2014-01-08.sagews

January 8, 2014

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NOTE: Andrew Ohana will teach the class on Jan 15 and Jan 17.

1.1 Syllabus

- hand out print copies
- go over in detail

2 Prime numbers

2, 3, 5, 7, 11, etc.
An integer $n > 1$ is prime if it is divisible only by itself.
Theorem: There are infinitely many primes.
Give proof on the board.

2.1 Counting primes

Prime counting function

$$\pi(x) = \#\{p : p \leq x \text{ is prime } \}$$

```
@interact
def f(t=10, PNT=False, Gauss=False):
    x = var('x')
    g = prime_pi.plot(0,t)
    if PNT:
        g += plot(x/(log(x)-1), (5,t), color='red')
    if Gauss:
        g += plot(Li, (2,t), color='darkgreen')
    show(g, gridlines=True, svg=True, frame=True)
```

2.2 Theorems and Conjectures

The Prime Number Theorem: The functions $\pi(x)$ and $x/\log(x)$ are asymptotic to each other, i.e., the limit as $x \to \infty$ of their quotient is 1.

Conjecture (The Riemann Hypothesis): For every $x > 2.01$, we have

$$|\pi(x) - \mathrm{Li}(x)| \leq \sqrt{x}\log(x).$$

2.2.1 Example

```
1e3 == 1000 # scientific notation
```

```
True
```

```
%time prime_pi(1e10)
```

```
455052511

CPU time: 0.13 s, Wall time: 0.13 s
```

```
%time prime_pi(1e11)
```

```
4118054813

CPU time: 0.90 s, Wall time: 0.90 s
```

```
@interact
def f(x=100000):
    if x >= 1e12:
        print "This will probably take too long so refusing"
        return
    a = prime_pi(x)
    b = Li(x)
    md("""
\\begin{eqnarray}
```

```
  \\pi(x) &=& %s    \\\\
  \\text{Li}(x) &=& %s\\\\
  \\pi(x) - \\text{Li}(x) &=& %s\\\\
  \\sqrt{x}\\log(x) &=& %s
\\end{eqnarray}
"""%(a, b, a-b, N(sqrt(x)*log(x)))  )
```

3 Largest KNOWN Prime

As of October 2013, the largest known prime number is $2^{57,885,161}1$, a number with 17,425,170 digits. see `http://en.wikipedia.org/wiki/Largest_known_prime_number`

This is a Mersenne prime. It is the 48th Mersenne prime, i.e., of the form $2^p - 1$.

```
%time p = 2^57885161
```
```
CPU time: 0.01 s, Wall time: 0.00 s
```

```
%time s = str(p)
```
```
CPU time: 14.68 s, Wall time: 14.67 s
```

```
len(s)
```
```
17425170
```

```
s[:10] + '...' + s[-10:]
```
```
'5818872662...1724285952'
```

Prizes still

150,000 thousand dollars to the first individual or group who discovers a prime number with at least 100,000,000 decimal digits

`https://www.eff.org/awards/coop`