{\partial D}{\partial t}\right]\cdot ds$

∴ INTEGRAL FORM : [DERIVATION]

① $\boxed{\oint_S \vec{D}\cdot d\vec{s} = \int_V \rho\, dv}$

W.K.T :- $\vec{\nabla}\cdot\vec{D} = \rho$

Apply volume integral both side

$\iiint (\vec{\nabla}\cdot\vec{D})\, dV = \int_V \rho\, dv$

Apply Gauss Divergence Theorem :- (Surface Integral ⟶ Volume Integral, Surface Integral ⟵ Volume Integral)

$$\boxed{\oint_S \vec{D}\cdot d\vec{s} = \int_V \rho\, dv}$$

Hence Prooved

$$\boxed{\int_l \vec{H}\cdot d\vec{l} = \int_S \left(\vec{J}+\frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{s}}$$

W.k.t from Maxwell's IV Differential Equation

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Taking Surface Integral

$$\int_S (\vec{\nabla}\times\vec{H})\cdot d\vec{s} = \int_S \left(\vec{J}+\frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{s}$$

Apply Stokes Theorem :-

$$\boxed{\int_l \vec{H}\cdot d\vec{l} = \int_S \left(\vec{J}+\frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{s}}$$ Hence Proved

Prove that :- $\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$ → Gauss Law of Electrostatics

→ Maxwell's Ist Eqn in Differential form

$$\boxed{\oint_S \vec{E}\cdot d\vec{s} = \frac{q}{\epsilon_0}}$$

$$\oint_S \epsilon_0 \vec{E}\cdot d\vec{s} = q = \int_V \rho\, dv \qquad \left\{ q = \int_V \rho\, dv \right\}$$

$$\oint_S \vec{D}\cdot d\vec{s} = \int_V \rho\, dv \qquad \left\{ \vec{D} = \epsilon_0 \vec{E} \right\}$$

Apply Gauss Divergence Theorem :-

$$\int_V (\vec{\nabla}\cdot\vec{D})\, dv = \int_V \rho\, dv$$

$$\int_V \left[(\vec{\nabla}\cdot\vec{D}) - \rho\right] dv = 0$$

$$\vec{\nabla}\cdot\vec{D} - \rho = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$$

Maxwell IIIrd Eq^n in Differential form:-

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ → It is based on Faraday's Law of E·M·I.

$$\boxed{emf = \frac{-d}{dt}\phi_B} \rightarrow (I)$$

But $$\boxed{emf = \oint_l \vec{E}\cdot\vec{dl}} \rightarrow (II)$$

By compairing eq (I) & eq (II)

$$\oint_l \vec{E}\cdot\vec{dl} = -\frac{d}{dt}\phi_B$$

$$B = \frac{\phi_B}{A}$$

$$\phi_B = B\cdot A$$

$$\oint_l \vec{E}\cdot\vec{dl} = \frac{-d}{dt}\oint_S \vec{B}\cdot\vec{dS}$$

$$\oint_l \vec{E}\cdot\vec{dl} = -\oint_S \frac{\partial \vec{B}}{\partial t}\cdot\vec{dS}$$

By stokes curl Theorem :-

$$\oint_S (\vec{\nabla}\times\vec{E})\cdot\vec{dS} = -\oint_S \frac{\partial \vec{B}}{\partial t}\cdot\vec{dS}$$

$$\oint_S \left[(\vec{\nabla}\times\vec{E}) + \frac{\partial \vec{B}}{\partial t}\right]\cdot\vec{dS} = 0$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ Hence Proved

Maxwell's IV^{th} Eq^n in Differential form :-

Requirements :-
① continuity Eq^n
② Ampere's Law
③ Vector identity
④ Stokes curl Theorem
⑤ Displacement Current

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

This eq^n is based on Ampere's circuital Law :-

$$\int_l \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$\boxed{\int_l \vec{H} \cdot \vec{dl} = I} \quad \left[\because \vec{B} = \mu_0 \vec{H}\right] \longrightarrow ①$$

$$\int_l \vec{H} \cdot \vec{dl} = \int_S \vec{J} \cdot \vec{dS} \quad \left[\because I = \int_S \vec{J} \cdot \vec{dS}\right]$$

Apply stokes curl theorem :-

$$\int_S (\vec{\nabla} \times \vec{H}) \cdot \vec{dS} = \int_S \vec{J} \cdot \vec{dS}$$

$$\int_S \left[(\vec{\nabla} \times \vec{H}) - \vec{J}\right] \cdot \vec{dS} = 0$$

$$\vec{\nabla} \times \vec{H} - \vec{J} = 0$$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J}} \longrightarrow ⑪$$

To check validity of eq ⑪ [Use eq^n of continuity]

$$\boxed{\vec{\nabla} \cdot \vec{J} = \frac{-\partial \rho}{\partial t}}$$

Taking divergence of eq ⑪

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J}$$

from vector identity :- $\boxed{\text{Div}(\text{curl} \vec{H}) = 0}$

Hence $\boxed{\vec{\nabla} \cdot \vec{J} = 0}$

But from eq^n of continuity

$$\boxed{\vec{\nabla} \cdot \vec{J} \neq 0}$$

Because $\vec{\nabla}\cdot\vec{J} = 0$, Hence we can say eq (11) is not valid eq^n.

To make eq (11) valid Maxwell's Introduced the concept of Displacement current.

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \vec{J}_d} \quad \text{(111)}$$

$\vec{J}_d \rightarrow$ Displacement current Density

Taking Divergence of Eq (3):

$$\text{Div}\left(\text{curl } \vec{H}\right) = \text{Div}\cdot\vec{J} + \text{Div}\cdot\vec{J}_d$$

$$0 = \vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{J}_d$$

$$\boxed{\vec{\nabla}\cdot\vec{J} = -\vec{\nabla}\cdot\vec{J}_d}$$

That means $\vec{\nabla}\cdot\vec{J} \neq 0$ for eq (3)

Hence from eq^n of continuity it is a valid equation.

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \vec{J}_d}$$

from eq^n of continuity :-

$$-\frac{\partial \rho}{\partial t} = -\vec{\nabla}\cdot\vec{J}_d$$

$$\vec{\nabla}\cdot\vec{J}_d = \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{D}\right)$$ [from Maxwell's Ist eq^n]

$$\vec{\nabla}\cdot\vec{J}_d = \vec{\nabla}\cdot\frac{\partial \vec{D}}{\partial t}$$

$$\boxed{\vec{J}_d = \frac{\partial \vec{D}}{\partial t}}$$

Put the value of J_d in eq ③

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}} \longrightarrow ③$$

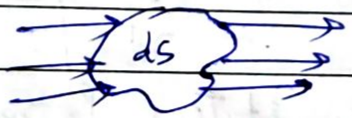
This is Maxwell's IV^{th} Differential Equation.

$$\boxed{\int_l \vec{B}\cdot\vec{dl} = \mu_0(I+I_d)} \longrightarrow \text{Modified Ampere's circuital Law}$$

Maxwell's Second Eq^n [Gauss Law of Magnetostatics Derivation]

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$$\phi_B = \oint_S \vec{B}\cdot\vec{ds} = 0 \longrightarrow ①$$



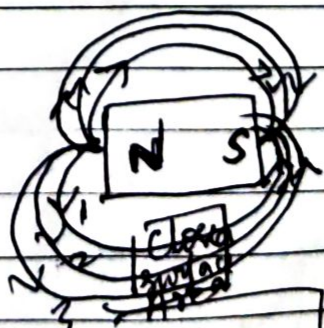
* Mag. flux across closed surface is always equal to zero.

$$\boxed{B = \frac{\phi_B}{\text{Area}}}$$

By applying Gauss Divergence Theorem in eq ①

$$\int_V (\vec{\nabla}\cdot\vec{B})\,dV = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0} \longrightarrow ②$$



$$\boxed{\vec{\nabla}\vec{B} = 0}$$

Physical Significance of Continuity Eq^n & Eq^n of Continuity for Steady state :-

① Signify the principle of charge conservation.

② Current Density ($\vec{J}$) represents charge flowing per unit time per unit area in continuity Eq^n.

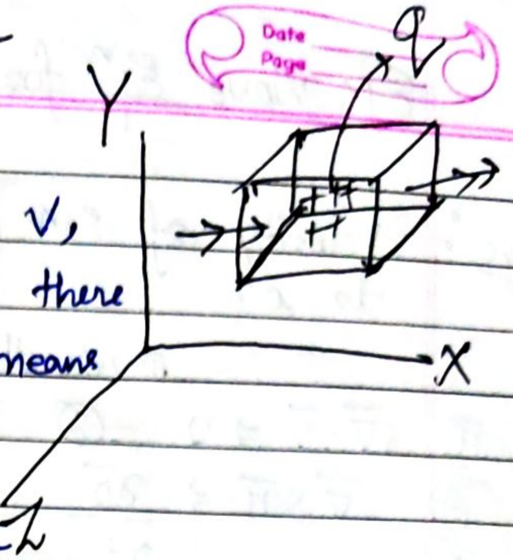
③ Divergence of current density is equal to the negative rate of change of volume charge density.

$$\boxed{\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}} \rightarrow \text{from eq}^n \text{ of continuity}$$

for steady state ρ remains constant for time varying mag. field.

So, $\boxed{\vec{\nabla}\cdot\vec{J} = 0} \rightarrow$ Eq^n of continuity for steady state

EQUATION OF CONTINUITY :-



Let q charges in a cube of volume V, come out from the surface. If there is a flow of charge that means current will be there. As the current flows there will be decrease in charges present inside the cube wrt time

Here :- $\boxed{I = -\frac{dq}{dt}} \longrightarrow (1)$

where $q \Rightarrow \int_V \rho \, dv$

& $I \Rightarrow \oint_S \vec{J} \cdot \vec{ds} \longrightarrow (2)$

from the law of conservation of charge :

$$\oint_S \vec{J} \cdot \vec{ds} \Rightarrow -\frac{dq}{dt}$$

$$\oint_S \vec{J} \cdot \vec{ds} \Rightarrow -\frac{d}{dt} \int_V \rho \, dv$$

By Gauss divergence theorem :-

$$\int_V (\vec{\nabla} \cdot \vec{J}) \, dv \Rightarrow -\frac{d}{dt} \int \rho \, dv$$

$$\int_V (\vec{\nabla} \cdot \vec{J}) \, dv \Rightarrow \int -\frac{\partial \rho}{\partial t} \, dv$$

$$\int_V \left[(\vec{\nabla} \cdot \vec{J}) + \frac{\partial \rho}{\partial t} \right] dv \Rightarrow 0$$

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} \Rightarrow 0$$

Eq^n of Continuity

$$\boxed{\vec{\nabla} \cdot \vec{J} \Rightarrow -\frac{\partial \rho}{\partial t}}$$

for stationary currents, $\boxed{div J \Rightarrow 0}$

EM Wave Eq^n for free space in terms of $\vec{E}$:

Prove :- Velocity of EM wave in free space is equal to c.

Maxwell's Eq^n in free space

① $\vec{\nabla}\cdot\vec{D} = 0$, ② $\vec{\nabla}\cdot\vec{B} = 0$, ③ $\vec{\nabla}\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

④ $\vec{\nabla}\times\vec{H} = \dfrac{\partial \vec{D}}{\partial t}$

$$\boxed{\begin{matrix}\sigma = 0 \\ J = \sigma\vec{E} = 0 \\ \rho = 0\end{matrix}}$$

$\left[\begin{matrix}\vec{D} = \epsilon_0\vec{E} \\ \text{in free space}\end{matrix}\right]$

from Maxwell's eq ③ in free space :-

$$\vec{\nabla}\times\vec{E} = \frac{-\partial \vec{B}}{\partial t} \longrightarrow ①$$

Take curl of eq ①

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \frac{-\partial}{\partial t}(\vec{\nabla}\times\vec{B}) \longrightarrow ②$$

from vector identity :-

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \vec{\nabla}^2\vec{E}$$

from eq ② :-

$$\nabla(\vec{\nabla}\cdot\vec{E}) - \vec{\nabla}^2\vec{E} = \frac{-\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$\because$ $\vec{\nabla}\cdot\vec{D} = 0$ or $\vec{\nabla}\cdot\vec{E} = 0$ for free space

& $\vec{B} = \mu_0\vec{H}$

$$0 - \vec{\nabla}^2\vec{E} = -\mu_0\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H})$$

from eq ④ in free space

$$\vec{\nabla}\times\vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla}^2\vec{E} = \mu_0\frac{\partial}{\partial t}\left(\frac{\partial \vec{D}}{\partial t}\right)$$

$$\boxed{\vec{\nabla}^2\vec{E} = \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}} \rightarrow ③ \quad \left\{\begin{matrix}\because \vec{D} = \epsilon_0\vec{E} \\ \text{free space}\end{matrix}\right\}$$

(ii) for General wave Expression :-

$$\boxed{\vec{\nabla}^2 \varphi = \frac{1}{v^2} \frac{\partial^2 \varphi}{\partial t^2}} \longrightarrow (4)$$

By comparing eq (3) & eq (4)

$$\frac{1}{v^2} = \mu_0 \epsilon_0$$

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}} \longrightarrow (5)$$

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \frac{\sqrt{4\pi}}{\sqrt{4\pi}} = \frac{1}{\sqrt{4\pi\epsilon_0}} \frac{\sqrt{4\pi}}{\sqrt{\mu_0}}$$

$$v = \sqrt{9\times10^9} \times \frac{\sqrt{4\pi}}{\sqrt{4\pi\times10^{-7}}} = \sqrt{9\times10^9\times10^7}$$

$$v = \sqrt{9\times10^{16}}$$

$$\boxed{v = 3\times10^8 \ m/s}$$

$$\boxed{v = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c}$$

EM Wave Eq^n for free space in terms of $\vec{H}$:-

for free space :- $\boxed{\sigma = 0 \quad \rho = 0 \\ \vec{J} = 0}$

① $\vec{\nabla} \cdot \vec{D} = 0$

② $\vec{\nabla} \cdot \vec{B} = 0$

③ $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

④ from Maxwell's 4^{th} Eq^n in free space :-

$$\boxed{\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}} \longrightarrow ①$$

Take curl of eq ① :-

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{D}) \longrightarrow ②$$

from vector identity :-

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{H}) - \vec{\nabla}^2 \vec{H}$$

So from eq ② :-

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{H}) - \vec{\nabla}^2 \vec{H} = \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{D}) \longrightarrow ③$$

from Maxwell's eq ② :-

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\text{So } \vec{\nabla} \cdot \vec{H} = 0 \quad \{\because \vec{B} = \mu_0 \vec{H}\}$$

$$0 - \vec{\nabla}^2 \vec{H} = \epsilon_0 \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{E}) \quad \{\because \vec{D} = \epsilon_0 \vec{E}\}$$

$$-\vec{\nabla}^2 \vec{H} = \epsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial \vec{B}}{\partial t}\right) \quad \left\{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\right\}$$

$$\boxed{\vec{\nabla}^2 \vec{H} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t^2}} \quad \{\vec{B} = \mu_0 \vec{H}\} \longrightarrow ④$$

To find velocity of EM wave in free space,

$$\vec{\nabla}^2 \varphi = \frac{1}{v^2} \frac{\partial^2 \varphi}{\partial t^2} \longrightarrow (5)$$

compare eq (4) & (5)

$$\frac{1}{v^2} = \epsilon_0 \mu_0 \quad \text{or} \quad \boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c}$$

Hence velocity of EM wave in free space is equal to speed of light.

Maxwell's Eq^n for $\boxed{\text{free Space}}$:-

In diff form

1. $\vec{\nabla} \cdot \vec{D} = \rho$
2. $\vec{\nabla} \cdot \vec{B} = 0$
3. $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
4. $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

No conduction current in free space $\boxed{\sigma = 0}$

& $\vec{J} = \sigma \vec{E} = 0$

$\boxed{\vec{J} = 0}$ $\boxed{\rho = 0}$

$\boxed{\vec{D} = \epsilon_0 \vec{E}}$ $\boxed{\vec{B} = \mu_0 \vec{H}}$

Maxwell's Eq^n for free space :-

1. $\boxed{\vec{\nabla} \cdot \vec{D} = 0}$ or $\vec{\nabla} \cdot \vec{E} = 0$
2. $\boxed{\vec{\nabla} \cdot \vec{B} = 0}$ or $\vec{\nabla} \cdot \vec{H} = 0$
3. $\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$

   $\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$
4. $\vec{\nabla} \times \vec{H} = 0 + \frac{\partial \vec{D}}{\partial t}$

   $\boxed{\vec{\nabla} \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$

To find velocity of EM wave in free space.

$$\vec{\nabla}^2 \varphi = \frac{1}{V^2}\frac{\partial^2 \varphi}{\partial t^2} \longrightarrow (5)$$

compare eq (4) & (5)

$$\frac{1}{V^2} = \epsilon_0 \mu_0 \quad \text{or} \quad \boxed{V = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c}$$

Hence velocity of EM wave in free space is equal to speed of light.

Maxwell's Eq^n for free Space :-

In diff form

1. $\vec{\nabla}\cdot\vec{D} = \rho$
2. $\vec{\nabla}\cdot\vec{B} = 0$
3. $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$
4. $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

No conduction current in free space $\boxed{\sigma = 0}$

& $\vec{J} = \sigma\vec{E} = 0$

$\boxed{\vec{J} = 0}$ $\boxed{\rho = 0}$

$\boxed{\vec{D} = \epsilon_0 \vec{E}}$ $\boxed{\vec{B} = \mu_0 \vec{H}}$

Maxwell's Eq^n for free space :-

(1) $\boxed{\vec{\nabla}\cdot\vec{D} = 0}$ or $\vec{\nabla}\cdot\vec{E} = 0$

(2) $\boxed{\vec{\nabla}\cdot\vec{B} = 0}$ or $\vec{\nabla}\cdot\vec{H} = 0$

(3) $\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$

$\vec{\nabla}\times\vec{E} = -\mu_0\frac{\partial \vec{H}}{\partial t}$

(4) $\vec{\nabla}\times\vec{H} = 0 + \frac{\partial \vec{D}}{\partial t}$

$\boxed{\vec{\nabla}\times\vec{H} = \epsilon_0\frac{\partial \vec{E}}{\partial t}}$

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Velocity of EM wave in a medium :-

for free space EM wave eqⁿ :- $\boxed{V = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \Rightarrow c} \rightarrow (1)$

in a medium, the velocity of EM wave from eq (1)

$$\boxed{V = \frac{1}{\sqrt{\mu \epsilon}}} \rightarrow (2)$$

from relation :- $\mu_r = \frac{\mu}{\mu_0}$, $\epsilon_r = \frac{\epsilon}{\epsilon_0}$

$$\boxed{\mu = \mu_r \mu_0} \ \& \ \boxed{\epsilon \Rightarrow \epsilon_r \epsilon_0}$$

Put these values in eq (2)

$$\boxed{V = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}} \Rightarrow \frac{c}{\sqrt{\mu_r \epsilon_r}}}$$

for non-magnetic material :-

$$\mu_r = 1 = \frac{\mu}{\mu_0}$$

$$V = \frac{c}{\sqrt{\epsilon_r}} \quad \text{or} \quad \boxed{\frac{c}{V} \Rightarrow \sqrt{\epsilon_r}}$$

$$\boxed{\eta = \sqrt{\epsilon_r}}$$

POYNTING VECTOR & POYNTING THEOREM

Poynting Theorem :- ~~To find the power in a uniform plane~~

* The sum of the time rate of change of electromagnetic energy within a certain volume and the time rate of energy flowing through out the boundary surface is equal to power transferred into the Electromagnetic field.

$$\boxed{-\int_V E\cdot J\,dv = \frac{d}{dt}\int_V\left[\frac{1}{2}(B\cdot H + E\cdot D)\right]dv + \oint_S (E\times H)\,ds}$$

Poynting Theorem also known as Work energy theorem or the energy conservation law in electromagnetism.

POYNTING VECTOR :- The main feature of EM waves is that it can transport energy from one point to another point. The rate of energy flow per unit area in a plane EM waves is defined by a vector $\vec{S}$ called Poynting Vector.

$\boxed{\text{Unit :- Watt}/m^2}$

$$\boxed{\vec{S} = \vec{E}\times\vec{H}} \longrightarrow (1)$$

$$\boxed{\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})}$$

from eq (1), it is clear that the direction of Poynting vector $\vec{S}$ is perpendicular to both $\vec{E}$ & $\vec{H}$, while $\vec{S}$ must be along $\vec{K}$.

:- SOME IMPORTANT FORMULAES :-

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① **Poynting Theorem :-**

$$\boxed{S \text{ or } P = |\vec{E} \times \vec{H}| = EH \sin 90^\circ}$$

② **Poynting Vector :-**

$$\boxed{S \text{ or } P \Rightarrow \frac{\text{Power Transferred}}{\text{Area}}}$$

③ **Relation b/w E & H :-**

Wave Impedance (Z_0)

$$\boxed{\frac{E}{H} \Rightarrow \sqrt{\frac{\mu_0}{\epsilon_0}} \Rightarrow 376.7\,\Omega \approx 377\,\Omega}$$

④ **Amplitude of Electric field & Magnetic field :-**

$$\boxed{E_0 \Rightarrow E_{avg}\sqrt{2} \quad \& \quad H_0 \Rightarrow H_{avg}\sqrt{2}}$$

⑤ Velocity of light (in free space) $c \Rightarrow \frac{1}{\sqrt{\mu_0 \epsilon_0}} \Rightarrow 3 \times 10^8 \ m/s$

⑥ Velocity of EM wave propagating through non-conducting medium

$$V \Rightarrow \frac{1}{\sqrt{\mu \epsilon}} \Rightarrow \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} \Rightarrow \frac{c}{\sqrt{\mu_r \epsilon_r}}$$

⑦ Attenuation Constant (conducting medium)

$$\boxed{\alpha \Rightarrow \sqrt{\frac{\omega \mu \sigma}{2}}}$$