Analyzing Algorithms

COP 3503
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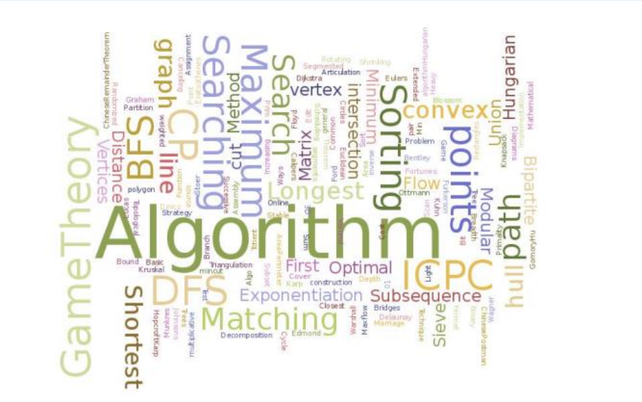

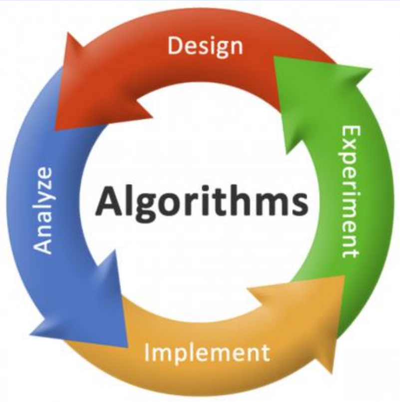

Remember The Sorting Problem?
Lets look at one of the solutions.
Insertion Sort

What is the running time of Insertion-Sort in the Best Case scenario?

- Array input is sorted already. $t_j = 1$

$$T(n) = c_1 n * c_2(n-1) + 0 * (n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} t_j + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7 \sum_{j=2}^{n} (t_j - 1) + c_8(n-1)$$

$$T(n) = c_1 n * c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} t_j + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7 \sum_{j=2}^{n} (t_j - 1) + c_8(n-1)$$

$$T(n) = (c_1 + c_2 + c_4 + c_8) * n - (c_2 + c_4 + c_8) + c_5 \sum_{j=2}^{n} t_j + (c_6 + c_7) \sum_{j=2}^{n} (t_j - 1)$$

$$\sum_{j=2}^{n} t_j = \sum_{j=2}^{n} 1 = \underbrace{1 + 1 + 1 + \cdots + 1}_{\text{(n-1) times}} = n - 1$$

$$\sum_{j=2}^{n} (t_j - 1) = \sum_{j=2}^{n} 0 = 0$$

$$T(n) = (c_1 + c_2 + c_4 + c_8)n - (c_2 + c_4 + c_8) + c_5(n-1)$$
$$T(n) = (c_1 + c_2 + c_4 + c_5 + c_8)n - (c_2 + c_4 + c_5 + c_8)$$
$$T(n) = an + b$$ Linear Function

What is the running time of Insertion-Sort in the Worst Case scenario?

- Array input is reverse order. $t_j = j$

$$\sum_{j=2}^{n} t_j = \sum_{j=2}^{n} j = 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2} - 1 = \frac{n^2 + n - 2}{2}$$

Woah we have an Arithmetic Series: $\boxed{1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}}$

$$\sum_{j=2}^{n} t_j - 1 = \sum_{j=2}^{n} j - 1 = 1 + 2 + 3 + \cdots + (n-1) = \frac{(n-1)n}{2} = \frac{n^2 - n}{2}$$

$$T(n) = (c_1 + c_2 + c_4 + c_8) * n - (c_2 + c_4 + c_8) + c_5 \frac{n^2 + n - 2}{2} + (c_6 + c_7) \frac{n^2 - n}{2}$$

$$T(n) = \left(\frac{c_5}{2} + \frac{c_6}{2} + \frac{c_7}{2}\right) n^2 + \left(c_1 + c_2 + c_4 + c_8 + \frac{c_5}{2} - \frac{c_6}{2} - \frac{c_7}{2}\right) n - (c_2 + c_4 + c_8 + c_5)$$

$T(n) = an^2 + bn + c$ Quadratic Function

$$T(n) = \Theta(n^2)$$

Important! An algorithm is considered more efficient than another algorithm if its worst-case running time has a smaller order of growth.

Order of Growth

- Determining how long an algorithm runs in terms of some input.
- Drop the lower-order terms
- Ignore the constant in the leading term

Let's look at some examples of Order of Growth