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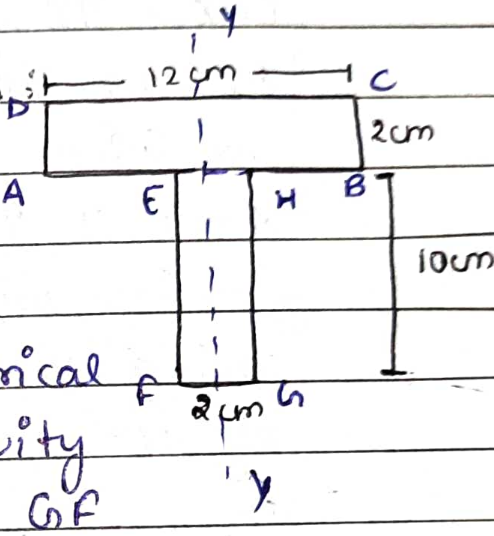
EM Assignment
Unit - 3

→ Centroid and Moment of Inertia :-



Q1 Find Centre of gravity of T-section.
Let there are two rectangles ABCD and EFGH present in given T-section.
Also, given section is also symmetrical about y-y axis. so, Centre of gravity G lie on this axis and take GF as the lowest line for reference axis.
then,

$ar(ABCD)(a_1) = 12 \times 2 = 24 \, cm^2$.

y_1 (distance of G_1 of ABCD from GF) $= 1 + 10$
$= 11 \, cm$.

$ar(EFGH)(a_2) = 2 \times 10 = 20 \, cm^2$.

y_2 (distance of G_2 of EFGH from GF) $= 5 \, cm$.

so, By taking Moment of all small areas about Y-Y axis :-

$\bar{y}$ (distance of G from GF of whole section) :-

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$\bar{y} = \frac{24(11) + 20(5)}{24 + 20} = \frac{264 + 100}{44}$$

$$\boxed{\bar{y} = 8.272 \, cm.}$$ Ans

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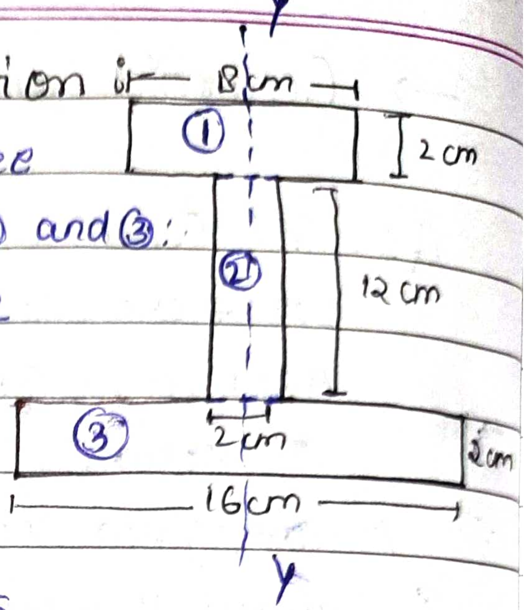
Q2 Find centre of gravity of I-section if -
Divide the given section in three rectangular section as ①, ② and ③: and Given section is symmetric about Y-Y axis, so centre of gravity will also lie on this axis & take the lowest line of section as reference axis.



so,

$a_1 = 8(2) = 16\,cm^2.$

y_1 (distance of CG of ① from lowest line) $= 1 + 12 + 2$
$= 15\,cm.$

$a_2 = 12(2) = 24\,cm^2.$

y_2 (distance of CG of ② from lowest line) $= 6 + 2.$
$= 8\,cm.$

$a_3 = 2(16) = 32\,cm^2.$

y_3 (distance of CG of ③ from lowest line) $= 1\,cm.$

then,
$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$\bar{y} = \frac{16(15) + 24(8) + 32(1)}{16 + 24 + 32}$$

$$\bar{y} = \frac{240 + 192 + 32}{72}$$

$$\bar{y} = \frac{464}{72}$$

$$\boxed{\bar{y} = 6.44\,cm}$$ Ans

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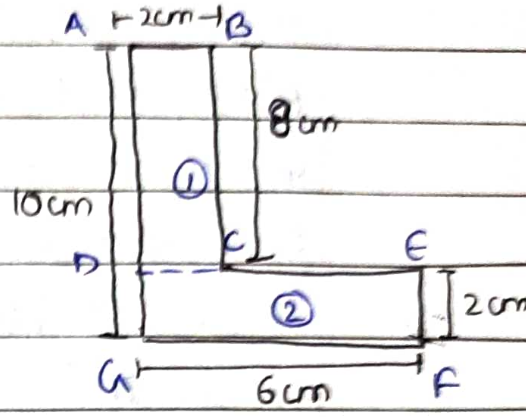
Q3. (a) Find centre of gravity of L-section :-

Given section divide into two rectangular section ① and ② and taking AG as Y-Y axis of reference and GF as X-X axis of reference.



then,

For $\bar{y}$:-

$a_1 = 8(2) = 16\ cm^2$

y_1 (distance of G of ① from GF) = $4+2 = 6\ cm$

$a_2 = 6(2) = 12\ cm^2$

y_2 (distance of G of ② from GF) = $1\ cm$.

so, $\bar{y} = \dfrac{a_1y_1 + a_2y_2}{a_1+a_2}$

$$\bar{y} = \frac{16(6) + 12(1)}{16+12} = \frac{96+12}{28}$$

$$\bar{y} = \frac{108}{28} \; ; \; \boxed{\bar{y} = 3.857\ cm} \text{ Ans}$$

For $\bar{x}$:-

$a_1 = 8(2) = 16\ cm^2$

x_1 (distance of G of ① from AG) = $1\ cm$.

$a_2 = 6(2) = 12\ cm^2$.

x_2 (distance of G of ② from AG) = $3\ cm$.

then, $\bar{x} = \dfrac{a_1x_1 + a_2x_2}{a_1+a_2} = \dfrac{16(1) + 12(3)}{16+12}$

$$\bar{x} = \frac{16+36}{28} \Rightarrow \boxed{\bar{x} = 1.857\ cm} \text{ Ans}$$

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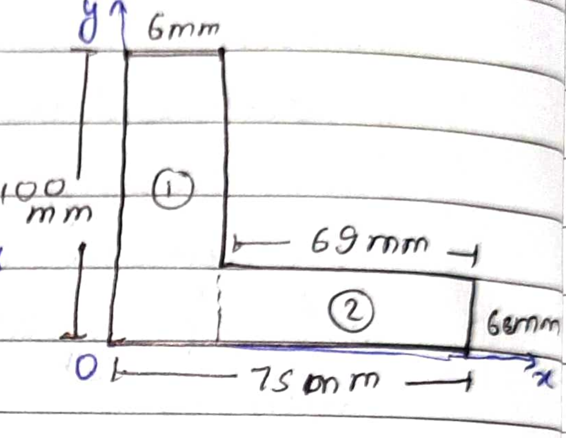
Q3(b) Find moment of inertia of given L- section about centroidal XX and YY axis :-



First, find location $\bar{x}$ and $\bar{y}$
so, given section divided into two rectangular sections ① and ② and take OY and OX as axis of reference.

then,

For $\bar{x}$:-

$a_1 = 6(100) = 600\ mm^2$; $x_1 = 3mm$.

$a_2 = 69(6) = 414 mm^2$; $x_2 = \frac{69}{2} + 6 = 40.5 mm$

then, $\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1+a_2} = \frac{600(3) + 414(40.5)}{600 + 414}$

$\boxed{\bar{x} = 18.31 mm}$

For $\bar{y}$:-

$a_1 = 6(100) = 600 mm^2$; $y_1 = 50mm$.

$a_2 = 69(6) = 414 mm^2$; $y_2 = 3mm$.

then, $\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2} = \frac{600(50) + 414(3)}{600+414}$

$\boxed{\bar{y} = 30.81 mm}$

Now, finding Moment of Inertia about centroidal X-X axis :-

$I_{xx_1} = (I_{G_1})_x + a_1h_1^2$ (By parallel axis theorem).

$I_{xx_1} = \frac{bd^3}{12} + a_1(y_1 - \bar{y})^2$

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$$I_{xx_1} = \frac{6(100)^3}{12} + 600(50 - 30.81)^2$$

$$I_{xx_1} = 720.95 \times 10^3 \, mm^4 \quad \text{—— ①}$$

similary, $I_{xx_2} = (I_{G_2})_x + a_2 h_2^2$.

$$= \frac{bd^3}{12} + a_2(y_2 - \bar{y})^2$$

$$I_{xx_2} = \frac{69 \times 6^3}{12} + 414(3 - 30.81)^2$$

$$I_{xx_2} = 321.42 \times 10^3 \, mm^4 \quad \text{—— (11)}$$

for I_{xx}:- add ① and ② :-

$$I_{xx} = I_{xx_1} + I_{xx_2}.$$

$$I_{xx} = 720.95 \times 10^3 + 321.42 \times 10^3.$$

$$\boxed{I_{xx} = 1042.378 \times 10^3 \, mm^4} \text{ Ans}$$

Now, To find Moment of Inertia about centroidal Y-Y axi's :-

$$I_{yy_1} = (I_{G_1})_y + a_1(x_1 - \bar{x})^2$$

$$= \frac{100(6)^3}{12} + 600(3 - 18.31)^2$$

$$I_{yy_1} = 142.437 \times 10^3 \, mm^4 \quad \text{—— ①}$$

$$I_{yy_2} = (I_{G_2})_y + a_2(x_2 - \bar{x})^2$$

$$= \frac{6 \times (69)^3}{12} + 414(40.5 - 18.31)^2$$

$$I_{yy_2} = 368.1 \times 10^3 \, mm^4 \quad \text{—— ②}$$

Add ① and ② :-

$$I_{yy} = I_{yy_1} + I_{yy_2} \Rightarrow 142.437 \times 10^3 + 368.1 \times 10^3$$

$$\boxed{I_{yy} = 510.537 \times 10^3 \, mm^4} \text{ Ans}$$

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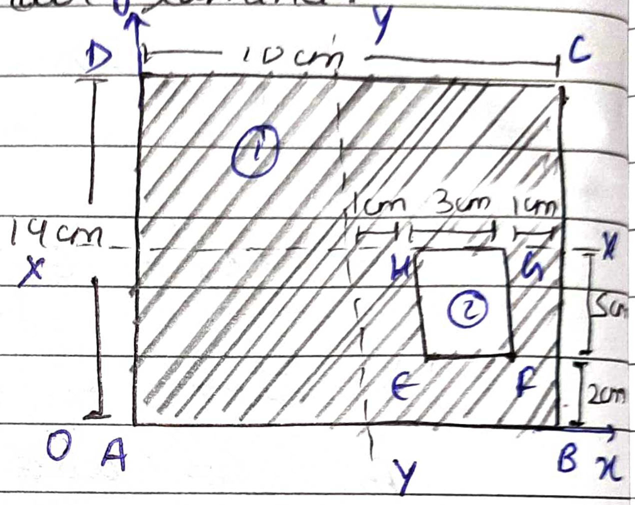
Q4. From a rectangular lamina ABCD 10cm × 14 cm a rectangular hole of 3cm × 5cm is cut as shown find the C.O.G of remainder lamina?



① To find C.O.G of remainder lamina about OY axis :-

a_1 (ar ABCD)

$= 10(14) = 140 \text{ cm}^2$.

x_{g_1} (distance of G of ① from O-Y axis):-

$x_1 g_1 = 5 \text{ cm}$.

a_2 (ar EFGH Hollow) $= 3(5) = 15 \text{ cm}^2$.

x_{g_2} (distance of G of ② from OY axis):-

$$x_2 = \frac{3}{2} + 1 + 5 = 6 + \frac{3}{2} = 7.5 \text{ cm}$$

so, $\bar{x} = \dfrac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$ (-ve sign is due to Hollow part).

$$\bar{x} = \frac{140(5) - 15(7.5)}{140 - 15}$$

$$= \frac{700 - 112.5}{125} = 4.7 \text{ cm}$$

$\boxed{\bar{x} = 4.7 \text{ cm.}}$ Ans

② To find C.O.G of remainder lamina about OX axis

$a_1 = 140 \text{ cm}^2$; $y_1 = 7 \text{ cm}$.

$a_2 = 15 \text{ cm}^2$; $y_2 = \frac{5}{2} + 2 = \frac{9}{2} = 4.5 \text{ cm}$

then, $\bar{y} = \cancel{\dfrac{a_1 y_1 (+) a_2 y_2}{a_1 - a_2}} = \dfrac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$

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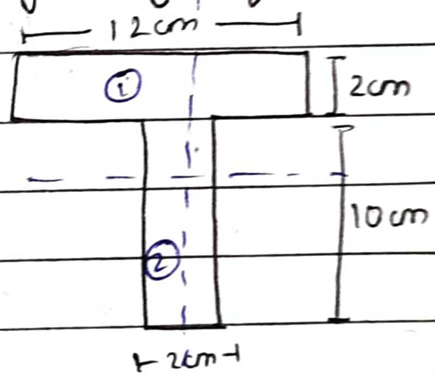
$$\bar{y} = \frac{140(7) - 15(4.5)}{140-15}$$

$$\bar{y} = \frac{980 - 67.5}{125}$$

$$\boxed{\bar{y} = 7.33\ cm} \quad \underline{Ans}$$

Q5 For the T-section shown, determine the moment of Inertia of section about the horizontal and vertical axes, passing through the centre of gravity of section?

C.G of this fig. is (0, 8.272) {already calculated}



Moment of inertia about YY :-

$$= \frac{a_1 d_1^2}{12} + \frac{a_2 d_2^2}{12}$$

$$= \frac{24(12)^2}{12} + \frac{20(2)^2}{12}$$

$I_{yy} = 288 + 6.67 = 294.67\ cm^4$. Ans

about XX :-

$$= \frac{a_1 d_1^2}{12} + a_1 x_1^2 + \frac{a_2 d_2^2}{12} + a_2 x_2^2$$

$$= \frac{24(2)^2}{12} + 24(2.728)^2 + \frac{20(10)^2}{12} + 20(3.272)^2$$

$= 186.607 + 166.67 + 214.119$

$$\boxed{I_{xx} = 567.3966\ cm^4} \quad \underline{Ans}$$

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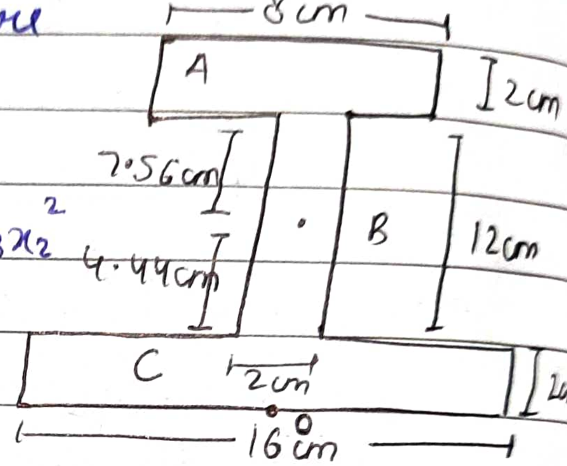
Q6. For the I figure of Q2. Find moment of inertia about XX axis passing through C.G.?

As per Q2. C.G of this figure is at (0, 6.44).



Moment of inertia about XX :-

$= \frac{a_A d_1^2}{12} + a_A x_1^2 + \frac{a_B d_2^2}{12} + a_B x_2^2$

$+ \frac{a_C d_3^2}{12} + a_C x_3^2$

$= \frac{16}{12}(2)^2 + 16(8.56)^2 + \frac{24}{12}(12)^2 + 24(1.56)^2 +$

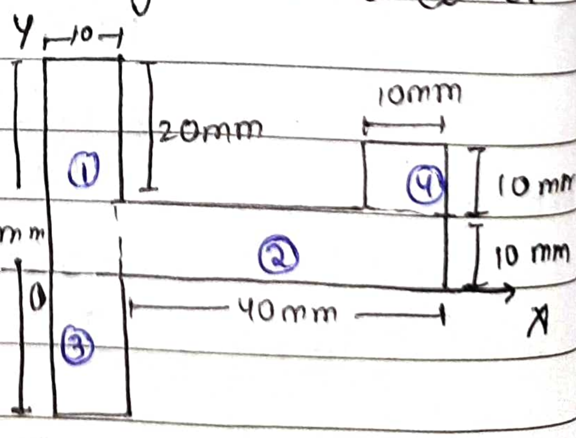
$\frac{32}{12}(2)^2 + 32(5.44)^2.$

$= 5.33 + 1472.37 + 288 + 58.4064 + 10.67$
$+ 946.99$

$= 2481.77 \text{ cm}^4$ **Ans**

Hence, Moment of inertia about XX $= 2481.77 \text{ cm}^4$.

Q7. Find C.G of area as shown in fig. wrt to co-ordinate axes.



Given fig. divided into 4 sections.

then, $a_1 = 10 \times 30 = 300 \text{ mm}^2$

$x_1 = 5\text{mm}$, $y_1 = 15\text{mm}$.

$a_2 = 10(40) = 400 \text{ mm}^2$

$x_2 = 20 + 10 = 30\text{mm}$, $y_2 = 5\text{mm}$.

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$a_3 = 10(20) = 200 mm^2$, $x_3 = 5mm$, $y_3 = -10mm$

$a_4 = 10(10) = 100 mm^2$, $x_4 = 5+10+30 \Rightarrow 45mm$.

$y_4 = 10+5 = 15mm$

then,

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4}{a_1+a_2+a_3+a_4}$$

$$\bar{y} = \frac{300(15) + 400(5) + 200(-10) + 100(15)}{300+400+200+100}$$

$$\bar{y} = \frac{4500 + 2000 + (-2000) + 1500}{1000}$$

$\boxed{\bar{y} = 6mm}$ Ans

similarly,

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4}{a_1+a_2+a_3+a_4}$$

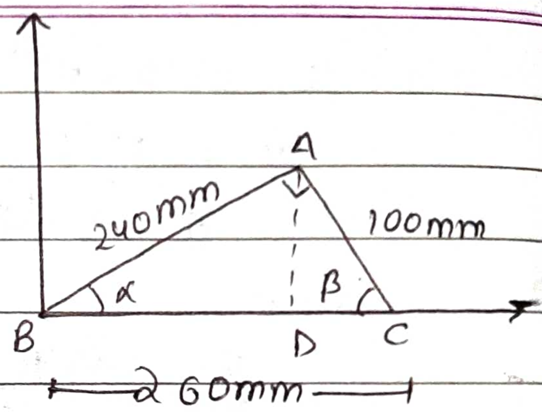
$$\bar{x} = \frac{300(5) + 400(30) + 200(5) + 100(45)}{1000}$$

$$\bar{x} = \frac{1500 + 12000 + 1000 + 4500}{1000}$$

$\boxed{\bar{x} = 19mm}$ Ans

Q8. A thin homogeneous wire is bent into a trianglular shape ABC such that AB = 240mm, BC = 260mm and AC = 100mm. Locate the C.G of wire wrt co-ordinate axes. Angle at A is right angle.

First determine angle α and β, using sine rule :-



$$\frac{100}{\sin\alpha} = \frac{240}{\sin\beta} = \frac{260}{\sin 90}$$

$$\sin\alpha = \frac{100(1)}{260}$$

$$\boxed{\alpha = 22.62^\circ}$$

$$\sin\beta = \frac{240(1)}{260} = \frac{240}{260}$$

$$\boxed{\beta = 67.38^\circ}$$

For $\bar{x}$:- $\bar{x} = \frac{L_1x_1 + L_2x_2 + L_3x_3}{L_1 + L_2 + L_3}$; $L_1 = AB = 240$,

x_1 = distance of C.G of AB from Y-axis.

$= \frac{240}{2}\cos\alpha = \frac{240}{2} \times \cos 22.62 = 110.77$ mm.

$L_2 = BC = 260$mm, x_2 = distance of C.G of BC from Y-axis $= 130$.

$L_3 = AC = 100$mm, x_2 = distance of C.G of AC from Y-axis =

$BD + \frac{100\cos\beta}{2}$

$= 240\cos\alpha + 50\cos\beta$

$= 240.77$ mm.

then,

$$\bar{x} = \frac{240(110.77) + 260(130) + 100(240.77)}{240 + 260 + 100}$$

$\boxed{\bar{x} = 140.77 \text{ mm}}$ Ans

$$\bar{y} = \frac{L_1 y_1 + L_2 y_2 + L_3 y_3}{L_1 + L_2 + L_3}$$ $\Rightarrow$ where, $y_1 = \frac{240}{2} \sin\alpha$

$= 120 \sin(22.62)$

$y_2 = 0$ $\qquad = 46.154\ mm.$

$y_3 = \frac{100}{2} \sin\beta = 50 \sin(67.38) = 48.154 mm.$

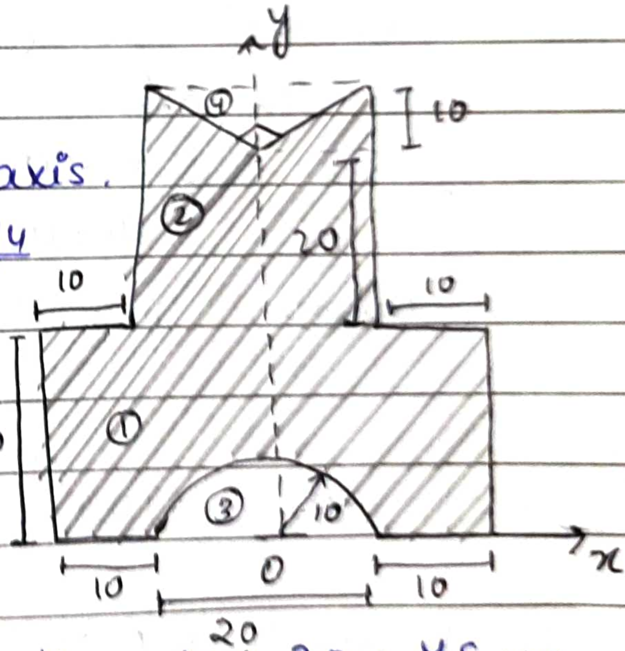
$$\bar{y} = \frac{240(46.154) + 260(0) + 100(46.154)}{600}$$

$\boxed{\bar{y} = 26.154\ mm.}$ Ans

Q9 Determine the C.G of uniform plane lamina shown in fig. All the dimension are in cm.

Given fig is symmetrical about Y-Y axis.

so, C.G will lie on this axis.



$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4}{a_1 + a_2 + a_3 + a_4}$$

where,

$a_1 = 40(30) = 1200 cm^2.$

$y_1 = \frac{30}{2} = 15cm.$

$a_2 = 30(20) = 600\ cm^2$, $y_2 = 15 + 30 = 45\ cm.$

$a_3 = -\frac{\pi(10)^2}{2} = -50\pi$, $y_3 = \frac{4r}{3\pi} = \frac{40}{3\pi}$

$a_4 = -\frac{20(10)}{2} = -100$; $y_4 = 60 - \frac{10}{3} = \frac{170}{3}.$

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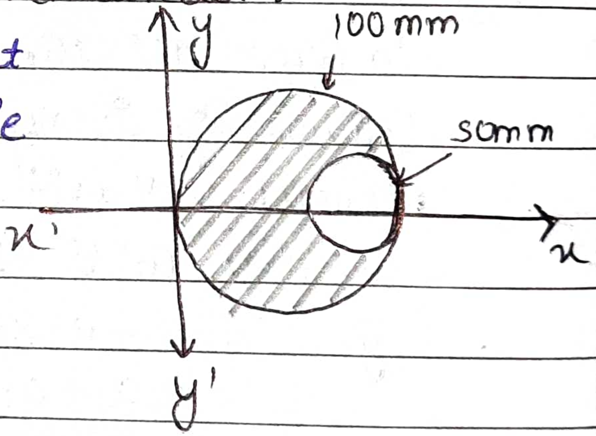
$\therefore \quad \bar{y} = \dfrac{1200(15) + 600(45) - 50\pi \times \frac{40}{3\pi} - 100\left(\frac{170}{3}\right)}{1200 + 600 - 50\pi - 100}$

$\bar{y} = \dfrac{18000 + 27000 - 666.7 - 5666.7}{1700 - 50\pi}$

$\boxed{\bar{y} = 25.06\,cm}$ from origin O Ans

Q10 From a circular plate of diameter 100mm a circular part of diameter 50mm is cut as shown in fig. Find the centroid of remainder.

Given fig is symmetrical about xx' axis so, C.G will lie on this axis.



$\bar{x} = \dfrac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$

where, $a_1 = \underline{\pi (100/2)^2}$

$= 2500\pi.$

$x_1 = \dfrac{100}{2} = 50mm.$

$a_2 = -\pi \left(50/2\right)^2 = -625\pi.$ (due to Hollow)

$x_2 = \dfrac{50}{2} + 50 = 75$ mm.

$\bar{x} = \dfrac{2500\pi(50) - 625\pi(75)}{2500\pi - 625\pi}$

$\boxed{\bar{x} = 41.67\,mm. \text{ and } \bar{y} = 0}$ Ans