Explaining teaching score with age

MODELING WITH DATA IN THE TIDYVERSE

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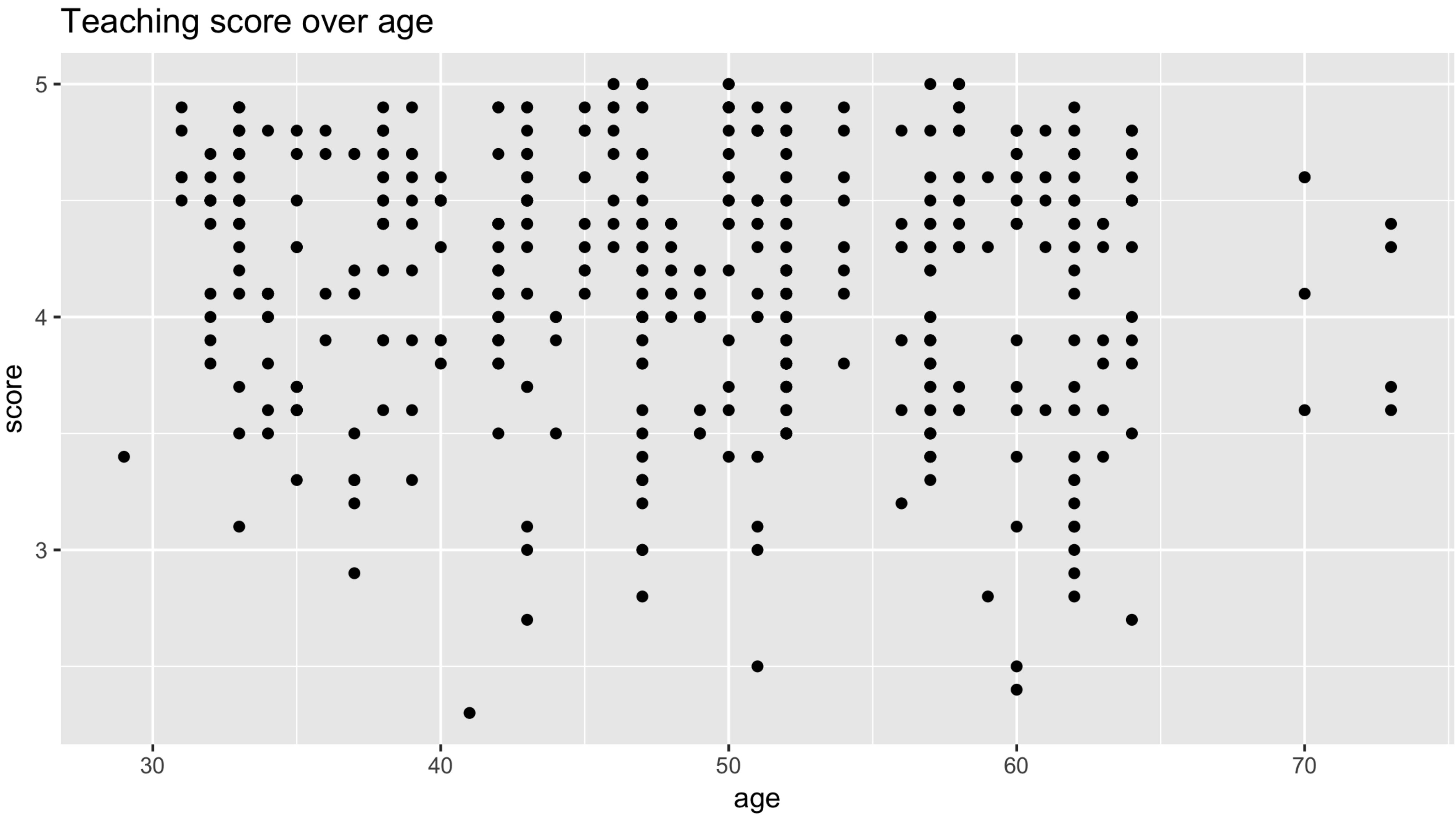
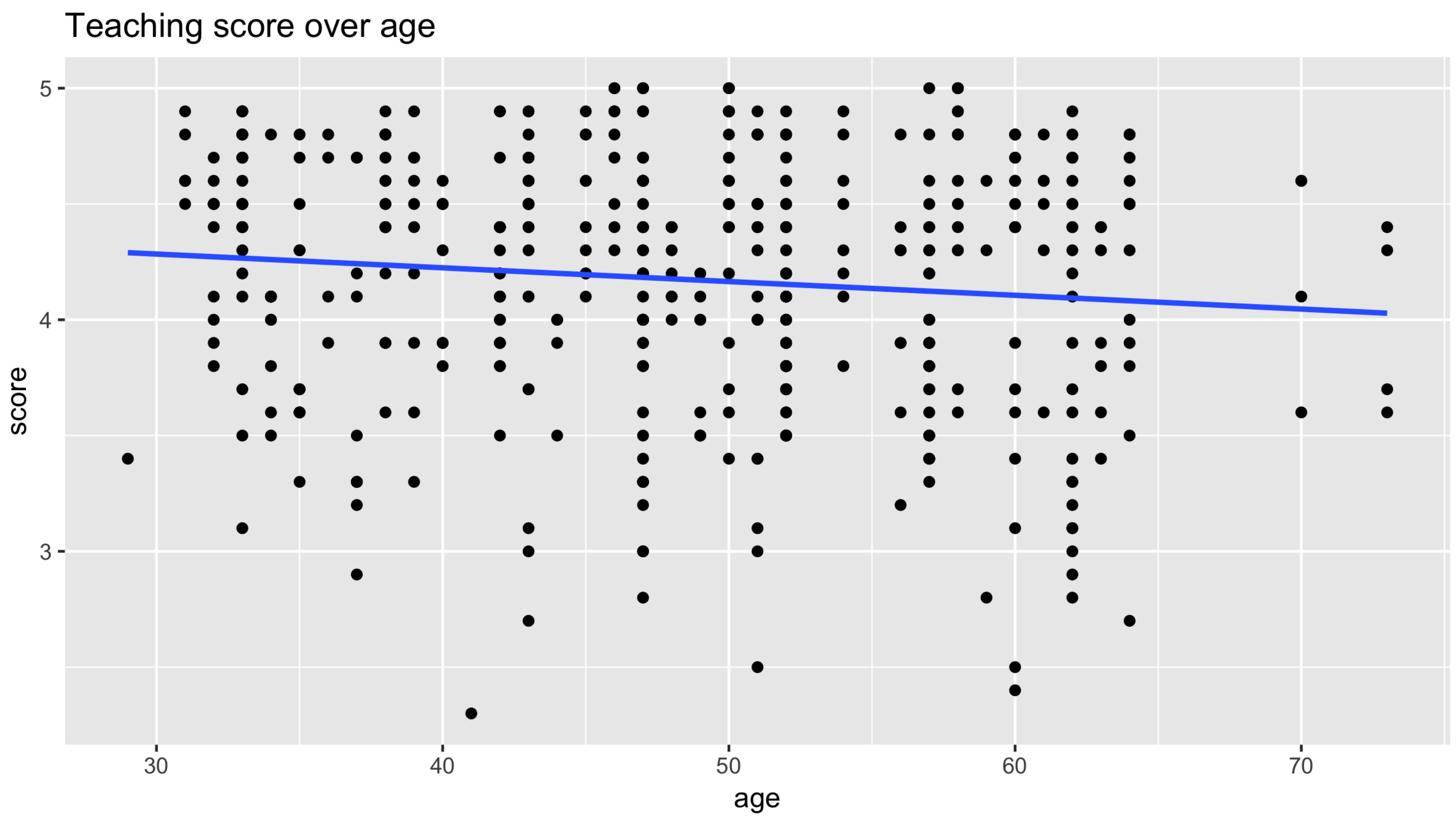
Refresher: Exploratory data visualization



Regression line

```r
# Code to create scatterplot
ggplot(evals, aes(x = age, y = score)) +
  geom_point() +
  labs(x = "age", y = "score",
       title = "Teaching score over age")
# Add a "best-fitting" line
ggplot(evals, aes(x = age, y = score)) +
  geom_point() +
  labs(x = "age", y = "score",
       title = "Teaching score over age") +
  geom_smooth(method = "lm", se = FALSE)
```

Regression line



Refresher: Modeling in general

- **Truth:** Assumed model is $y = f(\vec{x}) + \epsilon$
- **Goal:** Given y and $\vec{x}$, fit a model $\hat{f}(\vec{x})$ that *approximates* $f(\vec{x})$, where $\hat{y} = \hat{f}(\vec{x})$ is the *fitted/predicted* value for the *observed* value y

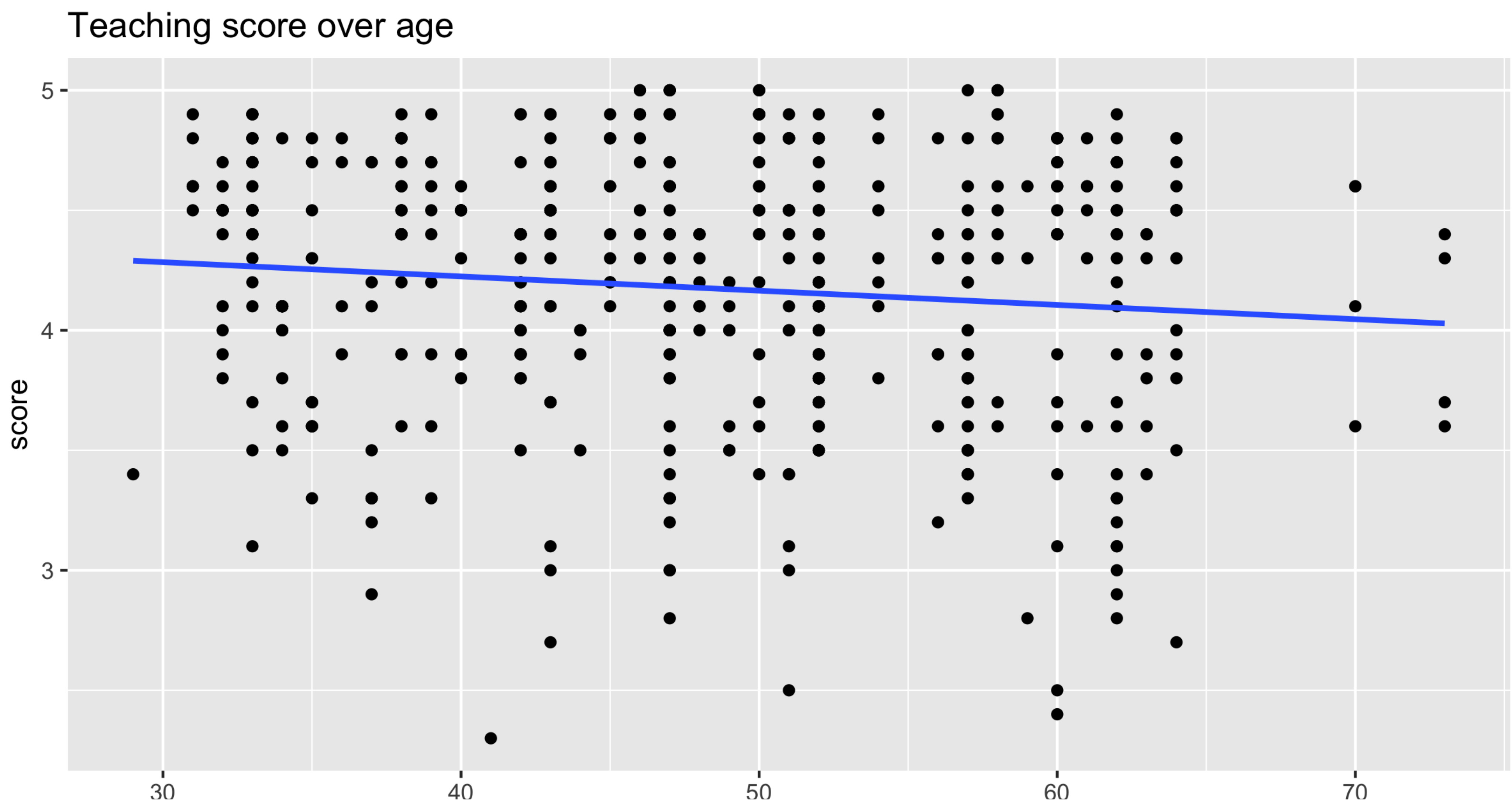
Modeling with basic linear regression

- **Truth:**
  - Assume $f(x) = \beta_0 + \beta_1 \cdot x$
  - *Observed* value $y = f(x) + \epsilon = \beta_0 + \beta_1 \cdot x + \epsilon$
- **Fitted:**
  - Assume $\hat{f}(x) = \hat{\beta}_0 + \hat{\beta}_1 \cdot x$
  - *Fitted/predicted* value $\hat{y} = \hat{f}(x) = \hat{\beta}_0 + \hat{\beta}_1 \cdot x$

Back to regression line

Equation for fitted blue regression line:

$$\hat{y} = \hat{f}(\vec{x}) = \hat{\beta}_0 + \hat{\beta}_1 \cdot x$$



Computing slope and intercept of regression line

Using the formula form `y ~ x`:

```r
# Fit regression model using formula of form: y ~ x
model_score_1 <- lm(score ~ age, data = evals)
# Output contents
model_score_1
```

```
Call:
lm(formula = score ~ age, data = evals)

Coefficients:
(Intercept)          age
   4.461932    -0.005938
```

Computing slope and intercept of regression line

Using the formula form `y ~ x`, which is akin to $\hat{y} = \hat{f}(\vec{x})$

```
# Fit regression model using formula of form: y ~ x
model_score_1 <- lm(score ~ age, data = evals)

# Output regression table using wrapper function:
get_regression_table(model_score_1)
```

```
# A tibble: 2 x 7
  term      estimate std_error statistic p_value...
  <chr>        <dbl>     <dbl>     <dbl>   <dbl>...
1 intercept    4.46      0.127     35.2    0...
2 age         -0.006     0.003     -2.31   0.021...
```

Let's practice!

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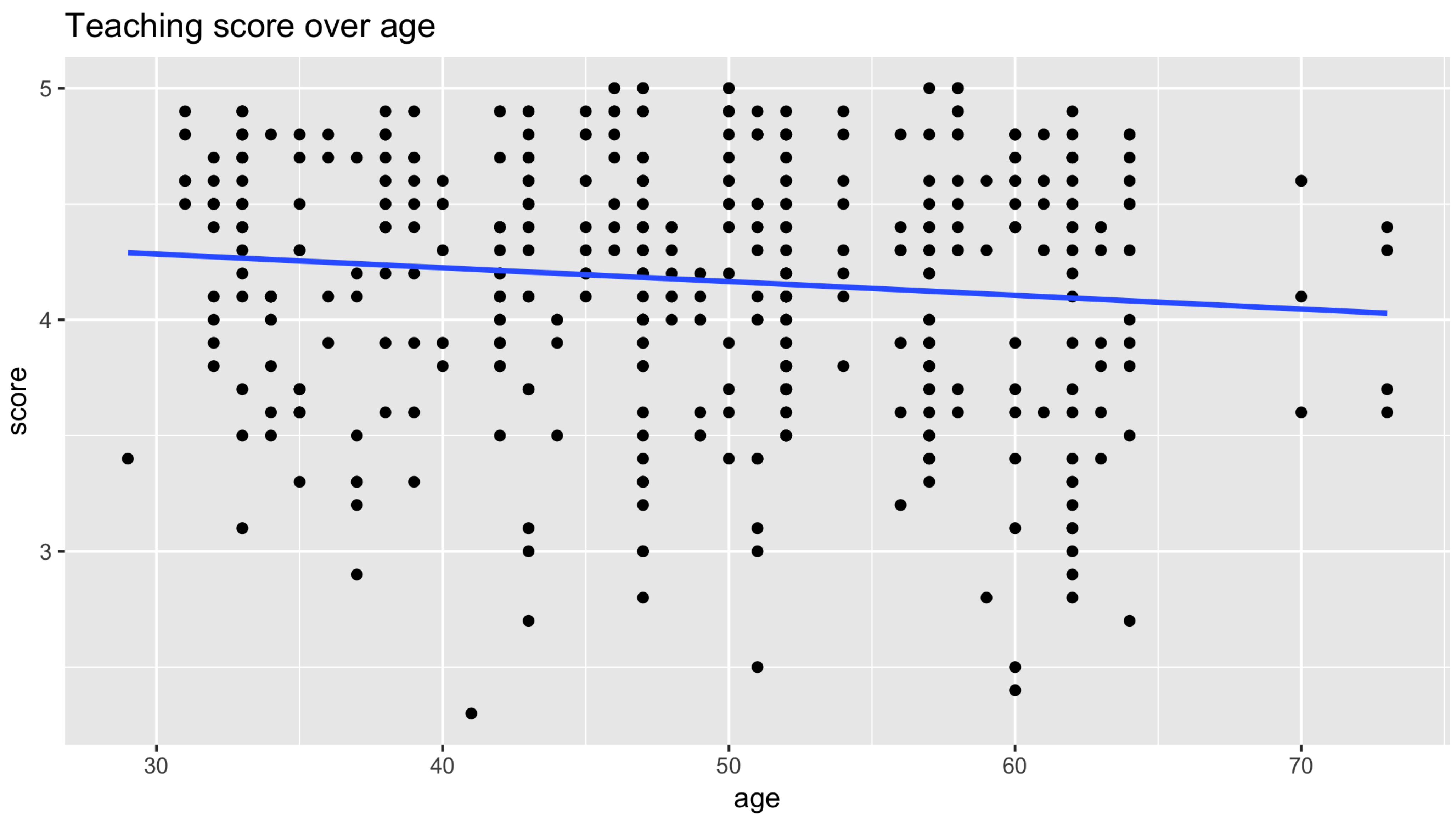
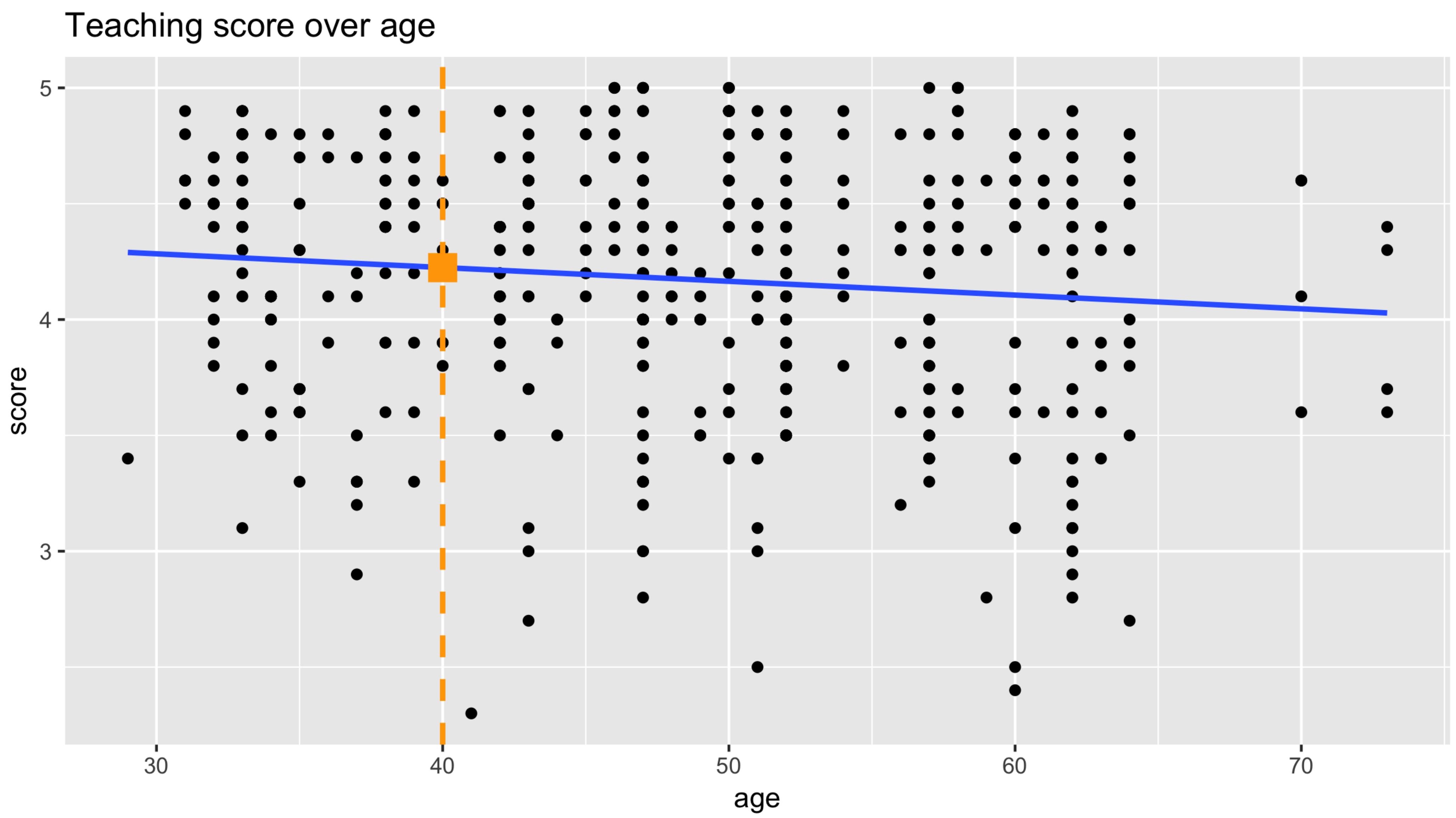
Refresher: Regression line



New instructor prediction



Refresher: Regression table

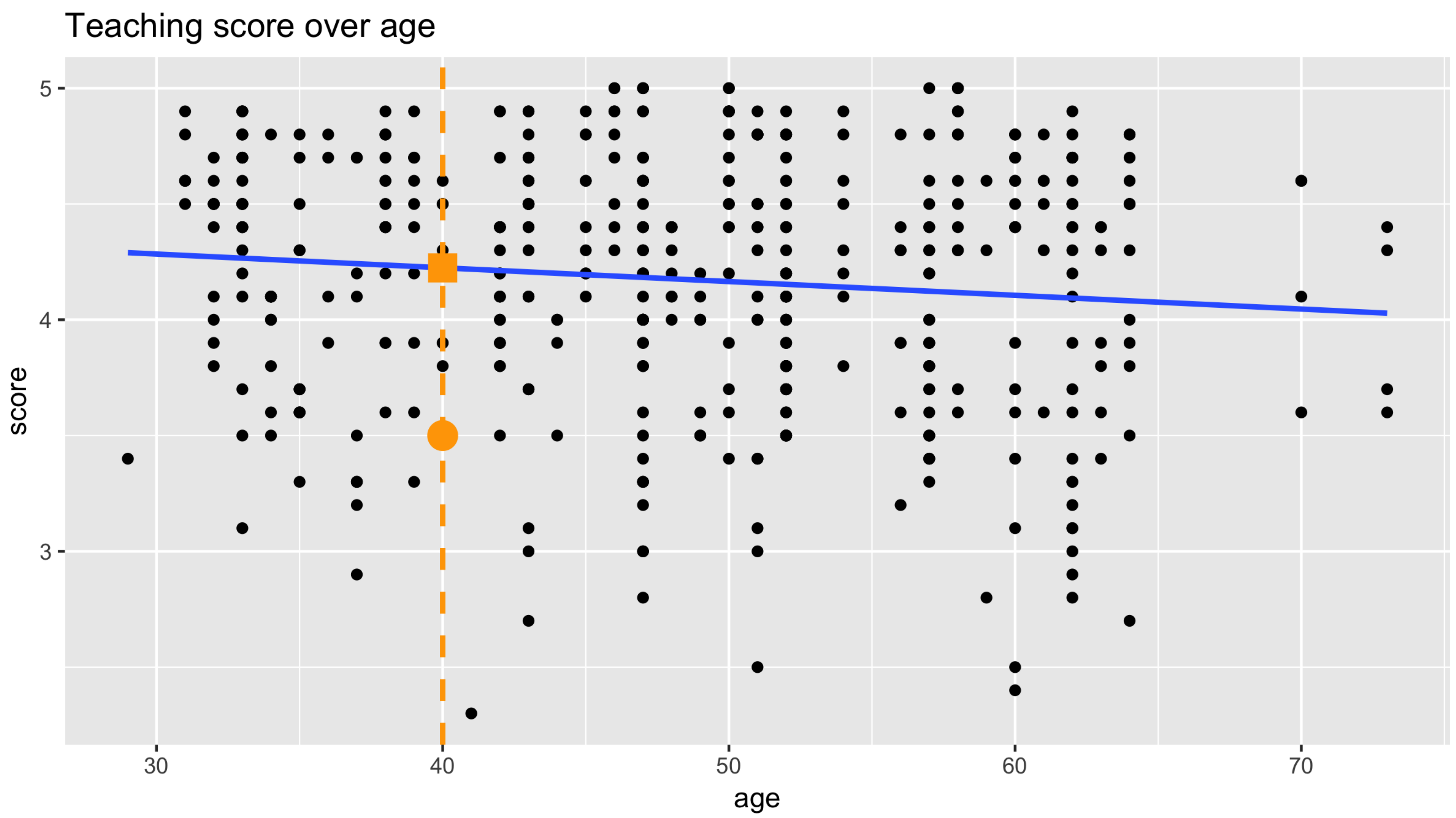
```r
library(ggplot2)
library(dplyr)
library(moderndive)

# Fit regression model using formula of form: y ~ x
model_score_1 <- lm(score ~ age, data = evals)

# Output regression table using wrapper function
get_regression_table(model_score_1)
```

```
# A tibble: 2 x 7
  term      estimate std_error statistic p_value lower_ci...
  <chr>        <dbl>     <dbl>     <dbl>   <dbl>    <dbl>...
1 intercept    4.46      0.127     35.2    0        4.21...
2 age         -0.006     0.003     -2.31   0.021   -0.011...
```

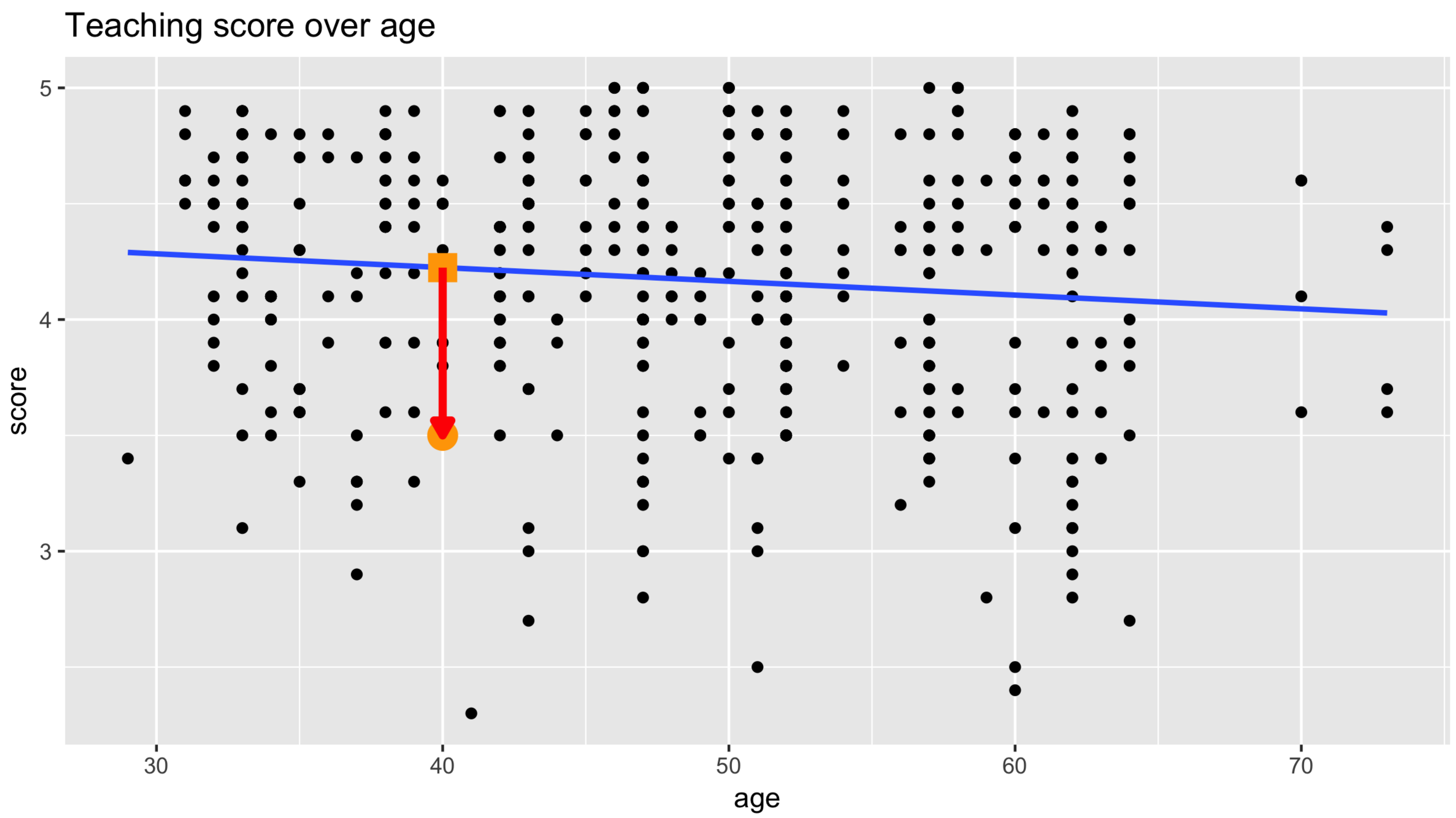
Predicted value

- Predictive regression models in general:
  $\hat{y} = \hat{f}(x) = \hat{\beta}_0 + \hat{\beta}_1 \cdot x$
- Our predictive model: $\widehat{\text{score}} = 4.46 - 0.006 \cdot \text{age}$
- Our prediction: $4.46 - 0.006 \cdot 40 = 4.22$

Prediction error



Prediction error



Residuals as model errors

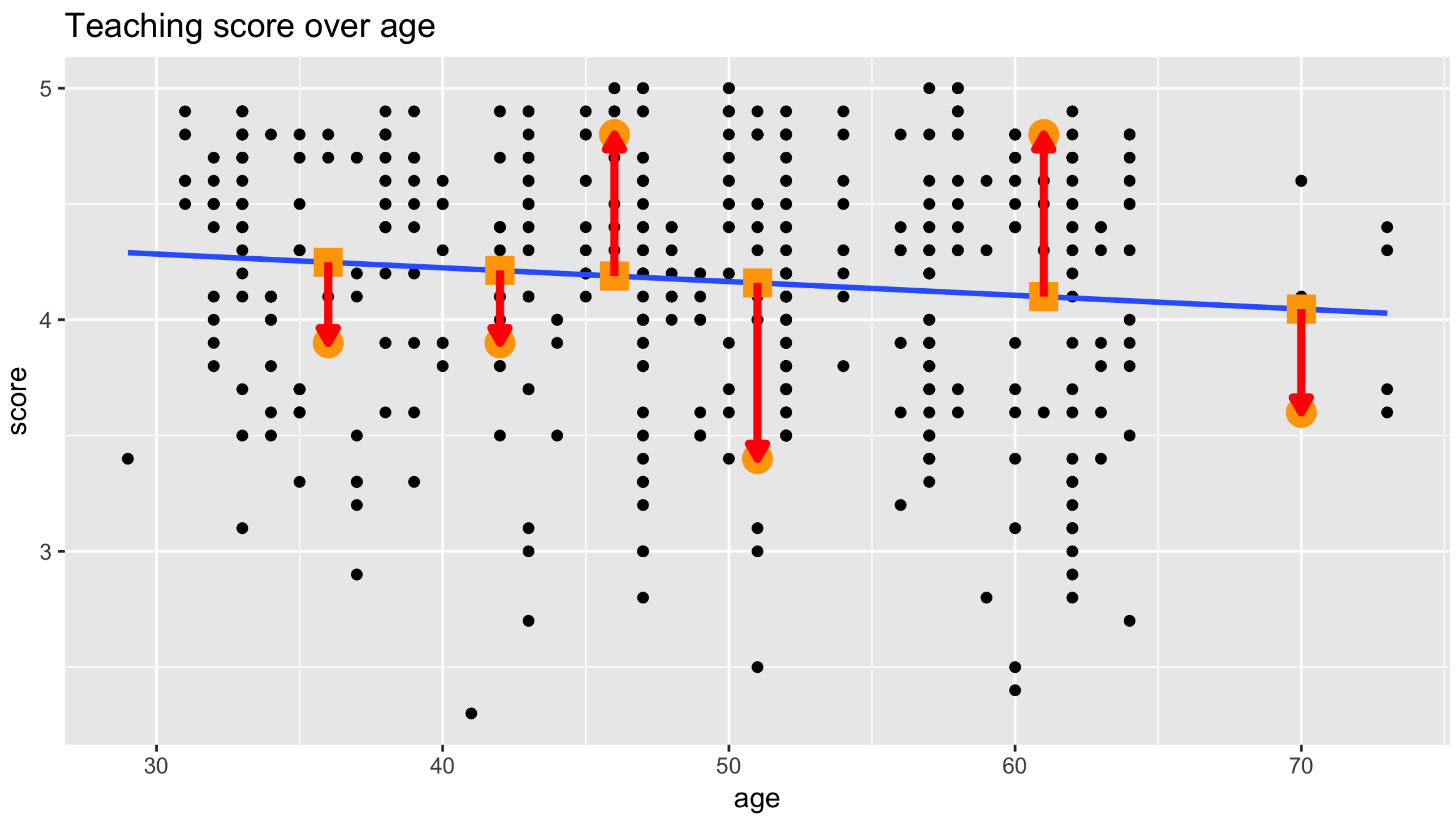
- Residual = $y - \hat{y}$
- Corresponds to ϵ from $y = f(\vec{x}) + \epsilon$
- For our example instructor: $y - \hat{y} = 3.5 - 4.22 = -0.72$
- In linear regression, they are on average 0.

Computing all predicted values

```r
# Fit regression model using formula of form: y ~ x
model_score_1 <- lm(score ~ age, data = evals)
# Get information on each point
get_regression_points(model_score_1)
```

```
# A tibble: 463 x 5
      ID score   age score_hat residual
   <int> <dbl> <dbl>     <dbl>    <dbl>
 1     1   4.7    36      4.25    0.452
 2     2   4.1    36      4.25   -0.148
 3     3   3.9    36      4.25   -0.348
 4     4   4.8    36      4.25    0.552
 5     5   4.6    59      4.11    0.488
```

"Best fitting" regression line



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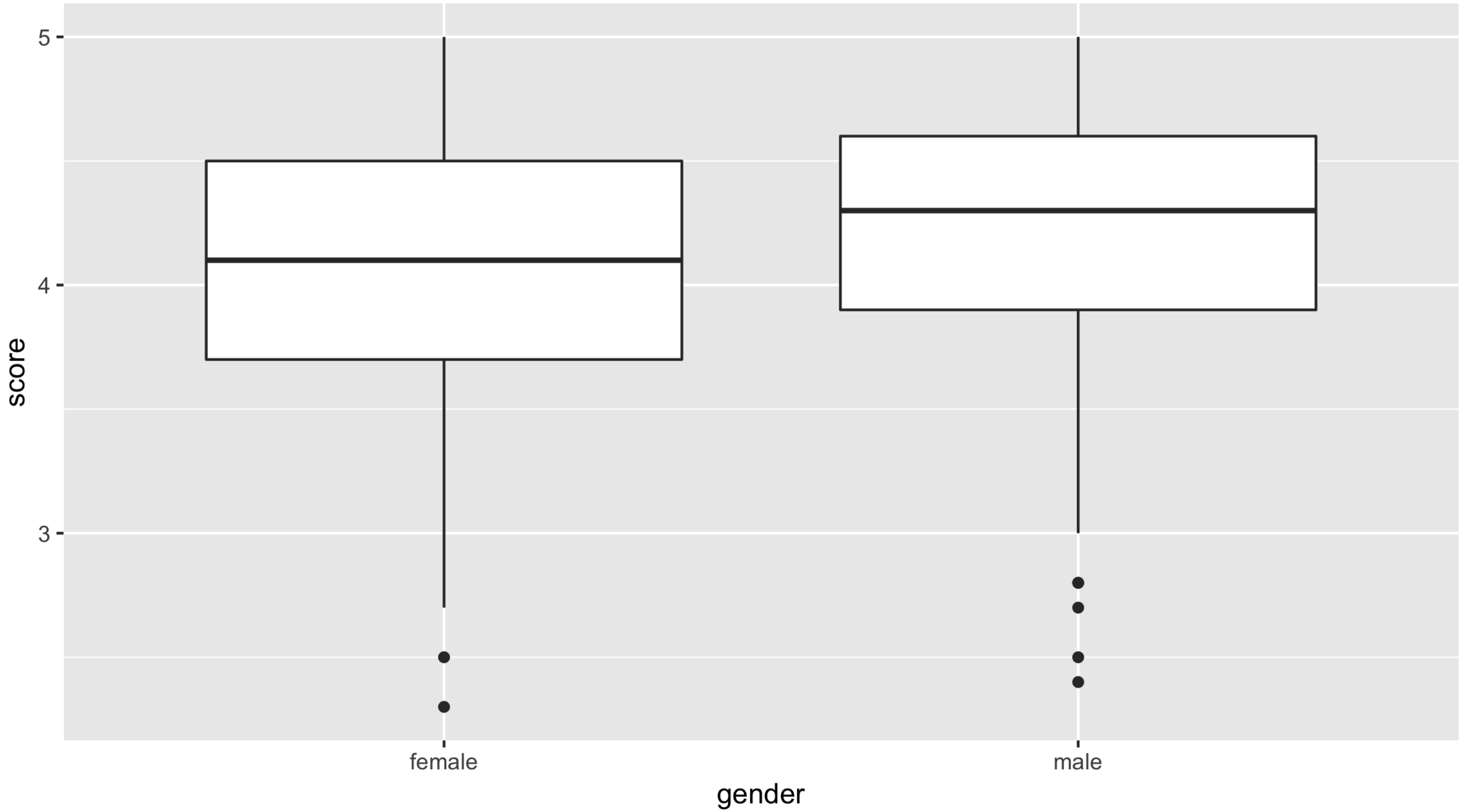
Exploratory data visualization

```
library(ggplot2)
library(dplyr)
library(moderndive)

ggplot(evals, aes(x = gender, y = score)) +
  geom_boxplot() +
  labs(x = "gender", y = "score")
```

Boxplot of score over gender

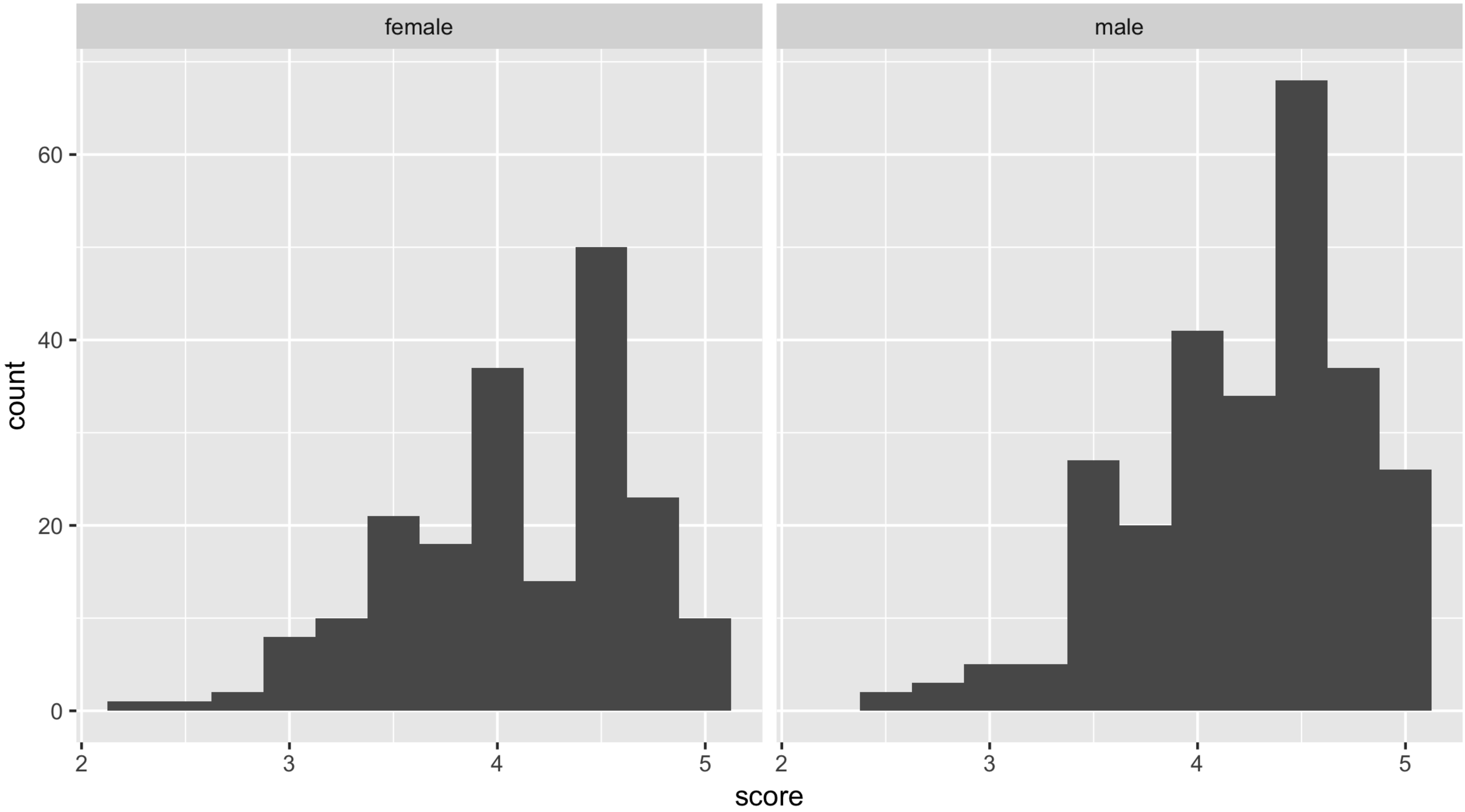


Facetted histogram

```r
library(ggplot2)
library(dplyr)
library(moderndive)

ggplot(evals, aes(x = score)) +
  geom_histogram(binwidth = 0.25) +
  facet_wrap(~gender) +
  labs(x = "gender", y = "score")
```

Facetted histogram



Fitting a regression model

```
# Fit regression model
model_score_3 <- lm(score ~ gender, data = evals)

# Get regression table
get_regression_table(model_score_3)
```

```
# A tibble: 2 x 7
  term       estimate std_error statistic p_value...
  <chr>         <dbl>     <dbl>     <dbl>   <dbl>...
1 intercept     4.09      0.039    106.     0...
2 gendermale    0.142     0.051      2.78   0.006...
```

Fitting a regression model

```r
# Compute group means based on gender
evals %>%
  group_by(gender) %>%
  summarize(avg_score = mean(score))
```

```
# A tibble: 2 x 2
  gender avg_score
  <fct>      <dbl>
1 female      4.09
2 male        4.23
```

A different categorical explanatory variable: rank

```
evals %>%
  group_by(rank) %>%
  summarize(n = n())
```

```
# A tibble: 3 x 2
  rank             n
  <fct>        <int>
1 teaching       102
2 tenure track   108
3 tenured        253
```

Let's practice!

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Group means as predictions

```r
library(ggplot2)
library(dplyr)
library(moderndive)

evals %>%
  group_by(gender) %>%
  summarize(mean_score = mean(score), sd_score = sd(score))
```

```
# A tibble: 2 x 3
  gender mean_score sd_score
  <fct>       <dbl>    <dbl>
1 female       4.09    0.564
2 male         4.23    0.522
```

Computing all predicted values and residuals

```r
# Fit regression model:
model_score_3 <- lm(score ~ gender, data = evals)

# Get information on each point
get_regression_points(model_score_3)
```

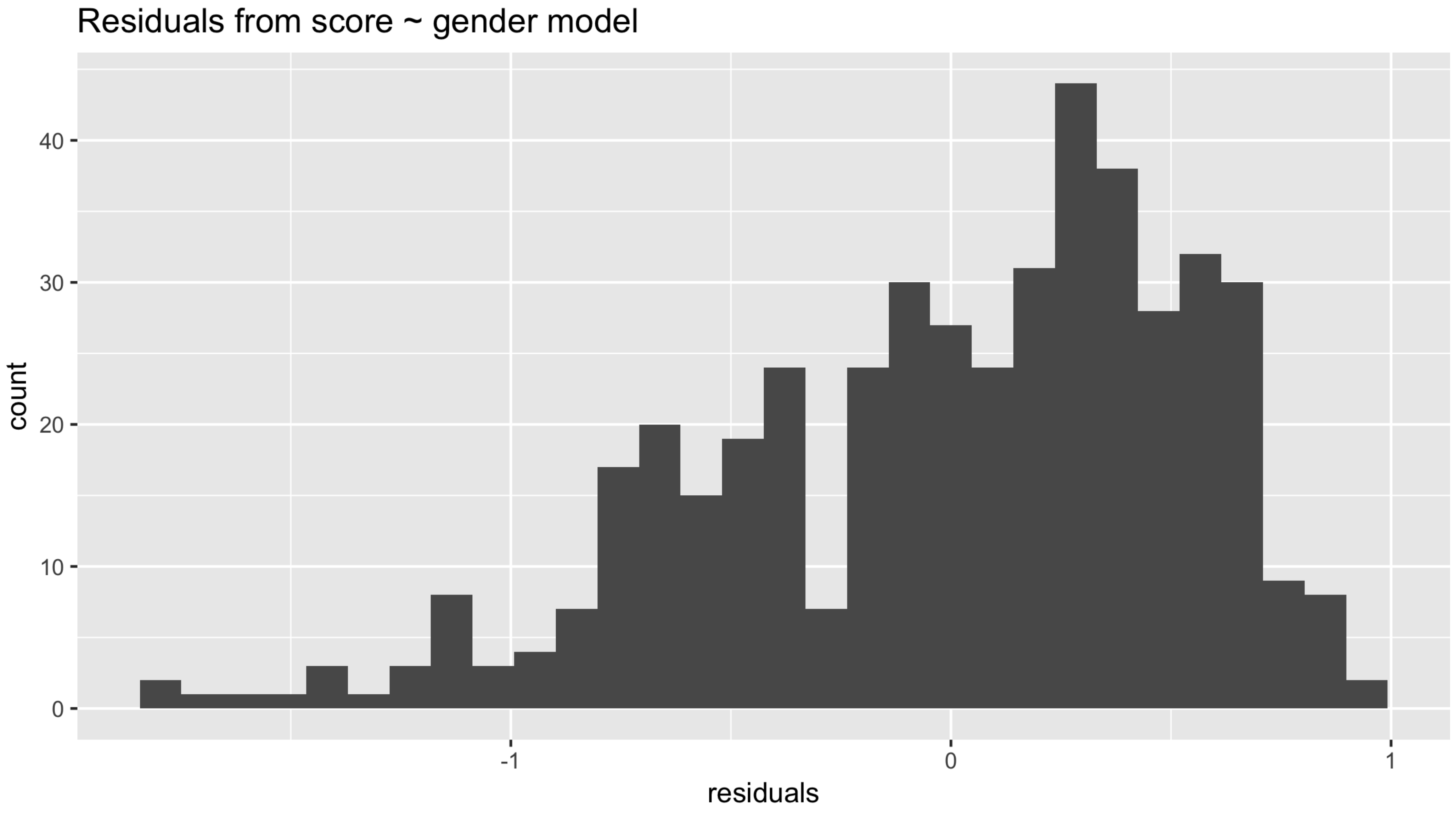
```
# A tibble: 463 x 5
      ID score gender score_hat residual
   <int> <dbl> <fct>      <dbl>    <dbl>
 1     1   4.7 female      4.09    0.607
 2     2   4.1 female      4.09    0.007
 3     3   3.9 female      4.09   -0.193
 4     4   4.8 female      4.09    0.707
 5     5   4.6 male        4.23    0.366
 6     6   4.3 male        4.23    0.066
```

Histogram of residuals

```
# Fit regression model
model_score_3 <- lm(score ~ gender, data = evals)

# Get regression points
model_score_3_points <- get_regression_points(model_score_3)
model_score_3_points
# Plot residuals
ggplot(model_score_3_points, aes(x = residual)) +
  geom_histogram() +
  labs(x = "residuals",
       title = "Residuals from score ~ gender model")
```

Histogram of residuals



Let's practice!

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