PROBLEM 1

We are given a dynamic process guided by the fuzzy logic control system with the following two fuzzy control rules:

Rule 1 If x is A_1 and y is B_1 Then z is C_1

Rule 2 If x is A_2 and y is B_2 Then z is C_2

Where x_0 and y_0 are the sensor readings for the linguistic input variables x and y and z is the consequent linguistic variable. The fuzzy predicates for the linguistic variables are given by A_1, A_2, B_1, B_2, C_1 and C_2, which membership functions are as:

$$\mu_{A_1}(x) = \begin{cases} \dfrac{x-2}{3} & 2 \le x \le 5 \\ \dfrac{8-x}{3} & 5 < x \le 8 \end{cases} \qquad \mu_{A_2}(x) = \begin{cases} \dfrac{x-3}{3} & 3 \le x \le 6 \\ \dfrac{9-x}{3} & 6 < x \le 9 \end{cases}$$

$$\mu_{B_1}(y) = \begin{cases} \dfrac{y-5}{3} & 5 \le y \le 8 \\ \dfrac{11-y}{3} & 8 < y \le 11 \end{cases} \qquad \mu_{B_2}(y) = \begin{cases} \dfrac{y-4}{3} & 4 \le y \le 7 \\ \dfrac{10-y}{3} & 7 < y \le 10 \end{cases}$$

$$\mu_{C_1}(z) = \begin{cases} \dfrac{z-1}{3} & 1 \le z \le 4 \\ \dfrac{7-z}{3} & 4 < z \le 7 \end{cases} \qquad \mu_{C_2}(z) = \begin{cases} \dfrac{z-3}{3} & 3 \le z \le 6 \\ \dfrac{9-z}{3} & 6 < z \le 9 \end{cases}$$

Further assume that at time t_1 we are reading the sensor values at $x_0(t_1) = 4$ and $y_0(t_1) = 8$. Using the Mamdani inferencing system and the Mean of Maximum (MOM) defuzzifcation1 strategy, find the final control output at time t_1. What is the value of the control output when we use the largest of maximum (lom) defuzzification strategy?

Solution-1

The provided rules states that

Rule 1: If x is A_1 and y is B_1,
Then z is C_1.

Rule 2: If x is A_2 and y is B_2,
Then z is C_2.

here, x and y are the antecedents/input

And, z is the consequent/output.

Plotting the values of x Antecedent

$$\mu_{A_1}(x) = \begin{cases} \frac{x-2}{3} & 2 \le x \le 5 \\ \frac{8-x}{3} & 5 < x \le 8 \end{cases}$$

$$\mu_{A_2}(x) = \begin{cases} \frac{x-3}{3} & 3 \le x \le 6 \\ \frac{9-x}{3} & 6 < x \le 9 \end{cases}$$

Thus, here for x antecedent, there are two descriptors A_1 and A_2. Both of these have two equations of line each. Hence, to calculate x-axis values, we need to substitute y-axis values as $[0,1]$ which is also called, Membership function. Range.

For A_1 ① line Equation;

$$\mu_{A_1}(x) = \frac{x-2}{3}$$

At, $\mu_{A_1}(x) = 0$, we get,

$$\Rightarrow 0 = \frac{x-2}{3} \Rightarrow x = 2.$$

At $\mu_{A_1}(x) = 1$, we get,

$$\Rightarrow \frac{x-2}{3} = 1 \Rightarrow x = 5$$

For A_1 (2) line Equation,

At $\mu_{A_1}(x) = 0$, we get,

$$\Rightarrow \frac{8-x}{3} = 0 \Rightarrow x = 8$$

At $\mu_{A_1}(x) = 1$, we get,

$$\Rightarrow \frac{8-x}{3} = 1 \Rightarrow x = 5$$

Similarly;

For A_2 (1) line Equation,

At $\mu_{A_2}(x) = 0$, we get,

$$\Rightarrow \frac{x-3}{3} = 0 \Rightarrow x = 3$$

At $\mu_{A_2}(x) = 1$, we get,

$$\Rightarrow \frac{x-3}{3} = 1 \Rightarrow x = 6$$

For A_2 ② line Equation,

At $\mu_{A_2}(x) = 0$, we get,

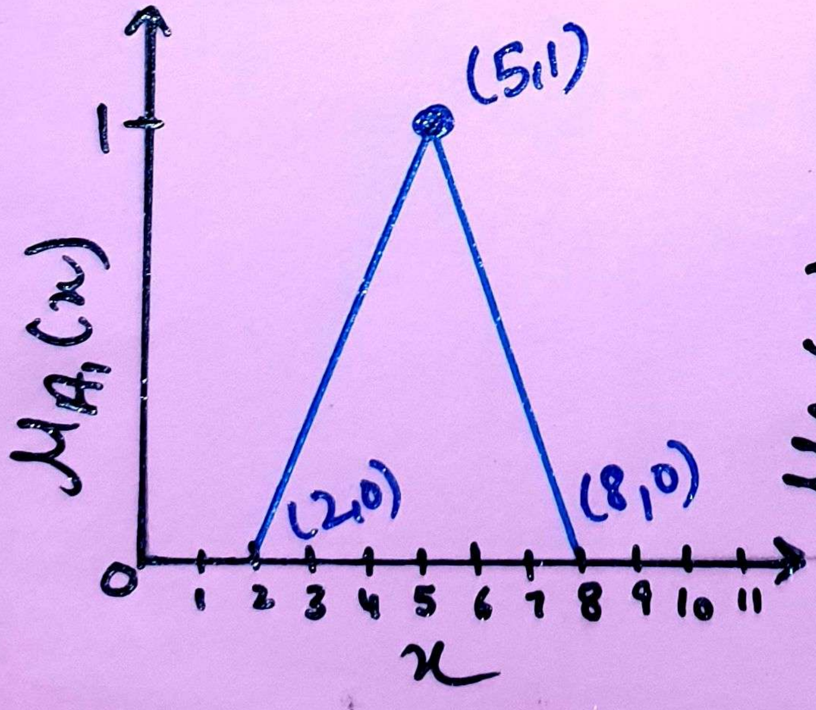
$\Rightarrow \dfrac{9-x}{3} = 0 \Rightarrow x = 9$

At $\mu_{A_2}(x) = 1$, we get,

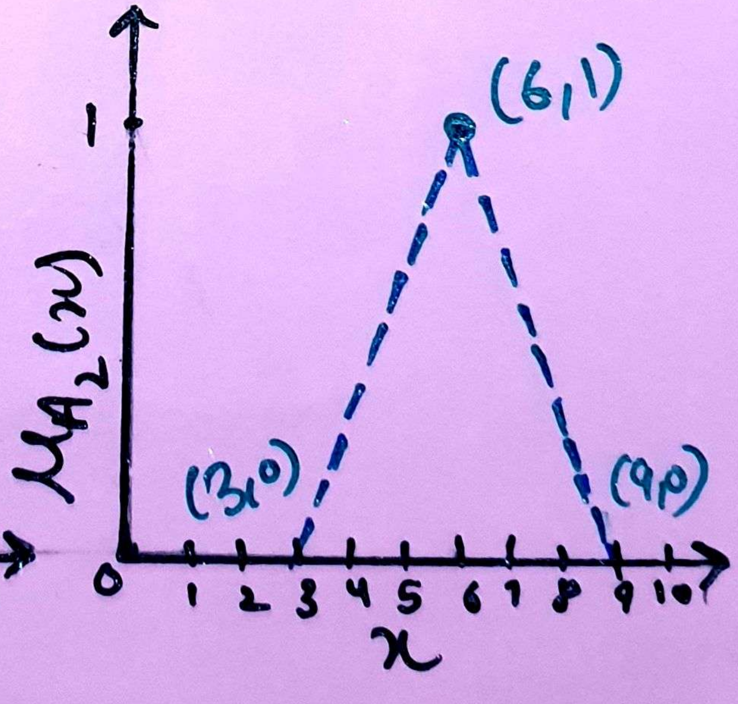
$\Rightarrow \dfrac{9-x}{3} = 1 \Rightarrow x = 6$

Plotting x Antecedent with A_1 and A_2 :→

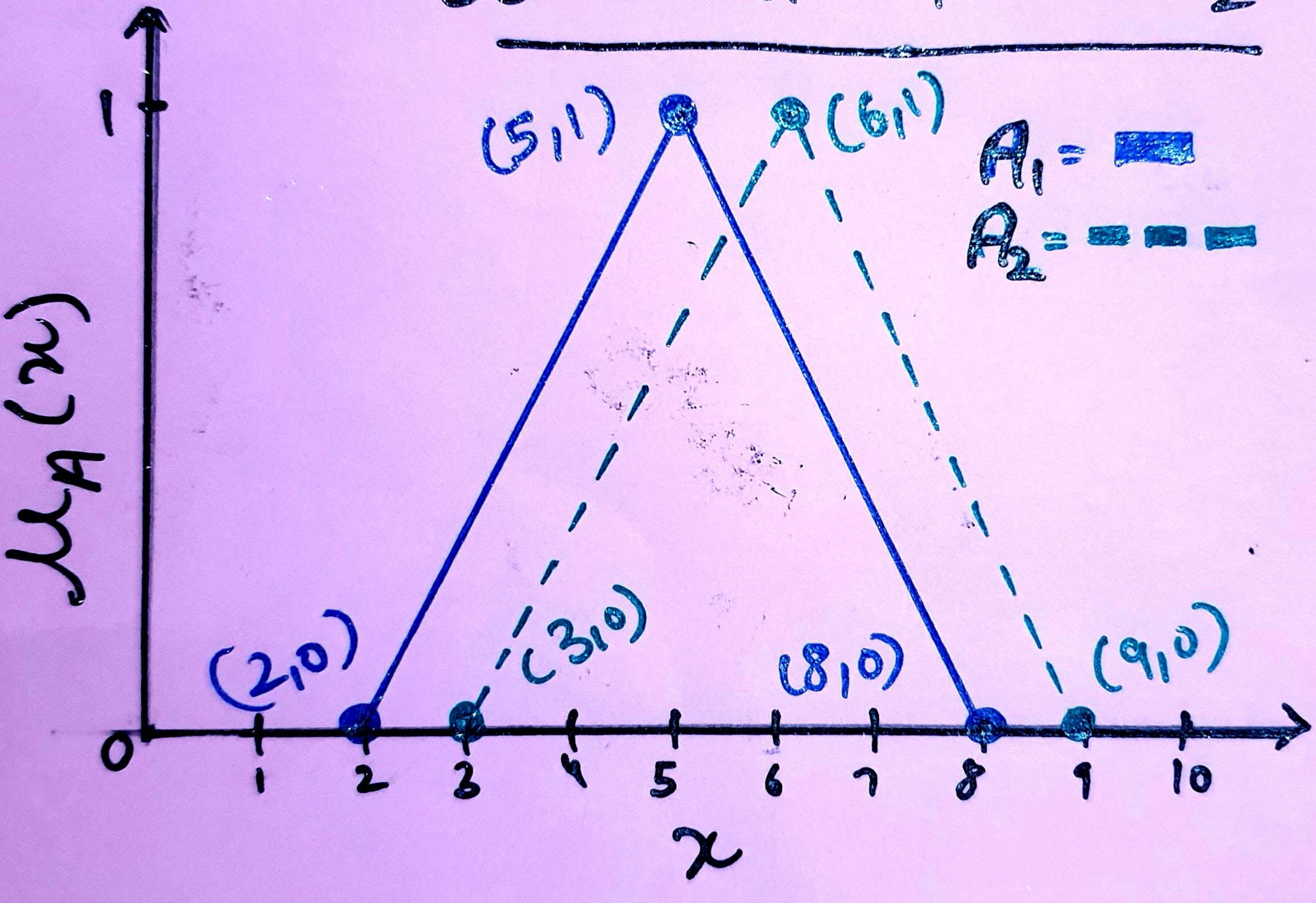
For A_1



For A_2



Combined A_1 and A_2



Similarly, we can find x-axis points of all equations, inputs and outputs by substituting y-axis as 0 and 1 on line equation.

Also, in a simple way, they can be plotted and x-axis values for remaining input and output can be calculated through their range values. So, for plotting Y and Z input and output, respectively, we will use their maximum and minimum range values as x-axis maximum and minimum values for that particular descriptor in both antecedent and consequent.

Plotting the values of y- antecedent.

$$\mu_{B_1}(y) = \begin{cases} \frac{y-5}{3} & 5 \le y \le 8 \\ \frac{11-y}{3} & 8 < y \le 11 \end{cases}$$

$$\mu_{B_2}(y) = \begin{cases} \frac{y-4}{3} & 4 \le y \le 7 \\ \frac{10-y}{3} & 7 < y \le 10 \end{cases}$$

Thus, here for y antecedent, there are two descripters B_1 and B_2. Both of these have two equations of line each. Hence, to calculate x-axis values on. Graph of y antecedent, we will use B_1 range as points on. x-axis as [5, 8] and [8, 11]. Similarly,

B_2 range as points on x-axis as $[4,7]$ and $[7,10]$. Even if we calculate it by $[0,1]$ substitution of membership function, the answer would be same.

Therefore,

for first line of B_1, points are $(5,0)$ and $(8,1)$ on x-axis and y-axis.

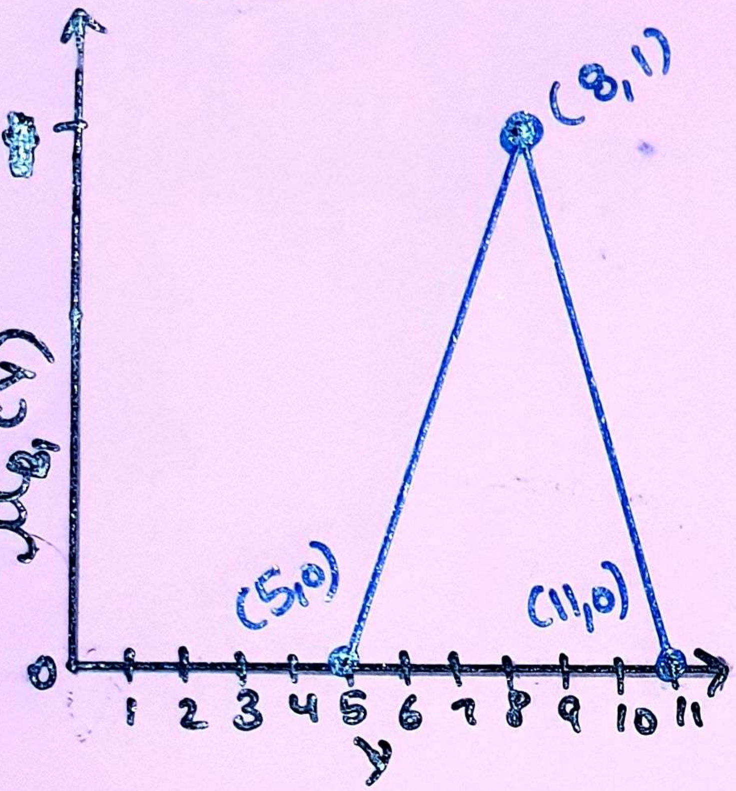
for second line of B_1, points are $(8,1)$ and $(11,0)$.

for first line of B_2, points are $(4,0)$ and $(7,1)$.

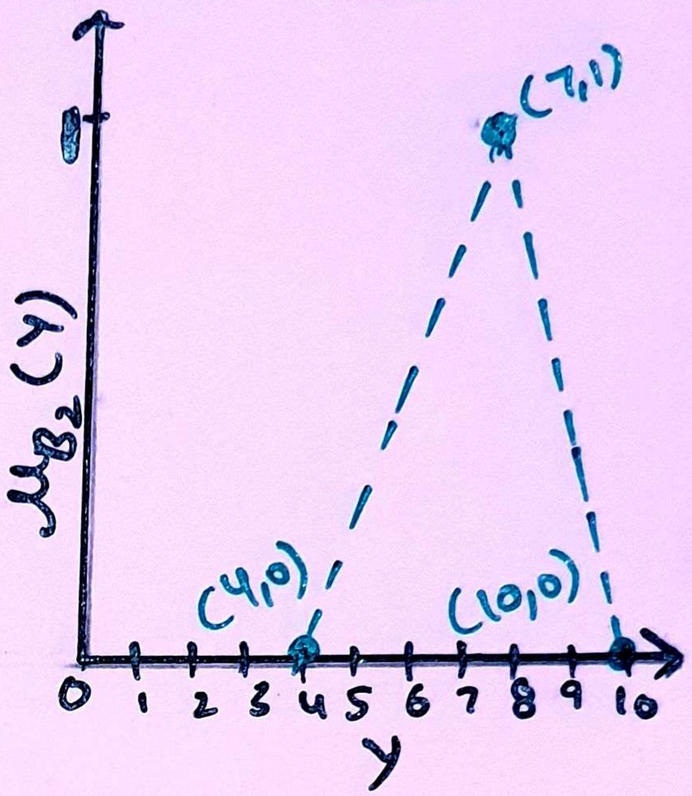
for second line of B_2, points are $(7,1)$ and $(10,0)$.

(where, (x,y) is points on x-axis and y-axis)

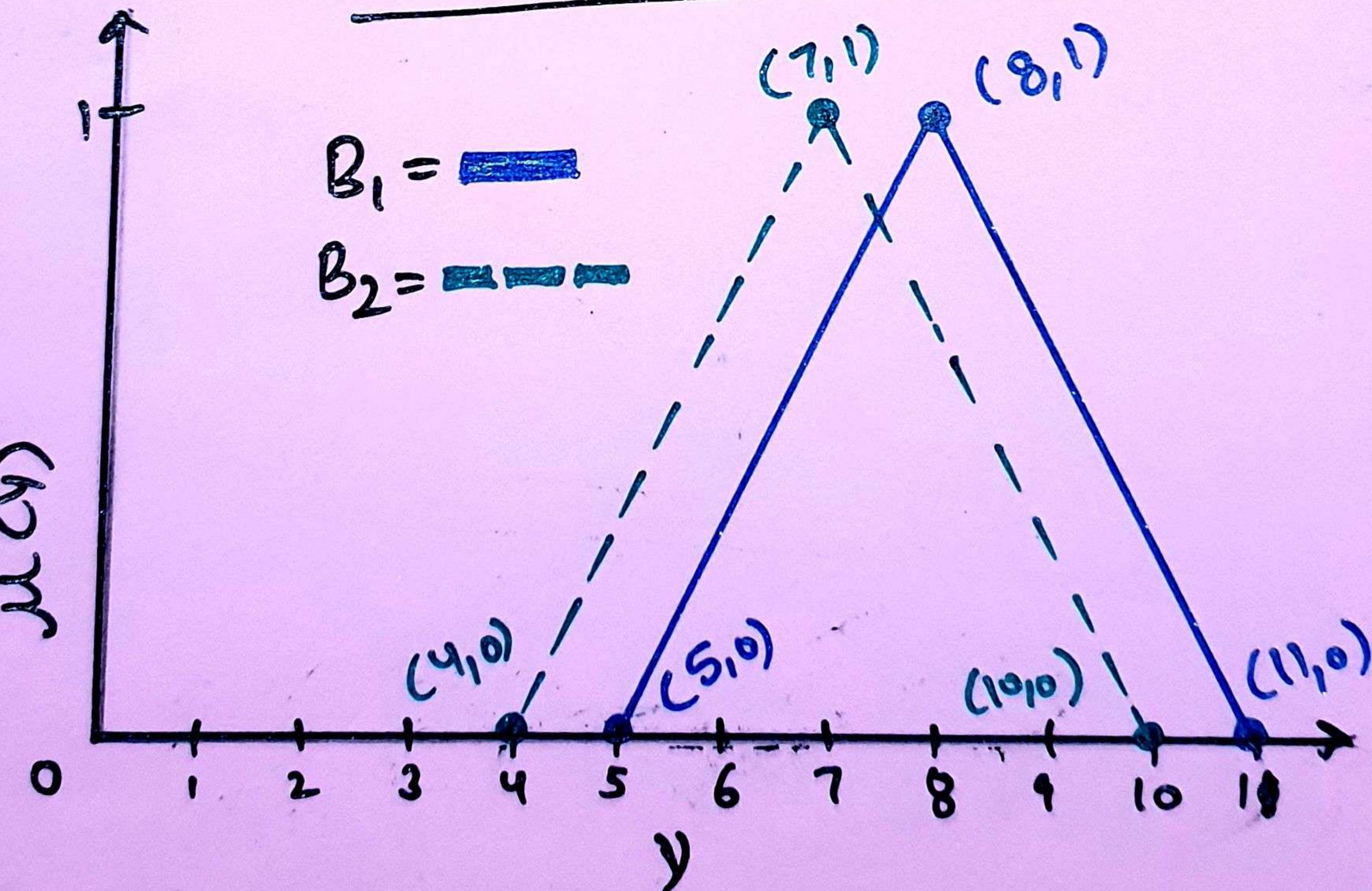
For B_1



For B_2



Combined B_1 and B_2.



Plotting the values of z Consequent.

$$\mu_{c_1}(z) = \begin{cases} \dfrac{z-1}{3} & 1 \le z < 4 \\ \dfrac{7-z}{3} & 4 < z \le 7 \end{cases}$$

$$\mu_{c_2}(z) = \begin{cases} \dfrac{z-3}{3} & 3 \le z \le 6 \\ \dfrac{9-z}{3} & 6 < z \le 9 \end{cases}$$

Thus, here for z-consequent, there are two states c_1 and c_2. Both of these have two equations of line each. Hence, to calculate x-axis values / z values on graph of z consequent, we will use c_1 range as points on z (x-axis) as [1,4] and [4,7]. Similarly, C_2 range as points on 3 (x-axis) as [3,6] and [6,9]. Here also, these values can be calculated by substituting membership function values as 0 and 1 for each line.

Therefore,

For first line of C_1, points are (1,0) and (4,1).

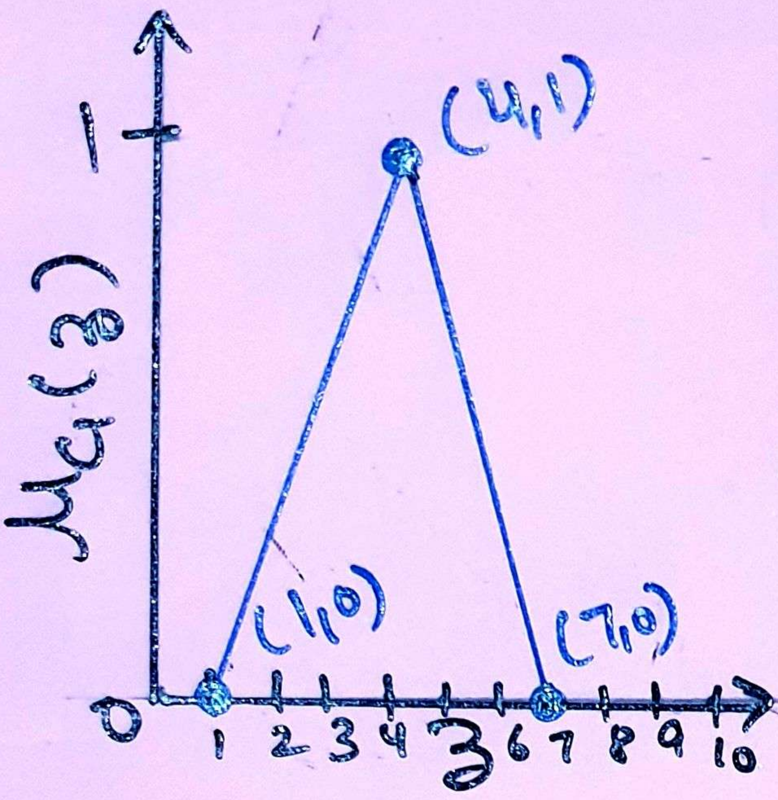
for Second line of C_1, points are (4,1) and (7,0).

for first line of C_2, points are (3,0) and (6,1)

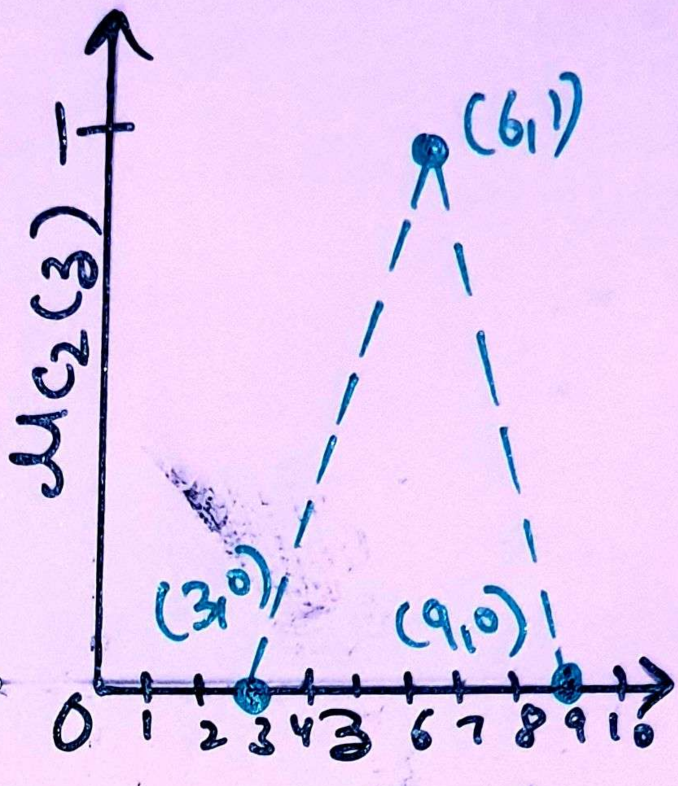
for second line of C_2, points are (6,1) and (9,0).

(where (x,y) is represented as points on x-axis and y-axis)

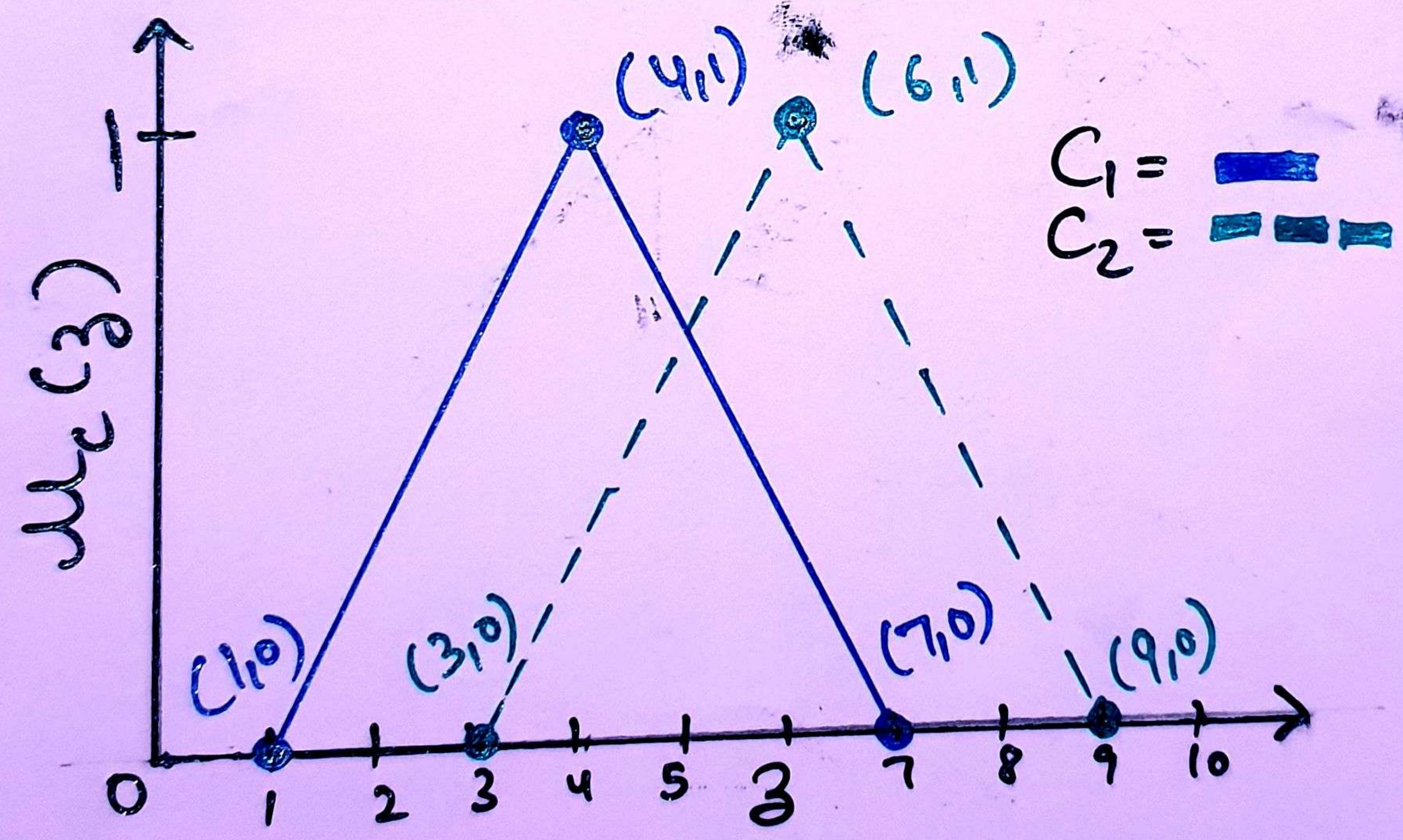
For C_1



For C_2



Combined C_1 and C_2.

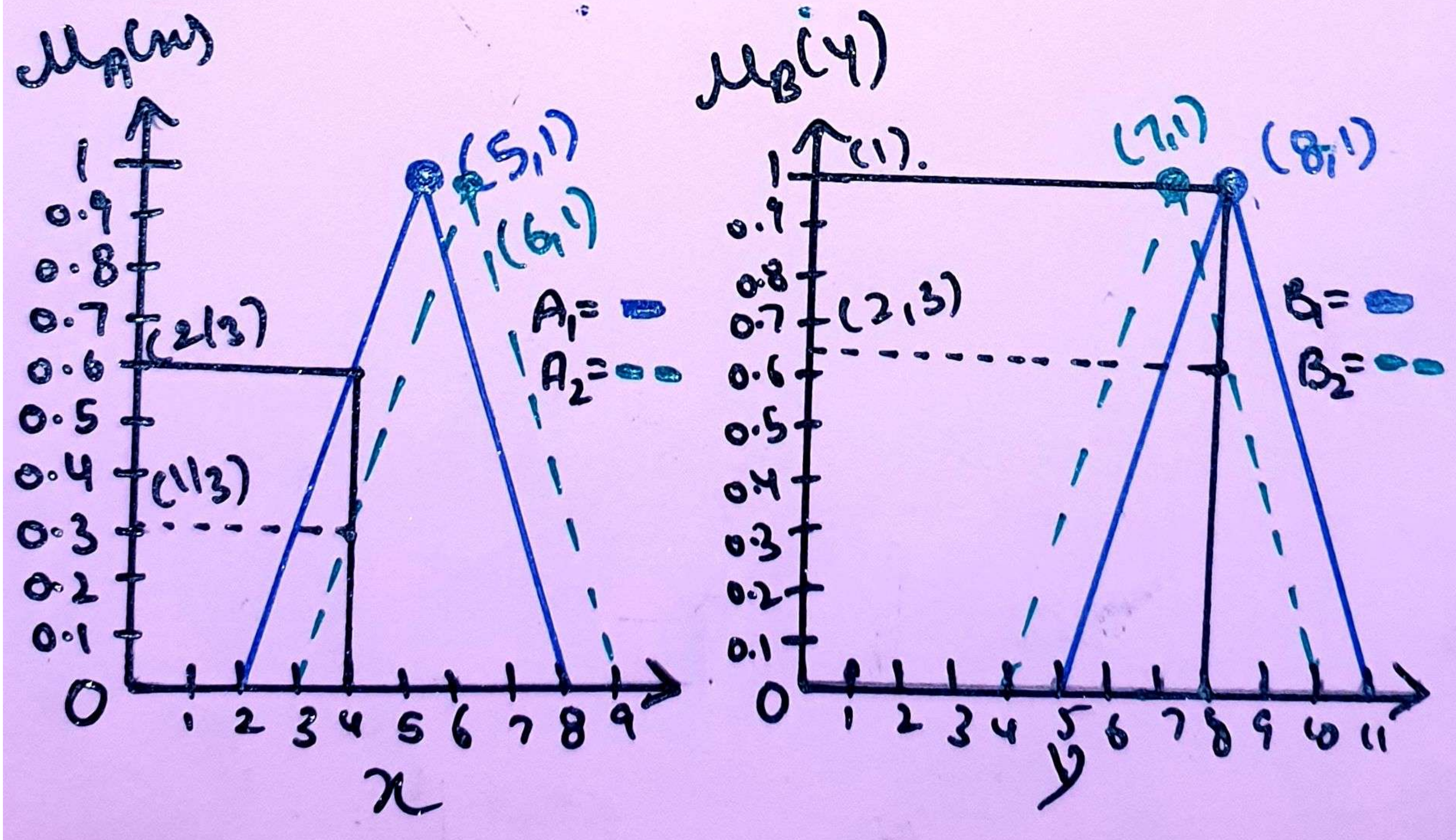


So, given the sensor readings at time t_1 are:

$x_0(t_1) = 4$ and $y_0(t_1) = 8$

Here, Aim is to find $z_0(t_1)$, using Mean of Maxima (MoM) defuzzification strategy.

Therefore plotting the provided x and y inputs in graph as;



Calculating $\mu_A(x)$ and $\mu_B(y)$.

So, here as. x-axis = 4 intersects at line 1 of A_1.

Thus, $\mu_{A_1}(x) = \frac{x-2}{3}$

$$\Rightarrow \mu_{A_1}(x) = \frac{4-2}{3} \Rightarrow \frac{2}{3} = \mu_{A_1}(x).$$

Also, for A_2, x-axis = 4 intersects at line 1 of A_2.

Thus, $\mu_{A_2}(x) = \frac{x-3}{3} = \frac{4-3}{3}$

$$\Rightarrow \mu_{A_2}(x) = 1/3.$$

Similarly,

as $y = 8$, intersects at $\mu_{B_1}(y) = 1$.

and $y = 8$, intersects at line 2 of B_2.

Thus, $\mu_{B_2}(y) = \frac{10-y}{3} = \frac{10-8}{3}$

$\Rightarrow \mu_{B_2}(y) = 2/3$.

Hence,

$\mu_{A_1}(x) = 2/3$.

$\mu_{A_2}(x) = 1/3$.

$\mu_{B_1}(y) = 1$

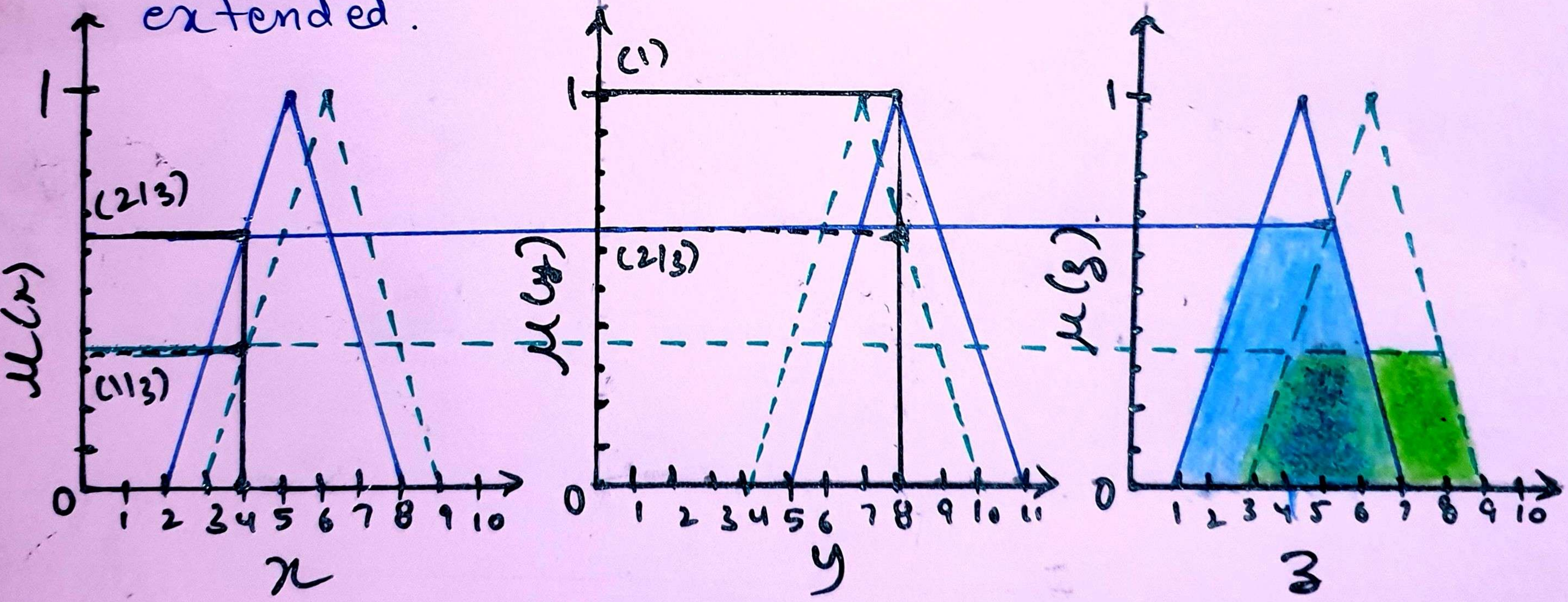
$\mu_{B_2}(y) = 2/3$.

Applying Rules.

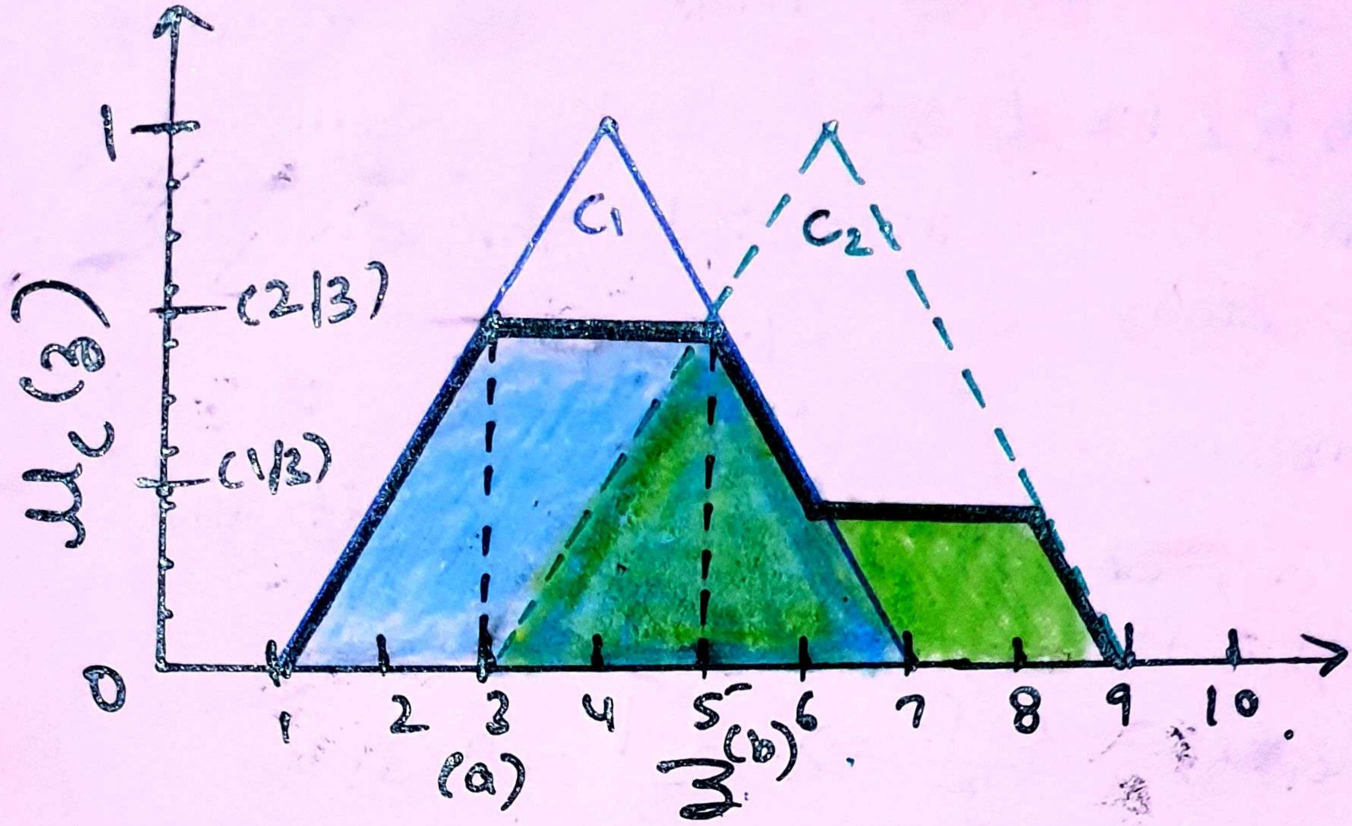
As Rule 1 and Rule 2 has 'and' operation between x and y antecedent. Therefor for C_1, minimum of A_1 and B_1 will be consirdered. and, for C_2, minimum of A_2 and B_2 will be consindered. So, implementing these rules will provide the following graph.

Plotting and extending lines according to Rules, to get fuzzy set of output (z).

Here, Among $\mu_{A_1}(x) = 2/3$ and $\mu_{B_1}(y) = 1$, $\mu_{A_1}(x)$ is minimum. Also, among $\mu_{A_2}(x) = (1/3)$ and $\mu_{B_2}(y) = 2/3$, $\mu_{A_2}(x)$ is minimum. So, minimum value lines are extended.



Mean of Maxima (MoM)



Thus, here in order to find $C_0(t_1)$ using MoM defuzzification method.

First, we will calculate maximum among $\mu_{c_1}(3) = 2/3$ and $\mu_{c_2}(3) = 1/3$, which returns $\mu_{c_1}(3) = 2/3$ as maximum membership function value

Second Step is to find 3 values.

where $\mu_{c_1}(z) = 2/3$ intersects on z. Although, it is visible from graph that. MoM lies in range $[a, b]$ i.e $[3, 5]$. But, again it can be calculated through line equations of. C_1..

Therefore,
for a, as it intersects at ① line. of C_1.. Hence,

$$\mu_{c_1}(z) = \frac{z-1}{3} \Rightarrow \frac{z-1}{3} = 2/3.$$

$$\Rightarrow z = 3 \text{ . i.e } a = 3 .$$

for b, as it intersects at ② line of C_1. Hence,

$$\mu_{c_1}(z) = \frac{7-z}{3} \Rightarrow \frac{7-z}{3} = 2/3 .$$

$$\Rightarrow z = 5 \quad \text{i.e } b = 5 .$$

<u>Third step</u> is calculate MoM. which is.

$$\text{MoM} = \frac{1}{n}\sum_{a}^{b} z_i .$$

$$\Rightarrow MOM = \frac{a + \ldots + b}{n}.$$

$$\Rightarrow MOM = \frac{3+4+5}{3}.$$

$$\Rightarrow \boxed{MOM = 4}$$

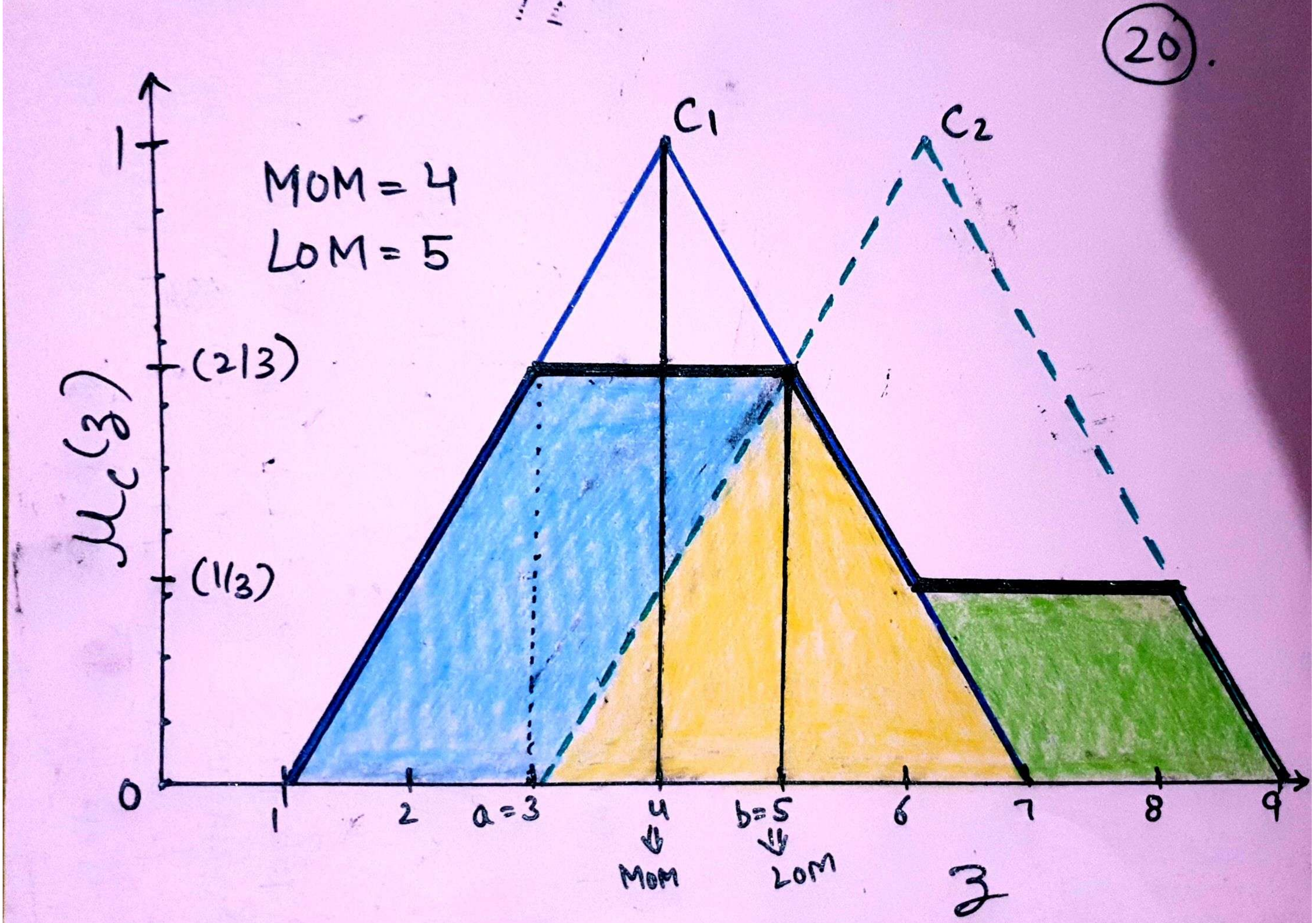
Therefore defuzzified crisp value via MOM is 4.

Largest of Maxima (LOM).

Here, defuzzified value by LOM will be the one, which returns the largest domain or z value where $\mu(z)$ is maximum (2/3).

So, in maxima range [3,5], 5 is the defuzzified value by Largest of Maxima method.

i.e $\boxed{LOM = 5}$.



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MOM = 4
LOM = 5
C1
C2
μc(z)
1
(2/3)
(1/3)
0
1
2
a=3
4
b=5
6
7
8
9
MOM
LOM
z