# RINQ 1A7F

May 19, 2025

```
[1]: !pip install biopython
```

Defaulting to user installation because normal site-packages is not writeable Requirement already satisfied: biopython in ./.local/lib/python3.11/site-packages (1.85)
Requirement already satisfied: numpy in ./.local/lib/python3.11/site-packages (from biopython) (1.26.4)
DEPRECATION: qiskit-nature 0.5.0 has a non-standard dependency specifier qiskit-terra>=0.22.\*. pip 24.0 will enforce this behaviour change. A possible replacement is to upgrade to a newer version of qiskit-nature or contact the author to suggest that they release a version with a conforming dependency specifiers. Discussion can be found at https://github.com/pypa/pip/issues/12063

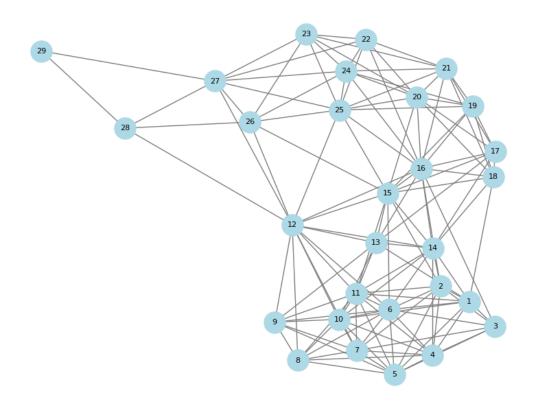
```
[2]: import numpy as np
     import networkx as nx
     import matplotlib.pyplot as plt
     from Bio import PDB
     from Bio.PDB import PDBList
     # Function to download PDB file from Protein Data Bank
     def fetch_pdb(pdb_id):
         """Fetches a PDB file from the Protein Data Bank given a PDB ID."""
         pdbl = PDBList()
         pdb_file = pdbl.retrieve_pdb_file(pdb_id, file_format='pdb')
         return pdb_file
     # Function to calculate the distance between two residues
     def calculate_distance(residue_i, residue_j):
         """Calculates the Euclidean distance between the C-alpha atoms of two_{\sqcup}
      ⇔residues."""
         try:
             atom_i = residue_i["CA"].coord
             atom_j = residue_j["CA"].coord
```

```
return np.linalg.norm(atom_i - atom_j)
    except KeyError:
        return float('inf')
# Function to construct a Protein-Residue Interaction Network
def construct_protein_residue_network(pdb_id, interaction_cutoff=8.0):
    """Constructs a residue interaction network from an online PDB file."""
   pdb_path = fetch_pdb(pdb_id)
   parser = PDB.PDBParser()
    structure = parser.get_structure("protein", pdb_path)
   G = nx.Graph()
   for chain in structure.get_chains():
        for residue_i in chain:
            for residue_j in chain:
                if residue_i != residue_j:
                    distance = calculate_distance(residue_i, residue_j)
                    if distance < interaction_cutoff:</pre>
                        G.add_edge(residue_i.get_id()[1], residue_j.get_id()[1])
   return G
# Example usage
pdb id = "1A7F" # Example PDB ID
interaction_cutoff = 8.0 # Distance cutoff in Angstroms
G = construct_protein_residue_network(pdb_id, interaction_cutoff)
# Extract residue indices from the graph
residues = list(G.nodes()) # Extract nodes (residue IDs) from the network
# Function to create adjacency matrix
def create_adjacency_matrix(G, residues):
    """Creates an adjacency matrix from the interaction network."""
   residues = list(set(residues)) # Ensure uniqueness
   adjacency_matrix = nx.to_numpy_array(G, nodelist=residues)
   return adjacency_matrix
# Create adjacency matrix using the corrected residue list
adj_matrix = create_adjacency_matrix(G, residues)
print("Adjacency Matrix:")
print(adj_matrix)
# Save adjacency matrix to file
np.savetxt("adjacency_matrix.txt", adj_matrix, fmt="%d")
```

```
# Draw the network
plt.figure(figsize=(8, 6))
nx.draw(G, with_labels=True, node_color='lightblue', edge_color='gray', __
 ⇒node_size=500, font_size=8)
plt.title("Protein-Residue Interaction Network")
plt.show()
Structure exists: '/home/smohtash/a7/pdb1a7f.ent'
Adjacency Matrix:
[[0.\ 1.\ 1.\ 1.\ 1.\ 1.\ 0.\ 0.\ 0.\ 1.\ 1.\ 0.\ 1.\ 1.\ 0.\ 0.\ 0.\ 1.\ 0.\ 0.\ 0.\ 0.\ 0.\ 0.
 0. 0. 0. 0. 0.]
[1. 0. 1. 1. 1. 1. 1. 0. 0. 0. 1. 0. 0. 1. 1. 1. 0. 0. 0. 0. 0. 0. 0. 0.
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[1. 1. 0. 1. 1. 1. 1. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0.
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[1. 1. 1. 1. 1. 0. 1. 1. 1. 1. 1. 1. 0. 1. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.]
[0. 1. 1. 1. 1. 1. 0. 1. 1. 1. 1. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
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[1. 0. 0. 1. 1. 1. 1. 1. 1. 0. 1. 1. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
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[1. 1. 0. 1. 1. 1. 1. 1. 1. 0. 1. 1. 1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
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[1. 0. 0. 0. 0. 0. 0. 0. 1. 1. 1. 1. 0. 1. 1. 1. 1. 0. 0. 0. 0. 0. 0. 0.
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[1. 1. 0. 1. 0. 1. 0. 0. 0. 1. 1. 1. 1. 0. 1. 1. 1. 1. 0. 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.]
1. 1. 0. 0. 0.]
1. 0. 0. 0. 0.]
0. 0. 0. 0. 0.]
[1. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 1. 1. 1. 0. 1. 1. 1. 0. 0. 0.
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```

1. 0. 0. 0. 0.] 1. 0. 0. 0. 0.] 1. 0. 1. 0. 0.] 1. 1. 1. 0. 0.] 1. 1. 1. 0. 0.] [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 1. 1. 0. 0. 1. 1. 1. 1. 1. 1. 0. 1. 1. 0. 0.] [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 1. 1. 1. 0. 1. 1. 0.] 1. 1. 0. 1. 1.] 0. 1. 1. 0. 1.] 0. 0. 1. 1. 0.]]

#### Protein-Residue Interaction Network

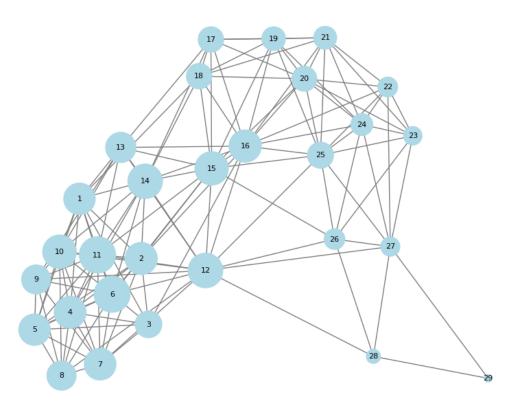


```
[3]: def compute_eigenvector_centrality(G):
         Computes the eigenvector centrality of the protein-residue interaction \Box
      \neg network.
         Parameters:
         G (networks.Graph): The protein-residue interaction network.
         dict: A dictionary of nodes with their eigenvector centrality scores.
         try:
             centrality = nx.eigenvector_centrality(G, max_iter=1000, tol=1e-6)
             return centrality
         except nx.NetworkXError as e:
             print(f"Eigenvector centrality calculation failed: {e}")
             return None
     # Compute eigenvector centrality
     eigenvector_centrality = compute_eigenvector_centrality(G)
     # Print top residues by eigenvector centrality
     print("Top Residues by Eigenvector Centrality:")
     sorted_centrality = sorted(eigenvector_centrality.items(), key=lambda item:
      →item[1], reverse=True)
     for residue, score in sorted centrality[:10]: # Print top 10 residues
         print(f"Residue {residue}: {score:.4f}")
     # Visualization: Draw network with node sizes based on eigenvector centrality
     plt.figure(figsize=(8, 6))
     node_sizes = [5000 * eigenvector_centrality[node] for node in G.nodes()] #__
      ⇒Scale node sizes
     nx.draw(G, with_labels=True, node_color='lightblue', edge_color='gray',
             node_size=node_sizes, font_size=8)
     plt.title("Protein-Residue Interaction Network (Eigenvector Centrality)")
     plt.show()
    Top Residues by Eigenvector Centrality:
    Residue 11: 0.2880
    Residue 6: 0.2841
    Residue 12: 0.2624
    Residue 14: 0.2618
    Residue 10: 0.2464
    Residue 15: 0.2453
    Residue 16: 0.2273
    Residue 2: 0.2265
```

Residue 4: 0.2230

### Residue 7: 0.2202

## Protein-Residue Interaction Network (Eigenvector Centrality)



```
# import numpy as np
# import pennylane as qml
# from pennylane import numpy as pnp

# Define adjacency matrix A for a sample graph (example)
# A = adj_matrix
# n = A.shape[0] # Number of nodes
# d = np.sum(A, axis=1) # Degree sequence
# d_norm = d / np.linalg.norm(d) # Normalized degree vector
# d_matrix = np.outer(d_norm, d_norm) # dd T matrix

# Define penalty parameters
# PO = 1 / np.sqrt(n)
# P1 = tau * n

# def construct_qubo(A, PO, P1, tau):
# """Constructs QUBO matrix Q."""
# n = A.shape[0]
```

```
C = (1 - 2 * tau) * np.eye(n) + np.ones((n, n)) - np.eye(n)
      Q = -P0 * (A @ d_matrix @ A + A @ d_matrix @ A @ A) + P1 * C
      return Q
# # Define tau (number of most influential nodes to identify)
# tau = 1
\# Q = construct \ qubo(A, PO, P1, tau)
# # Convert QUBO to cost Hamiltonian
\# coeffs = []
\# ops = []
# for i in range(n):
    coeffs.append(Q[i, i])
      ops.append(qml.PauliZ(i))
# for i in range(n):
     for j in range(i + 1, n):
#
         coeffs.append(Q[i, j])
          ops.append(qml.PauliZ(i) @ qml.PauliZ(j))
# cost_hamiltonian = qml.Hamiltonian(coeffs, ops)
# # Define mixer Hamiltonian
# mixer hamiltonian = qml.Hamiltonian(
      [1.0] * n, [qml.PauliX(i) for i in range(n)]
# )
# # Define QAOA circuit with sampling enabled
# dev = qml.device("default.qubit", wires=n, shots=1024)
# @qml.qnode(dev)
# def gaoa_circuit(params):
      """QAOA circuit for optimizing the QUBO problem."""
      for i in range(n):
         qml.Hadamard(wires=i)
     for p in range(len(params) // 2):
          qml.templates.ApproxTimeEvolution(cost hamiltonian, params[2 * p], 1)
          qml.templates.ApproxTimeEvolution(mixer_hamiltonian, params[2 * p + columnian)]
 \hookrightarrow 1], 1)
    return qml.expval(cost_hamiltonian)
# # Define cost function for optimization
# def cost_function(params):
# return qaoa_circuit(params)
# # Optimize using classical method
# opt = qml.GradientDescentOptimizer(stepsize=0.1)
# params = pnp.random.uniform(-np.pi, np.pi, size=(6,), requires_grad=True) #_1
\hookrightarrow For p=3
```

```
# for _ in range(100):
#     params = opt.step(cost_function, params)

# # Print results
# print("Optimized parameters:", params)
# print("Final cost value:", cost_function(params))
```

```
[5]: # import numpy as np
     # # Run the optimized QAOA circuit
     # binary_solution = qaoa_circuit(params)
     # # Ensure binary_solution is an array
     # binary_solution = np.array(binary_solution)
     # # If it's a scalar (0-D array), reshape it into a 1-D array
     # if binary solution.ndim == 0:
           binary_solution = np.array([binary_solution])
     # # Debugging: Print the raw output
     # print("Raw output from QADA circuit:", binary_solution)
     # print("Binary solution shape:", binary_solution.shape)
     # # Convert -1,1 Pauli-Z output to binary (0,1)
     # binary_solution = [(1 \text{ if } x == -1 \text{ else } 0) \text{ for } x \text{ in binary_solution.flatten})]
     # # Extract influential nodes
     # selected nodes = [i for i, value in enumerate(binary_solution) if value == 1]
     # # Print the extracted nodes
     # print("Most influential nodes:", selected_nodes)
```

# [6]: pip install dwave-ocean-sdk

Defaulting to user installation because normal site-packages is not writeable Requirement already satisfied: dwave-ocean-sdk in ./.local/lib/python3.11/site-packages (8.2.0)
Requirement already satisfied: dimod==0.12.18 in ./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (0.12.18)
Requirement already satisfied: dwave-cloud-client==0.13.3 in ./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (0.13.3)
Requirement already satisfied: dwave-gate==0.3.3 in ./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (0.3.3)
Requirement already satisfied: dwave-hybrid==0.6.13 in ./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (0.6.13)
Requirement already satisfied: dwave-inspector==0.5.2 in ./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (0.5.2)

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Requirement already satisfied: dwave-networkx==0.8.16 in
./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (0.8.16)
Requirement already satisfied: dwave-optimization==0.5.1 in
./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (0.5.1)
Requirement already satisfied: dwave-preprocessing==0.6.7 in
./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (0.6.7)
Requirement already satisfied: dwave-samplers==1.4.0 in
./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (1.4.0)
Requirement already satisfied: dwave-system==1.29.0 in
./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (1.29.0)
Requirement already satisfied: dwavebinarycsp==0.3.1 in
./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (0.3.1)
Requirement already satisfied: minorminer==0.2.17 in
./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (0.2.17)
Requirement already satisfied: penaltymodel==1.2.0 in
./.local/lib/python3.11/site-packages (from dwave-ocean-sdk) (1.2.0)
Requirement already satisfied: numpy>=1.17.3 in ./.local/lib/python3.11/site-
packages (from dimod==0.12.18->dwave-ocean-sdk) (1.26.4)
Requirement already satisfied: requests<3,>=2.25 in /apps/spack/negishi/apps/ana
conda/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
requests[socks]<3,>=2.25->dwave-cloud-client==0.13.3->dwave-ocean-sdk) (2.31.0)
Requirement already satisfied: urllib3<3,>=1.26 in ./.local/lib/python3.11/site-
packages (from dwave-cloud-client==0.13.3->dwave-ocean-sdk) (2.4.0)
Requirement already satisfied: pydantic<3,>=2 in ./.local/lib/python3.11/site-
packages (from dwave-cloud-client==0.13.3->dwave-ocean-sdk) (2.9.2)
Requirement already satisfied: homebase<2,>=1.0 in ./.local/lib/python3.11/site-
packages (from dwave-cloud-client==0.13.3->dwave-ocean-sdk) (1.0.1)
Requirement already satisfied: click<9,>=7.0 in /apps/spack/negishi/apps/anacond
a/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from dwave-
cloud-client==0.13.3->dwave-ocean-sdk) (8.1.7)
Requirement already satisfied: python-dateutil<3,>=2.7 in
./.local/lib/python3.11/site-packages (from dwave-cloud-client==0.13.3->dwave-
ocean-sdk) (2.9.0.post0)
Requirement already satisfied: plucky<0.5,>=0.4.3 in
./.local/lib/python3.11/site-packages (from dwave-cloud-client==0.13.3->dwave-
ocean-sdk) (0.4.3)
Requirement already satisfied: diskcache<6,>=5.2.1 in
./.local/lib/python3.11/site-packages (from dwave-cloud-client==0.13.3->dwave-
ocean-sdk) (5.6.3)
Requirement already satisfied: packaging>=19 in /apps/spack/negishi/apps/anacond
a/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from dwave-
cloud-client==0.13.3->dwave-ocean-sdk) (23.1)
Requirement already satisfied: werkzeug<4,>=2.2 in /apps/spack/negishi/apps/anac
onda/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from dwave-
cloud-client==0.13.3->dwave-ocean-sdk) (2.2.3)
Requirement already satisfied: typing-extensions<5,>=4.5.0 in
./.local/lib/python3.11/site-packages (from dwave-cloud-client==0.13.3->dwave-
ocean-sdk) (4.13.2)
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Requirement already satisfied: authlib<2,>=1.2 in ./.local/lib/python3.11/site-
packages (from dwave-cloud-client==0.13.3->dwave-ocean-sdk) (1.5.1)
Requirement already satisfied: importlib_metadata>=5.0.0 in /apps/spack/negishi/
apps/anaconda/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
dwave-cloud-client==0.13.3->dwave-ocean-sdk) (7.0.1)
Requirement already satisfied: orjson>=3.10 in ./.local/lib/python3.11/site-
packages (from dwave-cloud-client==0.13.3->dwave-ocean-sdk) (3.10.15)
Requirement already satisfied: networkx in ./.local/lib/python3.11/site-packages
(from dwave-hybrid==0.6.13->dwave-ocean-sdk) (3.4.2)
Requirement already satisfied: Flask<4,>=2.2 in /apps/spack/negishi/apps/anacond
a/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from dwave-
inspector==0.5.2->dwave-ocean-sdk) (2.2.5)
Requirement already satisfied: scipy>=1.7.3 in ./.local/lib/python3.11/site-
packages (from dwave-system==1.29.0->dwave-ocean-sdk) (1.15.3)
Requirement already satisfied: fasteners>=0.15 in ./.local/lib/python3.11/site-
packages (from minorminer==0.2.17->dwave-ocean-sdk) (0.19)
Requirement already satisfied: cryptography in /apps/spack/negishi/apps/anaconda
/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
authlib<2,>=1.2->dwave-cloud-client==0.13.3->dwave-ocean-sdk) (42.0.2)
Requirement already satisfied: Jinja2>=3.0 in /apps/spack/negishi/apps/anaconda/
2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
Flask<4,>=2.2->dwave-inspector==0.5.2->dwave-ocean-sdk) (3.1.3)
Requirement already satisfied: itsdangerous>=2.0 in /apps/spack/negishi/apps/ana
conda/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
Flask<4,>=2.2->dwave-inspector==0.5.2->dwave-ocean-sdk) (2.0.1)
Requirement already satisfied: zipp>=0.5 in /apps/spack/negishi/apps/anaconda/20
24.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
importlib_metadata>=5.0.0->dwave-cloud-client==0.13.3->dwave-ocean-sdk) (3.17.0)
Requirement already satisfied: annotated-types>=0.6.0 in
./.local/lib/python3.11/site-packages (from pydantic<3,>=2->dwave-cloud-
client==0.13.3->dwave-ocean-sdk) (0.7.0)
Requirement already satisfied: pydantic-core==2.23.4 in
./.local/lib/python3.11/site-packages (from pydantic<3,>=2->dwave-cloud-
client==0.13.3->dwave-ocean-sdk) (2.23.4)
Requirement already satisfied: six>=1.5 in ./.local/lib/python3.11/site-packages
(from python-dateutil<3,>=2.7->dwave-cloud-client==0.13.3->dwave-ocean-sdk)
(1.17.0)
Requirement already satisfied: charset-normalizer<4,>=2 in /apps/spack/negishi/a
pps/anaconda/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
requests<3,>=2.25->requests[socks]<3,>=2.25->dwave-cloud-client==0.13.3->dwave-
ocean-sdk) (2.0.4)
Requirement already satisfied: idna<4,>=2.5 in /apps/spack/negishi/apps/anaconda
/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
requests<3,>=2.25->requests[socks]<3,>=2.25->dwave-cloud-client==0.13.3->dwave-
ocean-sdk) (3.4)
Requirement already satisfied: certifi>=2017.4.17 in /apps/spack/negishi/apps/an
aconda/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
requests<3,>=2.25->requests[socks]<3,>=2.25->dwave-cloud-client==0.13.3->dwave-
```

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ocean-sdk) (2024.2.2)
Requirement already satisfied: PySocks!=1.5.7,>=1.5.6 in /apps/spack/negishi/app
s/anaconda/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
requests[socks]<3,>=2.25->dwave-cloud-client==0.13.3->dwave-ocean-sdk) (1.7.1)
Requirement already satisfied: MarkupSafe>=2.1.1 in /apps/spack/negishi/apps/ana
conda/2024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
werkzeug<4,>=2.2->dwave-cloud-client==0.13.3->dwave-ocean-sdk) (2.1.3)
Requirement already satisfied: cffi>=1.12 in /apps/spack/negishi/apps/anaconda/2
024.02-py311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
cryptography->authlib<2,>=1.2->dwave-cloud-client==0.13.3->dwave-ocean-sdk)
(1.16.0)
Requirement already satisfied: pycparser in /apps/spack/negishi/apps/anaconda/20
24.02-pv311-gcc-8.5.0-lr57z2f/lib/python3.11/site-packages (from
cffi>=1.12->cryptography->authlib<2,>=1.2->dwave-cloud-client==0.13.3->dwave-
ocean-sdk) (2.21)
DEPRECATION: qiskit-nature 0.5.0 has a non-standard dependency specifier
qiskit-terra>=0.22.*. pip 24.0 will enforce this behaviour change. A possible
replacement is to upgrade to a newer version of qiskit-nature or contact the
author to suggest that they release a version with a conforming dependency
specifiers. Discussion can be found at
https://github.com/pypa/pip/issues/12063
```

Note: you may need to restart the kernel to use updated packages.

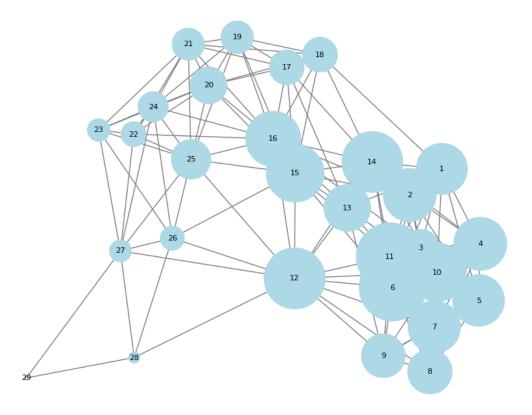
```
[7]: import networkx as nx
    import numpy as np
    from dimod import BinaryQuadraticModel, SimulatedAnnealingSampler
     # Define constants
    A = adj_matrix
    n = len(A)
    tau = 5 # Desired number of top nodes
    PO = 1 / np.sqrt(n) # Based on the paper
    P1 = 50 * n
                         # Stronger penalty to enforce the constraint
    def calculate_d_hat(adj_matrix):
        d = np.sum(adj_matrix, axis=1)
        d_hat = d / np.linalg.norm(d)
        return d_hat
    d_hat = calculate_d_hat(A)
     # Construct C matrix
    I = np.eye(n)
```

```
U = np.ones((n, n)) - I
     C = (1 - 2 * tau) * I + U
     # Construct Q matrix
     Q = -P0 * (A @ np.outer(d_hat, d_hat) @ A) \setminus
         - PO * (A @ np.outer(d_hat, d_hat) @ A @ A) \
         + P1 * C
     # Convert to BQM
     bqm = BinaryQuadraticModel.from_numpy_matrix(Q)
     # Simulated annealing
     sampler = SimulatedAnnealingSampler()
     response = sampler.sample(bqm, num_reads=10000, beta_range=(0.1, 4.0))
     # Filter solutions with exactly tau selected nodes
     valid_solutions = []
     for sample, energy in response.data(['sample', 'energy']):
         if sum(sample.values()) == tau:
             valid_solutions.append((sample, energy))
     if valid_solutions:
         best_sample, best_energy = min(valid_solutions, key=lambda x: x[1])
         top nodes = [node + 1 for node, val in best sample.items() if val == 1]
         print("Best valid sample:", best sample)
         print("Energy:", best_energy)
         print("Top nodes:", top_nodes)
     else:
         print("No valid solution with exactly selected nodes.")
    /tmp/ipykernel_2958978/3260906070.py:31: DeprecationWarning:
    BQM.from_numpy_matrix(M) is deprecated since dimod 0.10.0 and will be removed in
    0.12.0. Use BQM(M, "BINARY") instead.
      bqm = BinaryQuadraticModel.from_numpy_matrix(Q)
    Best valid sample: {0: 0, 1: 0, 2: 0, 3: 0, 4: 0, 5: 0, 6: 0, 7: 0, 8: 0, 9: 0,
    10: 1, 11: 0, 12: 0, 13: 1, 14: 1, 15: 1, 16: 1, 17: 0, 18: 0, 19: 0, 20: 0, 21:
    0, 22: 0, 23: 0, 24: 0, 25: 0, 26: 0, 27: 0, 28: 0}
    Energy: -36553.90811916324
    Top nodes: [11, 14, 15, 16, 17]
[8]: import numpy as np
     import matplotlib.pyplot as plt
     import networkx as nx
     from scipy.linalg import expm
     def compute_estrada_centrality_manual(G):
```

```
Computes Estrada centrality by evaluating the diagonal of the matrix_{\sqcup}
  \rightarrow exponential of A.
    Parameters:
    G (networkx.Graph): The protein-residue interaction network.
    Returns:
    dict: A dictionary of nodes with their Estrada centrality scores.
    A = nx.to_numpy_array(G) # Adjacency matrix
    E = expm(A)
                               # Matrix exponential e^A
    estrada_scores = np.diag(E) # Extract diagonal elements
    return {node: estrada_scores[i] for i, node in enumerate(G.nodes())}
# --- Compute Estrada Centrality ---
estrada_centrality = compute_estrada_centrality_manual(G)
# --- Print Top Residues by Estrada Centrality ---
print("Top Residues by Estrada Centrality:")
sorted_estrada = sorted(estrada_centrality.items(), key=lambda item: item[1],__
 ⇔reverse=True)
for residue, score in sorted_estrada[:10]: # Top 10
    print(f"Residue {residue}: {score:.4f}")
# --- Visualization: Network Colored by Estrada Centrality ---
plt.figure(figsize=(8, 6))
node_sizes = [5000 * estrada_centrality[node] / max(estrada_centrality.
 ⇒values()) for node in G.nodes()]
nx.draw(G, with_labels=True, node_color='lightblue', edge_color='gray',
        node_size=node_sizes, font_size=8)
plt.title("Protein-Residue Interaction Network (Estrada Centrality)")
plt.show()
Top Residues by Estrada Centrality:
Residue 11: 2599.6729
Residue 6: 2538.6080
Residue 12: 2113.2986
```

Residue 11: 2599.6729
Residue 6: 2538.6080
Residue 12: 2113.2986
Residue 14: 2101.2938
Residue 10: 1924.6021
Residue 15: 1891.9855
Residue 16: 1723.2204
Residue 4: 1594.3536
Residue 2: 1592.7368
Residue 7: 1555.0057

# Protein-Residue Interaction Network (Estrada Centrality)



```
[9]: import networkx as nx
import numpy as np
from dimod import BinaryQuadraticModel, SimulatedAnnealingSampler

# --- Input: Adjacency matrix ---
A = adj_matrix # Use your preloaded or computed adjacency matrix here
n = len(A)
tau = 1
P0 = 1 / np.sqrt(n)
P1 = 120 * n

# --- Step 1: Degree normalization ---
def calculate_d_hat(adj_matrix):
    d = np.sum(adj_matrix, axis=1)
    d_hat = d / np.linalg.norm(d)
    return d_hat

d_hat = calculate_d_hat(A)
```

```
# --- Step 2: Construct matrix exponential approximation E = I + A + 0.5 A^2 + 10.5 A^
   → (1/6) A ^3 ---
 A2 = A @ A
 A3 = A2 @ A
 E = np.eye(n) + A + 0.5 * A2 + (1/6) * A3
 # --- Step 3: Build low-rank Estrada projection: E * d hat * d hat ^T * E ---
 d_outer = np.outer(d_hat, d_hat)
 projection = E @ d_outer @ E # Rank-1 projection of spectral walk weights
 # --- Step 4: Constraint matrix ---
 I = np.eye(n)
 U = np.ones((n, n)) - I
 C = (1 - 2 * tau) * I + U
 # --- Step 5: Final QUBO matrix ---
 Q = -P0 * projection + P1 * C
 # --- Step 6: Convert QUBO to BQM ---
 bqm = BinaryQuadraticModel.from_numpy_matrix(Q)
 # --- Step 7: Simulated annealing to solve QUBO ---
 sampler = SimulatedAnnealingSampler()
 response = sampler.sample(bqm, num_reads=10000, beta_range=(0.1, 4.0))
 # --- Step 8: Filter valid -sparse solutions ---
 valid_solutions = []
 for sample, energy in response.data(['sample', 'energy']):
          if sum(sample.values()) == tau:
                  valid_solutions.append((sample, energy))
 if valid solutions:
         best_sample, best_energy = min(valid_solutions, key=lambda x: x[1])
         top nodes = [node + 1 for node, val in best sample.items() if val == 1]
         print("Best valid sample:", best_sample)
         print("Energy:", best_energy)
         print("Top nodes (Estrada centrality):", top_nodes)
         print("No valid solution with exactly selected nodes.")
/tmp/ipykernel_2958978/3045418004.py:38: DeprecationWarning:
BQM.from_numpy_matrix(M) is deprecated since dimod 0.10.0 and will be removed in
0.12.0. Use BQM(M, "BINARY") instead.
    bqm = BinaryQuadraticModel.from_numpy_matrix(Q)
Best valid sample: {0: 0, 1: 0, 2: 0, 3: 0, 4: 0, 5: 0, 6: 0, 7: 0, 8: 0, 9: 0,
10: 0, 11: 0, 12: 0, 13: 0, 14: 0, 15: 0, 16: 0, 17: 0, 18: 0, 19: 0, 20: 0, 21:
0, 22: 0, 23: 0, 24: 1, 25: 0, 26: 0, 27: 0, 28: 0}
```

Energy: -3829.6263915935997

Top nodes (Estrada centrality): [25]

[]: