UNIT 4 QUANTUM MECHANICS

LECTURE 4

What we learned so far about Quantum mechanics?

1. We had a short walk down the memory lane (1900-1927)

   - ✓ Classical mechanics, relativistic mechanics, quantum mechanics and quantum field theory
   - ✓ Hertz, Planck, Einstein, Bohr, Crompton, Raman, de Broglie, Heisenberg, Schrödinger, Dirac, Pauli, Born
   - ✓ Development of quantum mechanics

2. Classical mechanics Explained

Newton's corpuscular concepts of light-particles
Maxwell's wave concepts of light from EM theory
Reflection, refraction –explained through particle concept-ray optic
Interference,, diffraction, polarization– wave nature

It was all about light!

What we learned so far about Quantum mechanics?

2. How QM concept helped in overcoming classical limitation?

$$I_\nu d\nu = \frac{8\pi\nu^2}{c^3} kT \, d\nu$$

$$I_\nu d\nu = \frac{A\nu^3}{c^4} e^{-B\nu/T} \, d\nu$$

$$I_\nu d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1}$$

Black body radiation ,
Wien and Rayliegh-Jean formula,
UV catastrophe
Planck's quantum oscillator,

$$E_k = h\nu - h\nu_0$$

Photoelectric effect,
Hertz's discovery
Einstein's photoelectric equation,
The name photon

Crompton effect-scattering of light by electron
Raman effect-vibration spectra of molecules upon photon irradiation

All these phenomenon were successfully explained by QM

What we learned so far about Quantum mechanics?

3. Characteristic properties of a wave : **ν and λ**

4. Characteristic properties of a particle: **p and E**

5. Radiation-particle duel nature

$$p = mc = \frac{h}{\lambda}$$

6. Matter –wave duel nature

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

De Broglie's hypothesis; connecting the wave nature with particle nature through the Planck's constant..

Used Einstein's famous mass-energy relation $E=mc^2$

What we learned so far about Quantum mechanics?

7. Characteristics of matter wave

8. Wave velocity, group velocity and particle velocity

$$v_p = \frac{\omega}{k} \qquad v_g = \frac{\Delta\omega}{\Delta k} \qquad \therefore v_g = -\frac{\lambda^2}{2\pi}\frac{\partial\omega}{\partial\lambda}$$

v particle velocity

non-dispersive- $v_p = v_g$
normal-dispersive- $v_p > v_g$
anomalous dispersive mediums- $v_p < v_g$

9. Relationship between v_g and v_p & v_g and v

$$v_g = v_p - \lambda\frac{dv_p}{d\lambda}$$

dispersion

$$v_g = v$$

Quick QUIZ

Quick Quiz Response on the 9/28/2018 Lecture

No	Question	Attempts	Right	Wrong
1	For non-dispersive medium phase velocity (v_p) is independent of the wavelength of the wave and hence group velocity v_g is	**59**	**35**	**24**
2	For dispersive medium phase velocity (v_p) is dependent of the wavelength of the wave and hence group velocity v_g is	**60**	**7**	**53**
3	Matter-wave is associated with moving particle. In that case, the particle velocity is equal to the group velocity. True or False?	**60**	**49**	**11**

For non-dispersive medium phase velocity (v_p) is independent on the wavelength of the wave and hence group velocity v_g is

a) $v_g > v_p$
b) $v_g < v_p$
c) $v_g = v_p$
d) none of the above

Ans: C

For dispersive medium phase velocity (v_p) is dependent on the wavelength of the wave and hence group velocity v_g is

a) $v_g > v_p$
b) $v_g < v_p$
c) $v_g = v_p$
d) none of the above

Ans: B

Matter-wave is associated with moving particle. In that case, the particle velocity is equal to the group velocity. True or False?

(a) True
(b) False

Ans: A

Who coined the name ‘quantum mechanics?

a) Planck
b) Einstein
c) Born
d) Bohr

The phrase "quantum mechanics" was coined (in German, **Quantenmechanik**) by the group of physicists including **Max Born**, **Werner Heisenberg**, and **Wolfgang Pauli**, at the University of Göttingen in the early 1920s, and was first used in Born's 1924 paper "Zur **Quantenmechanik**".

Ans: C

Which of the following phenomena can not be explained by the classical theory?

a) Photoelectric effect
b) Crompton effect
c) Raman effect
d) All of the above

Ans: d

Photoelectric effect involves

a) Free-electron
b) Bound electron
c) Both (a) and (b)
d) None of the above

Ans: C

Scheduled lectures

Lecture 1 Sept 26: Need of quantum mechanics, photoelectric effect,
Importance of quantum mechanics and quantum nature of light

Lecture 2 Sept 27 : Concept of de Broglie matter waves, wavelength of matter waves in different forms,
Wave/Dual nature of matter and relation between wavelength and momentum/energy

Lecture 3 Sept 28: Properties of matter wave, Concept of phase velocity and group velocity (qualitative),

Lecture 4 Oct. 3: Heisenberg uncertainty principle, Application of uncertainty principle
Uncertainty principle to calculate uncertainty in the measurements of physical quantities

Lecture 5 Oct. 4 *:* Wave function and its significance, Schrodinger time dependent and independent equations
Introduction to wave functions and concept of probability, basic principle in quantum physics, Probabilistic behavior of quantum physics

Lecture 6 Oct. 5 *:* Particle in a box (e.g., electron confined in a potential)
Energy of the particles/electrons is discrete and is quantized.

Heisenberg uncertainty principle

The duality relations lead naturally to an uncertainty relation: In physics it is the Heisenberg uncertainty principle

Particle nature $\dashrightarrow$ $$p = \frac{h}{\lambda}$$ $\dashleftarrow$ **Wave nature**

Wave-particle duality introduce the concept of uncertainty. This concept suggest that if the particle nature of the matter becomes certain, the wave nature becomes uncertain and vice versa.

There are three **conjugate variables** of great importance in **quantum mechanics**: position and momentum, angular orientation and angular momentum, and energy and time.

Heisenberg uncertainty principle

Concept of wave or wave packet associated with a moving atoms/subatomic particle introduce uncertainty. According to Heisenberg uncertainty principle it is impossible to measure the exact position and momentum(or velocity) of very small particles like, molecules, atoms or subatomic species like electron, proton, neutron etc…

1. $\Delta p \, \Delta x \geq \hbar$ Original statement of Heisenberg uncertainty principle

2. $\Delta E \, \Delta t \geq \hbar$ Time –Energy uncertainty principle

3. $\Delta L_\theta \, \Delta\theta \geq \hbar$ Angular momentum -Angular orientation uncertainty principle

Heisenberg uncertainty principle

1. Momentum (p) and position (x)

$$p = \frac{h}{\lambda}$$

We know the propagation constant, **k** related to the wavelength $\boldsymbol{\lambda}$ by

$$k = \frac{2\pi}{\lambda}$$

$$p = \frac{h}{\lambda} = \frac{h}{2\pi/k} = \hbar k$$

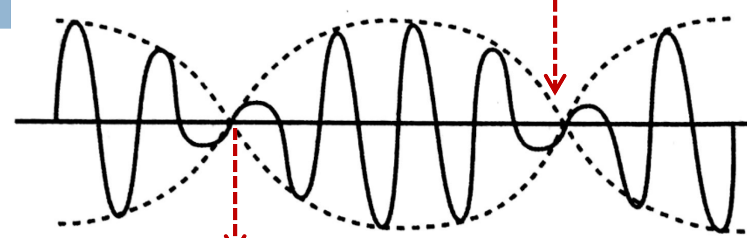
Where $\hbar = \frac{h}{2\pi}$

$$\Delta p = \hbar\, \Delta k \quad \dashrightarrow \quad (1)$$

Now we have to find a relation connecting Δk to Δx !! For that we have to go back to what we learned about **wave packet and group velocity** in the last class

Heisenberg uncertainty principle

$$y = 2a\,cos\left[\frac{\Delta\omega t}{2} - \frac{\Delta kx}{2}\right]\sin(\omega t - kx)$$



Group velocity of the de Broglie wave $V_g = V$, particle velocity. The position of the particle can be anywhere in the loop (dotted curve)

In the above equation y=0, corresponds to the node, in that case

$$cos\left[\frac{\Delta\omega t}{2} - \frac{\Delta kx}{2}\right] = 0 \quad \text{or} \quad \frac{\Delta\omega t}{2} - \frac{\Delta kx}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots. \frac{(2n+1)\pi}{2}$$

For the two consecutive nodes at ($y=0$, $x=x_1$) and ($x=x_2$. $y=0$) the above equation can be written as

Heisenberg uncertainty principle

$$\frac{\Delta\omega t}{2} - \frac{\Delta k x_1}{2} = \frac{(2n+1)\pi}{2} \text{ -------> } (2) \qquad \frac{\Delta\omega t}{2} - \frac{\Delta k x_2}{2} = \frac{(2n+3)\pi}{2} \text{ -------> } (3)$$

Upon subtracting Eq.(3)- Eq.2(2) we get $\frac{\Delta k(x_1 - x_2)}{2} = \pi$

but $x_1 - x_2 = \Delta x$ $\qquad \Delta k\,\Delta x = 2\pi$ -------> (4)

Substitute Eq. 4 in Eq.1 $\Delta p = \hbar \frac{2\pi}{\Delta x}$ $\qquad \Delta p\,\Delta x = 2\pi\hbar$

However, more accurate measurements show that the product of the uncertainties in momentum (Δp) and position (Δx) can not be less than $\hbar$

$\Delta p\,\Delta x \geq \hbar$ -------> (5) *This is Heisenberg uncertainty principle*

Heisenberg uncertainty principle: Energy and Time

1. Energy (E) and time (t)

Momentum, force, Work/Energy

$$F = ma = m\frac{dv}{dt} = \frac{d(mv)}{dt} = \frac{dp}{dt}$$

Force and momentum are also conjugate quantities

So the uncertainties in the measurement of force is related to the errors in p and t as follows

$$\Delta F = \frac{\Delta p}{\Delta t} \quad \text{or} \quad \Delta p = \Delta F \times \Delta t$$

But just now we proved $\Delta p \, \Delta x \geq \hbar$ substitute in the above eqn.

$$\frac{\hbar}{\Delta x} = \Delta F \times \Delta t$$

And upon re-arranging

Heisenberg uncertainty principle: Energy and Time

$$\frac{\hbar}{\Delta x} = \Delta F \times \Delta t \quad \Rightarrow \quad (\Delta x \Delta F) \times \Delta t = \hbar$$

But Force x distance is work, nothing but the energy $\Delta x \, \Delta F = \Delta E$

$$\Delta E \, \Delta t = \hbar$$

OR

$$\Delta E \, \Delta t \geq \hbar$$

Uncertainty principle represented in terms of energy and time

Heisenberg uncertainty principle: Angular momentum (L_θ) and angular orientation (θ)

A particle of mass m and velocity v making circular motion with radius r. Its angular momentum is given by

$$L_\theta = \text{mv r} = pr$$

In moving a distance x along the circle, the particle sweep an angle θ given by

$$\theta = \frac{x}{r} \qquad \Delta L_\theta = \Delta p\, r \qquad \Delta\theta = \frac{\Delta x}{r}$$

$$\Delta L_\theta\, \Delta\theta = \Delta p\, r\, \frac{\Delta x}{r} = \Delta p \Delta x$$

$$\Delta L_\theta\, \Delta\theta = \hbar$$

$$\Delta L_\theta\, \Delta\theta \geq \hbar$$

Uncertainty principle represented in terms of angular momentum and orientation

Applications of Heisenberg uncertainty principle

Applications of Heisenberg uncertainty principle are

1. Non existence of electron in the nucleus
2. Existence of proton, neutrons and α-particles in the nucleus
3. Binding energy of an electron in an atom
4. Radius of Bohr's first orbit
5. Energy of a particle in a box
6. Ground state energy of the linear harmonic oscillator
7. Radiation of light from an excited atom

Quick QUIZ

If I know the position of a subatomic particle precisely, then

a) I know nothing about the particle's momentum.
b) I know a little about the particle's momentum
c) The particle must be at rest.
d) The particle can't be at rest.

Electrons show diffraction effects with crystals because their de Broglie wavelength is

a) Similar to the spacing between atomic planes
b) Equal to the no. of atomic layers
c) Is very high compared to the spacing of atomic planes
d) None of the above

Most energetic photons are

a) alpha-rays
b) beta-rays
c) gamma-rays
d) x-rays

Quantum Mechanics

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