MATHEMATICAL PHYSICS

LAB-Report A8

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Theory

(a). We have m^{th} order differential eqn

$$\frac{d^m y}{dx^m} = f(x, y, y', y'', \cdots, y^{m-1}).$$

with initial conditions.

$$y(x_0) = \alpha_1$$
$$y'(x_0) = \alpha_2$$
$$y''(x_0) = \alpha_3$$
$$\vdots$$
$$y^{m-1}(x_0) = \alpha_{m-1}$$

To solve m^{th} order differential eqn., first we have to convert this to system of first order differential eqn.

$$y_1 = y.$$
$$y_2 = y' = y_1'$$
$$y_3 = y'' = y_2'$$
$$\vdots$$
$$y_{m-1} = y^{m-2} = y_{m-2}'$$
$$y_m = y^{m-1} = y_{m-1}' = f$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m' \end{bmatrix} = \begin{bmatrix} y_2 \\ y_3 \\ \vdots \\ f \end{bmatrix}$$

$$= \begin{bmatrix} x_0 & x_1 & x_2 & \cdots & x_n \\ y_1(x_0) & y_1(x_1) & \cdots & \cdots & y_1(x_f) \\ y_2(x_0) & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ y_m(x_0) & y_m(x_1) & y_m(x_2) & & y_m(x_f) \end{bmatrix}$$

$$= \begin{bmatrix} x_0 & x_1 & x_2 & \cdots & x_f \\ y(x_0) & y(x_1) & y(x_2) & \cdots & y(x_f) \\ y'(x_0) & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ y^{m-1}(x_0) & y^{m-1}(x_1) & y^{m-1}(x_2) & & y^{m-1}(x_f) \end{bmatrix}$$

(b) We have

$$y'' - 2y' + 2y = e^{2x} \sin x \quad \cdots (1)$$

first convert this above second order differential eqn into two first order eqn.

let $\frac{dy}{dx} = u \quad \cdots (2)$.

Using (2), eqn (1) becomes.

$$\frac{du}{dx} - 2u + 2y = e^{2x} \sin x$$

$$\frac{du}{dx} = e^{2x} \sin x + 2u - 2y \quad -- (3).$$

$f_1(x, y, u) \Rightarrow \frac{dy}{dx} = u$.

$f_2(x, y, u) \Rightarrow \frac{du}{dx} = e^{2x} \sin x + 2u - 2y \quad \cdots$

initial conditions,

$y(0) = -0.4 \qquad y'(0) = u(0) = -0.6$

for $0 \le x \le 1$

step size,

$$h = \frac{b-a}{N}$$

$$= \frac{1-0}{5} = 0.2.$$

RK 2 Method

$$y_{i+1} = y_i + \left(\frac{k_1 + k_2}{2}\right).$$

$$k_1 = h^* f(x_i, y_i). \qquad k_2 = h^* f(x_i + h, y_i + k_1).$$

Step 1

$$k_1 = h^* f_1(x_0, y_0, u_0)$$

$$= 0.2 \times f_1(0, -0.4, -0.6).$$

$$= 0.2 \times (-0.6) = -0.12$$

$$k_1 = h^* f_2(x_0, y_0, u_0).$$

$$= 0.2 \times f_2(0, -0.4, -0.6)$$

$$= 0.2 \times [0.8 - 1.2].$$

$$= -0.08$$

$$k_2 = h^* f_1(x_0 + h, y_0 + k_1, u_0 + k_1).$$

$$= 0.2 \times -0.68$$

$$= -0.136.$$

$$k_2 = h^* f_2(x_0 + h, y_0 + k_1, u_0 + k_1)$$

$$= 0.2 \times f_2(0.2, -0.2, -0.4).$$

$$= 0.2 \times (-0.0236\,2019)$$

$$= -0.00472404.$$

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so, using $y_{i+1} = y_i + \left(\frac{k_1+k_2}{2}\right)$

for f_1, $i=0$.

$$y_1 = y_0 + \left(\frac{k_1+k_2}{2}\right)$$

$$y_1 = -0.4 + \left(\frac{-0.12 + -0.136}{2}\right)$$

$$= -0.528.$$

for f_2, $i = 0$.

$$y_1 = y_0 + \left(\frac{k_1+k_2}{2}\right)$$

$$= -0.4 + \left(\frac{-0.08 - 0.00472404}{2}\right)$$

$$= -0.64236202.$$

~~Step 2~~ We started with initial value of

$y_0(0) = -0.4$ and $u_0(0) = -0.6$

Now we ~~find~~ have find the next term as seen above clearly,

$y_1 = -0.528$ $u_1 = -0.64236202$

to find the next term, we increase $x=0$ to $x = 0+h \Rightarrow x = 0.2$.

Step 2

$$k_1 = h^* f_1 (x_0+h,\ y_1,\ u_1)$$

$$= h^* (-0.64236202)$$
$$= -0.12847724$$

$$k_1 = h^* f_2 (x_0+h,\ y_1,\ u_1)$$

$$= h^* \cancel{(-0.62883086)}\ (0.06765578)$$
$$= 0.01353116.$$

$$k_2 = h^* f_1 (x_0+2h,\ y_1+k_1,\ \cancel{y}\ u_1+k_1).$$

$$= h^* (-0.62883086)$$
$$= -0.125576617$$

$$k_2 = h^* f_2 (x_0+2h,\ y_1+k_1,\ u_1+k_1)$$

$$= h^*\ 0.92194954$$
$$= 0.18438991.$$

So, next terms, $y_{i+1} = y_i + \frac{(k_1+k_2)}{2}$

$$y_2 = y_1 + \frac{(k_1+k_2)}{2}, \quad u_2 = u_1 + \frac{(k_1+k_2)}{2}$$

$$y_2 = -0.65511929 \qquad u_2 = -0.54340149.$$

Step3

$$k_1 = h^* f_1(x_0+2h, y_2, u_2)$$

$$= h^*(-0.54340149) = -0.1086803$$

$$k_1 = h^* f_2(x_0+2h, y_2, u_2)$$

$$= h^*(1.09010206) = 0.21802041$$

$$k_2 = h^* f_1(x_0+3h, y_2+k_1, u_2+k_1)$$

$$= h^*(-0.32538107)$$
$$= -0.06507621$$

$$k_2 = h^* f_2(x_0+3h, y_2+k_1, u_2+k_1)$$

$$= h^*(2.75151605)$$
$$= 0.55030321$$

So,

next terms are.

$$y_3 = y_2 + \frac{(k_1+k_2)}{2} \qquad u_3 = u_2 + \frac{(k_1+k_2)}{2}$$

$$y_3 = -0.74199754 \qquad u_3 = -0.15923968$$

Step4,

$$k_1 = h^* f_1(x_0+3h, y_3 + u_3).$$

$$= 0.2 \times -0.15923968 = -0.03184794$$

$$k_1 = h^* f_2(x_0+3h, y_3, u_3)$$
$$= 0.2 \times 3.04019477 = 0.60803895$$

$$k_2 = h^* f_1(x_0+4h, y_3+k_1, u_3+k_1)$$

$$= h^*(0.44879928) = 0.08975986$$

$$k_2 = h^* f_2(x_0+4h, y_3+k_1, u_3+k_1)$$

$$= h^*(5.99837749) = 1.1996755.$$

So, Next term,

$$y_4 = y_3 + \frac{(k_1+k_2)}{2} \qquad u_4 = u_3 + \frac{(k_1+k_2)}{2}$$

$$y_4 = -0.71304158 \qquad u_4 = 0.74461755.$$

Step 5

$$k_1 = h^* f_1(x_0+4h, y_4, u_4)$$

$$= h^*(0.7446755) = 0.14892351$$

$$k_1 = h^* f_2(x_0+4h, y_4, u_4)$$

$$= h^*(6.46840625) = 1.29368125.$$

$$k_2 = h^* f_1(x_0+5h, y_4+k_1, u_4+k_1)$$

$$= 0.40765976$$

$$k_2 = h^* f_2(x_0+5h, y_4+k_1, u_4+k_1)$$

$$= 2.28450201.$$

So, next term is.

$$y_5 = y_4 + \frac{(k_1+k_2)}{2} \qquad u_5 = u_4 + \frac{(k_1+k_2)}{2}$$

$$y_5 = -0.43474995 \qquad u_5 = 2.53370918.$$