UNIT 4 QUANTUM MECHANICS

LECTURE 3

LECTURE 1 - Revision

Need of quantum mechanics

To overcome the limitation of classical mechanics

Classical mechanics failed to explain….

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1) Stability of atom
2) Spectral distribution of black body radiation

   Planck's quantum hypothesis

3) Origin of discrete spectra of atoms
4) Photoelectric effect

   particle nature of light by Einstein

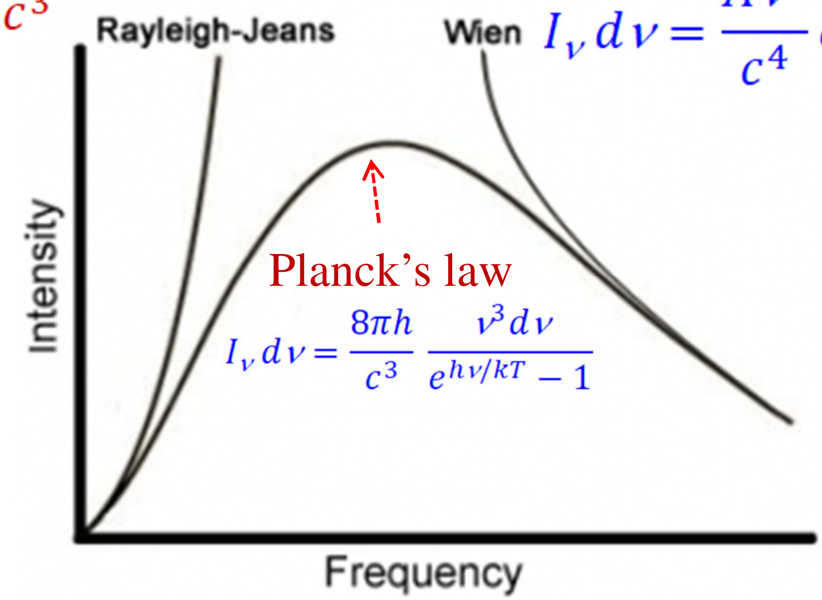
5) Crompton effect
6) Raman effect

LECTURE 1 - Revision

1. Black body radiation and Planck's hypothesis

$$I_\nu \, d\nu = \frac{8\pi \nu^2}{c^3} kT \, d\nu$$

$$I_\nu \, d\nu = \frac{A \nu^3}{c^4} e^{-B\nu/T} \, d\nu$$



$$I_\nu \, d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 \, d\nu}{e^{h\nu/kT} - 1}$$

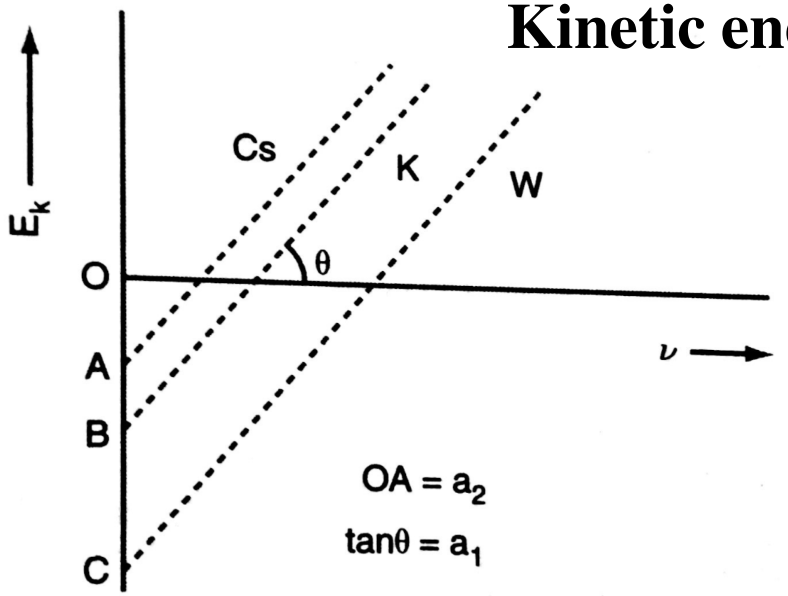
1. Atomic oscillator
2. Quanta of energy

- ✓ Rayleigh-Jeans can be deduced from Planck's law for low frequency (large wavelength) and high temperature
- ✓ Similarly Wien's law can be deduced from Planck's law for high frequency (low wavelength) and low temperature

LECTURE 1 - Revision

2. Photoelectric effect

Discovered by Hertz but explained by Einstein



Kinetic energy of the ejected electron E_k

$$E_k = h\nu - h\nu_0$$

Work function

Incident photon energy

This effect says about the emission/ejection of electrons from the metal surface upon irradiation with light waves

LECTURE 1 - Revision

- **W**e knew the wave nature of light or electromagnetic radiation with the help of Maxwell's equation- electromagnetic theory
- **T**hen assumed particle nature of light with the Planck's black body radiation
- **W**ith Einstein's photoelectric equation we experimentally proved particle nature of light

✓ *Importance of quantum mechanics and quantum/particle nature of light*

Wave nature of particles?? A mathematical relation connecting wavelength (λ) to momentum (p) De Broglie!

LECTURE 2 - Revision

Nature loves symmetry!!

1. Concept of de Broglie matter waves
   - *Dual nature of radiation…the fact that ignited* ***de Broglie's curiosity***
   - *Dual nature of matter -* ***de Broglie's imagination***

2. Wavelength of matter waves in different forms
   - *Relation between wavelength and momentum/energy…*

- E and p are the characteristics of the particle
- ν and λ are the characteristics of the wave

Can we find a mathematical formulation to connect these two?

LECTURE 2 - Revision

Dual nature of radiation

In the case of radiation (Plank's theory), we know Energy, $E = h\nu$

Now will go to Einstein special theory of relativity and that famous equation $E = mc^2$

De Broglie hypothesized that the two energies would be equal

$$mc^2 = h\nu = \frac{hc}{\lambda} \Longrightarrow mc = \frac{h}{\lambda}$$

But *mc* is nothing but the momentum of photon, $p = \frac{h}{\lambda}$

.. by mixing Einstein's famous matter-energy relation with Planck's famous quantum oscillator theory.. Wavelength of the wave is related to the momentum of its particle through the Planck's constant ..

LECTURE 2 - Revision

If a wave can be so then why not a particle?

de Broglie extended matter concept of radiation and applied to particles as well..

Because real particles do not travel at the speed of light, De Broglie used velocity (v) for the speed of light (c).

$$E = mv^2 = h\nu \Rightarrow mv = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

λ is the de Broglie wavelength of the matter wave of the particle moving with velocity v and momentum p

The **de Broglie wavelength** is the **wavelength**, λ, associated with a massive particle and is related to its momentum, p, through the Planck constant, h: In other words, you can say that matter also behaves like waves.

LECTURE 2 - Revision

1. If particle is accelerated through the kinetic energy

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}$$

2. If a charged particle having charge (q) is accelerated through electrostatic potential V

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

3. If the particle having mass (m) is accelerated by means of thermal energy

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$$

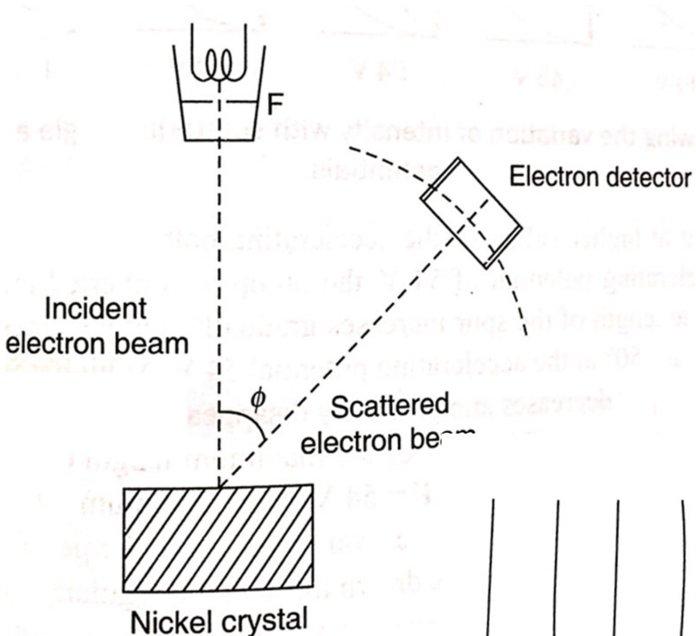
4. If the particle having rest mass (m_0) is moving with a velocity(v) comparable to the speed of light (c)

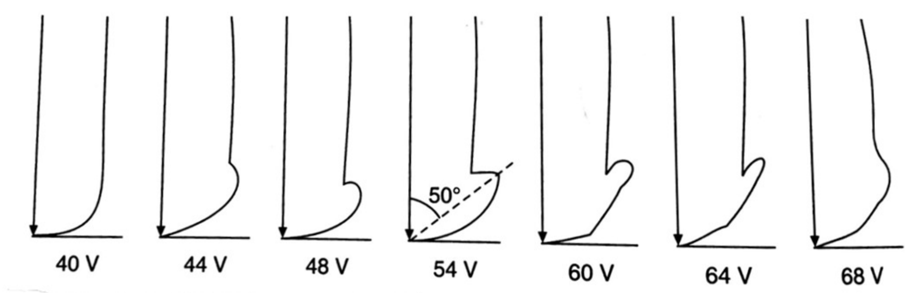
$$\lambda = \frac{h}{p} = \frac{h\sqrt{1-(v/c)^2}}{m_0 v}$$

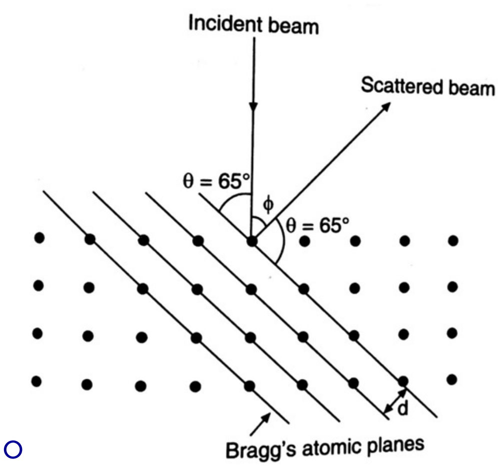
LECTURE 2 - Revision

Davisson-Germer Experiment

- Heated filament electron source
- Accelerated by applying voltage
- Intensity of the scattered electron measured
    - As function of accelerating voltage
    - As function of angle ϕ
- Plotted in form of polar diagram







Maximum scattering intensity is observed for ϕ=50°

$\theta+\phi+\theta=180°$

$2\,\theta+50°=180°;\ \theta=65°$

If it is due to electron diffraction (a wave phenomenon) then Bragg's law should be satisfied for the glancing angle θ=65°

$$2d\sin\theta = n\lambda$$

For nickel crystal d =0.91Å and for first order diffraction n=1

λ=2x 0.91Åx sin65°= **1.65Å**

Now we have to calculate the de Broglie wavelength of the electron accelerated with an voltage 54 V

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

Here q is e⁻, electron charge and V=54 V

$$\frac{6.625 \times 10^{-34}}{\sqrt{[2 \times 1.632 \times 10^{-19} \times 54 \times 9.1 \times 10^{-31}]}}$$

So de Broglie wavelength of electron for 54V acceleration is **1.67 Å**.

Comparable with the experimentally determined wavelength (1.65Å) of wave using wave diffraction experiment

Quick QUIZ

Quick Quiz Response on the 9/26/2018 Lecture

No	Question	Attempts	Right	Wrong
1	**Rayleigh-Jeans law is deduced from Planck's radiation formula under the condition of**	41	20	21
2	**Which of the following phenomena show the particle nature of light?**	41	33	8
3	**Wien law is deduced from Planck's radiation formula under the condition of**	41	17	24

Rayleigh-Jeans law is deduced from Planck's radiation formula under the condition of

$$I_\nu d\nu = \frac{8\pi\nu^2}{c^3} kT\, d\nu$$

a) **High frequency and low temperature**
b) **Low frequency and high temperature**
c) **High frequency and high temperature**
d) **Low frequency and low temperature**

$$I_\nu d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1}$$

Ans: B

Which of the following phenomena show the particle nature of light?

a) **Photoelectric effect**
b) **Interference**
c) **Diffraction**
d) **Polarization**

Ans: A

Wien law is deduced from Planck's radiation formula under the condition of

$$I_\nu d\nu = \frac{A\nu^3}{c^4} e^{-B\nu/T} d\nu$$

a) **High frequency and low temperature**

b) **Low frequency and high temperature**

c) **High frequency and high temperature**

d) **Low frequency and low temperature**

$$I_\nu d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1}$$

Ans: A

Quantum Mechanics

Lecture 1 Sept 26: Need of quantum mechanics, photoelectric effect,
Importance of quantum mechanics and quantum nature of light

Lecture 2 Sept 27 : Concept of de Broglie matter waves, wavelength of matter waves in different forms,
Wave/Dual nature of matter and relation between wavelength and momentum/energy

Lecture 3 Sept 28: Properties of matter wave, phase velocity and group velocity (qualitative),
Concept of phase velocity and group velocity(qualitative), dispersive medium

Lecture 4 Oct. 3*:* Heisenberg uncertainty principle,
Uncertainty principle to calculate uncertainty in the measurements of physical quantities

Lecture 5 Oct. 4 *:* Wave function and its significance Schrodinger time dependent and independent equations
Introduction to wave functions and concept of probability, basic principle in quantum physics, Probabilistic behavior of quantum physics

Lecture 6 Oct. 5 *:* Particle in a box (e.g., electron confined in a potential)
Energy of the particles/electrons is discrete and is quantized.

Properties of matter-wave

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

i. Lighter particles have large de Broglie wavelength than heavier one

ii. Smaller the velocity of the particle, the greater is the de Broglie wavelength associated with it

iii. Matter waves are generated only when particle is in motion. [v=0, $\lambda = \infty$]

iv. Matter waves are not electromagnetic ..i.e. independent of charge

v. Velocity of the matter-wave is not constant. It depends on the velocity of the particle, while velocity of the electromagnetic wave is constant

- Velocity of matter wave may be greater than the velocity of light. Difficult to believe and hence phase velocity and group velocity came into play..
- Wave-particle duality introduce the concept of uncertainty, This concept suggest that if the particle nature of the matter becomes certain, the wave nature becomes uncertain and vice versa.

phase and group velocities

1) **Velocities associated with de Broglie wave**
   i. Phase or wave velocity **(v_p)**
   ii. Group velocity **(v_g)**.. Wave packet
   iii. Particle velocity **(v)**

Analogy: city marathon runners

- Initially it would appear that all of them are running at the same speed. As time passes, group spreads out (disperses)
- because each runner in the group is running with different speed.
- If you think of phase velocity to be like the speed of an individual runner, then the group velocity is the speed of the entire group as a whole.

Phase, group & particle velocities

Phase velocity (v_p) of the wave is larger than the group velocity (v_g) of the waves?

It depends on the nature of the medium.

1) $v_p = v_g$ for non-dispersive medium- velocity not depend on wave length. Examples sound waves in air and electromagnetic waves in vacuum.

2) $v_p > v_g$ for normal-dispersive medium- electromagnetic radiation in medium where refractive depends on the wavelength and hence velocity of EM changes in the medium.

3) $v_p < v_g$ for anomalous-dispersive medium..This we see in matter-wave cases

Now we will see the relation for Phase and group velocities. And their relationship with the particle velocity (v)

Phase or Wave Velocity (v_p)

A wave travelling in the +x direction is given by

$$y = a \sin(\omega t - kx) \longrightarrow 1$$

Where a is the amplitude, ω (=2πν) is the angular frequency and k (=2π/λ) is the propagation constant

By definition the ratio of the angular frequency to the propagation constant is the phase velocity, $\mathbf{v_p}$

$$v_p = \frac{\omega}{k}$$

Now we will see why $\mathbf{v_p}$ it is called wave velocity also ?

In equation 1 (ωt-kx) is called the phase of the wave motion. And is a constant for plane wave

Phase or Wave Velocity (v_p)

$$\omega t - kx = constant$$

$$\frac{d}{dt}(\omega t - kx) = 0$$

$$\omega - k\frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k}$$

But dx/dt is the velocity of the wave.. And same as equation 1**. so phase velocity is nothing but the wave velocity**

$$v_p = \frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda} = \nu\lambda = c$$

For an electromagnetic wave in vacuum.

Phase or Wave Velocity (v_p)

Phase or Wave Velocity (v_p) for de Broglie wave

We have $v_p = \nu\lambda;\ E = h\nu;\ or\ \ \nu = E/h$ ---------------------> 2

According to de Broglie $\lambda = \frac{h}{p} = \frac{h}{mv}$ ---------------------> 3

From 2 and 3, phase velocity for the de Broglie wave

$$v_p = \nu\lambda = \frac{E}{h} \times \frac{h}{mv} = \frac{mc^2}{mv} \qquad \therefore v_p = \frac{c^2}{v}$$ ---------------------> 4

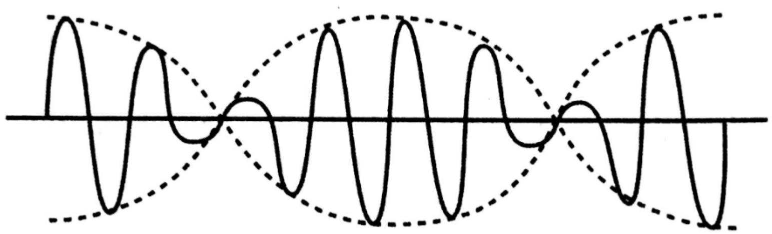
Since v << c, eqn.(4) implies that phase velocity of de Broglie wave of the particle is moving with velocity v is greater than c, speed of light!!

Group Velocity (v_g)

V_g, introduced to overcome the difficulty of $v_p > c$ of matter wave: Here each moving particle is associated with a group of wave or wave packet rather than a single wave.

$$y_1 = a\ \sin(\omega_1 t - k_1 x)$$

$$y_2 = a\ \sin(\omega_2 t - k_2 x)$$



$$y = y_1 + y_2 = a\ [\sin(\omega_1 t - k_1 x) + \sin(\omega_2 t - k_2 x)]$$

$$y = 2a\, sin\left[\frac{(\omega_1 + \omega_2)t}{2} - \frac{(k_1 + k_2)x}{2}\right] cos\left[\frac{(\omega_1 - \omega_2)t}{2} - \frac{(k_1 - k_2)x}{2}\right]$$

$$\omega = \frac{\omega_1 + \omega_2}{2} \qquad k = \frac{k_1 + k_2}{2} \qquad \Delta\omega = \omega_1 - \omega_2 \qquad \Delta k = k_1 - k$$

Group Velocity (v_g)

$$\therefore y = 2a\cos\left[\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right]\sin(\omega t - kx) \dashrightarrow \quad 5$$

Eqn.5 has two parts,

(1) A wave with angular frequency ω, propagation constant k and velocity v_p , given by

$$v_p = \frac{\omega}{k}$$

And is the phase velocity

(2) A nother wave with angular frequency $\Delta\omega$, propagation constant Δk and velocity v_g , given by

$$v_g = \frac{\Delta\omega}{\Delta k}$$

And is the group velocity.. Velocity of the wave packet.. Envelop showed by dotted lines in the figure

Group Velocity (v_g)

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{\partial\omega}{\partial k} = \frac{\partial(2\pi\nu)}{\partial\,(2\pi/\lambda)}$$

$$= \frac{\partial(\nu)}{\partial\,(1/\lambda)} = -\lambda^2 \frac{\partial\nu}{\partial\lambda}$$

So group velocity is given by

$$\therefore v_g = -\frac{\lambda^2}{2\pi}\frac{\partial\omega}{\partial\lambda}$$

Now we will the relation between v_p and v_g

relation between v_p and v_g

$$v_p = \frac{\omega}{k} \qquad v_g = -\frac{\lambda^2}{2\pi}\frac{\partial\omega}{\partial\lambda}$$

But $k = \frac{2\pi}{\lambda} \Rightarrow dk = -\frac{2\pi}{\lambda^2}d\lambda$

$$v_g = \frac{d\omega}{dk} = \frac{d(v_p k)}{dk}$$

and $\frac{k}{dk} = -\frac{\lambda}{d\lambda}$

$$v_g = v_p + k\frac{dv_p}{dk}$$

$$v_g = v_p + \left(-\frac{\lambda}{d\lambda}\right)dv_p \Rightarrow v_g = v_p - \lambda\frac{dv_p}{d\lambda} \dashrightarrow 6$$

That is group velocity is less than the phase velocity in a dispersive medium where v_p is a function of k or λ.. And for a no-dispervive medium v_p is independent of k or λ, equation 6 gives

$$v_g = v_p \quad \text{because} \quad \frac{dv_p}{d\lambda} = 0$$

relation between v_g and particle velocity (v)

Consider a material particle of rest mass m_0. Let its mass be m when moving with a velocity v. then it energy is given by

$$m = \frac{m}{\sqrt{1 - v^2/c^2}}$$

We know that

$$\omega = 2\pi\nu = \frac{2\pi E}{h} = \frac{2\pi mc^2}{h}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Substitute the value of m from the above into the last two equation of ω and k

relation between v_g and particle velocity (v)

$$\omega = \frac{2\pi m_0 c^2}{h\sqrt{1 - v^2/c^2}} \qquad k = \frac{2\pi m_0 v}{h\sqrt{1 - v^2/c^2}}$$

and differentiation with respect to the velocity of the particle v

$$\frac{d\omega}{dv} = \frac{2\pi m_0 v}{h\left(1 - v^2/c^2\right)^{3/2}} \qquad \frac{dk}{dv} = \frac{2\pi m_0}{h\left(1 - v^2/c^2\right)^{3/2}}$$

But Vg is defined as

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/_{dv}}{dk/_{dv}}$$

$$V_g = V$$

Quick QUIZ

For non-dispersive medium phase velocity (v_p) is independent of the wavelength of the wave and hence group velocity v_g is

a) $v_g > v_p$
b) $v_g < v_p$
c) $v_g = v_p$
d) none of the above

For dispersive medium phase velocity (v_p) is dependent of the wavelength of the wave and hence group velocity v_g is

a) $v_g > v_p$
b) $v_g < v_p$
c) $v_g = v_p$
d) none of the above

Matter-wave is associated with moving particle. In that case, the particle velocity is equal to the group velocity. True or False?

(a) True
(b) False

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