Quantum Winter Hackathon Question

Presented By: BQP, QCI, ISCFD

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Introduction

As the decade is coming to an end, there were many milestones broken over the past ten year - including the landing of the Curiosity rover, the SpaceX reusable rockets, and the discovery of gravitational waves. In order to achieve these milestones, scientists performed countless number of simulations to ensure the success of these projects. Performing simulations to understand and predict the world around us is deeply rooted in our technological history. Computer simulations have had a critical role in many industries, from landing humans on the moon to helping us minimize the spread of diseases.
All of these simulations are dictated by some governing equations - most notably, the partial differential equations (PDE). PDEs describe the characteristics or the behavior of the substance, which can be used to make predictions. One of such equations is the 1-D wave equation:

$$\frac{\partial u}{\partial t} + C\frac{\partial u}{\partial x} = 0$$

The wave equation can be used to simulate how information propagates across a medium with speed C to the right, if C > 0. The wave equation can be used to describe a variety of phenomena – from the spread of diseases and formation of new stars to describing how an aircraft flies.

Goals

1. Using the Finite Difference approach, discretize the 1-D wave equation with 2^{nd} order spatial accuracy and 1^{st} order time accuracy with the implicit time integration scheme.

2. Discretize the domain with 41 points between $x \in [0, 2]$, C = 1, and CFL = $\frac{C\Delta t}{\Delta x}$ = 2.0. Solve the 1-D wave equation up to 10 timesteps with initial condition:

$$u_0 = \begin{cases} 1 & x \leq 0.5 \\ 2 & x > 0.5 \end{cases}$$

Let the boundary condition be: u(x = 0,t) = 1.
3. Provide a plot of the L_1 norm of the numerical solution and the analytical solution,
$u = u_0(x - Ct)$

4. Show speedup between the classical algorithm and the Quantum Computing algorithm.