2014-02-14.sagews

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Contents

1 Lecture on Feb 14, 2014 Elliptic Curve Digital Signature Algorithm (ECDSA)

1.1 A very simple ECDSA implementation demo and test

Setup: Choose a prime number q, an elliptic curve E mod q, a point G of some prime order p, and define a set-theoretic map (in any way) $\phi : \mathbf{F}_q \to \mathbf{F}_p^*$. Choose a random secret $d \in \mathbf{F}_p^*$ and let $Q = dG$. The public key is (E, G, Q, p) and the private key is d.

```
q = next_prime(2^128); q
```

```
340282366920938463463374607431768211507
```

```
Fq = GF(q)
E = EllipticCurve(Fq, [3,4])
E.cardinality().factor()
```

```
2^2 * 5 * 17 * 1000830490943936657180023443038782793
```

```
# Found via -- P = E.random_point(); P
P = E([281642621541096348567721368996052493558, \
    321403994476306244071060762777806837 85])
```

```
factor(P.order())
```

```
2 * 1000830490943936657180023443038782793
```

```
G = 2*P; p = G.order(); p
Fp = GF(p)
```

```
1000830490943936657180023443038782793
```

```
def phi(x):
    a = Fp(x.lift())
    if a == 0:
        a = Fp(1)
    return a
```

```
d = Fp.random_element()
print "secret =", d
```

```
secret = 85509169948493851489056561321083269
```

```
Q = lift(d)*G    # lift(d) is integer in [0..p-1]
```

```
public_key = {'E':E, 'G':G, 'Q':Q, 'p':p}
```

Lets sign something

Hash: Hash the message m to an element $z \in \mathbf{F}_p^*$.

```
message = "This is math 480.  It's a very flexible class about various \
   things. -- William"
import hashlib
h = hashlib.sha1(message).hexdigest(); h
```

```
'e77f52876de8572f187bb226479f7268ec9464d0'
```

```
# But we need a number modulo p, so
z = hash(h) % p; z
```

```
4318374665117912394
```

Random Point: Choose a random $k \in \mathbf{F}_p^*$, and compute $kG \in E(\mathbf{F}_q)$.

```
k = Fp.random_element()
print "k =",k
kG = lift(k)*G
print "kG = ",kG
```

```
k = 34766806127427183032773852736175535 4
kG =  (19226206329851435713270349195607168317 5 : 22718889394229064756741627532430173032 8 :
1)
```

Compute Signature: Compute

$$r = \phi(x(k(G))) \in \mathbf{F}_p^* \quad \text{and} \quad s = \frac{z + rd}{k} \in \mathbf{F}_p.$$

```
r = phi(kG[0]); s = (z+r*d)/k
sig = (r,s)
print "sig =", sig
```

```
sig = (102609037278518954138990892625386919, 191169968480879734397280308320707618)
```

Hash: Hash message m to the same $z \in \mathbf{F}_p^*$ as above.

Verify: Compute the point

$$C = \frac{z}{s}G + \frac{r}{s}Q \in E(\mathbf{F}_q).$$

The signature is valid if $\phi(x(C)) = r$.

```
z
```

```
4318374665117912394
```

```
C = lift(z/s)*G + lift(r/s)*Q; C
```

```
(192262063298514357132703491956071683175 : 227188893942290647567416275324301730328 : 1)
```

```
phi(C[0]) #  == r  ? yep
```

```
102609037278518954138990892625386919
```

Prop: If (r, s) is valid, then this protocol concludes it is valid.

Proof: Since (r, s) is valid, we have $s = (z + rd)/k$, so $k = (z + rd)/s$. Thus

$$kG = \frac{z}{s}G + \frac{rd}{s}G = \frac{z}{s}G + \frac{r}{s}Q = C.$$

Lets sign another document

```
message2 = "This is a very flexible class about various things. -- \
   William"
h2 = hashlib.sha1(message2).hexdigest()
z2 = hash(h2) % p
r2 = phi(kG[0]); s2 = (z2+r2*d)/k
sig2 = (r2, s2)
print "sig2 =", sig2
```

```
sig2 = (102609037278518954138990892625386919, 384349446532996116240732504218379482)
```

And verify the signature

```
C2 = lift(z2/s2)*G + lift(r2/s2)*Q; C2
```

```
(192262063298514357132703491956071683175 : 227188893942290647567416275324301730328 : 1)
```

```
phi(C2[0])            # == r2 above.
```

```
102609037278518954138990892625386919
```

Question: What serious mistake did we just make?!

```
# Just looking at the signatures, we can easily compute this number:
print (z - z2)/(s-s2)

# Wait, that's actually k, which was some secret thing used in signing...\
    so?
k
```

```
347668061274271830327738527361755354
347668061274271830327738527361755354
```

```
# so!
(s*k-z)/r   # all known by attacker
```

```
85509169948493851489056561321083269
```

```
# Umh, that's the private key. Crap.
d
```

85509169948493851489056561321083269

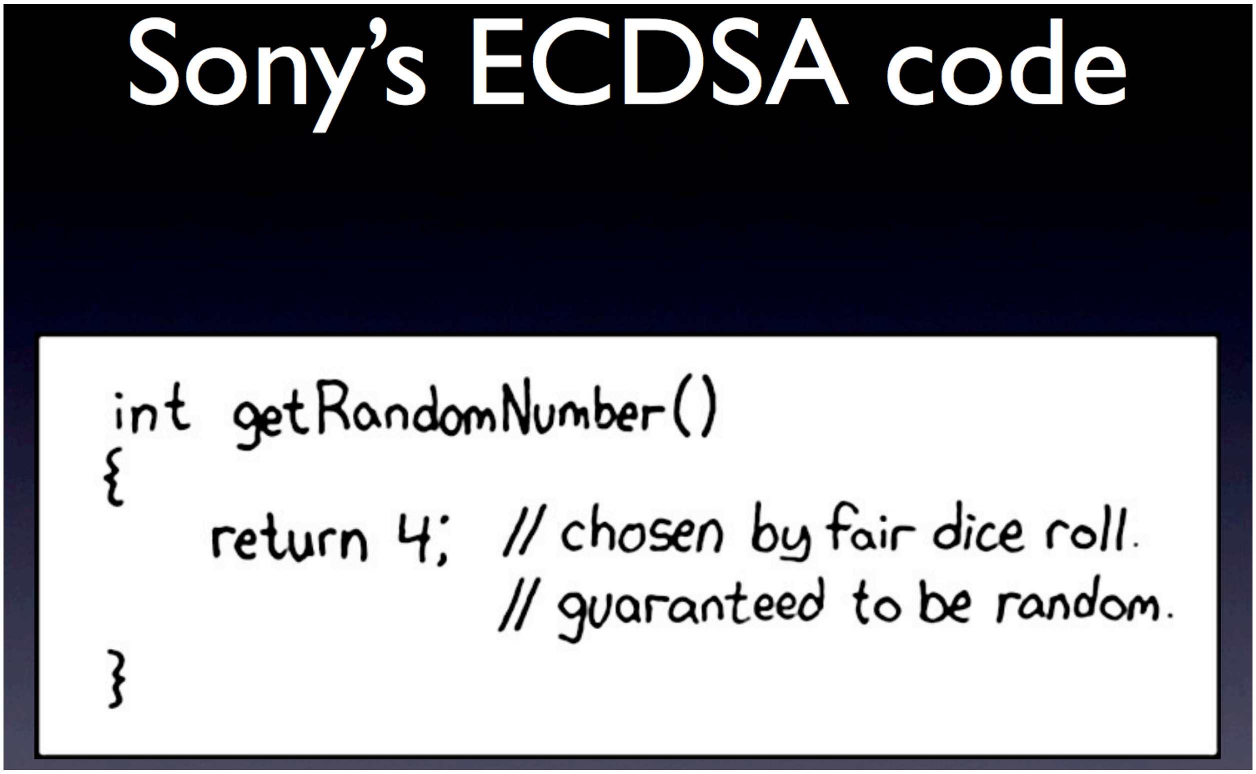
1.1.1 Our mistake

Our mistake was that we didnt generate a new random k. In general, if the ks arent really damn random ECDSA will be easily crackable.

1.2 ECDSA in PS3 an egrarious example

They used one single k, not changing it at all, leading to them being totally owned.


```
salvus.file('ps3-random.png')
```



`http://events.ccc.de/congress/2010/Fahrplan/attachments/1780_27c3_console_hacking_2010.pdf`

1.2.1 ECDSA in PS3 has since been fixed

Anyone still workin on the ECDSA code?

Question is in the title!

But seriously though. Is anyone working on the ECDSA code/private keys still,or has everyone given up and stuck to what they know?

I have a strange idea on how to get the keys/core files...

Could we not disassemble a PS3 motherboard very carefully and power individual components? Basically, couldnt we just dissect and isolate the ps3's guts,and then work from there?

I hope you guys answer my questions,

Bye for now!
(p.s if you ever read this Hellsing,show more pictures of your lovely b00biez :D)

Last edited by Brandono_99; 03-05-2013 at 04:48 PM. ***Reason:*** *Not enough manliness involved.*

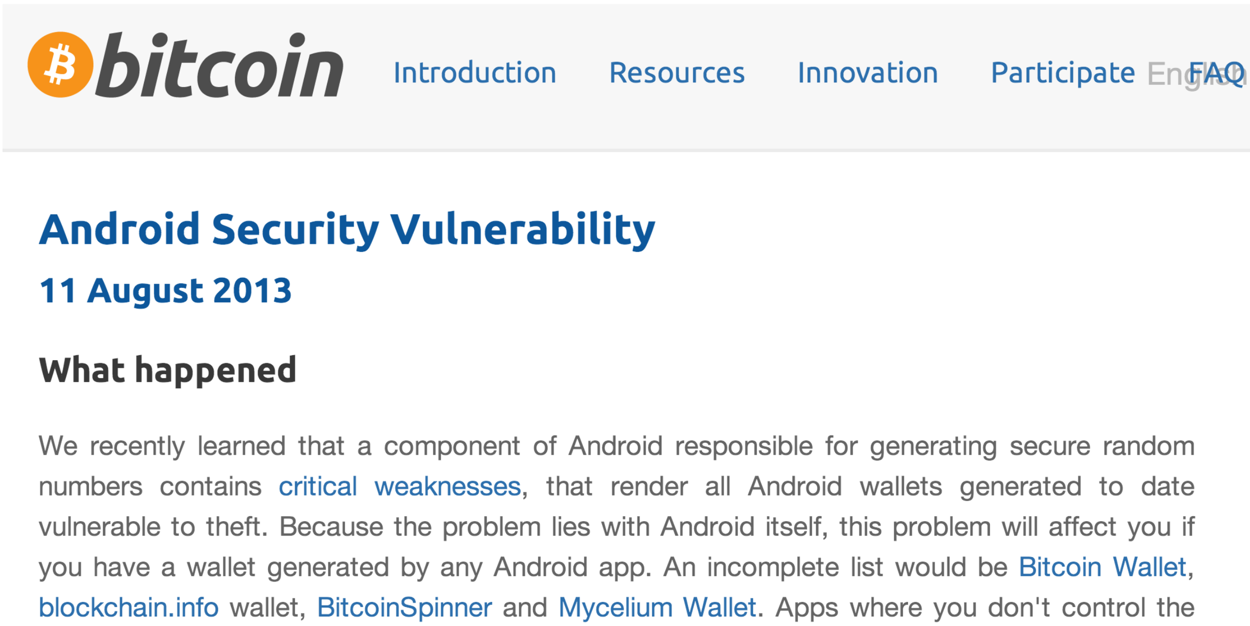
Reply With

the private keys are not in the ps3 !!!! they are in sony's office

1.3 ECDSA in Bitcoin

Yes, people have messed up the implementation of ECDSA here too, leading to theft

```
salvus.file('bitcoin-random.png')
```



1.4 The Bitcoin Elliptic Curve

Definition: `https://en.bitcoin.it/wiki/Secp256k1`

Discussion: `https://bitcointalk.org/?topic=2699.0`

ECDSA verification is the primary CPU bottleneck for running a network node. So if Koblitz curves do indeed perform better we might end up grateful for that in future

```
q =  2^256 - 2^32 - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1
is_prime(q)
```

```
True
```

```
# This is the elliptic curve "Secp256k1", where the "k" stands for "\
   Koblitz".
E = EllipticCurve(GF(q),[0,7]); E
```

```
Elliptic Curve defined by y^2 = x^3 + 7 over Finite Field of size
115792089237316195423570985008687907853269984665640564039457584007908834671663
```

www.math.washington.edu/~koblitz/

Neal Koblitz

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The photo was taken at Ticlio Pass, Peru.

```
%time p = E.cardinality(); p
```

```
115792089237316195423570985008687907852837564279074904382605163141518161494337
CPU time: 0.09 s, Wall time: 0.02 s
```

```
is_prime(p)
```

```
True
```

```
len(p.str(2))
```

```
256
```

```
s = '79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9 59F2815B 16\
    F81798'.replace(' ','').lower(); s
```

```
'79be667ef9dcbbac55a06295ce870b07029bfcdb2dce28d959f2815b16f81798'
```

```
x = E.base_field()(ZZ(s,base=16)); x
```

```
55066263022277343669578718895168534326250603453777594175500187360389116729240
```

```
G = E.lift_x(x)
G
-G
```

```
(55066263022277343669578718895168534326250603453777594175500187360389116729240 :
32670510020758816978083085130507043184471273380659243275938904335757337482424 : 1)
(55066263022277343669578718895168534326250603453777594175500187360389116729240 :
83121579216557378445487899878180864668798711284981320763518679672151497189239 : 1)
```

```
ZZ(G[1]).str(base=16)   # this is the one
ZZ(-G[1]).str(base=16)
```

```
'483ada7726a3c4655da4fbfc0e1108a8fd17b448a68554199c47d08ffb10d4b8'
'b7c52588d95c3b9aa25b0403f1eef75702e84bb7597aabe663b82f6f04ef2777'
```

```
G
```

```
(55066263022277343669578718895168534326250603453777594175500187360389116729240 :
32670510020758816978083085130507043184471273380659243275938904335757337482424 : 1)
```

```
G.order()
```

```
115792089237316195423570985008687907852837564279074904382605163141518161494337
```