2014-02-26.sagews

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1 Lecture on Feb 26, 2014

1.1 Continued fractions (last day)

- quadratic irrationals
- statistics of partial convergents

1.2 Statistics of Partial Convergents

Given a real number $\alpha \in \mathbf{R}$, we can consider its decimal expansion, which is a sequence of numbers 0,1,2,,9, which has a distribution.

```
@interact
def f(alpha=pi, bits=(53,10000), bins=50):
    v = stats.TimeSeries([int(k) for k in str(N(alpha,bits)) if k not in \
        ['.', '-']])
    v.plot_histogram(bins=bins).show(figsize=[8,3], axes=False, frame=\
        True, gridlines=True)
```

Unsolved Problem: Prove that the digits of π (or e or $\sqrt{2}$) are equidistributed, i.e., that in the limit the bars above area all the same height.

See `http://en.wikipedia.org/wiki/Normal_number` for more about this conjecture.

An issue with the above is that it depends on the choice of a base, in this case base 10, so it isnt so natural.

Theres also sequence of numbers associated to any real number α, namely its continued fraction. This doesnt depend on any arbitrary choice of base, and gives us another distribution. Whats it like?

```
@interact
def f(alpha=pi, bits=(53,10000), bins=100, max=50):
    c = continued_fraction_list(N(alpha,bits))
```

```
    v = stats.TimeSeries(c)
    print "Clipped:"
    v.clip_remove(max=max).plot_histogram(bins=bins).show(figsize=[8,3], \
       axes=False, frame=True, gridlines=True)
    print "\nUnclipped:"
    v.plot_histogram(bins=bins).show(figsize=[8,3], axes=False, frame=\
       True, gridlines=True)
    print "\ncontfrac=", c
```

1.3 Gauss-Kuzmin

Gauss figured out this distribution for a random real number: see `http://en.wikipedia.org/wiki/Gauss%E2%80%93Kuzmin_distribution`

Not every transcendental number has this distribution, e.g., e doesnt!

Unsolved Problem: Does the continued fraction expansion of π obey the Gauss-Kuzmin distribution?

For more about this problem, see `http://en.wikipedia.org/wiki/Khinchin%27s_constant`