: Electromagnetics:

maxwells electromagnetic equations are based upon the well known basic laws such as guss's law of electrostatics, gauss's law of magneto-statics, Faraday's law of electromagnetic induction and ampere's circutal law. when electric & magnetic fields are changing very rapidly in space and time then, the varying electric field gives or produces magnetic field & vice versa. we therefor consider electromagnatic fields by a set of equations, known as maxwell's equ^{n} of electromagnetics.

$$\oint_s \vec{D} \cdot d\vec{s} = \int_v \rho \cdot dv \quad \text{— (1)}$$ Gauss's law of electrastatics

$$\oint_s \vec{B} \cdot d\vec{s} = 0 \quad \text{— (2)}$$ Gauss's law of magneto-statics

$$\oint_l \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B} \cdot d\vec{s} \quad \text{— (3)}$$ Faraday's law of Induction

$$\oint_l \vec{H} \cdot d\vec{l} = \int_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{s}$$ Ampere's circutal law

closed path encloses open surface.
closed surface encloses volume.

Maxwell's equation:

$$\nabla \cdot \vec{D} = \rho \qquad \vec{\nabla} = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\nabla \vec{B} = 0 \qquad \vec{D} = \epsilon \vec{E}$$

ϵ Permitivity of free space

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{dt} \qquad \vec{B} = \mu_0 \vec{H}$$

μ_0 Permitibility of free space.

$$\vec{J} = \sigma \vec{E}$$

current density.

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Gauss's divergence theorem:

$$\oint_{s} \vec{A}\cdot d\vec{S} = \int_{v} (\nabla\cdot\vec{A})\cdot dv$$

Stoke's theorem:

$$\oint_{l} \vec{A}\cdot d\vec{l} = \int_{s} (\nabla\times\vec{A})\cdot d\vec{s}$$

derivation of maxwell's first equation:

Gauss's law of electrostatics, states that the electric flux over a hypothetical closed surface is $\frac{1}{\epsilon_0}$ times of the total charge with in the volume. mathematically it can be written as.

$$\oint_{s} \vec{E}\cdot d\vec{s} = \frac{1}{\epsilon_0}\int_{v} \rho\cdot dv \longrightarrow (1-A)$$

or

$$\oint_{s} \vec{D}\cdot d\vec{s} = \int_{v} \rho\cdot dv \quad \text{, since, } \vec{D} = \epsilon_0\vec{E}$$

$$\longrightarrow (1-B)$$

Now, using gauss's divergence theorem. in L.H.S. of eq^n (1-B) we get

$$\int_{v} (\nabla\cdot\vec{D})\cdot dv = \int_{v} \rho\cdot dv$$

$$\boxed{\nabla\cdot\vec{D} = \rho} \text{ Prooved}$$

Derivation of maxwell's second equation:

Gauss's law of magneto-statics states that the net magnetic flux passing through any closed surface is zero. it is known that magnetic monopoles does not exist, therefore any closed volume will always contain equal & opposite magnetic poles. thus, magnetic flux entering into the region is equal to the magnetic flux living to the region.

$$\oint_S \vec{B}\cdot d\vec{s} = 0$$

applying gauss' divergence theorem.

$$\int_V (\nabla\cdot\vec{B})\, dv = 0$$

or $$\boxed{\nabla\cdot\vec{B} = 0}$$

Derivation of maxwell's third equation:

maxwell's third eq^n is based upon faraday's law of induction, which states that an electric field is produced by changing magnetic flux.

"according to farraday's law of e.m.f. induced around a closed surface is equal to the negative times the rate of change of magnetic flux link with the circuit."

$$e.m.f = -\frac{\partial\phi_m}{\partial t} \quad \text{---} \ (1)$$

but the magnetic flux (ϕ_m) can be expressed in terms of magnetic flux density

$$\phi_m = \int_S \vec{B}\cdot d\vec{s} \quad \text{---} \ (2)$$

Now, substituting the value of ϕ_m from eq^n (2) in eq^n (1) we get,

$$e.m.f = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{s}$$

or

$$-\int_S \frac{\partial\vec{B}}{\partial t}\cdot d\vec{s} \quad \text{---} \ (3)$$

But, e.m.f can also be expressed in terms of electric field vector

$$e.m.f. = \oint_l \vec{E}\cdot d\vec{l} \quad \text{---} \ (4)$$

$$\boxed{\oint_l \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot d\vec{s}} \quad \text{---} \ (5)$$

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applying stokes theorm in L.H.S of eq^n ⑥

$$\int_s (\nabla \times \vec{E}) \cdot d\vec{s} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

or

$$\boxed{\nabla \times \vec{E} = -\frac{\partial B}{\partial t}}$$

Derivation of maxwell's fourth equation:

maxwell's fourth equ^n is the madified form of ampere circuital law. It is valid for both steady and time varying fied. and states that " magneto motive force around a closed path is equal to the sum of conduction current + displacement current. This signifies that conduction current as well as changing flux produces magnetic field.

Ampears circuital law states that the line integral of $\vec{H}$ around any closed path is equal to the total current with in that path.

$$\oint_l \vec{H} \cdot d\vec{l} = I \quad \text{—— ①}$$

I = conduction current + displacement current.

But the current may also be expressed in terms of current density J as

$$\int_s \vec{J} \cdot d\vec{s} = I \quad \text{—— ②}$$

Now from eq^n ① + ② we have

$$\oint_l \vec{H} \cdot d\vec{l} = \oint_s \vec{J} \cdot d\vec{s} \quad \text{—— ③}$$

Now, using stokes theorem R.H.S of eq^n ③ we get,

$$\int_s (\nabla \times \vec{H}) \cdot d\vec{s} = \int_s \vec{J} \cdot d\vec{s}$$

$$\oint_S (\text{curl}\,\vec{H} - \vec{J})\cdot d\vec{S} = 0$$

or $$\text{curl}\,\vec{H} = \vec{J} \quad \text{——} (4)$$

it can be shown that eq^n (4) is valid only for static charge. and insufficient for time varying field to show this. let us the divergence of eq^n (4)

$$\text{div}(\text{curl}\,\vec{H}) = \text{div}\,\vec{J}$$

$$\text{div}\,\vec{J} = 0 \quad \text{——} (5)$$

Now, from the continuity equation,

$$\text{div}\,\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\text{div}\,\vec{J} = -\frac{\partial \rho}{\partial t} \quad \text{——} (6)$$

therefore eq^n (5) can be valid if $\frac{\partial \rho}{\partial t} = 0$, that be, S/ charge density should be static.

Hence to include time varying fields maxwell suggested that ampear's law must be modified. The current density $\vec{J}$ should be replaced by $\vec{J} + \vec{J_d}$ where $\vec{J_d}$ is the current density for displacement current.

Now, eq^n (4) its form

$$\text{curl}\,\vec{H} = \vec{J} + \vec{J_d} = I \quad \text{——} (7)$$

Now taking the divergence of above eq^n (7). we get

$$\text{div}\,\vec{J} = -\text{div}\,\vec{J_d} \quad \text{——} (8)$$

Now, we compare abobe eq^{n}s (6) & (8) we get,.

$$\text{div}\,\vec{J_d} = \frac{d\rho}{dt} \quad \text{——} (9)$$

maxwell's first law

$$\nabla\cdot\vec{D} = \rho \quad \text{——} (10)$$

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Now, substituting the value of ρ from eq^n (10) in eq^n (9) we get,

$$\nabla \quad div\ \vec{J_d} = \frac{\partial}{\partial t}(\nabla \cdot \vec{D})$$

$$= N\ div\left(\frac{\partial \vec{D}}{\partial t}\right)$$

$$\vec{J_d} = \frac{\partial \vec{D}}{\partial t} \quad \text{—— (11)}$$

Now, substituting the value of total current. displacement current.

$$\boxed{curl\ \vec{H} = \nabla \times \vec{H} = \vec{J} + \vec{J_d} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

this the modified form of maxwell's fourth eq^n. the term which maxwell's added to ampear's law to include time varying fied is known as displacement current. it arises when electric displacement vector $\vec{D}$ change with time.

Characterstics of Displacement current:

1. Displacement current is a current in the sense it produces magnetic field.
2. the magnitude of displacement current is equal to the rate of change of electric displacement vector
3. Displacement current ~~is~~ solve the purpose to make the total current continuous. across the discontinuity in the conduction current

for a example- battry charging a capacitor produces closed current loop in the terms to total current.

Wave equation in free space conditions (E.M. wave equations. :-

$$\nabla \cdot \vec{D} = 0 \quad \text{—— (1)}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{—— (2)}$$

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$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \text{--- (3)}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{--- (4)}$$

conditions for free space.

$\rho = 0$, $\sigma = 0$, $\vec{J} = 0$

$\vec{B} = \mu_0 \vec{H}$, $\vec{D} = \epsilon_0 \vec{E}$

Now applying the conditions of free space, let down the eq^n (1) to eq^n (4) we get

$$\nabla \cdot \vec{E} = 0 \quad \text{--- (a)}$$

$$\nabla \cdot \vec{H} = 0 \quad \text{--- (b)}$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad \text{--- (c)}$$

$$\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{--- (d)}$$

Now, taking the curl of eq^n (c)

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \frac{\partial}{\partial t} (\nabla \times \vec{H})$$

$$\underbrace{\nabla (\vec{\nabla} \cdot \vec{E})}_{0} - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t} \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t}\right) - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$-\nabla^2 E = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

or

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \text{--- (9)}$$

$$\underbrace{\nabla (\nabla \cdot \vec{H})}_{0} - \nabla^2 \vec{H} = \epsilon_0 \frac{\partial}{\partial t} \left(-\mu_0 \frac{\partial \vec{H}}{\partial t}\right)$$

$$-\nabla^2 \vec{H} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

or

$$\nabla^2 \vec{H} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad \text{--- (5)}$$

general classical wave equation is given by -

$$\nabla^2\psi - \frac{1}{V^2}\frac{\partial^2\psi}{\partial t^2} = 0 \quad \text{—— (6)}$$

$$\frac{1}{V^2} = \mu_0\varepsilon_0$$

$$V = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad \mu_0 = 4\pi\times10^{-7}$$

$$\varepsilon_0 = 8.85\times10^{-12}$$

$$V = 2.998 = c$$

$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0 \quad \text{—— (7)}$$

$$\nabla^2\vec{H} - \frac{1}{c^2}\frac{\partial^2\vec{H}}{\partial t^2} = 0 \quad \text{—— (8)}$$

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0 \quad \text{—— (9)}$$

Relacing v by c in eq^n (4), (5), (6) we get (7), (8), (9)

Let us find the solu^n of above eq^n (7) + (8) for plane electromagnetic waves A plane electromagnetic wave is defined as the waves whose amplitude is same at any point in plane ⊥ to a specified dir^n. the solu^n of eq^n (7) + (8) may be written as-

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)} \quad \text{—— (10)}$$

$$\vec{H}(\vec{r},t) = \vec{H}_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)} \quad \text{—— (11)}$$

where E_0 and H_0 are the complex amplitudes of electric + magnetic fields which are constant in space and in time while k is propagation vector defined as-

$$\vec{K} = k.\hat{n} = \frac{2\pi}{\lambda}.\hat{n} = \frac{2\pi}{c} n.\hat{n}$$

$$\frac{\omega}{c}\hat{n} \quad \text{—— (13)} \qquad c = n\lambda,\ \lambda = \frac{c}{n}$$

$\vec{n}$ is the unit vector in the direction of propogation. application of free space condition to maxwell equation gives-

$$\vec{\nabla}.\vec{E} = 0$$

$$\vec{\nabla}.\vec{H} = 0$$

Let us first find the value of $\nabla.\vec{E}$ in the right of solu^n given in (10)

$$\vec{\nabla}.\vec{E} = \nabla\left[\vec{E}_0.e^{i(\vec{K}\cdot\vec{r} - \omega t)}\right]$$

$$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right].\left[\hat{i}E_{0x} + \hat{j}E_{0y} + \hat{k}E_{0z}\, e^{i(\vec{K}\cdot\vec{r} - \omega t)}\right]$$

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where $\vec{k} = \hat{i} k_x + \hat{j} k_y + \hat{k} k_z$

$\vec{r} = \hat{i} x + \hat{j} y + \hat{k} z$

$\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$

$$\left[\frac{\partial}{\partial x}(E_{0x}) + \frac{\partial}{\partial y} E_{0y} + \frac{\partial}{\partial z} E_{0z}\right] e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial}{\partial x}\left[E_{0x} e^{i(k_x x + k_y y + k_z z - i\omega t)}\right]$$

$$+ \frac{\partial}{\partial y}\left[E_{0y} e^{i(k_x x + k_y y + k_z z - i\omega t)}\right]$$

$$+ \frac{\partial}{\partial z}\left[E_{0z} e^{i(k_x x + k_y y + k_z z - i\omega t)}\right]$$

$$= E_{0x} e^{i(k_x x + k_y y + k_z z) - i\omega t} (ik_x)$$

$$+ E_{0y} e^{i(k_x x + k_y y + k_z z) - i\omega t} (ik_y)$$

$$+ E_{0z} e^{i(k_x x + k_y y + k_z z) - i\omega t} (ik_z)$$

$$= (E_{0x} ik_x + E_{0y} ik_y + E_{0z} ik_z) e^{i(k_x x + k_y y + k_z z) - i\omega t}$$

$$= \hat{i}(k_x E_{0x} + k_y E_{0y} + k_z E_{0z}) e^{i(\vec{k}\cdot\vec{r} - \omega t)}.$$

$$\vec{\nabla}\cdot\vec{E} = i(\vec{k}\cdot\vec{E_0} e^{i(\vec{k}\cdot\vec{r} - \omega t)})$$

$$\vec{\nabla}\cdot\vec{E} = i(\vec{k}\cdot\vec{E}) = 0 \quad \text{—— (13)}$$

$i \neq 0$ $\quad$ $\vec{k}\cdot\vec{E} = 0$

so, $\vec{k}$ and $\vec{E}$ are $\perp$ to each other

$$\vec{\nabla}\cdot\vec{H} = i(\vec{k}\cdot\vec{H}) = 0 \quad \text{—— (14)}$$

$i \neq 0$

$\vec{k}\cdot\vec{H} = 0$, so, $\vec{k}$ and $\vec{H}$ are $\perp$ to each other.

This conclusion indicates that electric and magnetic field vector $\vec{E}$ and $\vec{H}$ are perpendicular to the dir^n of the propagat $\vec{k}$. This iemplies that electromagnetic wave are transverse in nature.

In free space maxwell's third & fourt eq^n imposed the restriction eq^n 3^rd (c) and 3^rd (d)

$$\nabla\times\vec{E} = -\mu_0 \frac{\partial\vec{H}}{\partial t} \quad \text{—— 3 (c)}$$

$$\nabla\times\vec{H} = \epsilon_0 \frac{\partial\vec{E}}{} \quad \text{—— 3 (d)}$$

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Now, let us finding out the value of $\nabla \times \vec{E}$ and $\nabla \times \vec{H}$ in the light of wave eq^n solution 10, 11 eq^{n}s.

$$\nabla \times \vec{E} = \nabla \times \left[\vec{E_0}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right]$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \partial/\partial y & \partial/\partial z \\ E_{0x} e^{i(\vec{k}\cdot\vec{r} - \omega t)} & -- & -- \end{vmatrix}$$

$$\nabla \times \vec{E} = \hat{i}\left\{\frac{\partial}{\partial y}\left[E_{0z} e^{i(k_x x + k_y y + k_z z) - i\omega t}\right] - \frac{\partial}{\partial z}\left[E_{0y} e^{i(k_x x + k_y y + k_z z - \omega t)}\right]\right\}$$
$$+ \hat{j}\left\{\frac{\partial}{\partial z}\left[E_{0x} e^{i(k_x x + k_y y + k_z z - i\omega t)}\right] - \frac{\partial}{\partial x}\left[E_{0z} e^{i(k_x x + k_y y + k_z z - \omega t)}\right]\right\}$$
$$+ \hat{k}\left\{\frac{\partial}{\partial x}\left[E_{0y} e^{i(k_x x + k_y y + k_z z - i\omega t)}\right] - \frac{\partial}{\partial y}\left[E_{0x} e^{i(k_x x + k_y y + k_z z - \omega t)}\right]\right\}$$

$$\nabla \times \vec{E} = \hat{i}\left\{i E_{0z} k_y e^{i(\vec{k}\cdot\vec{r} - \omega t)} - i E_{0y} k_z e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right\}$$
$$+ \hat{j}\left\{i E_{0x} k_z e^{i(\vec{k}\cdot\vec{r} - \omega t)} - i E_{0z} k_x e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right\}$$
$$+ \hat{k}\left\{i E_{0y} \cdot k_x e^{i(\vec{k}\cdot\vec{r} - \omega t)} - i E_{0x} k_y e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right\}$$

$$\nabla \times \vec{E} = i\left\{\hat{i}\left[E_{0z} k_y - E_{0y} k_z\right] - \hat{j}\left[E_{0x} k_z - E_{0z} k_x\right] + \hat{k}\left[E_{0y} k_x - E_{0x} k_y\right]\right\} \times \left\{e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right\}$$

$$\nabla \times \vec{E} = i\left\{\vec{k} \times \vec{E_0}\right\} e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\nabla \times \vec{E} = i\left\{\vec{k} \times \vec{E}\right\}$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$= -\mu_0 \frac{\partial}{\partial t}\left[\vec{H_0}\, e^{-i(\vec{k}\cdot\vec{r} - \omega t)}\right]$$

$$= i\omega\mu_0 \left[\vec{H_0}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right] = i\omega\mu_0 \vec{H}$$

Now, equating the value.

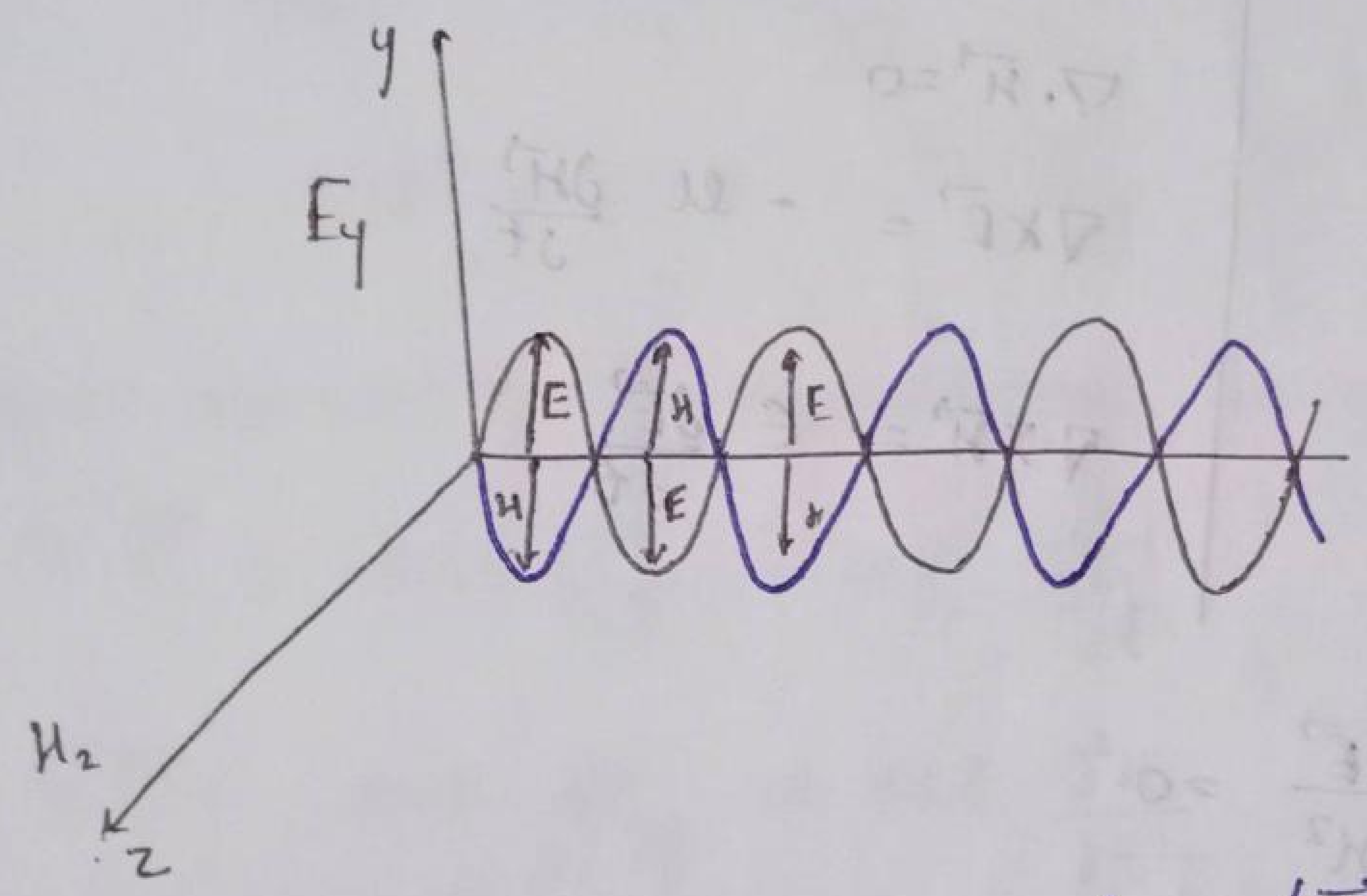
$$\cancel{i}\left\{\vec{k} \times \vec{E}\right\} = \cancel{i}\,\omega\mu_0 H$$

$$\boxed{\vec{k} \times \vec{E} = \omega\mu_0 \vec{H}} \quad \text{——— (A)}$$

$$\boxed{\vec{k} \times \vec{H} = -\epsilon_0 \omega \vec{E}} \quad \text{——— (B)}$$

from above equations (A) & (B) it is clear that the magnetic field vector $\vec{H}$ is perpendicular to both $\vec{E}$ and $\vec{k}$ and according to eq^n (B) $\vec{E}$ is perpendicular to both $\vec{k}$ & $\vec{H}$.

which concludes that electric and magnetic field vectors $\vec{E}$ and $\vec{H}$ are mutually perpendicular to each other and they are also perpendicular to the dir^n of propagation $\vec{k}$. which all terms conclude that the in a plane electromagnetic wave. $\vec{E}$, $\vec{H}$ and $\vec{k}$ forms a set of orthogonal vectors.



further from eq^n (A) $\quad \vec{H} = \frac{1}{\mu_0 \omega}(\vec{k} \times \vec{E})$

$$\vec{H} = \frac{k}{\mu_0 \omega}(\hat{n} \times \vec{E})$$

since $\vec{k} = k\hat{n}$

$$\vec{H} = \frac{1}{\mu_0 c}(\hat{n} \times \vec{E}) \qquad \text{since, } \frac{\omega}{k} = c$$

$$|\vec{H}| = \frac{1}{\mu_0 c}|\hat{n} \times \vec{E}|$$

$$H = \frac{1}{\mu_0 c} E \quad \text{or} \quad \frac{E}{H} = Z_0 = \mu_0 c = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$H = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\boxed{H = 376.6 \text{ oh.}}$$

Z_0 = impedence of wave.

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Q. obtain electromagnetic wave eq^n using maxwell's eq^n in an isotropic dielectric medium. and show that the speed of wave is less than the speed of wave in vaccum.

→ In an isotropic dielectric medium, the current density j and volume charge density ρ and conductivity σ are zero. apart from this

$\vec{D} = \epsilon \vec{E}$, $\vec{B} = \mu \vec{H}$, $\vec{J} = \sigma E = 0$

Now applying the conditions in maxwell's eq^n

$\nabla \cdot \vec{D} = \rho$	$\nabla \cdot \vec{E} = 0$
$\nabla \cdot \vec{B} = 0$	$\nabla \cdot \vec{H} = 0$
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{H} = -\mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

$$\nabla^2 \psi = -\frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

$$\frac{1}{v^2} = \mu \epsilon$$

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}}$$

$$\boxed{v = \frac{c}{\sqrt{\mu_r \epsilon_r}}}$$

Propogation of electromagnetic waves in conducting medium.

$$\left.\begin{aligned} \nabla \cdot \vec{D} &= \rho \quad &-(a) \\ \nabla \cdot \vec{B} &= 0 \quad &-(b) \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \quad &-(c) \\ \nabla \times \vec{H} &= J + \frac{\partial \vec{D}}{\partial t} \quad &-(d) \end{aligned}\right\} \text{——} (1)$$

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let us assum that medium is linear and isotopic conducting. and permeability ϵ, permitutty u. conductivity σ. However, the medium does not poses any charg / arrow other than that given by ohms law.

therfore the conditions are -

$$\vec{D} = \epsilon \vec{E}. \quad \vec{B} = \mu \vec{H} \qquad \text{—— (2)}$$
$$\vec{J} = \sigma \vec{E} \quad \rho = 0$$

$$\left.\begin{array}{ll} \nabla \cdot \vec{D} = 0 & \text{—— (a)} \\ \nabla \cdot \vec{H} = 0 & \text{—— (b)} \\ \nabla \times \vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t} & \text{—— (c)} \\ \nabla \times \vec{H} = \sigma \vec{E} + \epsilon \dfrac{\partial \vec{E}}{\partial t} & \text{—— (d)} \end{array}\right\} \text{—— (3)}$$

after aperating the equation, we get

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{H} = \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu\epsilon \frac{\partial^2 H}{\partial t^2}$$

Poynting theorem: it is the mathematical expression that relates energy transfer

mathematically this can be written as

$$\vec{P} = \vec{E} \times \vec{H}$$

unit :- energy per unit area x time

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{—— (A)}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \text{—— (B)}$$

Now, taking the dot product with A with $\vec{H}$ and of eqn B with $\vec{E}$, we have,

$$\vec{H} \cdot (\nabla \times \vec{E}) = \vec{H}\left(-\frac{\partial \vec{B}}{\partial t}\right) = H \cdot \left(-\mu \frac{\partial \vec{H}}{\partial t}\right) = -\mu\left(\vec{H} \cdot \frac{\partial \vec{H}}{\partial t}\right)$$

$$= -\frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2\right] \qquad \text{—— (C)}$$

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similarly,

$$E\cdot(\nabla\times\vec{H}) = \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t} = \vec{E}\cdot\vec{J} + \vec{E}\cdot\left(\epsilon\frac{\partial\vec{E}}{\partial t}\right)$$

$$= \frac{\partial}{\partial t}\left[\frac{1}{2}\epsilon E^2\right] \quad \text{—— (2) } D$$

Now, substracting eq^n D from eq^n C, we get

$$\vec{H}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot(\nabla\times\vec{H}) = \frac{-\partial}{\partial t}\left[\frac{1}{2}\mu H^2\right] - \vec{E}\cdot\vec{J} - \frac{\partial}{\partial t}\left[\frac{1}{2}\epsilon E^2\right]$$

$$\Downarrow$$

$$\nabla\cdot(\vec{E}\times\vec{H}) = -\left[\vec{E}\cdot\vec{J} + \frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2 + \frac{1}{2}\epsilon E^2\right)\right]$$

$$\nabla\cdot(\vec{E}\times\vec{H}) = -\vec{E}\cdot\vec{J} - \frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2 + \frac{1}{2}\epsilon E^2\right] \quad \text{—— (E)}$$

Now, considering the surface as bounds a volume V, and integrating the abobe relation over the volume V, we get,

$$\frac{-\partial}{\partial t}\int_V\left[\frac{\mu H^2}{2} + \frac{\epsilon E^2}{2}\right]\cdot dv - \int_V(\vec{E}\cdot\vec{J})\cdot dv = \int_V\nabla\cdot(\vec{E}\times\vec{H})\,dv$$

Now, using Guss's divergence theorem, we get

$$-\frac{\partial}{\partial t}\int_V\left(\frac{\mu H^2}{2} + \frac{\epsilon E^2}{2}\right)\cdot dv - \int_V(\vec{E}\cdot\vec{J})dv = \oint_S(\vec{E}\times\vec{H})\cdot d\vec{S}$$

I $\qquad\qquad$ II $\qquad\qquad$ III

about 1st term:-

$-\frac{\partial}{\partial t}\int\left(\frac{\mu H^2}{2} + \frac{\epsilon E^2}{2}\right)\cdot dv$, represents the rate of decrese of stored energy due to electric and magnetic energy.

The term II represents the rate at which EM energy is loss through. or is known as generalised about ... rate of flow of energy over the surface as enclosing the volume V.

Therefor, $(\vec{E} \times \vec{H})$ gives the rate of flow of energy through unit surface area enclosing the volume, be, $\boxed{\vec{P} = \vec{E} \times \vec{H}}$ which is denoted by $\vec{P}$.

2] Ques: if the magnitude of $\vec{H}$ in a plane wave is 1 A/m find the magnitude of $\vec{E}$ for a plane wave in free space,

$$Z_0 = \frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.76.$$

Ques: if the earth recieves 2 cal min^{-1} cm^{-2}. solar energy what are the peak values of electric and magnetic fields of radiation.

$$|\vec{P}| = |\vec{P}| = P = |\vec{E} \times \vec{H}| \quad \text{---(1)}$$

$$= EH \sin\theta = EH \sin 90 = EH$$

$$P = 2\ cal\ min^{-1}\ cm^{-2} = \frac{2 \times 4.2 \times 10^{4}}{60}\ J \cdot m^{-2} s^{-1}$$

$$Z_0 = \frac{E}{H} = 376.76, \quad \text{---(2)}.$$

from eq^n (1) + (2),

Ques: assuming that for a energy from a 1000 calculate the average value of the intencities of electric + magnetic field of radiation at the 2 m away from length.

$$P_{av} = \frac{1000}{4\pi r^2} = \frac{1000}{4\pi(2)^2}$$

$$P = EH.$$

$$\frac{E}{H} = 376.76.$$

from above, we can solve,

-: UNIT:-5 :- NANO MATERIALS

Nanoscience :- to study about the materials at nanaspace. such materials are atoms and moleculey. for ex. Hydrogen atom is 0.1 Nm presise red blood cells. is 500 Nm in size. and visible colour is 400-700 nm in size.

Nanascience: it is nothing but simply studing about materials, macromolecular space. in nanoscience, the properties differs significantly for those as Normal space.

Nanatechanology given nothing but simply the designing. characterization, production and application of structure and desing of system. The work is done by controling size and shape at nnometer scale.

Properties of nano materials are different at nanospace for the two main reasons-

1. Nano materials have relatively large surface area volume ratio as compared to the same mass of material produced in larger form.

for sphare, surface area = $4\pi r^2$

volume = $\frac{4}{3}\pi r^3$

ratio. = $\frac{3}{r}$

2. Quantum confinment effect :- It can begun to combinate the behaviour of matter at nanosphare effecting optical. electrical and magnetic behaviour other material due to quantum confinment effect changes as listed below, takes place,

i) opaque substance can become transparent, ex cupper
ii) innered material can become catalyst.
iii) stabil material can turn into cambustible, ex. alumin
iv) solid can turn into liquid at room temp. ex gold
v) insulator can become conductor's for silicon.

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:- Properties of nanomaterial :-

following are the properties of nanomaterial-

1. they are hard
2. they are exceptinally strong
3. they are ductile at high temp.
4. they are chemically very active.
5. they are wearing resistance.
6. they are erosion rstance.