AN AUGMENTED q-FACTOR MODEL

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BONAFIDE CERTIFICATE

Certified that this project report titled, AN AUGMENTED *q*-FACTOR MODEL is the bonafide work of Ishita Gupta who carried out the project under my supervision. Certified further, that to the best of my knowledge the work reported herein does not form part of any other project report of the basis of which a degree or award was conferred on an earlier occasion on this or any other candidate.

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ABSTRACT

In this study, we form an augmented q-factor model based on important firm characteristics and fundamentals such as size, value, profitability and investment. We compare this model with the performance of the existing Fama-French three factor model by conducting empirical tests on the Indian stock market. We adopt two different mechanisms of portfolio construction. The results show that the average return pattern in the Indian stock market is influenced by the factors mentioned above and that the augmented factor model performs better than the FFTF model. We also find that firm-specific characteristics also play a role in the overall performance of both the models.

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1 Introduction

1.1 Motivation

The fundamental concept of the risk-return relationship forms the basis for most asset pricing models today. Different substitutes are taken to mimic the underlying risk factors and generate the expected returns for a portfolio while considering these factors. The aim of any rational risk-averse investor will be to estimate and maximise the expected return of the portfolio for a given level of risk or to minimise the risk for a given level of return .

The stock market has given the retail investor a convenient platform to park their excess capital and earn high returns if invested judiciously. Many financial models have been built to estimate expected returns on stocks over a long period of time. Sharpe [1964] documented the linear relation between securities return and their market betas (βs), popularly known as the Capital Asset Pricing Model (CAPM) which became a cornerstone theory in finance. The CAPM became the first general equilibrium model for asset pricing postulating a strong linear cross sectional relationship between expected return and market beta. The model shows that this factor alone can explain the cross-section of average return on stocks. The standard CAPM model postulated that the only way to earn higher return is to bear higher risk if markets are efficient. However, over the past few decades, a number of studies indicated the presence of anomalies in the stock market which could generate above normal returns Sobti [2016]. It became clear that the CAPM alone fails to account for these anomalies and it indicated the presence of a missing factor over and above the market beta. This lead to the formation of multi-factor models like the Fama-French models which extended the idea of CAPM by incorporating more factors that accounted for the risk element in the stock market. Recent literature in asset pricing suggests that average stock returns are dependent on different risk factors and financial indicators of the company such as its profitability, earnings, size, value and investment.

Hou et al. [2015] constructs a new empirical model that largely summarises cross-section of average stock returns. The model is inspired by the investment-based asset pricing which is in turn built on the neoclassical q-theory of investment. In their q-factor model the expected return of an asset in excess of the risk-free rate, denoted $E(r_i) - r_f$, is described by the sensitivities of its returns to four factors: the market excess return which is inspired by CAPM, the difference between the return on a portfolio of small size stocks and the return on a portfolio of big size stocks (r_{ME}, where ME refers to Market Equity), the difference between the return on a portfolio of low investment stocks and the return on a portfolio of high investment stocks ($r_{I/A}$), and the difference between the return on a portfolio of high profitability (return on equity, ROE) stocks and the return on a portfolio of low profitability stocks (r_{ROE}).

Considering the above issues, the aim of this study is to explore such multi-factor models to estimate the expected stock returns based on data from the Indian stock market. The paper augments the Fama-French three factor (FFTF) model with factors based on investment and profitability based on definitions of Hou et al. [2015]. We then compare the results with the baseline (FFTF) model in order to empirically examine if the additional factors improve the estimation of expected returns.

1.2 Framework

The paper is divided into the following parts. Section 2 is the literature review where prior research is discussed and the we explore the basis of our model. Section 3 is the conceptual framework which discusses the factors and the underlying reasons for taking the same and builds the hypothesis for the paper. Section 4 contains the sources of data and stocks used. Section 5 explains the methodology for the factor portfolio constructions and the final portfolios used for calculation of returns. Section 6 discusses the statistical summary and empirical analysis. Finally, Section 7 concludes the results of the paper along with scope for further research.

2 Literature Review

Fama and French [1992] show that size and book-to-market combine to describe average returns in cross-sectional regressions, and that the relationship between the market beta and average returns is flat, even when beta is used alone. Fama and French [1993] proposes that the three-factor model replace the CAPM as the workhorse for estimating expected returns.The model consists of the market factor as specified by CAPM, a factor based on size or market equity and a factor based on book value to market ratio. They also develop small minus big (SMB) and high minus low (HML) factors which are mimicking the returns on size and book-to-market equity factors. Fama and French [1995] verify whether firm's earnings and returns have relation with size and value factors and find that there are similar size and book-to-market patterns in the covariation of fundamentals like earnings and sales. Fama and French [1996] show that, except for momentum, the three-factor model summarises the cross section of expected returns as of the mid- 1990s. The paper shows the power of three- factor model of capturing the average returns on portfolios constructed on size, BE/ME, P/E, cash flows/price, sales growth, short-term stock returns and long-term stock returns and document that the three-factor model is strong enough to capture the average returns on almost all portfolios.

Chan et al. [1998] evaluate different factors that include fundamental, technical, macroeconomic, statistical and market factors in capturing stock returns variations and document that fundamental factors, namely, size coupled with BE/ME, capture the covariations of stock returns. Chui and Wei [1998] confirm that for five emerging equity markets, the expected return on stocks is related to three risk factors, namely firm size, book-to-market equity and market risk. Fama and French [2006] reveal that companies with high BE/ME ratio give higher returns considering their profitability and investment. They also show that given book-to-market equity, high expectations on investment and profitability tend to provide high stock returns.

Previously, various studies have empirically examined the Fama-French three-factor model of stock returns for the Indian stock market. Connor and Sehgal [2001] test the CAPM as well as the three factor model and conclude that the three factors have a pervasive influence on random returns in the Indian stock market but the market factor clearly ranks first in explanatory power while there is no clear ranking of the other two factors. Balakrishnan [2016] verifies the presence of size, value and momentum effects and gets positive results in support of the presence of the factors. Kumar and Sehgal [2004] find a strong size effect in stock returns when alternative measures of market and non-market-based variables are used. However, a weak value effect is observed when P/E ratio is used as a proxy of firm value. Manjunatha and Mallikarjunappa [2018] use portfolios formed by using portfolios sorted on the basis of book-to-market equity ratio and conclude that the market factor has the highest explanatory followed by size in these portfolios.

Extensions of the Fama-French model have been discussed by various authors. Carhart [1997] augments the three-factor model with a momentum factor. Cooper et al. [2008] find that the firm total asset growth dominates other standard variables in predicting the cross-section of future returns and that in terms of t-statistics, it is more important than BM equity, firm capitalization, momentum among other firm-specific factors. They suggest that asset growth captures complex linkages among returns, size groups, and financing types, and motivates further study of why different components of asset growth are associated with variation in return effects across size groups.

Ball et al. [2016] studies a cash-based measure of operating profitability and provides evidence that cash-based profitability provides the stronger signal of future returns. In fact, it states that investors would be better off by just adding cash-based operating profitability to their investment opportunity set than by adding both accruals and profitability strategies. Hence, it serves as evidence for using operating cash flow as an indicator of expected growth.

Zhang [2017] reiterates the need to focus on firm's characteristics as firms do a good job in aligning investment policies with costs of capital, and this alignment drives many empirical patterns.

Stambaugh and Yuan [2017] combines the market and size factors with two 'mispricing' factors where rather than have each factor correspond to a single anomaly, as is typical, it constructs factors by combining stocks' rankings with respect to 11 prominent anomalies.

George et al. [2018] shows that the ratio of current price to 52-week high price contains information about future growth, information that helps explain the accrual and R&D-to-market anomalies. Lastly,Cooper et al. [2020] mentions the use of investment-to-assets ratio as an alternate to asset growth.

3 Conceptual Framework

As stated earlier, the paper augments the FFTF model with factors of investment and profitability based on definitions of Hou et al. [2015]. Formally, the augmented q-factor model can be stated as

$$E[r^i] - r^f = \beta^i_{Mkt}E[Mkt] + \beta^i_{Me}E[Me] + \beta^i_{Val}E[Val]\beta^i_{I/A}E[I/A] + \beta^i_{Roe}E[Roe] \quad (3.1)$$

Here, the $\beta^i s$ are the factor loadings and the $E[Mkt], E[Me]$, $E[Val]$, are the expected factor premiums as per the Fama-French three factor model and the $E[I/A]$, and $E[Roe]$ are the augmented q-factors from Hou et al. [2015].

3.1 Factors

The factors in the model can be defined as follows:

- Market Return (Mkt): Excess market return over the risk-free rate of return
- Size (Me): Stock prices times number of shares outstanding (Market Capitalisation)
- Value (Val): Book Value of equity divided by Market Value of Equity
- Investment, (I/A): Annual changes in total assets divided by lagged total assets

- Profitability, (ROE): Annual Income divided by previous year book equity

In the model the expected return of an asset in excess of the risk-free rate, denoted $E(r_i) - r_f$, is described by the sensitivities of its returns to five factors: the market excess return which is inspired by CAPM, the difference between the average return on a portfolio of small size stocks and the average return on a portfolio of big size stocks $E[ME]$, (where ME refers to Market Equity), the difference between the average return on a portfolio of high book-to-market equity ratio stocks and the average return on a portfolio of low book-to-market equity ratio stocks $E[Val]$, the difference between the average return on a portfolio of low investment stocks and the average return on a portfolio of high investment stocks $E[I/A]$, and the difference between the average return on a portfolio of high profitability (return on equity, ROE) stocks and the average return on a portfolio of low profitability stocks $E[ROE]$.

3.1.1 Relation between Investment and Profitability and Stock Returns

The investment and profitability factors have been referred to as q-factors as their construction is linked to the 'q-theory of investment', developed by James Tobin. The relationship between stock prices and firms' investment in physical capital is captured by this theory. Tobin argued that the rate of investment in any particular type of capital can be predicted by looking at the ratio of the capital's market value to its replacement cost. Intuitively, the higher is q, the higher the value of capital relative to its current purchase price will be, and the higher investment will be. In functionally efficient markets, investment opportunities are priced such that capital can be systematically allocated to the most productive uses Tobin [1984].

The *investment return* is the marginal rate at which a firm can transfer resources through time by increasing investment today and decreasing it at a future date, leaving its production plan unchanged at all other dates (Cochrane [1996]).

From the static model (Hou et al. [2015]), we know that,

$$E_0[r_{i1}^S] = \frac{E_0[\Pi_{i1}]}{1 + a(I_{i0}/A_{i0})} \tag{3.2}$$

where Π is an indicator of profit. This equation infers the inverse relationship between investment-asset ratio and stock returns and the direct relation between profitability and expected returns. Thus, keeping all other factors constant, it predicts that high investment stocks will earn lower returns than low investment stocks whereas high profitability stocks will earn higher returns than low profitability stocks.

The negative investment-return relation is conditional on expected profitability.

Investment is linked to profitability because firms that are more profitable tend to invest more than less profitable firms. Similarly, given investment-to-assets ratio, firms with high expected profitability should earn higher expected returns than firms with low expected profitability. Less financially distressed firms are more profitable, meaning higher expected profitability, and, all else equal, should earn higher expected returns than more financially distressed firms.

3.2 Research Hypothesis

The objective of this paper is to test that whether the augmented q-factor model with the additional q factors of investment and profitability performs better than the Fama French three factor by taking two different kinds of portfolio constructions:

1. The monthly equal-weighted returns of 54 portfolios formed on a quadruple sort on the basis of size, book-to-market ratio of the company, investment and profitability.
2. The monthly equal-weighted returns of 25 portfolios formed solely on the basis of book-to-market ratios (Manjunatha and Mallikarjunappa [2018]).

Hence, based on the available evidence on Fama and French [1992, 1993, 1995, 1996] and Hou et al. [2015] as well as Manjunatha and Mallikarjunappa [2018], the following null hypothesis is formulated.

H_0: The augmented model with the q factors of investment and profitability does not give any additional information on expected stock returns.

H_1: The augmented model with the q factors of investment and profitability gives additional information on expected stock returns.

4 Data

The sample data used for this study is composed of 253 companies of the S&P BSE 500 index for the years 2000-2020. Only the companies that had complete data for the relevant time period were included in the study bring the number of companies down to 253. The S&P BSE 500 is designed to be a broad representation of the Indian market. Consisting of the top 500 constituents in the S&P BSE AllCap, the index covers all major industries in the Indian economy.

The data covers 20 years monthly periods from July 2000 to June 2020. In order to factor into the earnings announcement in the form of financial statements and reports and its impact on stock prices, the portfolios are formed each year at the end of June. These portfolios are constructed on equal weight basis as suggested by Lakonishok et al. [1994] and the portfolio returns are found as an average of the returns of stocks in the portfolio.

The data with regard to company fundamentals, adjusted closing prices of shares and market capitalisation were taken from Centre for Monitoring Indian Economy (CMIE) Prowess. The risk free rate data of 91 day T-Bill rate was taken from Census and Economic Information Center (CEIC) database. Using the 91 day T-Bill rate as a substitute for risk-free return is a common practice observed in Agarwalla et al. [2014], Balakrishnan [2016], Bahl [2006], Sobti [2016], Manjunatha and Mallikarjunappa [2018]. The annualised figures were converted to monthly rates.

S&P BSE 500 was taken as the substitute for market returns. The market returns data was directly sourced from the BSE website. The adjusted share price series and market returns have been converted into return series using arithmetic returns. The accounting information has been obtained for the sample companies for the financial years 2000 to 2020. The financial year in India is from April of year t to March of calendar year $t + 1$. The fundamentals and the book value per share and number of shares outstanding for the sample companies are recorded in March-end of each year.

5 Methodology

The firm specific risk is diversified by grouping stocks of companies into portfolios for factor constructions. This leads to more accurate betas. We construct portfolios which are first independently sorted on size, value (book to market equity), investment and profitability. These are used to construct the factor premiums. We then form intersecting portfolios i.e. companies with the same characteristics in each variable are clubbed together to form a single portfolio. This allows us to evaluate the performance of the model independent of the attributes which may lead to abnormal returns and maintain uniformity in returns across the portfolio.Next, we form portfolios based on B/M ratios as per ranking.

5.1 Factor Construction

First, we form portfolios by ranking the sample stocks based on market capitalisation which is a proxy for **size**. At the end of June, every year starting from June 2000 till June 2019[1], the sample stocks are ranked and classified into two equally weighted portfolios (Lakonishok et al. [1994]) where the first portfolio consists of the top 50% of the stocks amongst the 253 companies. This portfolio is referred to as the 'Large' companies. The second portfolio is consists of the bottom 50% of the stocks amongst the 253 companies. This portfolio is referred to as the 'Small' companies. Thus, we have chosen the median as the breakpoint. It is important to note that in our sample, both the portfolios had almost an equal number of companies. We now compute equally weighted returns on the two portfolios for the first year of study that is from July 2000 (year t) to June 2001 ($t + 1$). Then, we revise the portfolio in the June of every year t. This portfolio revision is done every year for the whole study period. Finally, we calculate the average excess returns on the 'Small' portfolio over the 'Large' portfolio for each month of every year. In absolute terms, it is observed that small market capitalisation firms have more returns than large market capitalisation firms. This is because small size firms have more room to expand and grow while large size firms have more stable returns.[2]

Second, we form portfolios by ranking the sample stocks based on book to market equity ratio which is a proxy for **value of the firm**. Book equity refers to the share capital plus the retained earnings of the firm. At the end of March, every year, we do ranking of sample stocks based on B/M ratio and form three portfolios using equally-weighted approach. The first portfolio contains the top 30% of stocks in terms of B/M ratio. This portfolio is referred to as the 'High B/M' portfolio and these stocks are called as Value Stocks. The second portfolio contains the middle 40% of stocks in terms of B/M ratio. This portfolio is referred to as the 'Middle B/M' portfolio and these stocks are called as Neutral Stocks. The third portfolio contains the bottom 30% of stocks in terms of B/M ratio. This portfolio is referred to as the 'Low B/M' portfolio and these stocks are called as Growth Stocks. Thus, we have chosen 30-70-100 as our breakpoints (Fama and French [1993]). It is important to note that in our sample, all the three portfolios had almost an equal number of companies. We now compute equally weighted returns on the three portfolios for the first year of study that is from July 2000 (year t) to June 2001 ($t + 1$). Then, we revise the portfolio in the March of every year t. This portfolio revision is done every year for the whole study period. Finally, we calculate the average excess returns on the 'High' portfolio over the 'Low' portfolio for each month of every year. In absolute terms, it has been historically observed that value firms have outperformed growth firms.

Third, we form portfolios by ranking the sample stocks based on annual changes in total assets divided by lagged total assets which is a proxy for **investment growth**. At the end of March, every year, we do ranking of sample stocks based on IA and form three portfolios using equally-weighted approach.

[1]As mentioned earlier, June is used instead of March in order to account for change in investor behavior after company results are announced for the previous fiscal behaviour.

[2]It is important to note that these returns are not risk-adjusted as small firms may have much more risk than large firms and then the results may change dramatically.

The first portfolio contains the bottom 30% of stocks in terms of asset growth. This portfolio is referred to as the 'Low IA' portfolio and these stocks are referred to as conservative firms. The second portfolio contains the middle 40% of stocks in terms of asset growth. This portfolio is referred to as the 'Middle IA' portfolio. The third portfolio contains the top 30% of stocks in terms of asset growth. This portfolio is referred to as the 'High IA' portfolio and these stocks are referred to as aggressive stocks. Thus, we have chosen 30-70-100 as our breakpoints (Hou et al. [2015]). We now compute equally weighted returns on the three portfolios for the first year of study that is from July 2000 (year t) to June 2001 ($t+1$). Then, we revise the portfolio in the March of every year t. This portfolio revision is done every year for the whole study period. Finally, we calculate the average excess returns on the 'Low' portfolio over the 'High' portfolio for each month of every year. As such, firms with high long-term prior returns should invest more and earn lower expected returns than firms with low long-term prior returns (Hou et al. [2015]).

Lastly, we form portfolios by ranking the sample stocks based on Annual Income divided by previous year book equity which is a proxy for **profitability or ROE**. At the end of March, every year, we do ranking of sample stocks based on ROE and form three portfolios using equally-weighted approach. The first portfolio contains the top 30% of stocks in terms of ROE. This portfolio is referred to as the 'High ROE' portfolio and these firms are said to be highly profitable firms. The second portfolio contains the middle 40% of stocks in terms of asset growth. This portfolio is referred to as the 'Middle ROE' portfolio. The third portfolio contains the bottom 30% of stocks in terms of ROE. This portfolio is referred to as the 'Low ROE' portfolio and these stocks are low on profitability. Thus, we have chosen 30-70-100 as our breakpoints (Hou et al. [2015]). We now compute equally weighted returns on the three portfolios for the first year of study that is from July 2000 (year t) to June 2001 ($t + 1$). Then, we revise the portfolio in the March of every year t. This portfolio revision is done every year for the whole study period. Finally, we calculate the average excess returns on the 'High' portfolio over the 'Low' portfolio for each month of every year. As such, firms with higher profits are likely to generate higher returns for shareholders as compared to low profit firms.

Finally, the market factor (MKT) or market risk premium is the difference between the expected return on the market and the risk-free rate. This is the only factor that appears in CAPM and is common to all stocks irrespective of its attributes and therefore there is no need for decile division in this factor. The market returns are computed on S&P BSE 500 and the risk free returns are the monthly rates of the 91 day T-Bills.

5.1.1 Factor Returns

The cumulative returns of the size, value, investment, profitability and market portfolios are given in Figure 7.1 in the appendix. Over the period, July 2000 to June 2020, the cumulative returns of the market portfolio (Rm) was 296.93% and the cumulative market risk premium (MRP) was 164.4%. The cumulative return on the value factor was 82.83%. The size factor earned a cumulative return 286.01%. The investment factor earned a cumulative return of 115.45% and the profitability factor earned a cumulative return of 102.69% The correlations of the monthly factor returns is given in Table 7.1 in the appendix. The correlations across the factor-returns are low.

5.2 Portfolio Constructions

5.2.1 Intersecting Portfolios

We do a quadruple sort on the four factors of Size, Value, Investment and Profitability and form intersecting portfolios based on these attributes. There are 2 portfolios for size, 3 for value, 3 for investment and 3 for profitability as described earlier. Thus we get 2x3x3x3 = 54 portfolios for testing our model. Each portfolio refers to specific characteristics in each category and by evaluating them separately we maintain a uniformity across the portfolio, thus giving us more reliable betas. These portfolios can then be evaluated independently in terms of a single characteristic such as all Large Portfolios, or all High Value Portfolios to compare the performance of our model. The list of all portfolios[3] is shown in Table 5.1.

Table 5.1: Portfolios

No.	Portfolio	Size	Value	Investment	Profitability
1	LHHH	Large	High	High	High
2	LHHM	Large	High	High	Medium
3	LHHL	Large	High	High	Low
4	LHMH	Large	High	Medium	High
5	LHMM	Large	High	Medium	Medium
6	LHML	Large	High	Medium	Low
7	LHLH	Large	High	Low	High
8	LHLM	Large	High	Low	Medium
9	LHLL	Large	High	Low	Low
10	LMHH	Large	Medium	High	High
11	LMHM	Large	Medium	High	Medium
12	LMHL	Large	Medium	High	Low
13	LMMH	Large	Medium	Medium	High
14	LMMM	Large	Medium	Medium	Medium
15	LMML	Large	Medium	Medium	Low
16	LMLH	Large	Medium	Low	High
17	LMLM	Large	Medium	Low	Medium

[3]The descriptive statistics for the same are given in Table 7.3 the Appendix.

18	LMLL	Large	Medium	Low	Low
19	LLHH	Large	Low	High	High
20	LLHM	Large	Low	High	Medium
21	LLHL	Large	Low	High	Low
22	LLMH	Large	Low	Medium	High
23	LLMM	Large	Low	Medium	Medium
24	LLML	Large	Low	Medium	Low
25	LLLH	Large	Low	Low	High
26	LLLM	Large	Low	Low	Medium
27	LLLL	Large	Low	Low	Low
28	SHHH	Small	High	High	High
29	SHHM	Small	High	High	Medium
30	SHHL	Small	High	High	Low
31	SHMH	Small	High	Medium	High
32	SHMM	Small	High	Medium	Medium
33	SHML	Small	High	Medium	Low
34	SHLH	Small	High	Low	High
35	SHLM	Small	High	Low	Medium
36	SHLL	Small	High	Low	Low
37	SMHH	Small	Medium	High	High
38	SMHM	Small	Medium	High	Medium
39	SMHL	Small	Medium	High	Low
40	SMMH	Small	Medium	Medium	High
41	SMMM	Small	Medium	Medium	Medium
42	SMML	Small	Medium	Medium	Low
43	SMLH	Small	Medium	Low	High
44	SMLM	Small	Medium	Low	Medium
45	SMLL	Small	Medium	Low	Low
46	SLHH	Small	Low	High	High
47	SLHM	Small	Low	High	Medium
48	SLHL	Small	Low	High	Low
49	SLMH	Small	Low	Medium	High
50	SLMM	Small	Low	Medium	Medium
51	SLML	Small	Low	Medium	Low
52	SLLH	Small	Low	Low	High
53	SLLM	Small	Low	Low	Medium
54	SLLL	Small	Low	Low	Low

5.2.2 B/M Portfolios

The second set of portfolios are formed by taking stocks in the order of their book-to-market equity ratio. The data had a total of 253 stocks. Each year these 253 stocks were sorted in a descending order based on their book-to-market equity ratio. The top 10 stocks formed the first portfolio. The next 10 stocks formed the second portfolio and so on till the 25th portfolio. The top 8 portfolios were considered 'High' book-to-market equity, the next 9 were considered 'Medium' book-to-market equity, and the bottom 8 were considered 'Low' book-to-market equity.[4]

The augmented model and the FFTF were individually applied on both the set of portfolios in order to evaluate that how the model performance changes when stocks are sorted in a multi-dimensional space compared to a uni-dimensional space.

5.3 Model Specification

The augmented q-factor model is given by

$$R_{jt} = \alpha_t^j + \beta_{Mkt_t}^j E[Mkt_t] + \beta_{Me_t}^j E[Me_t] + \beta_{Val_t}^j E[Val_t] + \beta_{I/A_t}^j E[I/A_t] + \beta_{Roe_t}^j E[Roe_t] + \epsilon_t^j \tag{5.1}$$

Here, R_{jt} denotes the excess return over the risk-free rate for portfolio j in month t. The β_{Mkt}^j is the factor sensitivities of excess returns on market portfolio factor.β_{Me}^j, β_{Val}^j, $\beta_{I/A}^j$ and β_{Roe}^j are the respective factor sensitivities for the size factor portfolio, value factor portfolio, investment factor portfolio and profitability factor portfolio.

$E[Mkt_t], E[Me]_t$, $E[Val_t]$, are the expected factor premiums as per the Fama-French three factor model for the month t and the $E[I/A_t]$, and $E[Roe_t]$ are the expected factor premiums for the augmented q-factors at time t from Hou et al. [2015]. α_t^j is the abnormal mean return of portfolio j and ϵ_t^j is the mean-zero asset-specific return of portfolio j.

The Fama French three-factor model is given by

$$R_{jt} = \alpha_t^j + \beta_{Mkt_t}^j E[Mkt_t] + \beta_{Me_t}^j E[Me_t] + \beta_{Val_t}^j E[Val_t] + \epsilon_t^j \tag{5.2}$$

with the same definitions as above.

The Ordinary Least Squares method of estimation is used for econometric analysis. The Adjusted R squares are used for comparing the performance of the two models in the two cases of portfolio construction. Additionally, the p values are studied to see if the additional factors capture the cross-sectional variation in portfolio-excess-returns as suggested by Manjunatha and Mallikarjunappa [2018]. We also study the F values to check whether the factors together explain the cross-sectional variation in portfolio-excess-returns.

- In the case of intersecting portfolios, we have 54 portfolios. We run a regression for each portfolio. Here, R_{jt} denotes the excess return over the risk-free rate for each of the individual portfolio j in month t.

[4]The descriptive statistics for the same are given in Table 7.11 the Appendix.

- In the case of second case, portfolios, we have 25 portfolios of 10 stocks each. We run a regression for each portfolio. R_{jt} denotes the excess return over the risk-free rate for each of the individual portfolio j in month t.

6 Empirical Results and Analysis

The present study has been conducted by using a combination of five factors - market, size, value, investment and profitability to find out the extent to which these variables explain portfolio returns. The intercept and slope coefficient are tested using the p values and the overall fit of the regression is tested using the F-test at 5 per cent level of significance. The variables are significant if the corresponding p-values of their coefficient estimates are less than the level of significance. The detailed results for the case of 54 portfolios are given in Tables 7.8 and 7.7 and the detailed results for the case of 25 portfolios are given in Tables 7.13 and 7.15.

6.1 Intersecting Portfolios

The 54 portfolios were formed on the basis of intersections of a quadruple sort on the four factors of Size, Value, Investment and Profitability.

The p-values corresponding to the F-values in all the 54 portfolios came out to be significant for the augmented model as well as the FFTF model. Therefore, we can conclude that all the five factors together explain the portfolio returns.

The adjusted R squares for each of the 54 portfolios came out to be larger in case of the augmented factor model as compared to the FFTF model. On an average the adjusted R square was higher than the FFTF model by 2.55%.[1] We further analyse this by comparing the adjusted R square for large and small portfolios in both the models. On an average, the adjusted R square came out to be higher in the augmented factor model than the FFTF in small portfolios by 3.51% and in case of large portfolios it came out to be larger by 1.59%. Thus, we can conclude that the augmented model performed even better in the small size portfolios.

Let us now take a closer look at the performance of individual factors in all these portfolios.

[1] The detailed results are in Table 7.4 in the Appendix.

- The market return factor came out to be significant in all the 54 portfolios. This is a common result in many earlier studies. This fact reiterates the importance of CAPM and the excess market return factor in particular.

- The size factor was significant in 33 portfolios (61.11%). It especially performed well in small-size portfolios where it came out significant in all the 27 portfolios in both the augmented and FFTF model. However in the large portfolios, it has not performed extremely well with significance in only 6 out of 27 portfolios (22.22%). In case of value-based portfolio observations, the size factor has performed well in medium and low book-to-market equity portfolios with significance in 12 and 11 portfolios out of 18 each.

- The value factor was significant in 32 portfolios (59.26%). It performed well in small portfolios where it was significant in 17 on 27 portfolios while it performed exceedingly well in the case of high book-to-market equity portfolios where it was significant in 100% of the cases.

- The investment factor was significant in 31 portfolios (57.41%). It performed well in small portfolios where it was significant in 17 on 27 portfolios while it also performed well in high book-to-market portfolios with significance in 13 out of 18 portfolios.

- The profitability factor was significant in 28 portfolios (51.85%). It performed equally well in both small and large portfolios where it was significant in 14 on 27 portfolios while it also performed well in low book-to-market portfolios with significance in 12 out of 18 portfolios.

- If we take the significance level to be 10%, we find that the cases in which the Value and IA factors are significant increases especially in the low book-to-market equity portfolios from 38% and 50% to 55.56% and 66.67%. The cases in which the profitability factor is signficant increases especially in the high book to market portfolios as well as in small portfolios.

- In the FFTF model, we find that the market factor comes out significant in all 54 portfolios. The size factor as in the augmented factor model case performs well in small portfolios but not so well in large portfolios. It performs almost equally well in the high, low and medium book-to-market equity portfolios. The value factor overall performs only marginally better in the FFTF model as compared to the augmented model.

Thus, we can conclude that in the 54 portfolios, the augmented q-factor model performed better than the FFTF model. We also find that the q factors - investment and profitability have overall performed almost as good as the original Fama French three factors and in some cases even outperformed the same in explaining portfolio excess returns.

6.2 B/M Portfolios

The 25 portfolios were formed solely on the basis of book-to-market equity and each portfolio had 10 stocks each.

The p-values corresponding to the F-values in all the 25 portfolios came out to be significant for the augmented model as well as the FFTF model. Therefore, we can conclude that all the five factors together explain the portfolio returns and that the models fit the data better than the intercept-only model.

The adjusted R-squares came out almost equal in both the models and were in the range between 68.57% and 85.98%. The augmented q-factor model performed only marginally better with the average R-square of the augmented model being higher than the FFTF by only 0.117%.[2]

We will now look at the performance of individual factors in all these portfolios.

- In this case as well, the market return factor came out to be significant in all the 25 portfolios.
- The size factor was statistically significant in all the portfolios in both the models.
- The value factor performed well in high book-to-market equity portfolios as well as neutral book-to-market equity portfolios. However, it lost its statistical significance in the low book-to-market equity portfolios.
- The investment factor was significant in 6 portfolios (24%). It performed better in neutral and low book-to-market equity ratios.
- The profitability factor was not significant in any of the portfolios.
- If we take the significance level to be 10%, we see some change in the performance of the investment factor which increases from 6 (24%) to 8 (32%) while there is no change in the performance of the Value factor. In low book-to-market equity portfolios, the investment factor performed better than the value factor although the number of portfolios is too less to make a confident claim for the same.
- In the FFTF model also, we find that the market factor and the size factor were significant in all portfolios. The value factor again, performed well in high book-to-market equity portfolios as well as neutral book-to-market equity portfolios but lost its statistical significance in the low book-to-market equity portfolios.

Thus, we can conclude that both the models performed well in the the case of B/M portfolios. The notable observations come in the individual factor performances.

[2]The detailed results are in Table 7.12 in the Appendix.

The investment factor actually performed better than the value factor when we consider low book-to-market equity portfolios.

We observe that the market factor and size factor performed well across all B/M portfolios. The investment factor was significant in 8 out of 25 portfolios. The profitability factor performed very poorly in these portfolios. This indicates that profitability factor can perform well only when there is more consistency of firm characteristics across the portfolios. In a way, the behaviour for the Value factor and its loss of statistical significance in the case of low book-to-market equity portfolios is a result consistent with Manjunatha and Mallikarjunappa [2018]. In that study the value factor had no explanatory power in the case of low B/M portfolios. When all the BE/ME portfolios are considered together; market, size and value factors explain portfolio returns in 100%, 100% and 64% portfolios respectively in case of FFTF while the market, size, value, investment and profitability factors explain returns in 100%, 100%, 72%, 24% and 0% portfolios respectively.

7 Conclusion

In this study, we have formulated an augmented q-factor model which is an extension of the Fama-French three factor model. The additional q-factors were investment and profitability which are formed on the basis of definitions from Hou et al. [2015]. We check the efficiency of this model on the Indian stock market and also compare its performance with the baseline Fama-French three factor model. We adopted two mechanisms of portfolio construction - the first was an intersection of the quadruple sort on the four factors of size, value, investment and profitability and the second was a uni-dimensional sort solely on the basis of book-to-market equity ratios.

The overall explanatory power of the factor portfolios was assessed using adjusted R squares. The intercept and slope coefficient are tested using the p values and the overall fit of the regression is tested using the F-test at 5 per cent level of significance.

We find that the augmented q-factor model performed better than the FFTF model in the first case of portfolio construction and only marginally better than the latter in the second case of portfolio construction.

On individually analysing the factor performances, we find that the size, value and investment factors have some explanatory power in most portfolios. The models explain the returns of small size portfolios better than the large size portfolios and the returns of high value (book-to-market equity) portfolios better than low value portfolios. The market factor as given by CAPM was statistically significant in all portfolios.

The results we find using the models provide a good description of the cross section of average returns and can be used in applications like stock selection, portfolio selection and performance evaluation.

The future perspectives on this research could be extended to evaluate the performance of industry-specific portfolios to determine which factors are likely to explain and generate more returns. There are other methods of checking the efficiency and robustness of models like the Fama-Macbeth procedure. The factor definitions could be altered and new factors may be formed using combinations of market, firm specific factors and additional factors to enable a deeper understanding of factors model.

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List of Tables

Appendix

Factor Returns

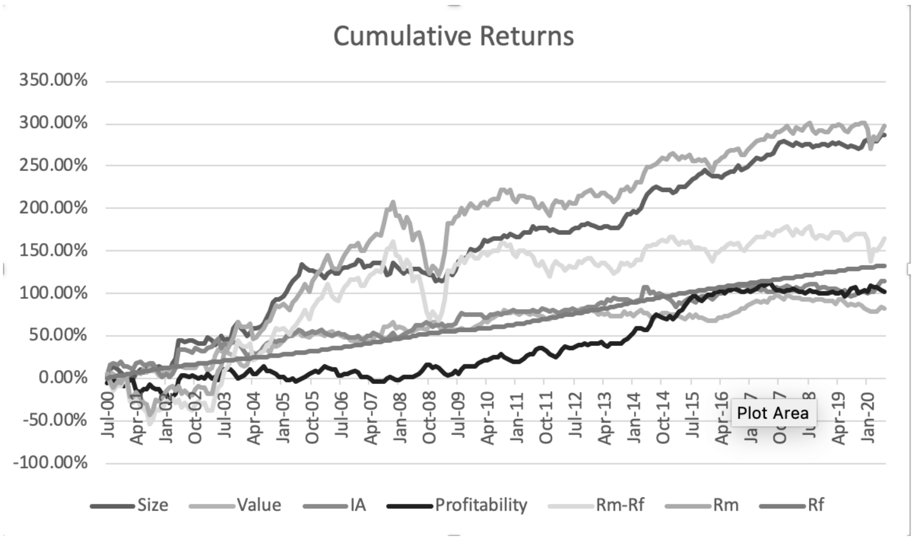

Figure 7.1: Cumulative returns for factors over 2000-2019

Table 7.1: Correlation between Factor Returns

	Size	Value	IA	Profitability	Rm-Rf
Size	1				
Value	0.0134	1			
IA	0.3698	0.1620	1		
Profitability	0.0071	0.0409	0.1125	1	
Rm-Rf	0.1761	0.1779	0.0526	-0.1847	1

Intersecting Portfolios

Table 7.2: Descriptive Statistics for 54 Portfolios

No.	Portfolio	Size	Value	Investment	Profitability	Average	Std Dev
1	LHHH	Large	High	High	High	1.07%	11.96%
2	LHHM	Large	High	High	Medium	1.36%	10.00%
3	LHHL	Large	High	High	Low	2.45%	11.46%
4	LHMH	Large	High	Medium	High	1.77%	12.17%
5	LHMM	Large	High	Medium	Medium	2.09%	10.09%
6	LHML	Large	High	Medium	Low	0.91%	10.27%
7	LHLH	Large	High	Low	High	3.24%	14.07%
8	LHLM	Large	High	Low	Medium	1.38%	12.54%
9	LHLL	Large	High	Low	Low	1.67%	11.84%
10	LMHH	Large	Medium	High	High	2.17%	8.17%
11	LMHM	Large	Medium	High	Medium	2.17%	10.24%
12	LMHL	Large	Medium	High	Low	1.01%	8.23%
13	LMMH	Large	Medium	Medium	High	1.88%	8.99%
14	LMMM	Large	Medium	Medium	Medium	2.09%	8.08%
15	LMML	Large	Medium	Medium	Low	1.57%	8.74%
16	LMLH	Large	Medium	Low	High	1.61%	8.90%
17	LMLM	Large	Medium	Low	Medium	1.99%	9.32%
18	LMLL	Large	Medium	Low	Low	1.75%	10.06%
19	LLHH	Large	Low	High	High	1.72%	8.92%
20	LLHM	Large	Low	High	Medium	1.48%	8.56%
21	LLHL	Large	Low	High	Low	1.79%	9.53%
22	LLMH	Large	Low	Medium	High	1.71%	7.09%
23	LLMM	Large	Low	Medium	Medium	1.70%	7.50%
24	LLML	Large	Low	Medium	Low	1.84%	8.52%
25	LLLH	Large	Low	Low	High	1.82%	8.77%
26	LLLM	Large	Low	Low	Medium	2.16%	9.39%
27	LLLL	Large	Low	Low	Low	1.99%	9.53%
28	SHHH	Small	High	High	High	3.83%	12.90%
29	SHHM	Small	High	High	Medium	2.97%	10.70%
30	SHHL	Small	High	High	Low	2.44%	14.21%
31	SHMH	Small	High	Medium	High	3.72%	12.24%
32	SHMM	Small	High	Medium	Medium	2.65%	10.15%
33	SHML	Small	High	Medium	Low	2.69%	11.36%
34	SHLH	Small	High	Low	High	3.73%	14.53%
35	SHLM	Small	High	Low	Medium	3.66%	11.14%
36	SHLL	Small	High	Low	Low	4.01%	11.41%
37	SMHH	Small	Medium	High	High	3.20%	10.67%
38	SMHM	Small	Medium	High	Medium	2.79%	10.62%
39	SMHL	Small	Medium	High	Low	2.26%	12.19%

40	SMMH	Small	Medium	Medium	High	3.15%	10.24%
41	SMMM	Small	Medium	Medium	Medium	2.29%	9.54%
42	SMML	Small	Medium	Medium	Low	2.67%	11.49%
43	SMLH	Small	Medium	Low	High	3.00%	10.65%
44	SMLM	Small	Medium	Low	Medium	3.39%	11.13%
45	SMLL	Small	Medium	Low	Low	2.20%	12.19%
46	SLHH	Small	Low	High	High	2.40%	11.96%
47	SLHM	Small	Low	High	Medium	2.46%	12.72%
48	SLHL	Small	Low	High	Low	1.70%	15.23%
49	SLMH	Small	Low	Medium	High	1.45%	13.28%
50	SLMM	Small	Low	Medium	Medium	1.45%	10.22%
51	SLML	Small	Low	Medium	Low	3.19%	15.58%
52	SLLH	Small	Low	Low	High	4.98%	21.91%
53	SLLM	Small	Low	Low	Medium	2.90%	11.00%
54	SLLL	Small	Low	Low	Low	2.55%	17.66%

Table 7.3: Descriptive Statistics - Intersecting Portfolios

	Portfolio	Average	Standard Deviation
Size	Large	1.79%	9.74%
	Small	2.88%	12.48%
Value	High	2.54%	11.84%
	Medium	2.29%	9.97%
	Low	2.18%	11.52%
IA	Low	2.67%	12.00%
	Medium	2.16%	10.31%
	High	2.18%	11.02%
ROE	High	2.58%	11.52%
	Medium	2.28%	10.16%
	Low	2.15%	11.64%

Empirical Results

Table 7.4: Adjusted R Squares for Intersecting portfolios

No.	Portfolio	Adjusted R Square-Augmented Factor model	Adjusted R Square-Fama-French	Difference
1	LHHH	47.53%	43.26%	4.27%
2	LHHM	52.40%	50.22%	2.18%
3	LHHL	55.44%	55.50%	-0.06%
4	LHMH	49.49%	45.45%	4.04%
5	LHMM	44.45%	44.33%	0.11%
6	LHML	64.91%	60.00%	4.90%
7	LHLH	57.95%	54.89%	3.06%
8	LHLM	62.82%	61.44%	1.38%
9	LHLL	61.24%	58.12%	3.12%
10	LMHH	57.35%	56.55%	0.80%
11	LMHM	67.27%	67.21%	0.06%
12	LMHL	70.26%	69.50%	0.76%
13	LMMH	62.31%	62.25%	0.07%
14	LMMM	78.14%	78.20%	-0.05%
15	LMML	77.72%	77.53%	0.19%
16	LMLH	33.45%	29.63%	3.82%
17	LMLM	57.48%	56.34%	1.14%
18	LMLL	67.59%	66.16%	1.43%
19	LLHH	77.25%	74.80%	2.45%
20	LLHM	75.90%	75.78%	0.12%
21	LLHL	73.48%	71.98%	1.50%
22	LLMH	69.74%	69.29%	0.45%
23	LLMM	84.22%	84.34%	-0.12%
24	LLML	69.98%	68.40%	1.58%
25	LLLH	56.60%	54.63%	1.97%
26	LLLM	73.03%	73.12%	-0.09%
27	LLLL	71.60%	67.76%	3.84%
		Average difference for Large Portfolios		**1.59%**

No.	Portfolio	Adjusted R Square-Augmented Factor model	Adjusted R Square-Fama-French	Difference
28	SHHH	64.70%	60.41%	4.29%
29	SHHM	73.09%	72.74%	0.34%
30	SHHL	65.73%	58.13%	7.60%
31	SHMH	67.90%	66.78%	1.11%
32	SHMM	73.72%	73.93%	-0.21%
33	SHML	71.80%	70.90%	0.90%
34	SHLH	54.82%	50.47%	4.35%

35	SHLM	79.71%	78.56%	1.15%
36	SHLL	67.31%	63.76%	3.56%
37	SMHH	69.60%	66.98%	2.62%
38	SMHM	71.94%	71.32%	0.62%
39	SMHL	60.58%	59.75%	0.84%
40	SMMH	75.92%	75.86%	0.06%
41	SMMM	79.03%	78.58%	0.45%
42	SMML	53.66%	52.97%	0.69%
43	SMLH	74.45%	73.01%	1.44%
44	SMLM	68.54%	66.98%	1.56%
45	SMLL	56.75%	52.83%	3.91%
46	SLHH	62.07%	54.74%	7.32%
47	SLHM	29.73%	29.24%	0.49%
48	SLHL	50.35%	37.47%	12.89%
49	SLMH	55.15%	50.44%	4.71%
50	SLMM	51.32%	51.43%	-0.12%
51	SLML	50.50%	42.29%	8.21%
52	SLLH	56.57%	38.92%	17.64%
53	SLLM	51.66%	49.50%	2.17%
54	SLLL	61.12%	54.89%	6.22%
	Average difference for Small Portfolios			**3.51%**
	Overall average difference in adjusted R squares			**2.55%**

Table 7.5: Significance at 5% - Augmented Model

	Intercept	Rm-Rf	Size	Value	IA	Profitability
Large Portfolios	5 18.52%	27 100.00%	6 22.22%	15 55.56%	14 51.85%	14 51.85%
Small Portfolios	2 7.41%	27 100.00%	27 100.00%	17 62.96%	17 62.96%	14 51.85%
Overall (%)	7 **12.96%**	54 **100.00%**	33 **61.11%**	32 **59.26%**	31 **57.41%**	28 **51.85%**

Table 7.6: Significance at 10%

	Intercept	Rm-Rf	Size	Value	IA	Profitability
Large Portfolios	5 18.52%	27 100.00%	8 29.63%	18 66.67%	16 59.26%	14 51.85%
Small Portfolios	5 18.52%	27 100.00%	27 100.00%	19 70.37%	18 66.67%	16 59.26%
Overall (%)	10 **18.52%**	54 **100.00%**	35 **64.81%**	37 **68.52%**	34 **62.96%**	30 **55.56%**

Table 7.7: 5% Significance table for FFTF

	Intercept	Rm-Rf	Size	Value
Large Portfolios	5 18.52%	27 100.00%	8 29.63%	16 59.26%
Small Portfolios	3 11.11%	27 100.00%	27 100.00%	17 62.96%
Overall (%)	8 **14.81%**	54 **100.00%**	35 **64.81%**	33 **61.11%**

Value-Based Portfolio Results

Table 7.8: Significance at 5% - Value-Based Portfolios

	Intercept	Rm-Rf	Size	Value	IA	Profitability
High	1 5.56%	18 100.00%	10 55.56%	18 100.00%	13 72.22%	9 50.00%
Medium	3 16.67%	18 100.00%	12 66.67%	7 38.89%	9 50.00%	7 38.89%
Low	3 16.67%	18 100.00%	11 61.11%	7 38.89%	9 50.00%	12 66.67%
Overall (%)	7 **12.96%**	54 **100.00%**	33 **61.11%**	32 **59.26%**	31 **57.41%**	28 **51.85%**

Table 7.9: Significance at 10% - Value-Based Portfolios

	Intercept	Rm-Rf	Size	Value	IA	Profitability
High	2 11.11%	18 100.00%	10 55.56%	18 100.00%	13 72.22%	11 61.11%
Medium	4 22.22%	18 100.00%	12 66.67%	9 50.00%	9 50.00%	7 38.89%
Low	4 22.22%	18 100.00%	13 72.22%	10 55.56%	12 66.67%	12 66.67%
Overall (%)	10 **18.52%**	54 **100.00%**	35 **64.81%**	37 **68.52%**	34 **62.96%**	30 **55.56%**

Table 7.10: Significance at 5% - Fama-French

	Intercept	Rm-Rf	Size	Value
High	2 11.11%	18 100.00%	11 61.11%	17 94.44%
Medium	3 16.67%	18 100.00%	12 66.67%	8 44.44%
Low	3 16.67%	18 100.00%	12 66.67%	8 44.44%
Overall (%)	8 **14.81%**	54 **100.00%**	35 **64.81%**	33 **61.11%**

B/M Portfolios

Table 7.11: Descriptive Statistics - B/M Portfolios

Portfolio	Means	Standard Deviation
1	3.56%	11.04%
2	3.93%	11.58%
3	2.98%	10.46%
4	2.91%	10.25%
5	2.72%	9.81%
6	2.78%	9.87%
7	2.71%	9.58%
8	2.25%	9.99%
9	2.45%	9.51%
10	2.46%	9.05%
11	2.11%	9.38%
12	2.59%	8.60%
13	2.47%	9.12%
14	2.47%	8.59%
15	2.00%	9.22%
16	2.24%	8.86%
17	2.43%	8.62%
18	2.28%	7.88%
19	2.18%	8.39%
20	1.72%	7.96%
21	2.22%	8.29%
22	1.87%	8.57%
23	1.48%	7.91%
24	1.41%	7.61%
25	2.02%	8.80%

Empirical Results

Table 7.12: Adjusted R Squares for B/M Portfolios

Portfolio	Adjusted R Square-Augmented Factor model	Adjusted R Square-Fama-French	Difference in Rsquares
1	68.57%	68.64%	-0.068%
2	72.98%	73.11%	-0.131%
3	82.64%	82.75%	-0.115%
4	79.69%	79.65%	0.040%
5	82.93%	82.95%	-0.025%
6	80.62%	80.38%	0.240%
7	80.11%	80.14%	-0.025%
8	77.00%	77.07%	-0.066%
9	84.43%	84.32%	0.117%
10	77.13%	77.17%	-0.041%
11	72.37%	71.54%	0.830%
12	78.94%	78.90%	0.039%
13	81.00%	80.53%	0.462%
14	79.92%	79.93%	-0.016%
15	81.18%	81.17%	0.001%
16	77.63%	76.66%	0.975%
17	78.02%	78.14%	-0.123%
18	78.70%	78.76%	-0.065%
19	79.23%	79.36%	-0.129%
20	82.04%	81.98%	0.057%
21	77.53%	77.10%	0.424%
22	84.34%	83.86%	0.474%
23	77.11%	77.16%	-0.047%
24	85.67%	85.66%	0.012%
25	85.98%	85.89%	0.095%
	Average difference		0.117%

Table 7.13: Significance at 5% - B/M Portfolios

	Intercept	Rm-Rf	Size	Value	IA	Profitability
High (1-8)	3 37.50%	8 100.00%	8 100.00%	8 100.00%	1 12.50%	0 0.00%
Neutral (9-17)	3 33.33%	9 100.00%	9 100.00%	8 88.89%	3 33.33%	0 0.00%
Low (18-25)	1 12.50%	8 100.00%	8 100.00%	2 25.00%	2 25.00%	0 0.00%
Overall (%)	7 **28.00%**	25 **100.00%**	25 **100.00%**	18 **72.00%**	6 **24.00%**	0 **0.00%**

Table 7.14: Significance at 10% - B/M Portfolios

	Intercept	Rm-Rf	Size	Value	IA	Profitability
High (1-8)	3 37.50%	8 100.00%	8 100.00%	8 100.00%	1 12.50%	0 0.00%
Neutral (9-17)	3 33.33%	9 100.00%	9 100.00%	8 88.89%	4 44.44%	1 11.11%
Low (18-25)	2 25.00%	8 100.00%	8 100.00%	2 25.00%	3 37.50%	0 0.00%
Overall (%)	8 **32.00%**	25 **100.00%**	25 **100.00%**	18 **72.00%**	8 **32.00%**	1 **4.00%**

Table 7.15: Significance at 5% - Fama French Model

	Intercept	Rm-Rf	Size	Value
High (1-8)	4 50.00%	8 100.00%	8 100.00%	8 100.00%
Neutral (9-17)	3 33.33%	9 100.00%	9 100.00%	7 77.78%
Low (18-25)	2 25.00%	8 100.00%	8 100.00%	1 12.50%
Overall (%)	9 **36.00%**	25 **100.00%**	25 **100.00%**	16 **64.00%**