UNIVERSITY EXAMINATIONS

October/November 2020

MAT1512

Calculus A

Examiners:
First: DR .S.B. MUGISHA
Second: DR Z. ALI

100 Marks
2 Hours

Closed book and online examination, which you have to write within 2 hours and submit online through the link: https://myexams.unisa.ac.za/portal

Use of a non-programmable pocket calculator is NOT allowed

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This examination allows attachment documents only as part of your submission.

Declaration: I have neither given nor received aid on this examination.

Answer All Questions and Submit within the stipulated timeframe.

Late submission will not be accepted.

This paper consists of 4 pages.

ALL CALCULATIONS MUST BE SHOWN.

QUESTION 1

(a) Determine the following limits (if they exist):

(i) $\lim\limits_{x\to -5} \dfrac{x^2+x-20}{3(x+5)}$ (3)

(ii) $\lim\limits_{t\to 0} \dfrac{\sin 5t}{t^2+4t}$ (3)

(iii) $\lim\limits_{x\to -\infty} \dfrac{3-|x|}{2|x|+1}$ (3)

(iv) $\lim\limits_{x\to 0} \dfrac{2x}{3-\sqrt{x+9}}$ (3)

(v) $\lim\limits_{x\to +\infty} \dfrac{2x+x^2+1}{1-x+2x^2}$ (3)

(b) (i) Use the Squeeze Theorem to show that

$$\lim_{x\to \infty} \frac{\sin(e^x)}{x} = 0 \qquad (3)$$

(ii) Hence, evaluate

$$\lim_{x\to \infty} \frac{\sin(e^x)}{\sqrt{x^2+2}} \qquad (3)$$

(c) Let the function f be defined as:

$$f(x) = \begin{cases} 4a & \text{if} \quad x \le -2 \\ 3x^2 & \text{if} \quad -2 < x \le 1 \\ x+b & \text{if} \quad x > 1 \end{cases}$$

Determine the values of the constants a and b so that f is continuous at $x=-2$ and $x=1$. (4)

[25]

QUESTION 2

(a) By the first principle of differentiation, find the derivative of $f(x) = \dfrac{2}{2x-1}$ at $x=1$. (5)

[TURN OVER]

(b) Find the derivative of the following functions by using the appropriate rules of differentiation:

(i) $y=\dfrac{1}{\sqrt{x}}\left(x^2-\dfrac{2}{x}\right)$ (3)

(ii) $g(x)=(\cos 5x)^{\sin\left(x^2\right)}$ (3)

(iii) $h(x)=\dfrac{\sin x}{1+\cos x}$ (3)

(iv) $F(x)=\displaystyle\int_{\sqrt{x}}^{x} t\sqrt{t^2+1}\,dt$ (3)

(c) Given $2xy+\pi\sin y=2\pi x$, find:

(i) $\dfrac{dy}{dx}$ by using implicit differentiation. (3)

(ii) the equation of the tangent and normal lines to the curve $2xy+\pi\sin y=2\pi x$ at the point $\left(1,\dfrac{\pi}{2}\right)$. (5)

[25]

QUESTION 3

(a) Determine the following integrals:

(i) $\displaystyle\int\left(x-\frac{2}{x^2}\right)\left(x+\frac{2}{x^2}\right)dx$ (3)

(ii) $\displaystyle\int e^{5x}\left(\frac{e^{2x}}{7}+\frac{3}{e^{3x}}\right)dx$ (3)

(iii) $\displaystyle\int\frac{1}{\left(4-\sqrt{3x}\right)^3}dx$ (4)

(iv) $\displaystyle\int_0^{\frac{\pi}{4}}(\tan x)^3(\sec x)^3\,dx$ (5)

(b) Let $f(x)=x^2-2$ and $g(x)=-|x|$, then

(i) Sketch the graphs of f and g on the same axes. (4)

(ii) Find the area enclosed by $f(x)=x^2-2$ and $g(x)=-|x|$. (6)

[25]

[TURN OVER]

QUESTION 4

(a) Solve the following Initial Value Problem:

$$\frac{dy}{dx} = \frac{\cos^2 y}{4x-3} \quad ; \qquad y(1) = \frac{\pi}{4}. \qquad (7)$$

(b) Let $F(x, y) = y\cos(x^2y^2) + y$, then

(i) find the first partial derivatives F_x and F_y. (6)

(ii) using b(i) above, find $\frac{dy}{dx}$. (6)

(iii) If $F(x, y) = 0$, then find $\frac{dy}{dx}$ using implicit differentiation to confirm your answer in part (b) (ii) above. (6)

[25]

TOTAL: [100]