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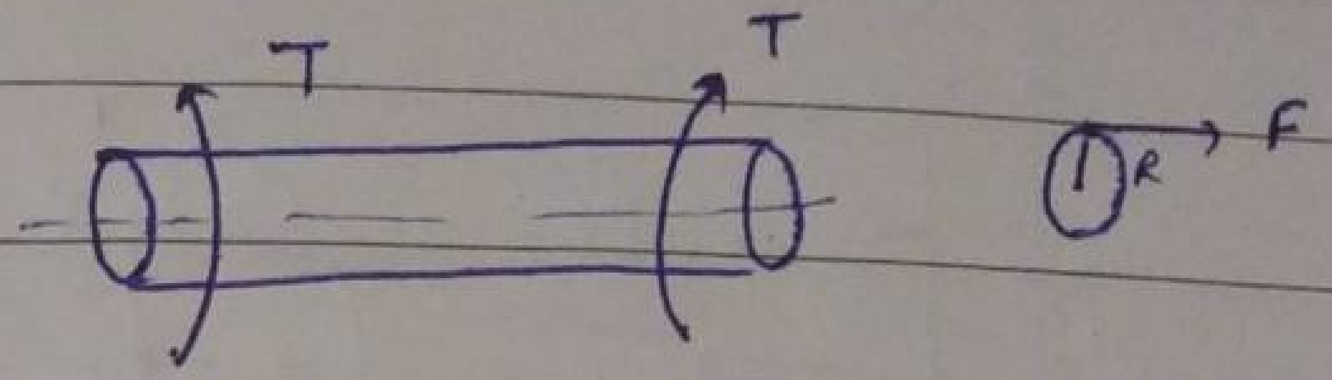
Theory of torsion:-

Torque (T) = tangencial force X Radius

F X R

$\boxed{T = F \times R}$



⇒ A shaft is said to be in torsion, when equal & opposite torques are applied at the two ends, of the saft.

Torsion ⇒ twisting moment

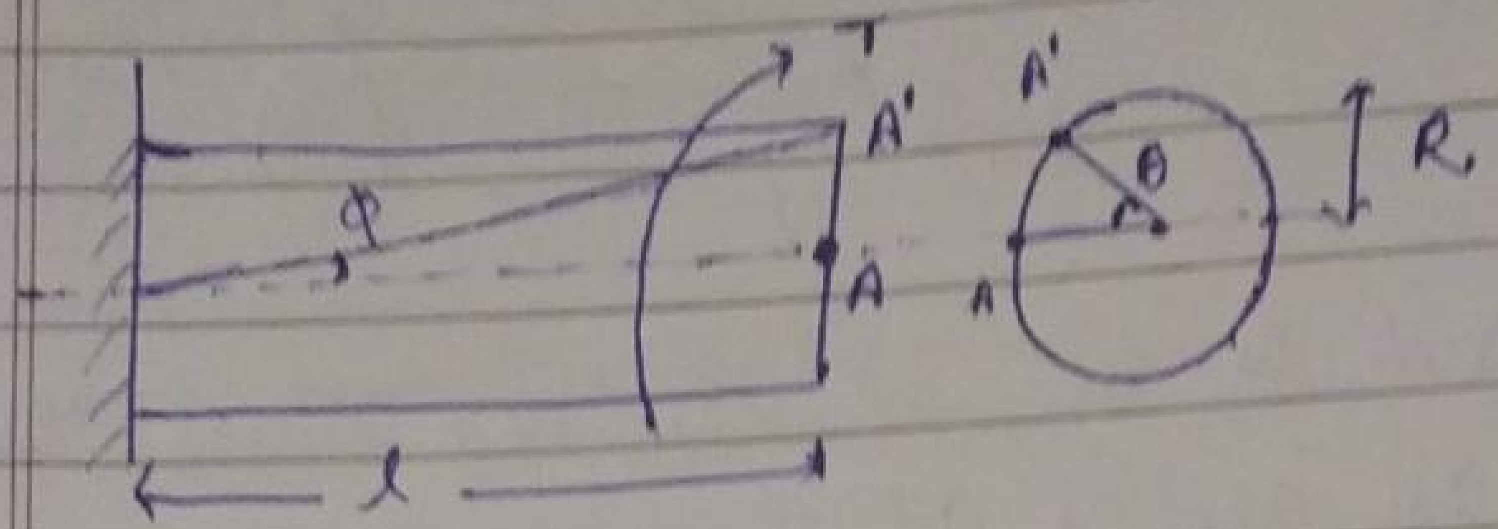
⇒ This twisting moment causes shear stresses and shear strain's in the material of shaft.

⇒ Assumptions used in tortional analysis :-

1. Material of shaft is Homogeneous and isotropic.
2. the twist along the shaft is uniform.
3. the Shaft is of uniform cross-section throughout.
4. load is within elastic limit.
5. Cross-section of shaft remains plane before & after twist.
6. No change occurs in length of shaft.

Torsional equation:- Derivation.

Q. prove that torsional equation is given by $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$



consider a shaft of length l and radius R.

$\tan\phi = \frac{AA'}{l}$ from cross-section.

$AA' = R\theta$ —(2)

since ϕ is very small angle,

$$\boxed{\phi = \frac{AA'}{l}} \text{ —(1)}$$

from eqⁿ (1) + (2)

$$\phi = \frac{R\theta}{l} \text{ —(3)}$$

ϕ = shear strain, τ = shear stress

$\frac{\tau}{\phi} = G$ (modulus of rigidity)

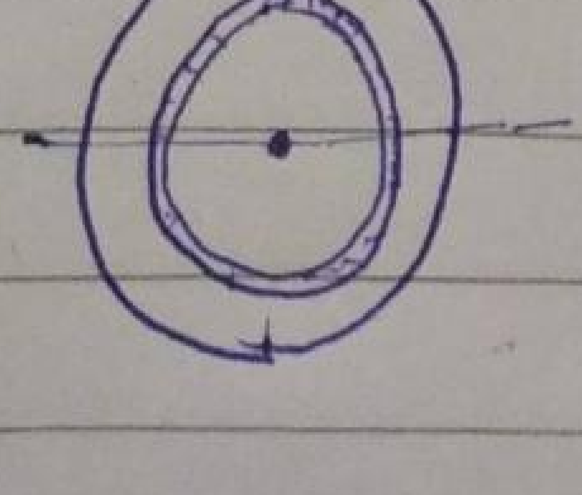
or $\phi = \frac{\tau}{G}$ —(4)

from eqˢ (3) + (4)

$$\frac{\tau}{G} = \frac{R\theta}{l} \quad \text{or} \quad \boxed{\frac{\tau}{R} = \frac{G\theta}{l}}$$

Proved → (A)

Now, analysing cross sectional view:-



thickness dr, radius = r

Radius of cross section = R

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Considering an elemental ring of radius r

$\therefore$ Resistive force developed by ring

$$dF = \text{shear stress} \times \text{area}$$

from eq^n A $\quad \tau = \frac{G\theta R}{l}$

$$dF = \tau \times dA$$

$$= \frac{G\theta r}{l} \times dA$$

Therefor resitive torque produced by ring,

$$dT = dF$$

$$= \frac{G\theta}{l} r \cdot dA \cdot r$$

$$dT = \frac{G\theta}{l} r^2 \cdot dA \qquad dA = 2\pi r \cdot dr$$

total Torque,

$$dT = \cancel{\frac{G\theta}{l}\int_0^{R} r^2 \cdot 2\pi r \cdot dr} \quad \frac{G\theta}{l}\int r^2 \cdot dA$$

$$\cancel{\frac{G\theta}{l} \times 2\pi \times \frac{R^3}{3}}$$

$$T = \frac{G\theta}{l} \cdot J$$

$\int r^2 \cdot dA =$ Polar moment of interia J

$$\boxed{\frac{T}{J} = \frac{G\theta}{l}} \quad \text{——} (B)$$

from eq^n (A) & (B)

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}} \quad \text{Torsional eq}^n$$

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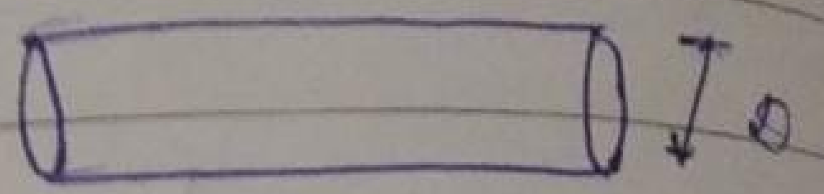
Ques:- the average torque transmitted by a shaft is 2255 N.m. the max torque is 146%. average torque. If the allowable shear stress in the shaft material is 45 N/mm², determine the suitable diameter of the shaft.



Given:

$T_{av} = 2255 \text{ N.m}$

$T_{max} = 146\%$ of av. torque

$\tau_{max} = 45 \times 10^6 \text{ N/m}^2$

We know that

$$\frac{T_{max}}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$$

$\boxed{J = \text{polar modulus}}$

$$T_{max} = \frac{146 \times 2255}{100} = 3292.3 \text{ N.m}$$

$$J = \frac{\pi D^4}{32} \quad \text{(for circular)}$$

$$R = \frac{D}{2},$$

$$\frac{3292.3}{\frac{\pi D^4}{32}} = \frac{45 \times 10^6}{D/2}$$

$$\text{or} \quad \frac{3292.3 \times 32}{\pi D^3} = \frac{45 \times 10^6 \times 2}{1}$$

$$D^3 =$$

$$\boxed{D = 71.9 \text{ mm}}$$

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Q: A solid shaft is subjected to a maximum torque of 15 MN.cm. Determine the diameter of the shaft, if the allowable shear stress and the twist are limited to 1 kN/cm² and 1° respectively for 210 cm length of shaft. G = 8 MN/cm²

given,

$T_{max} = 15 \times 10^6$ N.cm

$\tau = 1$ kN/cm² $= 10^3$ N/cm²

$\theta = 1° = \frac{\pi}{180}$ rad.

$l = 210$ cm

$G = 8 \times 10^6$ N/cm²

$D = ?$

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}}$$

θ = radi..

① using strength criteria (str.)

$$\frac{T}{J} = \frac{\tau}{R},$$

$$\frac{15 \times 10^6}{\frac{\pi D^4}{32}} = \frac{10^3}{\frac{D}{2}}$$

$$\Rightarrow \boxed{D = 42.43 \text{ cm}}$$

② Now using stiffness criteria,

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$= \frac{15 \times 10^6}{\frac{\pi D^4}{32}} = \frac{8 \times 10^6 \times \pi}{210 \times 180}$$

$$\boxed{D = 21.89 \text{ cm}}$$

for dimention always consider larger one therefore suitable diameter of shaft, $\boxed{D = 42.43 \text{ cm.}}$

and load minimum consider. [illegible]

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Ques:- In a tensile test, a test piece 25 mm in diameter, 200 mm gauge length streched 0.0975 mm under a pull of 50,000 N. In a torsion test, the same rod twisted 0.025 radian over a length of 200 mm when a torque of 400 N·m was applied, evaluate the poissons ratio and the three elastic modulus for the material.

$\mu = ?$ Given, $D = 25\,mm$

Young's $E = ?$ $l = 200\,mm$

rigidity $G = ?$ $P = 50{,}000\,N$

Bulk, $K = ?$ $\Delta l = 0.0975\,mm$

$\theta = 0.025$ rad.

$T = 400\,N\cdot m$

$= 400 \times 10^3\,N\cdot mm$

⇒ During tensile test

$$\Delta l = \frac{Pl}{AE}$$

$$E \Rightarrow \frac{Pl}{A\Delta l} = \frac{50{,}000 \times 200}{\frac{\pi}{4}(25)^2 \times 0.0975}$$

$$\boxed{E = 208941.87\ N/mm^2}$$

⇒ During torsion test

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$G = \frac{Tl}{J\theta} = \frac{400 \times 10^3 \times 200}{\frac{\pi}{32} D^4 \times 0.025}$$

$$\boxed{G = 83493.026\ N/mm^2}$$

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as, we know

$$E = 2G(1+\mu)$$

$$\mu = \left(\frac{E}{2G} - 1\right)$$

$$\boxed{\mu = 0.252}$$

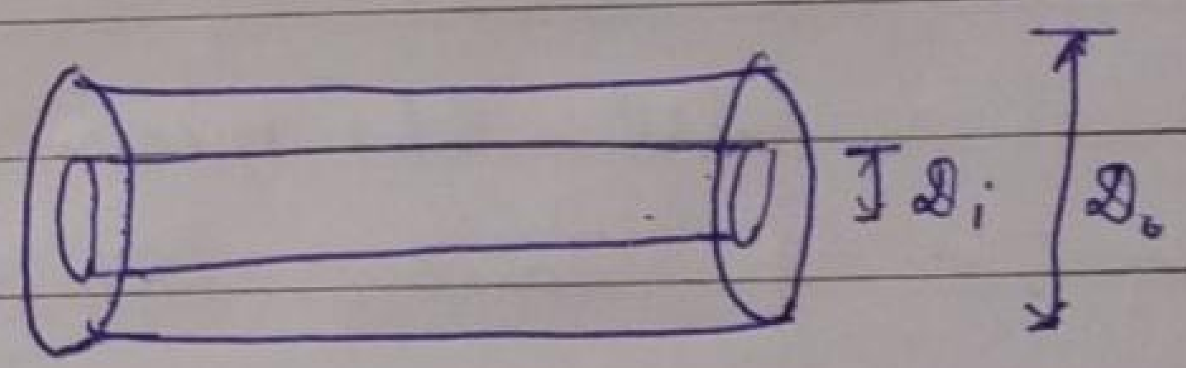
⇒ calculation for k

$$E = 3k(1-2\mu)$$

$$K = \frac{E}{3(1-2\mu)}$$

$$\boxed{K = 139294.58\ N/m^2}$$

Ques: A Hollow shaft of external diameter 120mm transmits 300kW power at 200 r.p.m. Determine the maximum internal diameter if the maximum stress in the shaft is not to exceed $60 N/mm^2$



given

$D_o = 120mm$ $\quad D_i = ?$

$P = 300kW$ $\quad \tau_{max} = 60 N/m^2$

$N = 200$ r.p.m

We know: $$P = \frac{2\pi N T}{60}$$

$$120 = \frac{2\pi \times 20\cancel{0} \times T}{6\cancel{0}}$$

$$\boxed{T = 14323.94\ N\cdot m}$$

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$$\frac{T}{J} = \frac{\tau_{max}}{R} \quad \text{---(1)}$$

where $J = \frac{\pi}{32}\left[D_o^4 - D_i^4\right]$, $R = \frac{D_o}{2}$

$$\frac{14332.97 \times 10^3}{\frac{\pi}{32}\left[D_o^4 - D_i^4\right]} = \frac{60}{D_o/2}$$

$$\boxed{D_i = 88.57\,mm}$$

Q: A solid steel shaft has to transmit 75 kW at 200 r.p.m. Taking allowable shear stress as 70 N/mm² find suitable diameter exceeds the mean by 30%.

given.
$P = 75\,kW$
$N = 200$ r.p.m.
$\tau_{max} = 70\,N/mm^2$
$D = ?$

let the mean torque is T

$T_{max} = T + \frac{30}{100}T = 1.3T$

$T_{max} = 1.3T$

we, know. $P = \frac{2\pi NT}{60}$

$$\boxed{T = 3580.98\,N\cdot m}$$

$T_{max} = 1.3 \times 3580.98$

$$\boxed{T_{max} = 4655.28\,N\cdot m}$$

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using the relation,

$$\frac{T_m}{J} = \frac{\tau_m}{R}$$

$$\frac{=96555.28}{\frac{\pi}{32} D^4} = \frac{70}{D/2}$$

$$\boxed{D = 69.70\ mm}$$

Qus:- Determine suitable diameters of hollow steel shaft whose internal diameter is 0.6 times it's external diameter. The shaft transmits 220 kw at 200 r.p.m. The maximum shear stress is 75 M Pa,

$D_i = 0.6 D_0$ — ①

$P = 220\ kW = 220 \times 10^3\ W$

$N = 200\ r.p.m.$

$\tau_m = 75\ MPa = 75 \times 10^6\ N/m^2$

$\theta = 1^\circ$

$l = 1m$, $G = 80 kN/mm^2$

$G = 80 \times 10^9\ N/mm^2$

$\theta = \frac{\pi}{180}$ rad.

⇒ using strength cirte

$$\frac{T}{J} = \frac{\tau}{R}$$

$$P = \frac{2\pi N T}{60}$$

$$\boxed{T = 10504.23\ Nm}$$

$$\frac{10504.23}{\frac{\pi}{32}\left[D_0^4 - D_i^4\right]} = \frac{75 \times 10^6 \times 2}{D_0}$$

810000 - 50625

759,355 $\times 10^5$

$$\boxed{D_0 = 93.58\ mm.}$$

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$$\boxed{D_o = 93.58\,mm}$$

$$D_i = 56.15\,mm$$

from, stifness criteria,

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$\frac{10504.23 \times 32}{\pi\left[D_o^4 - D_i^4\right]} = \frac{80 \times 10^3 \times \pi}{180 \times 1m}$$

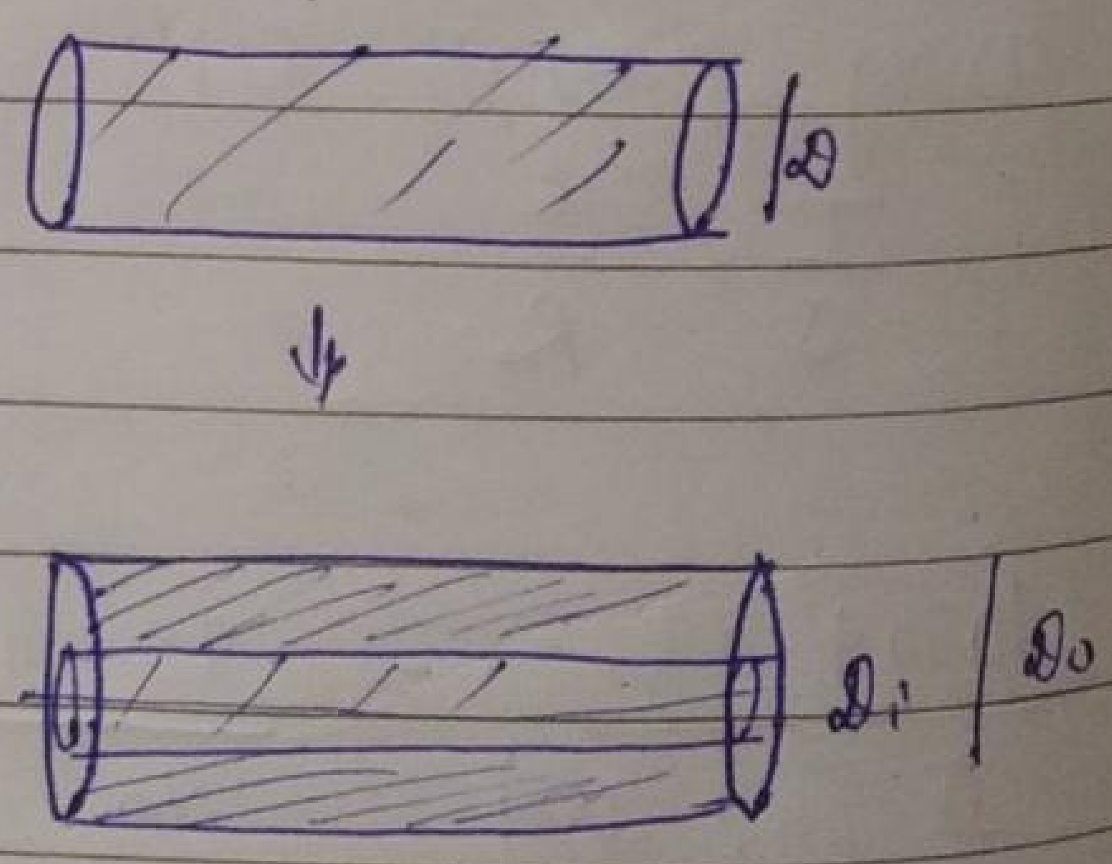
$$\boxed{\begin{aligned} D_o &= 96.8\,mm \\ D_i &= 58.08\,m. \end{aligned}}$$

suitabe diameter : $D_o = 96.8\,mm$
$D_i = 58.08\,m$

Q: A solid shaft of diameter 300 mm is proposed to be replaced by a hollow shaft of internal diameter equal to 0.7 times the external diameter. determine the external diameter of hollow shaft, if the same power is transmitted at the same level of stress.

$P_H = P_S$ H = Hollow
S = solid

$\tau_H = \tau_S$



as the power is same
$P_H = P_S$

$$= \frac{2\pi N T_H}{60} = \frac{2\pi N T_S}{60}$$

$$\boxed{T_H = T_S}$$

$D_i = 0.7\,D_o$
$D_o = ?$

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using strength criter -

$\frac{T}{J} = \frac{\tau}{R}$

$\Rightarrow T = \frac{\tau}{R} \cdot J$

$T = \frac{\tau \times 2}{D} \times \frac{\pi}{32} D^4$

$\boxed{T_S = \frac{\pi}{16} \tau D^3}$

for Hollow shaft,

$T = \frac{\tau}{R} \cdot J$

$= \frac{\tau 2}{D_o} \times \frac{\pi}{32} [D_o^4 - D_i^4]$

$T_H = \frac{\pi}{16} \frac{\tau}{D_o} [D_o^4 - D_i^4]$

$\cancel{\frac{\pi}{16}} \cancel{\tau} D^4 = \cancel{\frac{\pi}{16}} \frac{\cancel{\tau}}{D_o} [D_o^4 - D_i^4]$

$\frac{D_o^4 - (0.7 D_o)^4}{D_o} = (300)^3$

$\boxed{D_o = 328.74 \text{ mm}}$

$D_o = 328.74$mm an -

Que:- A shaft of hallow circular section has outer diameter 120mm, inner diameter 100 mm possible shear stress is 95 MPa. angle of twist not to exceed 3.6° in a length of 3m. The maximum value of power that can be transmitted. Take modulus of rigidity = 80 GPa.

$\omega = 2.r.p.s.$

$T_{max} = T + \frac{T \times 30}{100}$

$\boxed{T_m = 1.3T}$

$D_o = 120$mm

$D_i = 100$mm

$\tau_{max} = 95 \times 10^6$ Ma $= 95$ N/mm²

$\theta = 3.6° = 3.6 \times \frac{\pi}{180}$.

$P = ?$

$l = 3000$mm

$G = 80$G Pa

$\boxed{P_m = 25674.46 W}$