①

Heat Transfer → Heat Transfer is the transport of heat energy from one point in a medium to another or from one medium to another in the presence of a tempr gradient or tempr difference. The tempr diff b/w two points in the same medium or b/w two mediums which are in thermal contacts, is know as the driving force for heat transfer.

Modes of Heat transfer :- There are three modes of Heat Transfer.

① Conduction
② Convection
③ Radiation.

① Conduction :- Conduction is the only mode of heat transfer in a solid medium. It may also occur in a stagnant gaseous or liquid medium. The basic law of conduction is Fourier's Law.

② Convection → It is the transport of heat energy by the way of displacement of fluid element from one point to another point which is at tempr difference.

It is of two types.

ⓐ forced Convection.
ⓑ free Convection (Natural Convection)

③ Radiation → Radiation differs from other two modes of heat transfer mechanism in that it does not require presence of material or medium to take place.

Infact Energy transfer by radiation is fastest as speed of Light. It suffer no losses in the vaccum.

The rate of release of such Energy is proportional to the fourth power of the absolute tempr of the body, and the basic governing law is known as Stefan- Boltzmann law.

Mechanism of Heat Conduction → There are two mechanism for Heat transfer through Conduction in solid.

(a) flow of free Electron → mostly in metal.

(b) Lattic vibration movment → in nonmetal.

→ In Case of metal maxm Problem of heat is Conducted ~~by~~ via flow of free Electron and only small portion by lattice vibration

→ In Case of non metal the No of free electron are absent or very less and maxm amt of heat is Conducted via Lattice vibration movment.

Steady State Conduction in One-Dimension

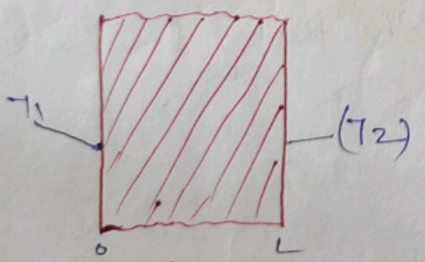
Conduction means transport of heat energy in a medium from a region at a higher tempr to a region at a lower tempr. without any macroscopic motion in the medium.

The diff in tempr b/w the region causes the flow of heat and is called the tempr driving force. Heat Conduction is also called the diffusion of heat.

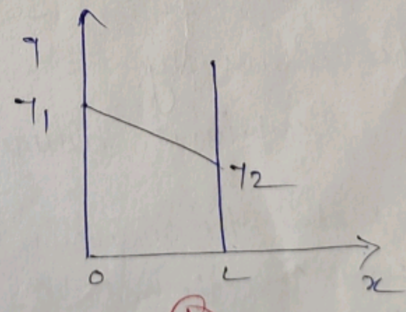
Fourier's Law → The basic law of heat Conduction in a medium was established by J.B.J. Fourier in 1822. The law states that " If two plane parallel surface each having an area A are seprated by a distance l are maintained at tempr (T_1) and (T_2) ($T_1 > T_2$) the rate of heat Conduction (Q) at steady state through the wall is given by

$$\boxed{Q = KA\frac{(T_1 - T_2)}{L}} \quad \text{—} (1)$$

K = thermal Conductivity



(a)
Wall



(B)
Tempr profile.

In differential form the fourier's low expressed as.

$$\boxed{q_x = -k\frac{dT}{dx}} \quad \text{—} (2)$$

(q_x) ⇒ heat flux
$\left(\frac{dT}{dx}\right)$ ⇒ Tempr gradient in x dir

where (q_x) is heat flux.

(the rate of heat Conduction in the x-dir per unit area normal to x-dir)

Thermal Conductivity :-

$$K = \frac{(q_x)}{-\left(\frac{dT}{dx}\right)} = \frac{(Q/A_x)}{-\frac{(\Delta T)}{(\Delta x)}} = \frac{\left(\frac{Watt}{m^2}\right)}{(°C)/(m)} \left(\frac{Watt}{m \cdot °C}\right)$$

$$\text{if}\{(\Delta T) = \Delta x = A_x = 1\} \Rightarrow$$

Thermal Conductivity is define as rate of heat flow through unit area unit thickness and unit tempr difference is maintained through the body.

Effect of tempr on thermal Conductivity →

1. Solid
   (a) Metal
   (b) Non-Metal.

Metal $\{K = K_{lattice} + K_{free}\}$

for metal: $\{K_{free} >>> K_{lattice}\}$

$$\boxed{K \approx K_{free}}$$

In Case of Metal as tempr (↑es) the molecular vibration ↑es (in turn ↓ing the mean free path of molecules) So they obstruct the flow of free electron thus reducing the Conductivity.

Exception is aluminium (K↑es, T↑es)

In Case of Non-metal the maximum portion of heat is Conducted by lattice vibration as tempr ↑es the molecule vibrate more frequently. hence K↑es with ↑ing the thermal Conductivity.

Gases → In Case of Gases as T↑es the velocity of molecule get ↑es hence the No of Collision get ↑es hence K↑es with Tempr.

~~Heat ...~~ Steady state heat ... (partially hidden) cross-sectional

(3) Liquid → from Experimental result It has shown that K decreases with increases in the Temp^r.

$\boxed{K \downarrow es, T \uparrow es}$

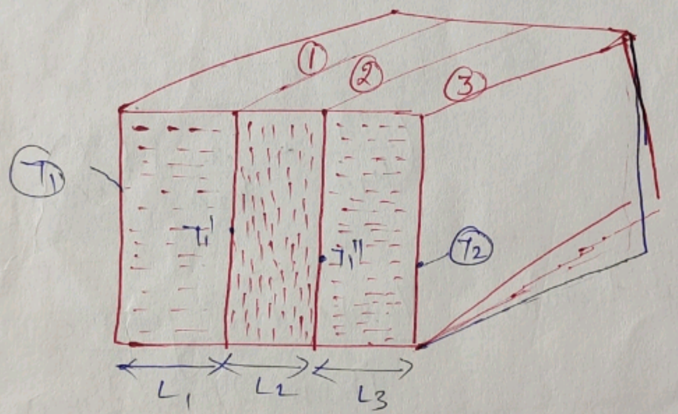
factor affecting thermal Conductivity

(1) Temp^r & pressure
(2) Phase evolved
(3) Void fraction or porosity in solid.

$\boxed{K_{Solid} > K_{liquid} > K_{Gas}}$

* If the thermal Conductivity of a solid is same in all direction, the material is Called isotropic. but there are some material in which Conductivity depend upon the dir as well. Such material are called anisotropic

Steady state Conduction of heat through a Composite solid :-



Let us Consider a Composite wall Consisting of three layers of materials 1, 2 & 3 and having thickness l_1, l_2 & l_3 having thermal Conductivity K_1, K_2 & K_3 respectively

The area of heat Conduction is Constant. Therefore rate of heat flow at steady state through the individual layer are equal

So

$$Q_1 = Q_2 = Q_3 = Q$$

The rates of heat flow through the wall as given by fouriers Law are as following.

Layer 1 $\Rightarrow$ $Q_1 = \frac{K_1 A(T_1 - T_1')}{l_1}$ —(1)

~~Heat Tr~~ Steady state heat Tr through the ...

Rate of Heat Tr through varying cross-sectional area ⇒

Layer 2 ⇒ $Q_2 = \dfrac{K_2 A (T' - T'')}{L_2}$ — (II)

Layer 3 ⇒ $Q_3 = \dfrac{K_3 A (T'' - T_2)}{L_3}$ — (III)

$\{Q_1 = Q_2 = Q_3 = Q\}$

from Equn (I) $T_1 - T' = \dfrac{Q L_1}{K_1 A}$ — (IV)

Equn (II) $T' - T'' = \dfrac{Q L_2}{K_2 A}$ — (V)

Equn (III) $T'' - T_2 = \dfrac{Q L_3}{K_3 A}$ — (VI)

Add Equn (IV) (V) & (VI)

$$(T_1 - T_2) = Q\left(\frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A}\right)$$

$$Q = \frac{(T_1 - T_2)}{\left(\frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A}\right)}$$

$$\boxed{\text{Rate of heat transfer} = \frac{\text{Tempr driving force}}{\text{Thermal resistance}}}$$

Thermal resistance of Layer (1) $R_1 = \left(\dfrac{L_1}{K_1 A}\right)$

" " " (2) $R_2 = \left(\dfrac{L_2}{K_2 A}\right)$

" " " (3) $R_3 = \left(\dfrac{L_3}{K_3 A}\right)$

Q-7

~~Heat TH~~ Steady state heat flow through the Composite Cylinder (5)

Rate of Heat flow through varying cross-sectional area → (4)

Lets Cancider a hallow cylinder of inside radius (r_i), outside radius (r_o) and length (L) as shown in fig. the inner and outer covered surface are maintaine

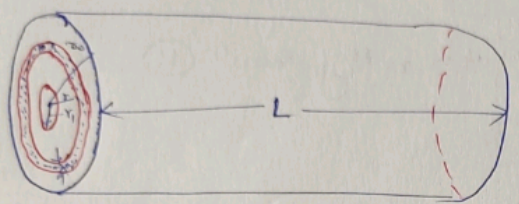
Assumption

1-D

Steady state

$q_g = 0$

$K =$ Constant



(Radial heat Conduction through a thin cylindrical shell)

Apply the heat balance over a thin cylindrical shell inside radius (r) & thickness (Δr)

~~Rate~~

(Heat in) - (Heat out) $\pm$ (Heat Apperance / Disapperance) = Heat rate of heat in the thin (C.V) cylindric shell

$$Q_r - Q|_{r+\Delta r} + q_g 2\pi r \Delta r L = 2\pi r \Delta r \rho c_p \left(\frac{\partial T}{\partial t}\right)$$

divided by (Δr)

$$\frac{(Q|_r - Q|_{r+\Delta r})}{\Delta r} + q_g \frac{2\pi r \cancel{\Delta r} \times L}{\cancel{\Delta r}} = \frac{2\pi r L \cancel{\Delta r} \rho c_p}{\cancel{\Delta r}} \frac{\partial T}{\partial t}$$

$$-\frac{dQ_r}{dr} + q_g 2\pi r \times L = 2\pi r L \rho c_p \left(\frac{dT}{\partial t}\right)$$

$$\frac{\partial}{\partial r}\left(K \cancel{2\pi r L} \frac{\partial T}{\partial r}\right) + q_g \cancel{2\pi r L} = \cancel{2\pi r \times L} \rho c_p \frac{\partial T}{\partial t}$$

$$\boxed{\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right) + (q_g/K) = \left(\frac{\rho c_p}{K}\right)\frac{\partial T}{\partial t}}$$

tical thickness :)

~~Heat~~ Steady state heat flow through the Composite cylinder. (5)

$$\boxed{\left(\frac{1}{r}\right)\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\left(\frac{q_g}{k}\right)=\frac{1}{\alpha}\left(\frac{\partial T}{\partial t}\right)} \quad \text{— (1)}$$

$\alpha \to$ thermal diffusivity

$\alpha = \left(\frac{k}{\rho c_p}\right)$

No heat generation $(q_g = 0)$

at steady state

$$\left[\left(\frac{1}{r}\right)\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=0\right] \quad \text{— (II)}$$

on integrating eqn (II)

$$r\frac{\partial T}{\partial r} = C_1$$

$$\left(\frac{\partial T}{\partial r}\right)=\left(\frac{C_1}{r}\right) \quad \text{— (III)}$$

$$T = C_1 \ln r + C_2] \quad \text{— (iv)}$$

B.C

$r = r_1 \quad T = T_1$

$r = r_2 \quad T = T_2$

$$T_1 = C_1 \ln r_1 + C_2 \quad \text{— (v)}$$

$$T_2 = C_1 \ln r_2 + C_2 \quad \text{— (vi)}$$

(iv) - (v)

$$(T_1 - T_2) = C_1 \ln(r_1/r_2)$$

$$C_1 = \frac{(T_1-T_2)}{\ln(r_1/r_2)} \quad \text{— (vii)}$$

from fourier law

$$Q = -K\,2\pi r L\frac{\partial T}{\partial r}$$

$$Q = -K\,2\pi r \times L \times \frac{C_1}{r}$$

$$= \frac{\cancel{-}K\,2\pi \cancel{r} \times L}{\cancel{r}}\;\frac{(T_1-T_2)}{\cancel{(-)}\ln(r_2/r_1)}$$

$$\boxed{Q = \frac{(T_1-T_2)}{\left(\frac{\ln(r_2/r_1)}{2\pi K L}\right)}} = \frac{(T_1-T_2)}{(\text{Resistance})}$$

We

deno

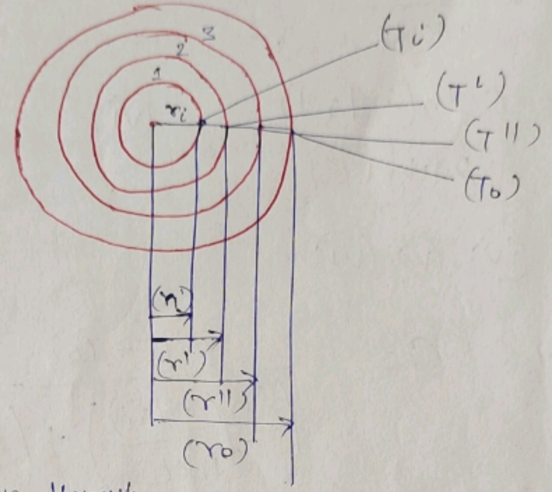
rad

cyl

To

Critical thickness ->

Steady state heat T/f through the Composite cylinder (5)



Heat Conduction through

We Consider an Composite cylinder wall Consisting of three layers denoted by 1, 2 and 3 having thermal Conductivity k_1, k_2, k_3 and inner radius r_i, r' and r'' respectively. The outer radius of Composite cylinder is (r_o). The tempr at these radial position T_i, T', T'', and T_o

The rate of heat T/f through layer (1)

$$Q_1 = \frac{2\pi K_1 L\,(T_i - T')}{\ln(r'/r_i)} \quad \text{—(I)}$$

$$Q_2 = \frac{2\pi K_2 L\,(T' - T'')}{\ln(r''/r')} \quad \text{—(II)}$$

$$Q_3 = \frac{2\pi K_3 L\,(T'' - T_2)}{\ln(r_o/r'')} \quad \text{—(III)}$$

at steady state the rate of heat flow through individual layer are equal.

$Q_1 = Q_2 = Q_3 = Q.$

Heat transfer Coefficient

(7)

Convective heat T/f Coefficient :- Convection refers to transport of heat in a fluid medium because of motion of fluid. Convective heat t/f is, therefore associated with bulk motion of

$$T_i - T' = \frac{Q \ln(r'/r_i)}{2\pi K_1 L} \quad \text{— (iv)}$$

$$T' - T'' = \frac{Q \ln(r''/r')}{2\pi K_2 L} \quad \text{— (v)}$$

$$T'' - T_o = \frac{Q \ln(r_o/r'')}{2\pi K_3 L} \quad \text{— (vi)}$$

adding (iv) (v) (vi)

$$Q = \frac{(T_i - T_o)}{\left(\frac{\ln(r'/r_i)}{2\pi K L}\right) + \left(\frac{\ln(r''/r')}{2\pi K L_2}\right) + \ln\left(\frac{r_o/r''}{2\pi L K_3}\right)}$$

Critical thickness :-

Heat transfer Coefficient

(7)

Convective heat T/f Coefficient :- Convection refers to transport of heat in a fluid medium because of motion of fluid. Convective heat t/f is, therefore associated with bulk motion of in the fluid medium.

H.T.C is a Phenomenologic Coeffient. It is based on the observation that the rate of Convective heat t/f is propotional to the area of heat t/f and tempr driving force.

$$Q \propto A \quad \& \quad Q \propto (T_s - T_o)$$

$$Q = hA(T_s - T_o) =$$

$$\Rightarrow \boxed{Q = hA\Delta T}$$

$$\boxed{h = \frac{Q}{A\,\Delta T}}$$

$\{h = H.T.C\}$ $\boxed{h = \frac{q}{\Delta T}}$ — (1) $\{q = \text{Heat flux}\}$

Eqn (1) is also known as Newton's law of Cooling.

Overall H.T.C

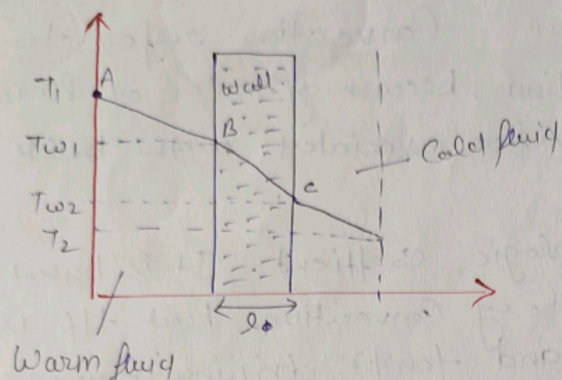
(a) Heat T/f b/w fluid seprated by a plane wall.

We Consider heat t/f from one fluid to another seprated by a plane wall of thickness l. The tempr of the bulk of fluid on two sides of the wall are T_1 and T_2 ($T_1 > T_2$) and heat T/f Coefficient b/w the wall and fluid are h_1 & h_2 respectively. The tempr of fluid interface are T_{w1} & T_{w2} the thermal Conductivity of material is (k_w)

Critical thickness ->

$$\left(\frac{1}{U}\right) = \left(\frac{1}{h_1} + \frac{1}{k_w/l} + \frac{1}{h_2}\right)$$ (8)

If the area of heat T/f is (A)



Rate of heat T/f from hot fluid to the wall $Q_1 = h_1A(T_1 - T_{w1})$ -(1)

Rate of heat T/f from through the wall $Q_2 = \frac{k_wA(T_{w1} - T_{w2})}{l}$ -(2)

Rat of heat T/f from the wall to the cold fluid $Q_3 = h_2A(T_{w2} - T_2)$ -(3)

the heat T/f take place at S.S through a constant area we can write.

$$Q_1 = Q_2 = Q_3 = Q \text{ (say)}$$

$$(T_1 - T_{w1}) = \frac{Q}{h_1A} \quad -(4)$$

$$(T_{w1} - T_{w2}) = \frac{Ql}{k_wA} \quad -(5)$$

$$(T_{w2} - T_2) = \frac{Q}{h_2A} \quad -(6)$$

Add (4) + (5) & (6)

$$(T_1 - T_2) = Q\left(\frac{1}{h_1A} + \frac{l}{k_wA} + \frac{1}{h_2A}\right) = \frac{Q}{UA}$$

$$Q = \frac{(T_1 - T_2)}{\left(\frac{1}{h_1A} + \frac{l}{k_wA} + \frac{1}{h_2A}\right)}$$

Where U is called overall heat T/f Coefficient.

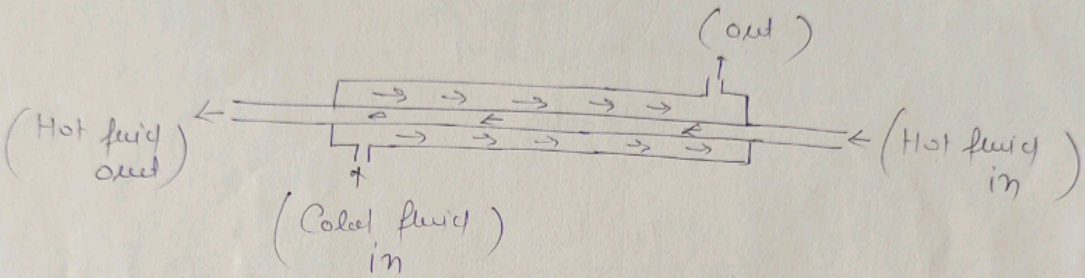
Critical thickness :-

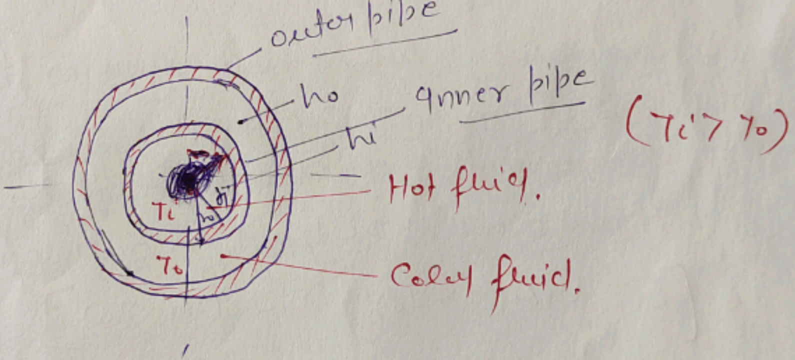
$$\left(\frac{1}{U}\right) = \left(\frac{1}{h_1} + \frac{1}{k\omega/l} + \frac{1}{h_2}\right)$$

$$\boxed{Q = UA(T_1 - T_2)}$$

Total thermal resistance $= (1/UA)$

(b) Heat T/f b/w fluid separated by cylindrical wall.





The rate of heat T/f from the hot fluid to the inner surface at $(r = r_i)$

$$Q_i = A_i h_i (T_i - T_{wi}) \quad (1)$$

The rate of heat T/f from the wall

$$Q_2 = \frac{(T_{wi} - T_{wo})}{\frac{\ln(r_o/r_i)}{2\pi k_w l}} \quad (2)$$

$$Q_3 = A_o h_o (T_{wo} - T_o) \quad (3)$$

~~Critical thickness~~

Critical Insulation thickness

Addition of insulation, reduces the rate of heat t/f

from Eqn (1) $(T_i - T_{wi}) = \dfrac{Q}{A_i h_i}$ — (4)

(2) $T_{wi} - T_{wo} = \dfrac{Q}{\left(\dfrac{2\pi L k_w}{\ln(r_o/r_i)}\right)}$ — (5)

(3) $T_{wo} - T_o = \dfrac{Q}{A_o h_o}$ — (6)

$A_i = 2\pi r_i \times L$ — (7)

$A_o = 2\pi r_o \times L$ — (8)

$$\left[Q = \frac{(T_i - T_o)}{\dfrac{1}{A_i h_i} + \dfrac{\ln(r_o/r_i)}{2\pi k_w L} + \dfrac{1}{h_o A_o}} \right] \quad \text{— (9)}$$

The inside and outside surface area of inner pipes are different so we define two overall coefficient

an overall H.T.C based on inside (U_i)

and overall H.T.C based on outside (U_o)

$$\left[Q = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o) \right] \quad \text{— (10)}$$

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{A_i \ln(r_o/r_i)}{2\pi k_w L} + \dfrac{A_i}{A_o}\dfrac{1}{h_o}} \quad \text{— (11)}$$

$$U_o = \frac{1}{\dfrac{A_o}{A_i h_i} + \dfrac{A_o \ln(r_o/r_i)}{2\pi k_w L} + \dfrac{1}{h_o}} \quad \text{— (12)}$$

$$Q = \frac{(T_i - T_o)}{R_T} = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o)$$

$$R_T = \frac{1}{U_o A_o} = \frac{1}{U_i A_i}$$

$$\boxed{U_i A_i = U_o A_o}$$

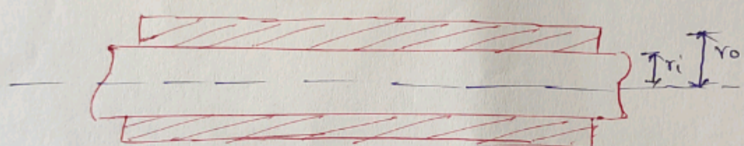
Heat T/f from Extended [illegible]

Critical thickness →

Critical Insulation thickness

Addition of insulation, reduces the rate of heat t/f

Concider a pipe carrying steam with a layer of insulation. The inner radius of the layer of insulation and corresponding tempr are (r_i) & (T_i) and outer radius of insulation is (r_o) & Tempr (T_o) & outside heat t/f coefficient is (h_o), thermal conductivity of insulating material is k. inner side heat T/f coefficient is (h_i) assumed to be very large (h_i)



$$Q = \frac{(T_i - T_o)}{\underbrace{\frac{\ln(r_o/r_i)}{2\pi K L}}_{R_{conduction}} + \underbrace{\frac{1}{A_o h_o}}_{R_{convection}}} = \frac{2\pi L (T_i - T_o)}{\frac{\ln(r_o/r_i)}{K} + \frac{1}{r_o h_o}}$$

Two resistance are there Conduction & Convection as r_o ↑es ($r_o - r_i$) called thickness of insulation get increases

We want to determine the outer radius of insulation for which the heat loss will be maxm

$$\frac{dQ}{dr_o} = \frac{-2\pi L (T_i - T_o)\left(\frac{1}{K r_o} - \frac{1}{h_o r_o^2}\right)}{\frac{\ln(r_o/r_i)}{K} + \frac{1}{r_o h_o}} = 0$$

the temp gradient can be determine by Eqn ⑥

$$Q = b\,\omega\,(T_w - T_o)(-k)\left[-m \sinh m(l-x) - \frac{h}{km}\,(m \cosh m(l-x))\right]$$

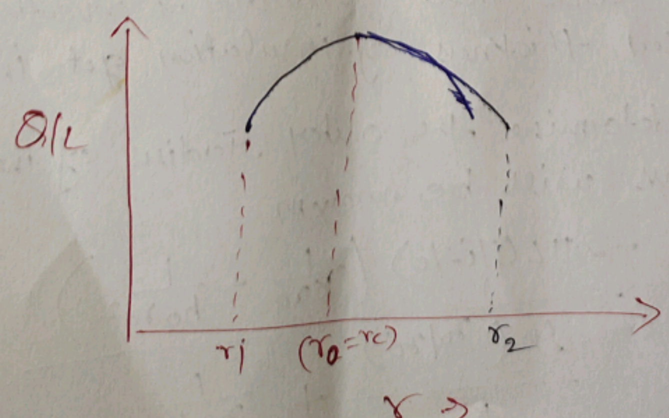
Heat T/F from Extended ...

$$\frac{1}{k r_0} - \frac{1}{h_0 r_0^2} = 0 \qquad \left\{\frac{d^2Q}{dr_0^2} < 0\right\}$$

$$\boxed{r_{0\,critical} = \frac{k}{h}}$$

Conductive resistance increases but ~~Conductive~~ Convective resistance decreases. initially with adding the insulation the rate of heat loss goes ↑es reaches maxm and therefore after ↓es.

"The critical thickness of insulation is the thickness of insulation up to which heat loss ↑es and reaches maxm and there after heat loss decreases."



the tempr gradient can be determine by Eqn ⑥

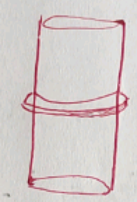
10

$$Q = b\omega(T_w - T_o)(-k)\left[-m\sinh m(l-x) - \frac{h}{k}(m\cosh m(l-x)\ldots\right]$$

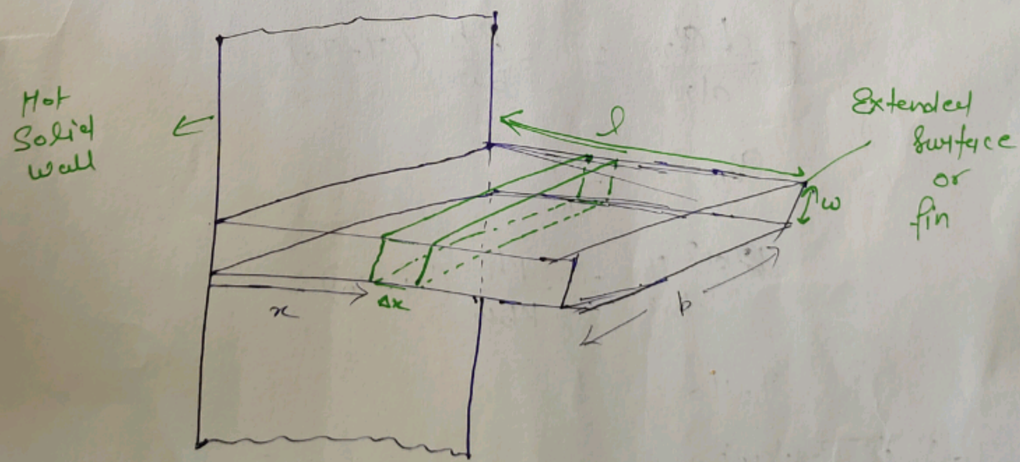
Heat T/f from Extended Surfaces:-(the fins)

9

The thermal Conductivity of gases are much smaller than those of Liquid. the gas phase heat t/f Coefficient is also much smaller than the liquid-phase Coefficient. In heat Exchanger device for heat Exchange b/w a gas and a liquid, the gas-side film will offer most of the thermal resistance, therefore a large heat t/f area will be required. and area can be ↑es by fixing ~~the~~ attachments like rectangular metal strips or annular ~~ring~~ rings Called fins.



We Concider a thin rectangular plate having a fin (l) distance from wall, thickness (ω) and width (b). The tempr of the wall is (T_ω) and ambient tempr (T_o)



the tempr gradient can be dete...

Heat loss to the ambient occurs from both surfaces (top & bottom) of the fin. As thickness of fin is small fin tempr varies only in the longitudinal direction and heat t/f occur through the fin by Conduction in same dir.

Let us Consider a thin section of thickness Δx of the fin at a distances x from the base. Abilable area for heat Conduction is $(b \times w)$ and the area of heat Loss by Convection from the exposed fin surface to the ambient is $(P \Delta x)$

$P \rightarrow$ Perimeter

$$P = 2(B+w)$$

Apply the heat balance over small Element.

(heat in) - (heat out) ± (genration or loss) = (rate of heat accumulation at S.S.)

$$b \times w\, q|_x - b \times w\, q|_{x+\Delta x} - P \Delta x\, h (T - T_0) = 0$$

divided by $(b \times w \times \Delta x)$

$$-\frac{d q_x}{dx} = \frac{Ph}{bw}(T - T_0)$$

$$q_x = -K \frac{dT}{dx}$$

$$\frac{d^2T}{dx^2} = \frac{Ph}{Kbw}(T - T_0)$$

$$\bar{T} = T - T_0$$

(T_0) is Constant

$$\frac{d^2 \bar{T}}{dx^2} = \frac{Ph}{Kbw}(\bar{T})$$

the temp gradient can be determine by eqn 6

$$Q = bw(T_w - T_0)(-k)\left[\frac{-m\sinh m(l-x) - \frac{h}{km}(m\cosh m(l-x))}{\cosh ml + \frac{h}{km}\sinh ml}\right]_{x=0} \quad -(7)$$

maxm rate of heat t/f

$$Q_{max} = (\text{total area})(H.T.C)(d\Delta T)$$

$$= (Pl + \cancel{bw})\, h\,(T_w - T_0) \quad -(8)$$

The fin efficiency η is defined as.

$$\cancel{\eta = \frac{Q_{actual}}{Q_{maxm}}}$$

$$\cancel{= \frac{bw\,km}{h(Pl+bw)} \cdot \frac{\sinh ml + \frac{h}{km}\cosh ml}{(\cosh ml + \frac{h}{km}\sinh ml)}}$$

~~At edge of the fin ($x = l$)~~ for most practical cases we assume fin is
If fin is adiabatic
($h \approx 0$) not zero

$$\left\{\frac{\theta}{\theta_b} = \left\{\left(\frac{T - T_0}{T_w - T_0}\right) = \frac{\cosh m(l-x)}{\cosh ml}\right\}\right\}$$

$$\left\{Q_{fin} = bw(T_w - T_0)\,km(\cancel{\sinh}\ \tanh ml)\right\}$$

$$\eta = \frac{bw\cancel{(T_w - T_0)}\,km\tanh mL}{Pl\,h\cancel{(T_w - T_0)}}$$

$m = \frac{KA\,\xi \tanh h\xi L}{hPl}$ ... $h\xi L]$

$$\frac{d^2\bar{T}}{dx^2} = \xi^2 \bar{T} \quad -(1) \qquad \xi^2 = \left(\frac{Ph}{Kbw}\right) ; \left\{\xi = \sqrt{\frac{Ph}{Kbw}}\right\}$$

$$\bar{T} = C_1 \cosh \xi x + C_2 \sinh \xi x \quad -(2)$$

to find C_1 & C_2 put B.C.

$x = 0 \quad (T = T_w), \quad \bar{T} = (T_w - T_0)$

$x = L \quad -K\frac{dT}{dx} = h(T - T_0), \quad -K\frac{d\bar{T}}{dx} = h\bar{T} \quad -(3)$

the rate at which heat is conducted from inside the solid to the boundary must be equal to the rate at which it is transported to the ambient medium by convection.

$$(C_1 = T_w - T_0) \quad -(4)$$

from Eqn (2) $\left(\frac{d\bar{T}}{dx}\right)_{(x=L)} = [C_1 \xi \sinh(\xi x) + C_2 \xi \cosh \xi x]$

$$-K[C_1 \xi \sinh(mL) + C_2 \xi \cosh(mL)] = h[C_1 \cosh mL + C_2 \sinh mL]$$

$$\left(\frac{-K}{h}\right)[C_1 m \sinh mL + C_2 m \cosh mL] = h[C_1 \cosh mL + C_2 \sinh mL]$$

$$C_2 = \frac{-C_1\left[\frac{h}{km}\cosh(mL) + \sinh(mL)\right]}{\cosh(mL) + \frac{h}{km}\sinh(mL)} \quad -(5)$$

$$\bar{T} = (T_w - T_0)\left[\frac{\cosh m(l-x) + \frac{h}{km}\sinh m(l-x)}{\cosh(mL) + \left(\frac{h}{km}\right)\sinh mL}\right]$$

$$\left[\frac{(T - T_0)}{(T_w - T_0)} = \frac{\cosh m(l-x) + \frac{h}{km}\sinh m(l-x)}{\cosh(mL) + \frac{h}{km}\sinh ml}\right] \quad -(6)$$

$Q_{(actual)} = $ (Cross-sectional area) (heat flux at fin base)

$$(b \times w) \times \left(-K\frac{dT}{dx}\Big|_{x=0}\right)$$

$$\eta = \frac{KA}{hPL}\xi \tanh \xi L$$

$$\eta = \frac{\cancel{\xi}\tanh \xi L}{\xi^{\cancel{2}} L} = \frac{\tanh \xi L}{\xi L}$$