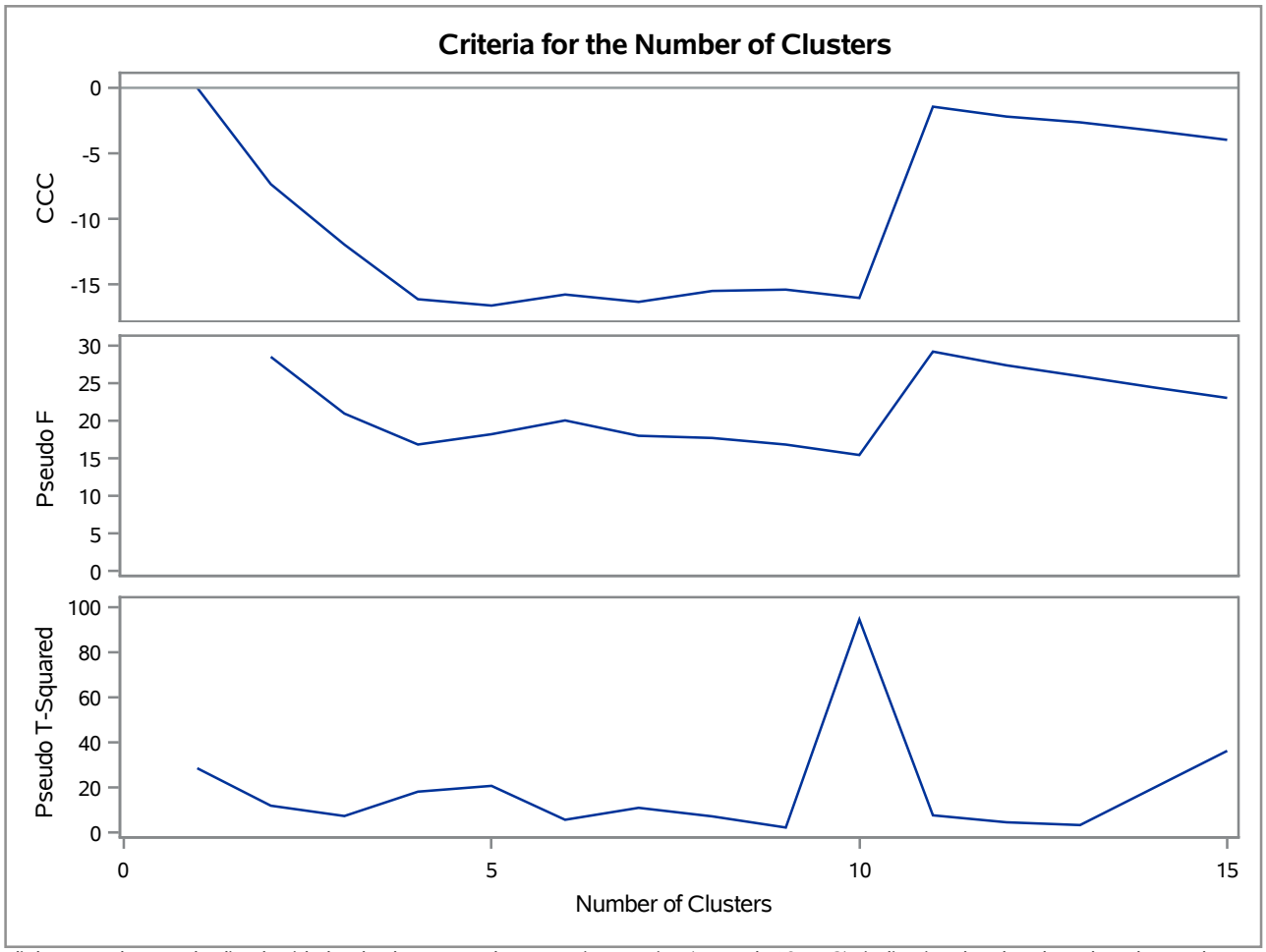
Exercise 1a

The CLUSTER Procedure
Average Linkage Cluster Analysis

Cluster History

Number of Clusters	Clusters Joined		Freq	Semipartial R-Square	R-Square	Approximate Expected R-Square	Cubic Clustering Criterion	Pseudo F Statistic	Pseudo t-Squared	Norm RMS Distance	Tie
15	CL21	CL18	173	0.0596	.618	.653	-4.0	23.0	36.2	0.9773	
14	OB189	OB190	2	0.0048	.614	.642	-3.3	24.4	.	1.0066	
13	CL27	OB208	3	0.0062	.608	.630	-2.6	25.9	3.3	1.0409	
12	CL15	OB163	174	0.0090	.599	.618	-2.2	27.4	4.5	1.0815	
11	OB108	CL29	4	0.0086	.590	.604	-1.4	29.2	7.6	1.1408	
10	CL12	CL16	196	0.1848	.405	.588	-16	15.4	94.6	1.1679	
9	OB164	CL13	4	0.0088	.396	.570	-15	16.8	2.2	1.2374	
8	CL10	CL14	198	0.0207	.376	.550	-16	17.7	7.1	1.2941	
7	CL8	CL17	201	0.0328	.343	.527	-16	18.0	10.9	1.3246	
6	CL7	OB202	202	0.0177	.325	.498	-16	20.0	5.6	1.4929	
5	CL6	CL11	206	0.0667	.258	.460	-17	18.2	20.7	1.5626	
4	CL5	CL9	210	0.0646	.194	.407	-16	16.8	18.1	1.6046	
3	CL4	OB185	211	0.0281	.166	.327	-12	21.0	7.3	1.8486	
2	CL3	OB107	212	0.0472	.119	.211	-7.4	28.5	11.9	2.3385	
1	CL2	CL182	214	0.1185	.000	.000	0.00	.	28.5	2.6108	

The CLUSTER Procedure
Average Linkage Cluster Analysis



Using average linkage on the standardized oxide levels, the CCC values remain negative (around -16 to -3), indicating that the glass data do not show extremely strong, well-separated clusters. However, the Pseudo F statistic reaches its largest value (~29.2) when there are about 11 clusters, and the Pseudo T2 statistic shows a very large spike (~94.6) at 10 clusters, followed by a sharp drop. This pattern (high Pseudo F at 11 and a big Pseudo T2 jump when merging from 11 to 10 clusters) suggests that a solution with 11 clusters is the most reasonable number of clusters.
Exercise 1b

The FREQ Procedure

Frequency

Table of CLUSTER by groupedtype

CLUSTER	groupedtype					
	buildingwindow	containers	headlamps	tableware	vehiclewindow	Total
1	140	8	2	7	17	174
2	0	0	21	1	0	22
3	0	2	0	0	0	2
4	4	0	0	0	0	4
5	0	0	3	0	0	3
6	1	2	0	0	0	3
7	0	0	2	0	0	2
8	0	1	0	0	0	1
9	0	0	1	0	0	1
10	0	0	0	1	0	1
11	1	0	0	0	0	1
Total	146	13	29	9	17	214

The 11 cluster solution partially matches the true glass types. One very large cluster (Cluster 1) contains a mixture of almost all glass types, which shows that most types share very similar chemical compositions and are difficult to separate. In contrast, the headlamp glass forms a very clean and distinct group (Cluster 2), meaning this type has a unique chemical composition. The remaining clusters are small (1-4 observations) and represent minor chemical variations or outliers.The clustering aligns well only for headlamps. The other glass types overlap too much chemically to form clean clusters.
Exercise 2a

The FREQ Procedure

CLUSTER	Frequency	Percent	Cumulative Frequency	Cumulative Percent
1	174	81.31	174	81.31
2	22	10.28	196	91.59
3	2	0.93	198	92.52
4	4	1.87	202	94.39
5	3	1.40	205	95.79
6	3	1.40	208	97.20
7	2	0.93	210	98.13
8	1	0.47	211	98.60
9	1	0.47	212	99.07
10	1	0.47	213	99.53
11	1	0.47	214	100.00

The ANOVA Procedure

Dependent Variable: RI

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	0.00007956	0.00007956	16.09	<.0001
Error	194	0.00095959	0.00000495		
Corrected Total	195	0.00103916			

The ANOVA Procedure

Levene's Test for Homogeneity of RI Variance ANOVA of Squared Deviations from Group Means					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
CLUSTER	1	2.4E-10	2.4E-10	2.37	0.1253
Error	194	1.964E-8	1.01E-10		

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for RI

Comparisons significant at the 0.05 level are indicated by ***.				
CLUSTER Comparison	**Difference Between Means**	**Simultaneous 95% Confidence Limits**		
1 - 2	0.0020184	0.0010258	0.0030109	***
2 - 1	-0.0020184	-0.0030109	-0.0010258	***

We fit an ANOVA model to see whether refractive index (RI) differs between the large clusters from Exercise 1 (clusters with 5 or more observations). Only Clusters 1 and 2 met this requirement. The ANOVA F-test is significant ($F = 16.09$, $p < 0.0001$), which means that including cluster explains more variation in RI than an error-only model. RI differs across the two major clusters. Levene's test for equal variances is not significant ($p = 0.1253$), so the assumption of equal variances is reasonable and the ANOVA model is appropriate. Overall, the model shows that cluster membership has a statistically meaningful effect on RI.
Exercise 2b
Tukey's test shows a significant difference in mean RI between Cluster 1 and Cluster 2. The 95% confidence interval for the difference does not include zero, confirming that these two clusters also differ in refractive index. This tells us that the cluster structure from Exercise 1 captures chemical differences that are also visible in RI, even though the clusters were formed using oxide levels and not RI directly.However, while the difference is statistically significant, the magnitude (around 0.002 units of RI) is small. This means that although RI helps distinguish the clusters, the model would not be very strong for predicting RI from cluster membership alone.
Exercise 3a

The STEPDISC Procedure

Stepwise Selection Summary

Step	Number In	Entered	Removed	Partial R-Square	F Value	Pr > F	Wilks' Lambda	Pr < Lambda	Average Squared Canonical Correlation	Pr > ASCC
1	1	Mg		0.5868	74.21	<.0001	0.41316290	<.0001	0.14670927	<.0001
2	2	Ca		0.3703	30.59	<.0001	0.26014860	<.0001	0.19583534	<.0001
3	3	K		0.3264	25.08	<.0001	0.17522991	<.0001	0.26452170	<.0001
4	4	Ba		0.1745	10.89	<.0001	0.14464505	<.0001	0.29830882	<.0001
5	5	Na		0.0994	5.66	0.0002	0.13026910	<.0001	0.31393957	<.0001
6	6	Al		0.0752	4.14	0.0030	0.12047871	<.0001	0.32328134	<.0001
7	7	Si		0.0894	4.98	0.0007	0.10970368	<.0001	0.32972706	<.0001
8	6		Mg	0.0435	2.31	0.0593	0.11469378	<.0001	0.32576868	<.0001

The stepwise discriminant analysis adds variables one at a time based on how strongly they help separate the glass groups. From the output, the oxides entered in this order: Mg, Ca, K, Ba, Na, Al, and Si. Each of these variables had a highly significant p-value (<0.05) meaning each oxide contributed in meaningful separation between glass types.The only variable removed was Mg at the final step because, after the other oxides were included, Mg no longer added new information.Overall, the analysis suggests that Ca, K, Ba, Na, Al, and Si are the most useful oxides for distinguishing between glass types, and these are the predictors we would use in the final model.

Exercise 3b

The DISCRIM Procedure
Test of Homogeneity of Within Covariance Matrices

Chi-Square	DF	Pr > ChiSq
1014.930198	84	<.0001

Since the Chi-Square value is significant at the 0.1 level, the within covariance matrices will be used in the discriminant function.
Reference: Morrison, D.F. (1976) Multivariate Statistical Methods p252.

The DISCRIM Procedure

Multivariate Statistics and F Approximations					
S=4 M=0.5 N=101					
Statistic	**Value**	**F Value**	**Num DF**	**Den DF**	**Pr > F**
Wilks' Lambda	0.11469378	25.55	24	712.88	<.0001
Pillai's Trace	1.30307472	16.67	24	828	<.0001
Hotelling-Lawley Trace	4.49331863	37.98	24	473.57	<.0001
Roy's Greatest Root	3.77726192	130.32	6	207	<.0001
NOTE: F Statistic for Roy's Greatest Root is an upper bound.					

The DISCRIM Procedure
Classification Summary for Calibration Data: WORK.GLASSID
Cross-validation Summary using Quadratic Discriminant Function

Number of Observations and Percent Classified into groupedtype						
From groupedtype	**buildingwindow**	**containers**	**headlamps**	**tableware**	**vehiclewindow**	**Total**
buildingwindow	41 28.08	1 0.68	3 2.05	4 2.74	97 66.44	146 100.00
containers	4 30.77	7 53.85	2 15.38	0 0.00	0 0.00	13 100.00
headlamps	0 0.00	1 3.45	27 93.10	0 0.00	1 3.45	29 100.00
tableware	0 0.00	0 0.00	0 0.00	9 100.00	0 0.00	9 100.00
vehiclewindow	4 23.53	0 0.00	0 0.00	1 5.88	12 70.59	17 100.00
Total	49 22.90	9 4.21	32 14.95	14 6.54	110 51.40	214 100.00
Priors	0.2	0.2	0.2	0.2	0.2	

Error Count Estimates for groupedtype						
	buildingwindow	**containers**	**headlamps**	**tableware**	**vehiclewindow**	**Total**
Rate	0.7192	0.4615	0.0690	0.0000	0.2941	0.3088
Priors	0.2000	0.2000	0.2000	0.2000	0.2000	

The test of homogeneity of covariance matrix is highly significant (Chi-Square = 1014.93, $p < 0.0001$), which means the different glass types do not share similar covariance structures. So, the correct model to use is Quadratic Discriminant Analysis (QDA) instead of LDA. QDA allows each glass group to have its own variability pattern.The MANOVA results support this choice and show that the selected oxides do a good job distinguishing between the glass types. All multivariate tests are highly significant ($p < 0.0001$). This means the chemical oxide composition differs strongly across the glass groups, and there is clear separation.The MANOVA confirms strong separation between glass types, and the covariance test tells us that QDA is the appropriate method to model these differences.
Exercise 3c

The DISCRIM Procedure
Classification Summary for Calibration Data: WORK.GLASSID
Cross-validation Summary using Quadratic Discriminant Function

Number of Observations and Percent Classified into groupedtype						
From groupedtype	**buildingwindow**	**containers**	**headlamps**	**tableware**	**vehiclewindow**	**Total**
buildingwindow	41 28.08	1 0.68	3 2.05	4 2.74	97 66.44	146 100.00
containers	4 30.77	7 53.85	2 15.38	0 0.00	0 0.00	13 100.00
headlamps	0 0.00	1 3.45	27 93.10	0 0.00	1 3.45	29 100.00
tableware	0 0.00	0 0.00	0 0.00	9 100.00	0 0.00	9 100.00
vehiclewindow	4 23.53	0 0.00	0 0.00	1 5.88	12 70.59	17 100.00
Total	49 22.90	9 4.21	32 14.95	14 6.54	110 51.40	214 100.00
Priors	0.2	0.2	0.2	0.2	0.2	

Error Count Estimates for groupedtype						
	buildingwindow	**containers**	**headlamps**	**tableware**	**vehiclewindow**	**Total**
Rate	0.7192	0.4615	0.0690	0.0000	0.2941	0.3088
Priors	0.2000	0.2000	0.2000	0.2000	0.2000	

The cross-validation results show that some glass types are much easier for the model to identify than others. Headlamps and tableware perform the best, with headlamps correctly classified about 93% of the time and tableware classified perfectly, suggesting that these groups have very distinct chemical oxide profiles. Containers and vehiclewindow have moderate accuracy, 54% and 71% respectively, their chemical compositions overlap a bit with other types and make them harder to separate. Buildingwindow has the poorest classification performance, with only about 28% correctly identified; most buildingwindow samples are misclassified as vehiclewindow, indicating that these two types have very similar oxide compositions. Overall, the model's cross-validation error rate of about 31% shows that the selected oxides do a good job in distinguishing some glass types.
Exercise 4a

The FREQ Procedure

Frequency	Table of groupedtype by newgroupedtype			
	newgroupedtype			
groupedtype	glassware	headlamps	window	Total
buildingwindow	0	0	146	146
containers	13	0	0	13
headlamps	0	29	0	29
tableware	9	0	0	9
vehiclewindow	0	0	17	17
Total	22	29	163	214

The building windows and vehicle windows have very similar chemical compositions, and containers and tableware also look very similar. When we combine the glass types, the frequency table confirms that the regrouping makes sense. All buildingwindow and vehiclewindow samples fall neatly into the new windowcategory, and all containers and tableware samples fall into the new glassware category. Headlamps remain their own separate group because they are chemically quite different from the others.Based on our earlier results (Exercise 3), this claim is reasonable. Building and vehicle windows behaved similarly in the classification model, which suggests their oxide levels are close. Containers and tableware also showed similar patterns and were frequently misclassified between each other, meaning they share chemical characteristics too.So, both the claims agree that these groups are similar enough to combine into broader categories.
Exercise 4b

The STEPDISC Procedure

Stepwise Selection Summary										
Step	Number In	Entered	Removed	Partial R-Square	F Value	Pr > F	Wilks' Lambda	Pr < Lambda	Average Squared Canonical Correlation	Pr > ASCC
1	1	Mg		0.5808	146.17	<.0001	0.41920299	<.0001	0.29039850	<.0001
2	2	Ca		0.3689	61.37	<.0001	0.26456685	<.0001	0.38642927	<.0001
3	3	Ba		0.2039	26.77	<.0001	0.21060867	<.0001	0.46384216	<.0001
4	4	K		0.1182	13.94	<.0001	0.18571396	<.0001	0.49414588	<.0001
5	5	Si		0.0517	5.64	0.0041	0.17611549	<.0001	0.50235161	<.0001

The DISCRIM Procedure
Test of Homogeneity of Within Covariance Matrices

Chi-Square	DF	Pr > ChiSq
566.066166	30	<.0001

Since the Chi-Square value is significant at the 0.1 level, the within covariance matrices will be used in the discriminant function.
Reference: Morrison, D.F. (1976) Multivariate Statistical Methods p252.

The DISCRIM Procedure

Multivariate Statistics and F Approximations					
S=2 M=1 N=102.5					
Statistic	**Value**	**F Value**	**Num DF**	**Den DF**	**Pr > F**
Wilks' Lambda	0.17611549	57.25	10	414	<.0001
Pillai's Trace	1.00470322	41.99	10	416	<.0001
Hotelling-Lawley Trace	3.65138708	75.34	10	307.76	<.0001
Roy's Greatest Root	3.34439415	139.13	5	208	<.0001
NOTE: F Statistic for Roy's Greatest Root is an upper bound.					
NOTE: F Statistic for Wilks' Lambda is exact.					

The DISCRIM Procedure
Classification Summary for Calibration Data: WORK.GLASSID2
Cross-validation Summary using Quadratic Discriminant Function

Number of Observations and Percent Classified into newgroupedtype				
From newgroupedtype	**glassware**	**headlamps**	**window**	**Total**
glassware	15 68.18	3 13.64	4 18.18	22 100.00
headlamps	5 17.24	23 79.31	1 3.45	29 100.00
window	12 7.36	2 1.23	149 91.41	163 100.00
Total	32 14.95	28 13.08	154 71.96	214 100.00
Priors	0.33333	0.33333	0.33333	

Error Count Estimates for newgroupedtype				
	glassware	**headlamps**	**window**	**Total**
Rate	0.3182	0.2069	0.0859	0.2037
Priors	0.3333	0.3333	0.3333	

When we repeat the discriminant analysis using the new three-group variable (glassware, headlamps, window), the stepwise procedure selects Mg, Ca, Ba, K, and Si as the most important oxides for separating the three categories. The test of equality of covariance matrices is highly significant ($X^2 = 566.07$, $p < 0.0001$), which means the groups do not share a common covariance structure so Quadratic Discriminant Analysis (QDA) is the appropriate model instead of LDA. The MANOVA results also strongly support chemical differences among the three new groups: Wilks' Lambda = 0.176, $F = 57.25$, $p < 0.0001$, with all multivariate tests agreeing.The cross-validation accuracy improves compared to Exercise 3. The new "window" group is classified extremely well, with 91.41% correctly identified. Headlamps are also highly accurate at 79.31% correct. Glassware is still the most difficult group, but even here the model correctly classifies 68.18% of samples. Overall, the total cross-validated error rate drops to 20.4%, which is a improvement over the original five-group model (31% error). This shows that merging the categories into broader categories produces cleaner group structure.

Exercise 4c

When we compare the new three-group model to the original five-group model, the classification performance improves. The improved accuracy comes from merging buildingwindow and vehiclewindow into one "window" group: accuracy for window glass rises from about 66% & 70% separately to 91% correctly classified in the combined model. Headlamps also improve slightly, increasing from about 79% to 93% correct. Glassware is the only group that becomes harder to classify, dropping from roughly 76% combined accuracy (containers & tableware) in Exercise 3 to about 68% in the three-group model. Overall, the total error rate improves from ~31% in Exercise 3 to ~20% in the new model, which means the reduced grouping leads to cleaner chemical separation and better predictive accuracy.