Chapter 10

Error Detection and Correction

Note

Data can be corrupted during transmission.

Some applications require that errors be detected and corrected.

10-1 INTRODUCTION

Let us first discuss some issues related, directly or indirectly, to error detection and correction.

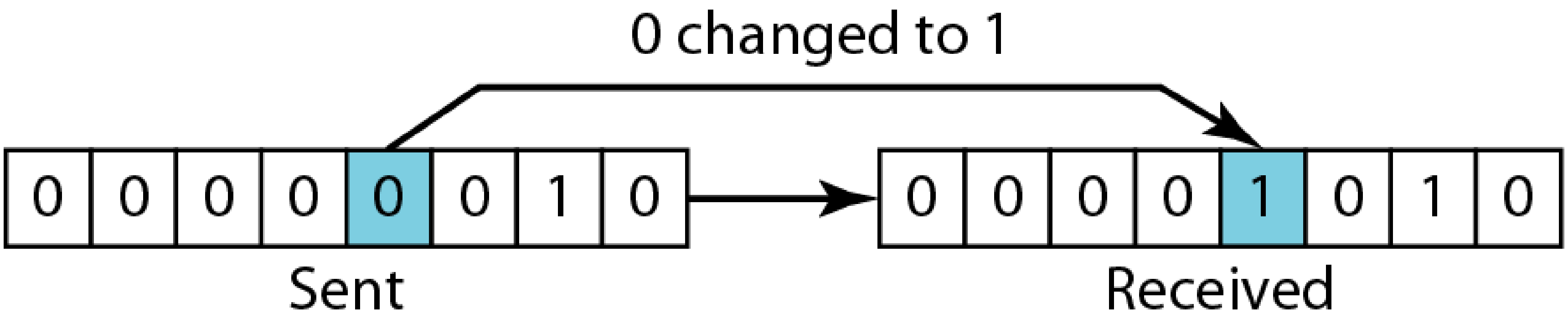
Topics discussed in this section:

Types of Errors
Redundancy
Detection Versus Correction
Forward Error Correction Versus Retransmission
Coding
Modular Arithmetic

Note

In a single-bit error, only 1 bit in the data unit has changed.

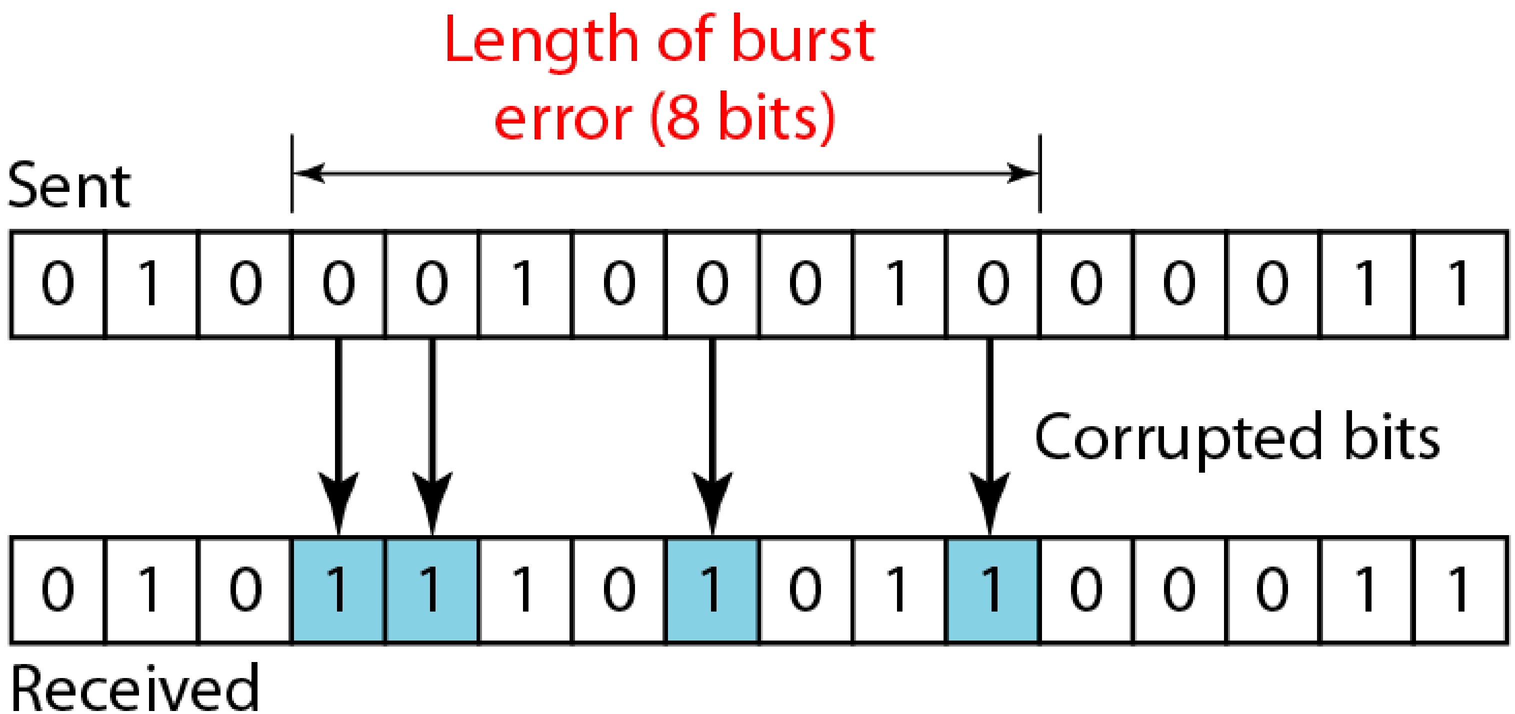
Figure 10.1 *Single-bit error*



Note

A burst error means that 2 or more bits in the data unit have changed.

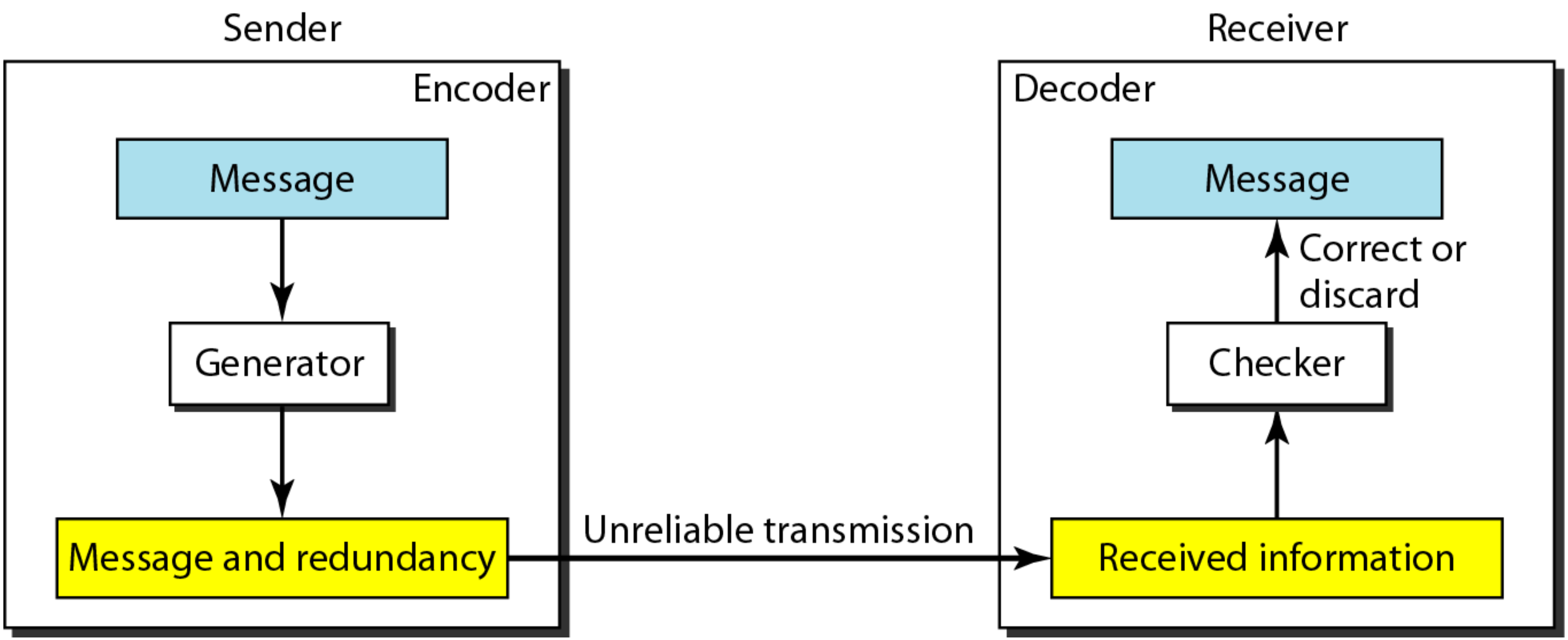
Figure 10.2 *Burst error of length 8*



Note

To detect or correct errors, we need to send extra (redundant) bits with data.

Figure 10.3 *The structure of encoder and decoder*



Note

In modulo-N arithmetic, we use only the integers in the range 0 to N −1, inclusive.

Figure 10.4 *XORing of two single bits or two words*

$$0 \oplus 0 = 0 \qquad\qquad 1 \oplus 1 = 0$$

a. Two bits are the same, the result is 0.

$$0 \oplus 1 = 1 \qquad\qquad 1 \oplus 0 = 1$$

b. Two bits are different, the result is 1.

$$\begin{array}{cccccc} & 1 & 0 & 1 & 1 & 0 \\ \oplus & 1 & 1 & 1 & 0 & 0 \\ \hline & 0 & 1 & 0 & 1 & 0 \end{array}$$

c. Result of XORing two patterns

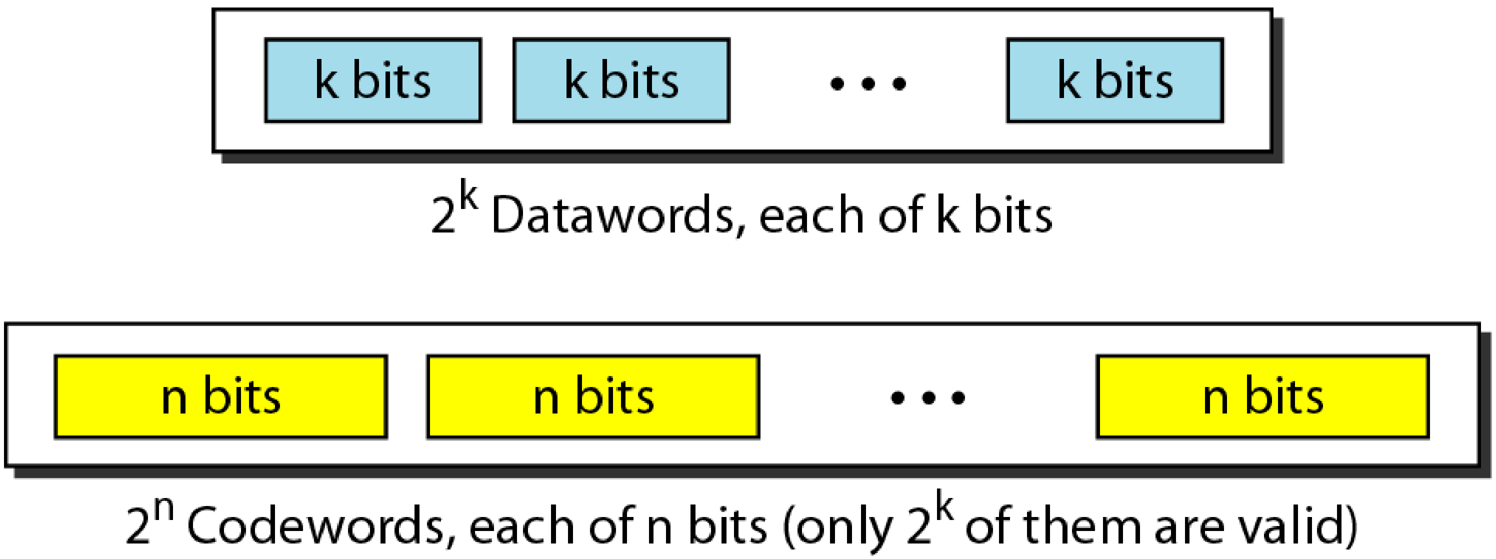
10-2 BLOCK CODING

In block coding, we divide our message into blocks, each of k bits, called datawords. We add r redundant bits to each block to make the length n = k + r. The resulting n-bit blocks are called codewords.

Topics discussed in this section:

Error Detection
Error Correction
Hamming Distance
Minimum Hamming Distance

Figure 10.5 *Datawords and codewords in block coding*



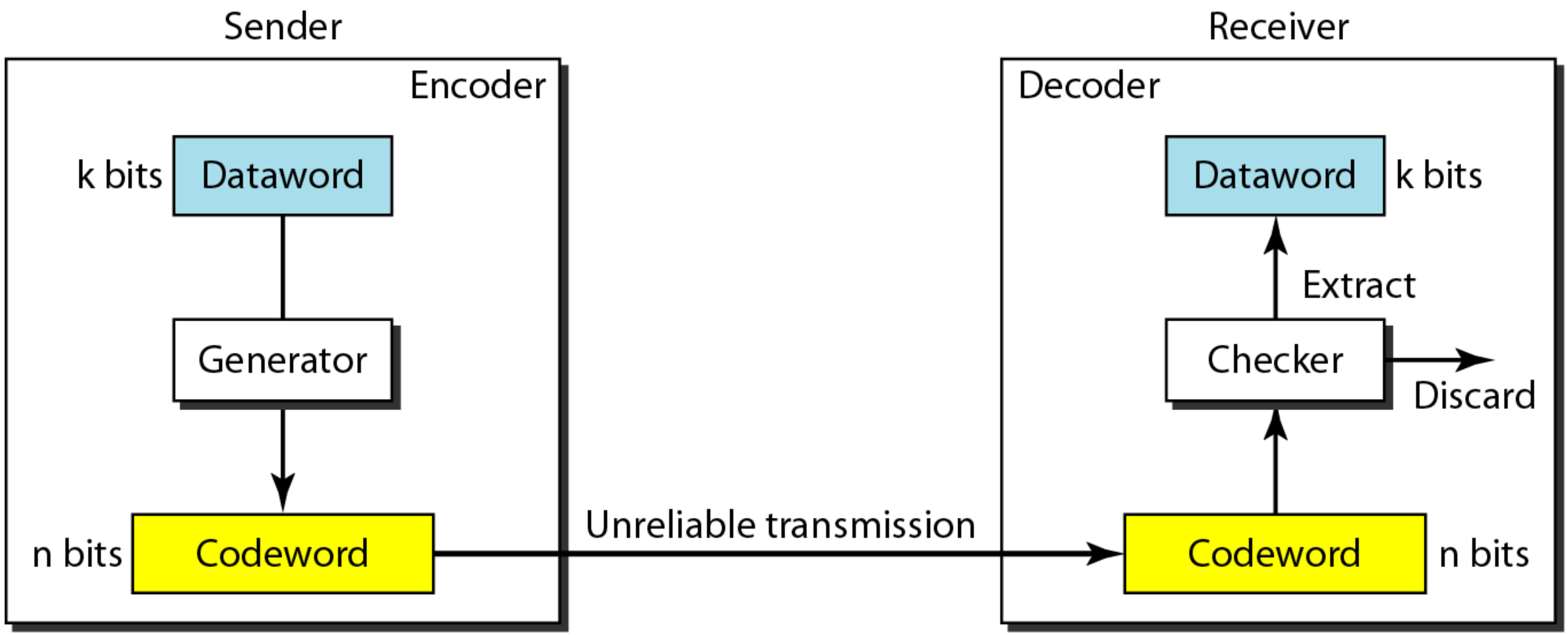
Example 10.1

The 4B/5B block coding discussed in Chapter 4 is a good example of this type of coding. In this coding scheme, $k = 4$ and $n = 5$. As we saw, we have $2^k = 16$ datawords and $2^n = 32$ codewords. We saw that 16 out of 32 codewords are used for message transfer and the rest are either used for other purposes or unused.

Error Detection

- Enough redundancy is added to detect an error.
- The receiver knows an error occurred but does not know which bit(s) is(are) in error.
- Has less overhead than error correction.

Figure 10.6 *Process of error detection in block coding*



Example 10.2

Let us assume that k = 2 and n = 3. Table 10.1 shows the list of datawords and codewords. Later, we will see how to derive a codeword from a dataword.

Assume the sender encodes the dataword 01 as 011 and sends it to the receiver. Consider the following cases:

1. The receiver receives 011. It is a valid codeword. The receiver extracts the dataword 01 from it.

Example 10.2 (continued)

2. ***The codeword is corrupted during transmission, and 111 is received. This is not a valid codeword and is discarded.***

3. ***The codeword is corrupted during transmission, and 000 is received. This is a valid codeword. The receiver incorrectly extracts the dataword 00. Two corrupted bits have made the error undetectable.***

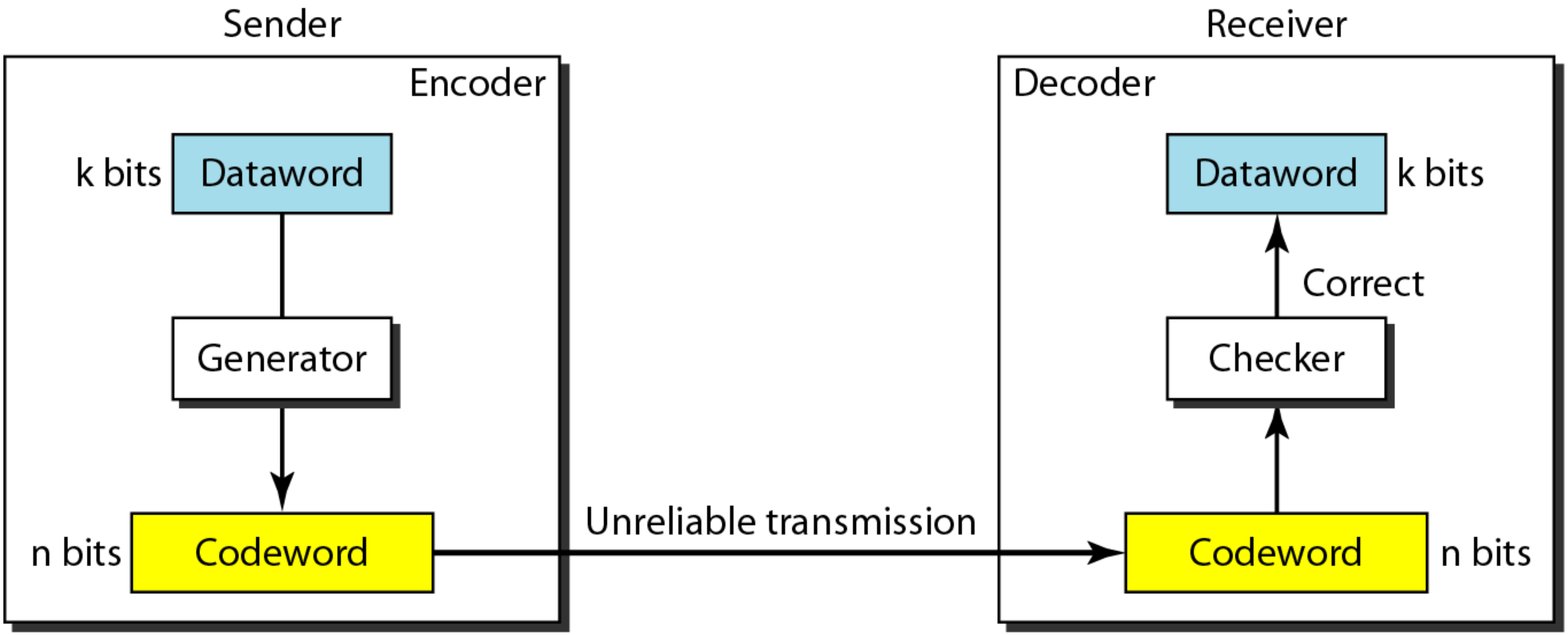
Table 10.1 ***A code for error detection (Example 10.2)***

Datawords	*Codewords*
00	000
01	011
10	101
11	110

Note

An error-detecting code can detect only the types of errors for which it is designed; other types of errors may remain undetected.

Figure 10.7 *Structure of encoder and decoder in error correction*



Example 10.3

Let us add more redundant bits to Example 10.2 to see if the receiver can correct an error without knowing what was actually sent. We add 3 redundant bits to the 2-bit dataword to make 5-bit codewords. Table 10.2 shows the datawords and codewords. Assume the dataword is 01. The sender creates the codeword 01011. The codeword is corrupted during transmission, and 01001 is received. First, the receiver finds that the received codeword is not in the table. This means an error has occurred. The receiver, assuming that there is only 1 bit corrupted, uses the following strategy to guess the correct dataword.

Example 10.3 (continued)

1. ***Comparing the received codeword with the first codeword in the table (01001 versus 00000), the receiver decides that the first codeword is not the one that was sent because there are two different bits.***

2. ***By the same reasoning, the original codeword cannot be the third or fourth one in the table.***

3. ***The original codeword must be the second one in the table because this is the only one that differs from the received codeword by 1 bit. The receiver replaces 01001 with 01011 and consults the table to find the dataword 01.***

Table 10.2 *A code for error correction (Example 10.3)*

Dataword	*Codeword*
00	00000
01	01011
10	10101
11	11110

Note

The Hamming distance between two words is the number of differences between corresponding bits.

Example 10.4

Let us find the Hamming distance between two pairs of words.

1. ***The Hamming distance d(000, 011) is 2 because***

$$\boxed{000 \oplus 011 \text{ is } 011 \text{ (two 1s)}}$$

2. ***The Hamming distance d(10101, 11110) is 3 because***

$$\boxed{10101 \oplus 11110 \text{ is } 01011 \text{ (three 1s)}}$$

Note

The minimum Hamming distance is the smallest Hamming distance between all possible pairs in a set of words.

Example 10.5

Find the minimum Hamming distance of the coding scheme in Table 10.1.

Solution

We first find all Hamming distances.

$d(000, 011) = 2$	$d(000, 101) = 2$	$d(000, 110) = 2$	$d(011, 101) = 2$
$d(011, 110) = 2$	$d(101, 110) = 2$		

The d_{min} in this case is 2.

Example 10.6

Find the minimum Hamming distance of the coding scheme in Table 10.2.

Solution

We first find all the Hamming distances.

$d(00000, 01011) = 3$	$d(00000, 10101) = 3$	$d(00000, 11110) = 4$
$d(01011, 10101) = 4$	$d(01011, 11110) = 3$	$d(10101, 11110) = 3$

The d_{min} in this case is 3.

Note

To guarantee the detection of up to s errors in all cases, the minimum Hamming distance in a block code must be $d_{min} = s + 1$.

Example 10.7

The minimum Hamming distance for our first code scheme (Table 10.1) is 2. This code guarantees detection of only a single error. For example, if the third codeword (101) is sent and one error occurs, the received codeword does not match any valid codeword. If two errors occur, however, the received codeword may match a valid codeword and the errors are not detected.

Example 10.8

Our second block code scheme (Table 10.2) has d_{min} = 3. This code can detect up to two errors. Again, we see that when any of the valid codewords is sent, two errors create a codeword which is not in the table of valid codewords. The receiver cannot be fooled.

However, some combinations of three errors change a valid codeword to another valid codeword. The receiver accepts the received codeword and the errors are undetected.

Figure 10.8 ***Geometric concept for finding d_{min} in error detection***

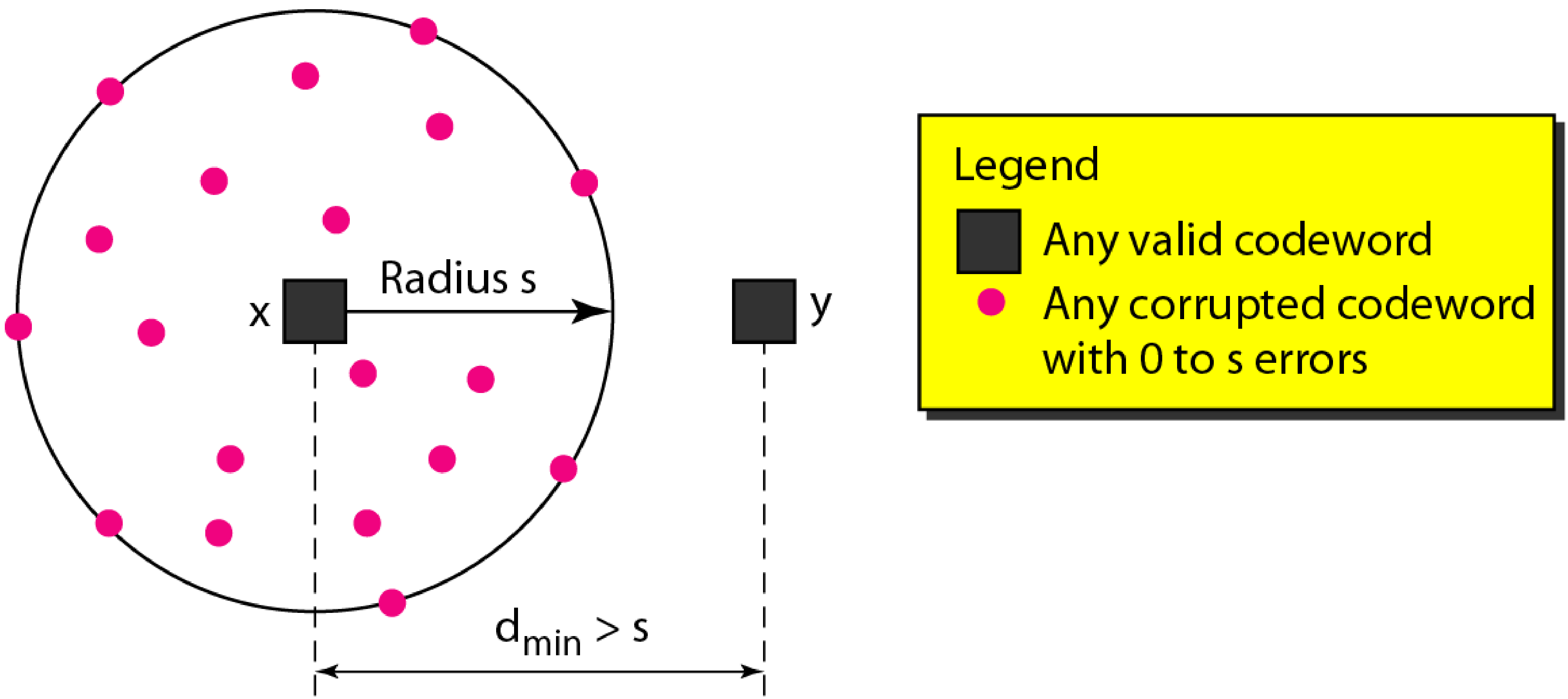
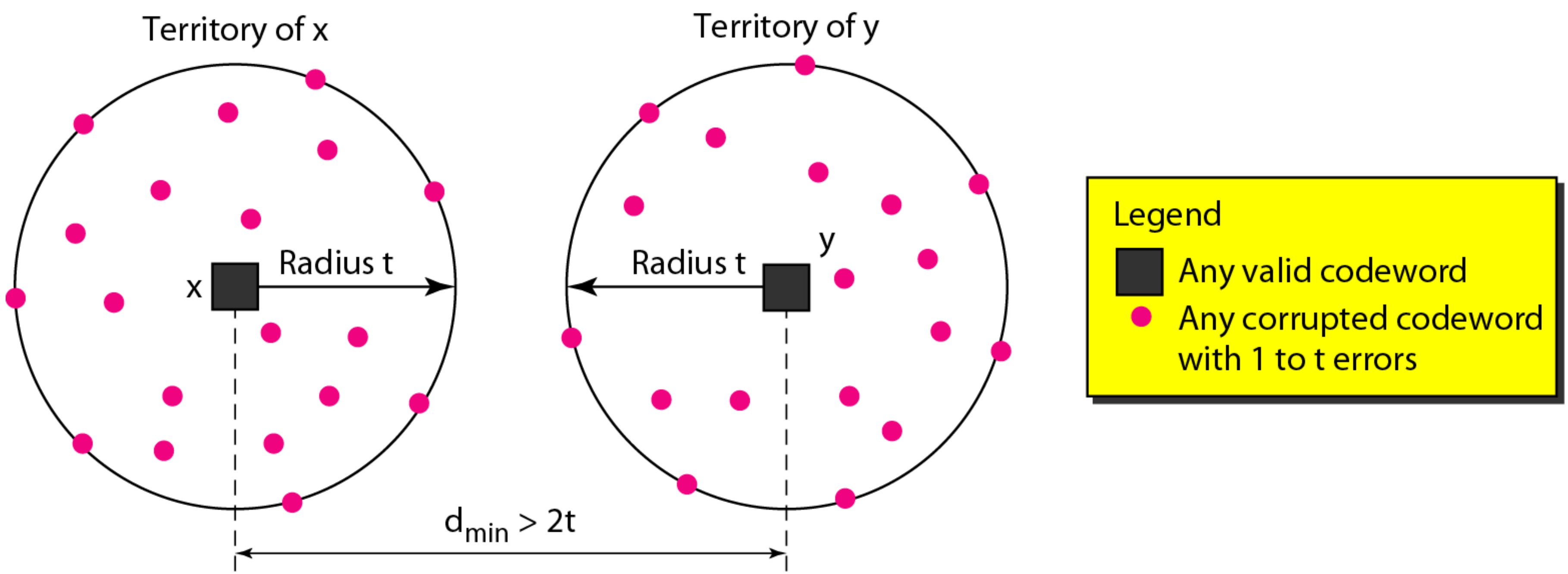


Figure 10.9 *Geometric concept for finding d_{min} in error correction*



Note

To guarantee correction of up to *t* errors in all cases, the minimum Hamming distance in a block code must be $d_{min} = 2t + 1$.

Example 10.9

A code scheme has a Hamming distance d_{min} = 4. What is the error detection and correction capability of this scheme?

Solution

This code guarantees the detection of up to three errors (s = 3), but it can correct up to one error. In other words, if this code is used for error correction, part of its capability is wasted. Error correction codes need to have an odd minimum distance (3, 5, 7, . . .).