due-01-29.sagews

January 22, 2014

Contents

1 Homework 2 Due Jan 29, 2014

1.1 Instructions

- Put your solutions in the empty space below the problem.
- Create a new cell by clicking on the horizontal cell dividers.
- If you put
  - If you then press shift-enter youll see the rendered math.
  - If you double click on the output then you can edit the input again.
  - This uses Markdown format, which you can learn about here: `http://daringfireball.net/projects/markdown/`
- Put this worksheet in a folder called homework in your project.
- When youre done, open the worksheet, and copy/paste the URL into an email to wstein@gmail.com with the subject math 480: homework 01-29.

1.2 Problems

1.2.1 Problem 1: The Freshman Dream

Prove that if a and b are integers and p is a prime, then $(a+b)^p \equiv a^p + b^p \pmod{p}$. You may assume that the binomial coefficient $\frac{p!}{r!(p-r)!}$ is an integer.

```
# For example:
(2+3)^7 % 7
(2^7 + 3^7) % 7
```

```
5
5
```

1.2.2 Problem 2: Beyond Fermats Little Theorem

1. Prove that if a is an integer and p is a prime, then $a^p \equiv a \pmod{p}$.
2. Is the same statement true with p replaced by any positive integer? I.e., without the hypothesis that p is prime?

1.2.3 Problem 3: Parametrizing Extended Euclidean Algorithm Representations

1. Prove that if x, y is a solution to $ax + by = d$, with $d = \gcd(a, b)$, then for all $c \in \mathbb{Z}$,

$$x = x + c \cdot \frac{b}{d}, \qquad y = y - c \cdot \frac{a}{d}$$

   is also a solution to $ax + by = d$.
2. Find two distinct solutions to $2014x + 3000y = 2$.
3. Prove that all solutions are of the above for some c.

1.2.4 Problem 4: Complete sets of residues

1. Find four complete sets of residues modulo 11, where each element of the ith set satisfies the ith condition:

   (1) nonnegative, (2) odd, (3) even, (4) prime.
2. Let n be any positive integer at all. Does there necessarily exist four complete set of residues modulo n, where each element of the ith set satisfies the ith condition:

   (1) nonnegative, (2) odd, (3) even, (4) prime.

(Your answer will be either a proof or an explicit counterexample for some specific value of n.)

1.2.5 Problem 5: When does n divide $(n-1)!$?

(Note: every explanation point here indicated a factorial, except this one!)

1. Which of the numbers 1,2,3,4,5,6 have the property that n divides $(n-1)!$?
2. Prove that if $n \geq 5$ is composite then n divides $(n-1)!$.

1.2.6 Problem 6: Your project

1. Use the +New button to create a folder called project.
2. Inside that folder, use +New to create a LaTeX document.
3. Put the title of your project and your name in the document.
4. Use the following commands to create a rough outline of your project:

   \tableofcontents

   \sectionName of Section \subsectionName of a subsection \subsubsectionName of a sub-subsection

   Each time you click the save button, the preview pane on the right should update within about 15 seconds. If there are errors, click on the errors/warnings button to see them.
5. Do a web search for something like basic latex tutorial, find something you like, and learn some LaTeX.